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SUNDANCE Memo #83 , LUMINARY Memo #36

TO: Distribution  
FROM: George W. Cherry  
DATE: July 30, 1968  
SUBJECT: Computing the Lag Angles which Prevent Beginning and  
Ending Overshoots During Attitude Maneuver Routine  
Supervised DAP Maneuvers  
Reference: R-557. GSOP for Manned LM Earth Orbital Missions  
Using Program SUNDANCE. Section 3. Digital Autopilot.  
Sub-section 3.2 Autopilot Control Modes, by Don W. Keene

The purpose of this memo is to derive the three lag angles given in Eq. (3.2-3) on page 3.2-5 of the reference and point out that with a wide deadband ( $5^{\circ}$ ) the DAP might pause for a while — on the order of a minute — to commence an attitude maneuver routine supervised maneuver. The main purpose here is to point out how released program SUNDANCE operates and bring up the question of whether LUMINARY should be mechanized the same way.

The problem Don solves with the lag angles is the problem of synchronizing the attitude motion of the reference spacecraft (the commanded angles) with the physically feasible motion of actual spacecraft (the actual achieved angles). If the reference vehicle gets too far ahead of the actual spacecraft while it is accelerating to the desired maneuver rate, the actual spacecraft will have to accelerate to a higher than desired maneuver rate (rate overshoot) in order to catch up with the reference spacecraft. As Don explains an inverse kind of fuel-wasting phenomenon occurs on stopping the maneuver.

The reference vehicle motion begins moving instantaneously at the desired rate when the reference maneuver begins. Obviously, the physical, real spacecraft motion will get behind. Therefore, Don biases the reference

angles. Don chooses the lag or bias angles so that the reference spacecraft is synchronized at the instant when the physical spacecraft could have achieved the desired rate if it had been smoothly and continuously accelerated by 2 jets about each axis since the beginning of the maneuver. Let us derive the bias (lag) angles.

Let  $t_a$  and other variables be defined as follows:

$\theta_a$  = angular orientation about the axis when the actual vehicle achieves the desired rate.

$t_i$  = time at the initiation of the maneuver.

$\theta_i$  = angular orientation about the axis when the maneuver is initiated.

Assuming that the vehicle is motionless at the beginning of the maneuver, we have

$$\theta_a = \theta_i + \alpha(t_a - t_i)^2/2 \quad (1)$$

$$\omega_D = \alpha(t_a - t_i) \quad (2)$$

Equations (1), (2) describe the potential actual motion. Now, how does the maneuver routine evolve the motion? In particular, where is the reference vehicle at  $t_a$ .

$\theta_r$  = reference vehicle angular orientation when the actual vehicle achieves the desired rate. (We want to make this equal to  $\theta_a$ .)

The reference vehicle starts moving instantaneously at  $\omega_D$  and it is not subsequently accelerated. However, we can assume that it starts behind the actual vehicle by  $\beta$ , the lag angle, so that from some point on the motions of the actual and reference vehicles are synchronous.

Then

$$\theta_r = \theta_i + \beta + \omega_D (t_a - t_i) \quad (3)$$

Setting Eqs. (3) and (1) equal to each other yields

$$\theta_a = \theta_r \quad (4)$$

$$\theta_i + \alpha (t_a - t_i)^2 / 2 = \theta_i + \beta + \omega_D (t_a - t_i) \quad (5)$$

Solving equation (5) for  $\beta$  and eliminating time by means of Eq. (2) yields

$$\beta = -\omega_D^2 / 2\alpha \quad (6)$$

Note that  $\alpha$  is a signed quantity in Eq. (6). (R-557 does not explicitly discuss the sign problem.) In effect, the sign of  $\beta$  must be the opposite of the sign of the respective  $\omega_D$  or  $\alpha$ .

The effect of the instantaneous introduction of  $\beta$  and  $\omega_D$  at the initiation of the maneuver is so to alter the phase plane and the desired phase plane point that the desired state goes to the point indicated by an asterisk in Figure 1. (The case shown is for positive  $\omega_D$ .) Note that the desired phase point is well within the  $5^\circ$  deadband and that the maneuver around the illustrated axis will not start until the desired phase point moves to the left outside the deadband. (In effect, the switching curves migrate to the right.) This effect is the result of the fact that the  $\omega_D$ ,  $\beta$  combination displaced the desired phase point along a parabola (indicated by the dotted line in Figure 1) parallel to the lower left-hand boundary of the coast region.

Assume now that the component of  $\omega_D$  in this plane is 0.1 deg/sec and that the deadband is 5 degrees. Fifty seconds will elapse before the accelerator begins. This effect could be removed by taking into account

the deadband, DB, when computing  $\beta$ , and thereby starting the maneuver at the heavy dot in Figure 1. The expression for  $\beta$  would be

$$\beta = -\omega_D^2 / 2\alpha + \text{sgn}(\alpha) |DB|$$

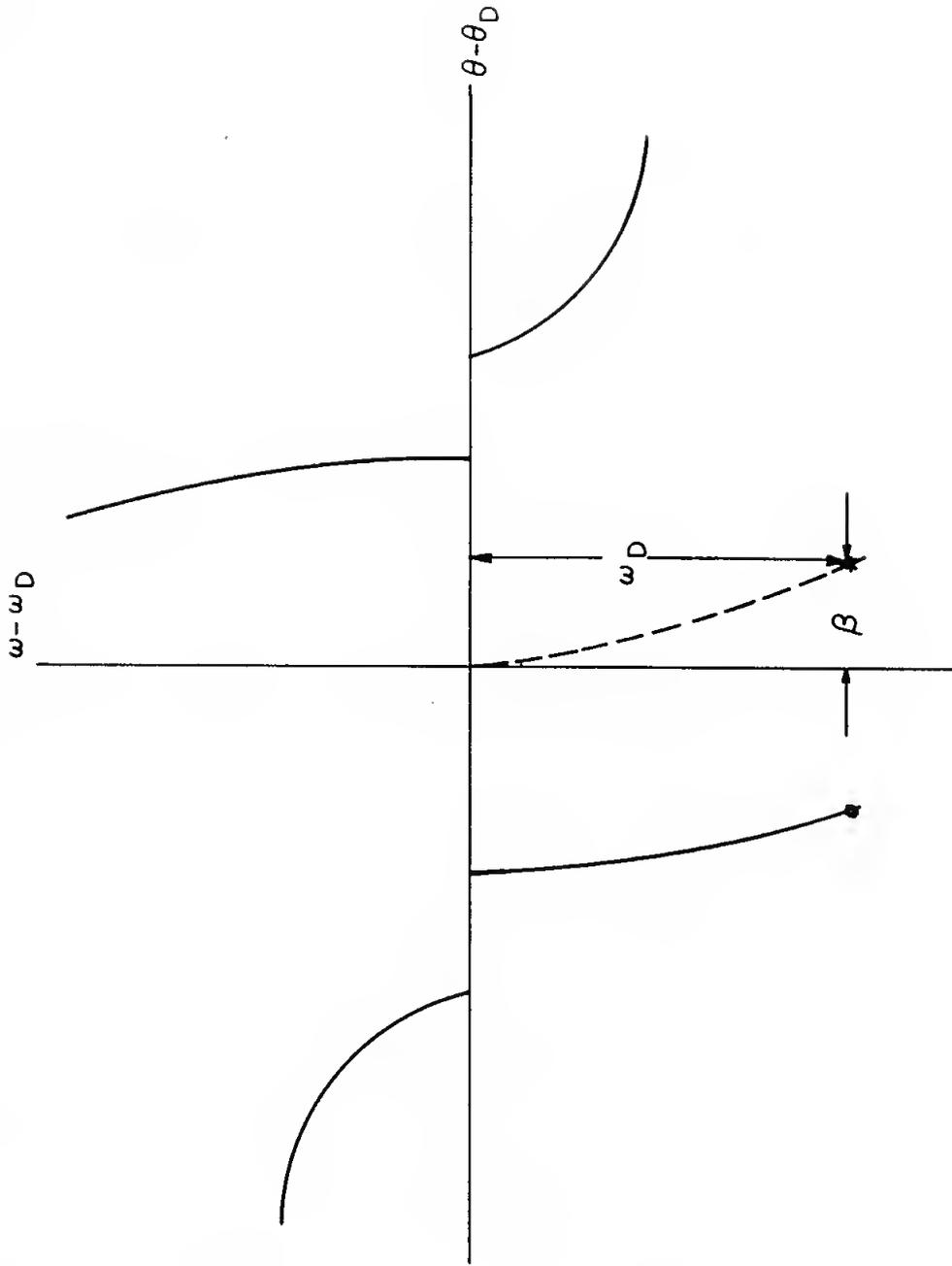


Fig. 1 Displacement of Phase Point When Maneuver Begins