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Space Guidance Analysis Memo #8-71 (Addendum 1)

TO: Distribution

FROM: William M. Robertson

DATE: April 20, 1971

SUBJECT: Incorporation of the Advance of Perihelion into Concise Analytic Approximations to the Sun's Geocentric Position Vector.

## SUMMARY

In Space Guidance Analysis Memo #8-71, the classical analytic conic approximation to the solar geocentric position vector (carried through fourth order in the earth's eccentricity) was given, together with explicit formulas for calculating the values of the five mean-element "constants" used in the approximation with respect to any mean equator and equinox coordinate system during the 20th Century. It was pointed out that the mean-element "constants" are not strictly constant, and that the largest rate of change among them (excluding the linear term in the mean anomaly which is already accounted for explicitly in the formulation) was the approximately 12 arc-second per year advance in the longitude of perihelion.

In this addendum, a simple method of accounting for the major part of the perihelion longitude advance is described. The resulting solar geocentric position approximation will be accurate over longer time spans than the approximation in the original memo, since the largest rate of change not explicitly accounted for among the mean-element "constants" is now the approximately 1/2 arc-second per year decrease in the obliquity of the ecliptic.

The notation and nomenclature of the original Space Guidance Analysis Memo are maintained in this addendum; for definitions and descriptions the original memo should be refered to.

## DETERMINATION OF THE MEAN-ELEMENT CONSTANTS

The obliquity $\epsilon$, the eccentricity $e_{EM}$, the mean anomaly $M_{0EM}$ at epoch, and the mean motion $n_{EM}$ all continue to be evaluated (with respect to a mean-of-date coordinate system or a 1950.0 coordinate system) by the formulae given in the original memo. However, the remaining mean-element, the longitude of perihelion, will now be evaluated at epoch by the formulae given in this section (the result will have the nomenclature $\tilde{\omega}_{0EM}$ in analogy with the mean anomaly $M_{0EM}$ at epoch), and this value will be linearly extrapolated to the actual time t when the solar position is desired by the formula given in the next section.

## Mean-of-Date Coordinate System

The longitude of perihelion of the earth-moon barycenter fully updated to the time $t_c$ but linearly back-dated to the epoch $t_0$ is:

$$\boxed{\begin{aligned}\tilde{\omega}_{0EM} &= 101.220833^{\circ} + 0.470684\ D - (0.32328/36525)(t_c - t_0) \\ &\quad + 3.39(10)^{-5}\ D^2 + 7.0\ (10)^{-8}\ D^3\end{aligned}}$$

This expression contains the precession correction to the time-point $t_c$; since precession is occuring much faster (about 50 arc-seconds per year) than the largest neglected rate of change in the mean-element constants (1/2 arc-second per year decrease in the obliquity), the time-point $t_c$ should be chosen to correspond to the desired coordinate system as described in the original memo.

## 1950.0 Coordinate System

The longitude of perihelion of the earth-moon barycenter fully updated to the time $t_c$ but linearly back-dated to the epoch $t_0$ is:

$$\boxed{\begin{aligned}\tilde{\omega}_{0EM} &= 102.08053^{\circ} + 0.32328\ T + 1.5(10)^{-4}\ T^2 - (0.32328/36525)(t_c - t_0) \\ &= 102.08053^{\circ} + (0.32328/36525)(t_0 - 2433282.5) + 1.5(10)^{-4}\ T^2\end{aligned}}$$

No precessional terms are contained in this expression. As in the original memo, the time-point $t_c$ should be selected as the mid-point of the time interval over which the solar position approximation is desired so that the mean-element constants obtained will be very nearly the average values of the elements over the time interval.

## DETERMINATION OF THE SOLAR POSITION

It is desired to find the solar position at the arbitrary time t. The following sequence of computations is performed, using the mean-element constants ($\epsilon$, $e_{EM}$, $\tilde{\omega}_{0EM}$, $M_{0EM}$, $n_{EM}$), where $\tilde{\omega}_{0EM}$ is determined by one of the formulae of the previous section of this addendum.

The longitude of perihelion of the earth-moon barycenter at the arbitrary desired time t is:

$$\boxed{\tilde{\omega}_{EM}(t) = \tilde{\omega}_{0EM} + (0.32328^{\circ}/36525)(t - t_0)}$$

This of course will differ from the constant denoted by $\tilde{\omega}_{EM}$ in the original memo (except when $t = t_c$ exactly).

The mean anomaly of the earth moon barycenter at the arbitrary desired time t is:

$$\boxed{M_{EM} = M_{0EM} + n_{EM}(t - t_0)}$$

The true anomaly $f_{EM}$ of the earth-moon barycenter at the arbitrary desired time t is evaluated by the same formula given in the original memo (bottom of page 5).

The true longitude of the earth-moon barycenter at the arbitrary desired time t is:

$$\boxed{L_{EM} = f_{EM} + \tilde{\omega}_{EM}(t)}$$

where $\tilde{\omega}_{EM}(t)$ is the perihelion longitude expression given above in this section.

All the remaining expressions in the solar position determination section of the original memo continue to be valid without change for the "improved" approximation of this addendum.

## APPLICATION: ANALYTIC COMPUTATION OF "LOSSEM" SOLAR CONSTANTS

In Solar Ephemerides Subroutine in the Lunar Module Guidance Computer has always used the solar position unit vector approximation:

$$\underline{1}_{SUN} = (\cos(LOS),\ K_1 \sin(LOS),\ K_3 \sin(LOS))$$

where

$$LOS = LOS_0 + LOS_R(t - t_0) - C \sin(OMEGA_C(t - t_0) + PHASE_C)$$

By comparison with the expressions in the previous section of this addendum and in the original memo, it may be seen that the five constants ($LOS_0$, $LOS_R$, $C$, $OMEGA_C$, $PHASE_C$) may be approximated by the relations:

| | |
|---|---|
| $LOS_0$ | $= \tilde{\omega}_{0EM} - 180^\circ + M_{0EM}$ |
| $LOS_R$ | $= n_{EM} + \left(\frac{0.32328}{36525}\right)$ |
| $C$ | $= -\left(2\, e_{EM} - \frac{1}{4}\, e_{EM}^{\ 3}\right)$ |
| $OMEGA_C$ | $= n_{EM}$ |
| $PHASE_C$ | $= M_{0EM}$ |

or

| | |
|---|---|
| $LOS_0$ | $= \tilde{\omega}_{0EM} + M_{0EM} - 180^\circ$ |
| $LOS_R$ | $= n_{EM} + \left(\frac{0.32328}{36525}\right)$ |
| $C$ | $= +\left(2\, e_{EM} - \frac{1}{4}\, e_{EM}^{\ 3}\right)$ |
| $OMEGA_C$ | $= n_{EM}$ |
| $PHASE_C$ | $= M_{0EM} - 180^\circ$ |

and the obliquity constants are given by:

$$K_1 = \cos \epsilon$$
$$K_3 = \sin \epsilon$$

The units of the term (0.32328/36525) in $LOS_R$ are degrees per ephemeris day, and the units of C are radians.

The addition of the term (0.32328/36525) to $n_{EM}$ in the $LOS_R$ expression provides the linear correction for the advance of the longitude of perihelion described earlier in this addendum. This correction amounts to about 12 arc-seconds per year. It should be noted that the correction is a secular (i.e. non-periodic) one; if the correction is not included the error in the approximation steadily accumulates. On the other hand, the equations for $\underline{1}_{SUN}$ and LOS given at the beginning of this section already have a periodic error with an amplitude of about $0.025^{\circ}$ (90 arc-sec) due to the neglect of the $\sin 2M_{EM}$ in the true anomaly expression. However, the inclusion of the secular correction term keeps the overall error from becoming too large (i.e. drifting) over a long time span.

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March 22, 1971