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Memo

TO: Distribution
FROM: Allan Klumpp
DATE: March 11, 1966
SUBJECT: More on Coriolis

Over the past several weeks there have been several written and innumerable verbal responses to the Coriolis memo of 7 January 1966. Among the equations and manipulations of the referenced memo which led to the contradiction, hardly one escaped being assigned to the total responsibility. Depending upon the approach taken, any one of the steps or equations could be singled out as erroneous. But not on the basis of principle, only on the pragmatic basis that the contradiction becomes apparent to the examiner at that point. One explanation was based on principle. In the opinion of this writer, the principle is the key.

If a contradiction results from manipulating equations according to certain postulates, then it seems proper to question the postulates, not the manipulations. The postulates involved in the previous memo are:

1. A vector seen by an observer in one reference frame is equal to a vector seen by another observed in another reference frame angularly displaced from the first frame if the vectors are representatives of the same physical phenomenon. (Physical vector concept)

2. The time derivative of a vector with respect to a reference frame is defined as the limit as the time differential approaches zero of the change of the vector relative to the frame during the time differential, divided by the time differential.

3. The time integral of a vector with respect to a reference frame is defined as the limit as the time differential approaches zero of the summation produced by multiplying the vector of a given time by the time differential and adding for a succession of times covering the integration

interval, where each successive time is advance by the time differential.

4. If equal vectors are integrated with respect to the same frame over the same time interval, then the results are equal.

5. If equal vectors are added to or subtracted from equal vectors, then the resulting vectors are equal.

If a contradiction results from manipulations according to these postulates, then it would seem the postulates are inconsistent. Professor Hollister of the Department of Aeronautics and Astronautics, MIT, has singled out postulate 3 stating, "the integral of a physical vector is not defined". In the absence of postulate 3 the other postulates apparently are consistent in that, to this writer's knowledge, no contradiction can be produced by manipulating equations according to the remaining postulates.

The integration attempted in the memo, Ref. 1, could not properly be defined because the vector form of equations does not explicitly relate vectors to any frame. Thus the vectors were not related to the frames in which the integration was attempted.

The choice of a postulate to be discarded seems arbitrary. In fact it seems unreasonable to build a mathematical structure in which differentiation is defined, postulate 2, but integration is not, postulate 3. It would seem more reasonable to this writer to replace postulate 1 by a new postulate which does relate vectors to their reference frames.

It can be argued that postulate 1 as written above is unreasonable. The purpose of a reference frame is to define the direction of the vector. Even though the same physical phenomenon is represented in two reference frames, it does not seem reasonable to say the same vector is involved, or that the vectors are equal, because the directions with respect to angularly displaced frames are not the same.

This argument can be used to rework postulate 1 such that a consistent mathematical structure can be built upon all the postulates, 1 through 5. The reworked postulate 1 might read:

A vector seen by an oversver in one reference frame is equal to the vector resulting from transforming a vector seen by another observer in another reference frame angularly displaced from the first frame if the vectors seen by the observers are representatives of the same physical phenomenon and if the transformation correctly accounts for the relative angular displacement of the reference frames.

By the original postulate (1) it may be written for a given physical vector observed with respect to two reference frames B and S

$$\underline{V}_S = \underline{V}_B$$

whereas by the reworked postulate 1 this relationship must be written

$$\underline{V}_S = T\,\underline{V}_B$$

where T is the transformation between the frames.

If one chooses a consistent set of postulates excluding postulates 3 and 4, then the Coriolis equation as customarily written is consistent with them. If one chooses a consistent set of postulates including postulates 3, 4, and the reworked postulate 1, then the Coriolis equation as customarily written is inconsistent with the postulates. It may be made consistent by writing it as follows:

$$\underline{G}_s = T\,(\underline{G}_B + \underline{\omega}_B * \underline{G}_B)\,,$$

where T is the time varying transformation to space coordinates from body coordinates.

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Klumpp (12)

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