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SGA Memo # 20 - 65

TO: SGA Distribution

FROM: Larry Brock

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SUBJECT: Notes for Ephemeris Programs for the AGC

I Introduction

Some aspects of the ephemeris problem are discussed here as a possible aid for the development of the necessary Apollo Guidance Computer programs. These are: 1) the choice of reference coordinate system, 2) the position and velocity of the moon and sun, 3) the transformation between the reference coordinate system and a system fixed on the earth, and 4) the transformation between the reference coordinates and moon fixed coordinates.

II Reference Coordinate System

The choice of a reference coordinate system is largely arbitrary. The coordinate system presently being used is defined by the line of intersection of the mean equatorial plane and the mean orbit of the earth (the ecliptic) at the nearest beginning of the Besselian year*. For example, the coordinate system used for a mission that begins in the interval from July 1, 1966 through June 30, 1967 would use the mean equator and equinox at the beginning of the Besselian year 1967 (denoted by 1967.0) which occurs January 1.041. Of course if a launch date and alternate launch date occur near the end of June, the same coordinate system is used for the entire mission. An overlap of about two months is provided on the present ephemeris tapes. A list of the time of the beginnings of Besselian years is on page 435 of Ref. 1.

Rectangular coordinates are defined so that the x axis is along the ascending mode of the ecliptic on the equator (the equinox), the z axis is along the mean pole, and the z axis completes a right handed set.

Some advantages of using a coordinate system referred to the mean equinox at the nearest beginning of the year are the following:

* Ref. 1, page 30

1. The precession of the earth can generally be neglected in computing the earth oblateness acceleration. Since the reference equinox is always within six months of the mean equinox of date, the maximum deviation in the pole, due to precession is about 10'' which gives an error of approximately $.5 \times 10^{-4}$ times the oblateness perturbation. If the epoch is as much as 10 years away, the error would be about 10^{-3} times the oblateness perturbation which is about the size of the third and fourth order term and would have to be accounted for.

2. The precession of the pole must be considered in transforming between earth fixed and reference coordinates. If the reference equinox is near the mean equinox of date then this transformation is small and many simplifications can be made. In fact, precession can be described by small angle rotations about the y and z axes. (See sec. IV).

3. The coordinate system referred to the beginning of the Besselian year is commonly used and is one in which much of the information from the Nautical Almanac Office is available.

The primary disadvantage of this coordinate system (or any other system) is that it may not be consistent with the system choosen by someone else with whom information must be transferred.

Another possible choice for coordinate system is one referred to the true pole at some instant midway in a mission. The transformation between the reference system and the earth fixed system will be much smaller and the computer program would be somewhat simpler. The primary disadvantages are that a different system would be necessary for every mission requiring that star positions, the lunar ephemeris, etc. be different for each mission and also that this system is even less likely to agree with other peoples choice.

III The Position and Velocity of the Moon and Sun

The standardized NASA ephemeris for the moon and sun is available on magnetic tape from the Jet Propulsion Laboratory[2, 3]. Mr. David Latimer of the Digital Computation Group is familiar with these tapes and has a program for converting them to the proper format. The information is in rectangular coordinates and is referred to the 1950.0 coordinates. Modified second and fourth differences are also included.

The ephemeris of the moon is a new computer solution of Brown's series. The vector position and velocity of the moon is given in terms of earth radii. The proper earth radius for converting to kilometers is given in Ref. 4 page 7. The ephemeris data will also have to be transformed from 1950.0 coordinates to the reference coordinates using the Euler angles on page 30 of Ref. 1. Mode 4 of the program MOONPOSSUBRJPL can be used to do this transformation. (If the nearest beginning of the Besselian year is used as the epoch of the reference coordinates, the ephemeris files that have already been transformed can be used).

The ephemeris is given in terms of ephemeris time. A correction has to be made to universal time before it can be used in the ephemeris. This difference (ΔT = E.T. - U.T.) is presently determined by observations of the moon. It is known for the present time only by extrapolating past valves and is accurate to about .1 sec. In the future the Naval Observatory advises that they are planning to coordinate ephemeris with their atomic clock. They will then be able to give ΔT at any time to an accuracy of about .01 sec. Provision should be made for updating the valve of ΔT stored in the AGC before each mission.

The most promising method of storing the lunar ephemeris for the AGC is to fit the data with a polynomial (See Ref. 5). An eight order polynomial fitted so as to give the minimum maximum error gives about 1 mile accuracy. Mr. Latimer has a program for making this kind of fit. Other possible methods of storing the data would be to use harmonic series instead of power series or to refit the first few terms of Brown's series.

IV Transformation Between Earth Fixed and Reference Coordinates

It will be necessary to have available in the Apollo computer in the reference coordinate system, the vector position of various places on the earth (e.g. landmarks, DSIF stations, landing sites). In order to obtain these position vectors in reference coordinates, the AGC must carry the necessary vectors in earth fixed coordinates and then transform these vectors at the desired times to the reference coordinates. The transformation must account for three effects; 1) the daily rotation of the earth about its axis; 2) the nutation of the earth's axis and 3) the precession of the earth's axis.

The earth fixed coordinate system is defined with the z axis along the earth's angular rotation vector, the x axis in the meridian of Greenwich, and the y axis completing a right handed set.

The rotation of the earth about its axis relative to the mean equinox is given by the definition of universal time. Universal time is defined* as 12 hours plus the Greenwich hour angle of a point on the equator whose right ascension, measured from the mean equinox of date, is:

$$R_u = 18.64606556 + 2400.051261\ T_u + .000025805\ T_u^{\ 2} \tag{1}$$

where R_u is an angle in units of hours (i.e. 24 hours equals 360^o) and T_u is the number of Julian centuries of 36525 days of universal time elasped since the epoch of Greenwich mean noon of 1900 January 0.

What is needed for the on board transformation is the angle between the Greenwich meridian and the fixed equinox of date. It is thus necessary to find the valve of R_u for the epoch of the reference coordinates and then add the change in R_u from the epoch of the reference coordinate system to the current time. The rate of change of R_u relative to the fixed coordinates will be the time derivative of Eq. (1) minus the precision of the equinox in the equational plane. This gives for the angle A about the z axis between the reference quinox and the Greenwich meridian,

$$A = 24\ t_u - 12 + R_{uo} + R_{u}\ \ (t_u - t_{uo}) \tag{2}$$

where A is an angle in units of hours, t_u is the current universal time in days, Ruo is $R_u(t_{uo})$, t_{uo} is (neglecting ΔT) the epoch of the reference coordinates, and R_{rv} is

$$\frac{d\ R_v\ (t_{vo})}{d\ t}$$

minus precession which given

$$R_{ur} = .0657074857 \text{ hrs/day} \tag{3}$$

The fact that the time dependent term cancels out of R_{ur} means that the earth rotation rate is constant which is consistant with the definition of universal time. The angle A is illustrated in Fig. 1.

The second effect that must be considered is the nutation of the pole which

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* Ref. 1 page 73

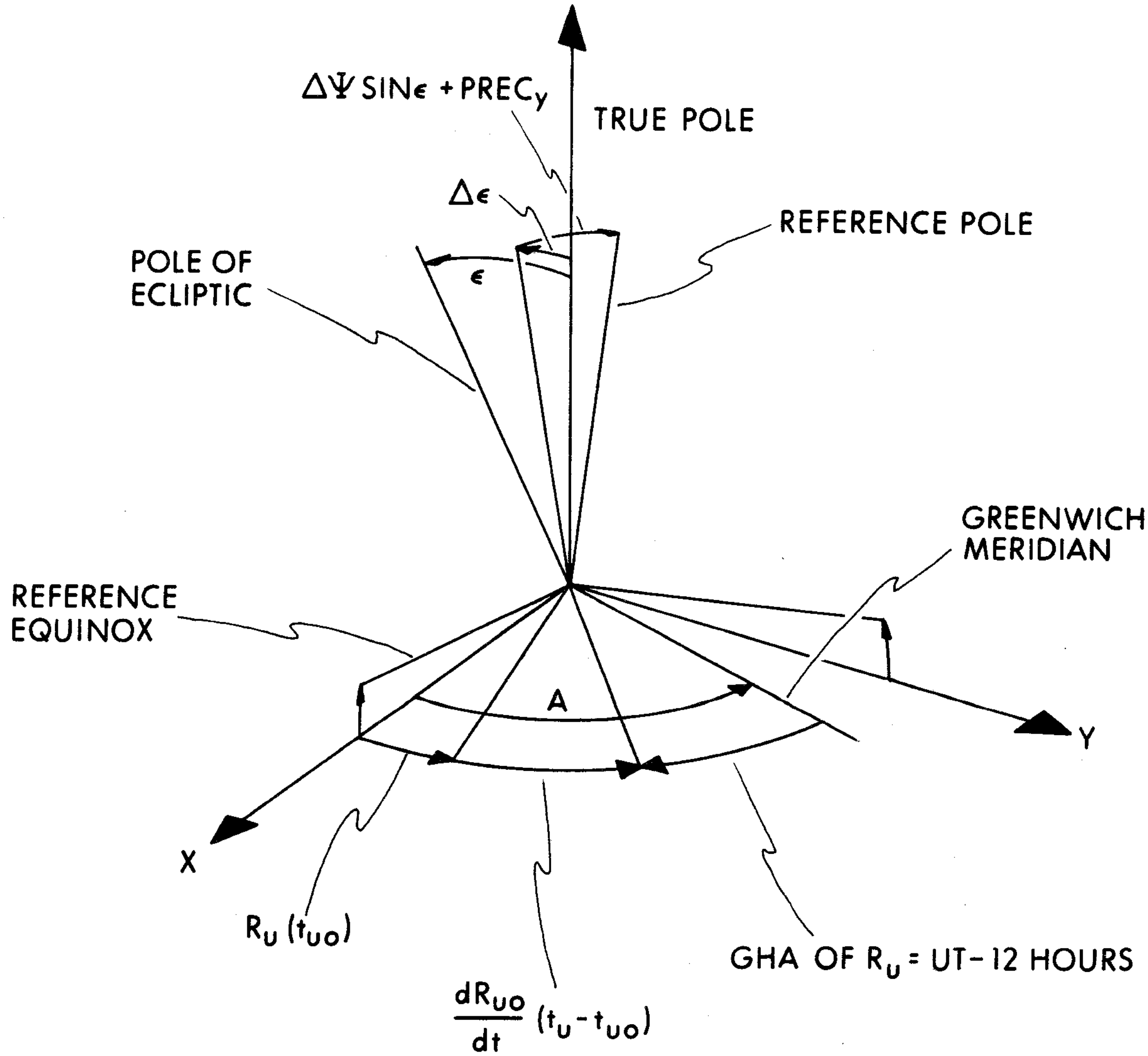

Fig. I

Transformation Between Earth Fixed and Reference Coordinate describes the position of the mean pole relative to the true pole. The mean pole is obtained by rotating the true pole about the x axis by the nutation in the obliquity ($\Delta\epsilon$) and about the negative y axis by the nutation in longitude times the sine of the obliquity ($\Delta\psi \sin \epsilon$). The nutation in obliquity and longitude is computed from a series similar to the Brown's series for the position of the moon, The Nautical Almanac Office uses all terms of $0''.0002$ or greater. These series are given on pages 44 and 45 of Ref. 1. There are 69 terms for $\Delta\psi$ and 40 terms for $\Delta\epsilon$. For AGC purposes the nutation can be approximated by a polynomial that best fits the true nutation over the duration of a mission. The number of terms required will be determined by the accuracy desired. The principal harmonics of the nutation are a 19 year oscillation with an amplitude of about 9'' a half year oscillation with an amplitude of 0.5'' and a 14 day oscillation with an amplitude of 0.1''. The nutation can be approximated by a linear time term giving an accuracy for a two week mission of about 0.1'' (or about 10 ft. on the surface of the earth). If greater accuracy is required, second or third order terms would have to be added.

The third effect which must be considered is the precession of the pole of the earth about the pole of the ecliptic. This precession has a period of about 26,000 years. The mean pole and equinox at one time relative to the mean pole and equinox at another time is given exactly in terms of Euler angles, on page 30 of Ref. 1 and the approximate transformation matrix is given on page 34. If the reference coordinates are within a half a year of the current mean pole and equinox then all of the terms in the transformation matrix can be neglected except for the first order small angle rotations about the y and z axes. The neglected terms give errors smaller than 1 ft. at the surface of the earth. The rotation about the z axis is already accounted for in the calculation for the daily rotation of the earth which leaves only the small rotation about y.

The nutation and precession can thus be combined into small angle rotations A_x about the x axis of $\Delta\epsilon$ and A_y about the negative y axis of $\Delta\psi \sin \epsilon$ plus $prec_y$ where

$$A_x = A_{xo} + A_{xr}(t - t_o) = \Delta\epsilon \tag{4}$$

$$A_y = A_{xo} + A_{yr}(t - t_o) = \Delta\psi \sin \epsilon + prec_y$$

A_{xo}, A_{xr}, A_{yo}, and A_{yr} are chosen so as to best fit the nutation and precession over interval of the mission. The valves of $\Delta\psi$ and $\Delta\epsilon$ are available at half day intervals from the JPL tapes along with the second and fourth differences. The obliquity can be assumed constant for a two week flight. The precession is approximately

$$\text{prec}_y = (.00971690 - .00000414\ T_o)\ \text{rad/tropical century}$$

where T_o is the tropical centuries since 1950.0

The total transformation matrix for converting from earth fixed coordinates to reference coordinates is thus

$$\begin{bmatrix} 1 & 0 & A_y \\ 0 & 1 & A_x \\ -A_y & -A_x & 1 \end{bmatrix} \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

V Transformation between Moon Fixed and Reference Coordinates

It is also necessary for the AGC to transform lunar landmarks and landing sites from moon fixed coordinates to the reference coordinates. The orientation of the moon is described by two rotations. The first, called optical libration, describes the mean rotation of the moon relative to the reference coordinates. The second rotation, called physical libration, is a set of small angle rotations (2 min.) about each axis which gives the approximate orientation of the true moon relative to the mean moon. The total transformations required are described in detail in Ref. 6 so they will not be discussed further here. It should be remembered that the development in Ref. 6 transforms lunar positions from moon fixed coordinates to ecliptic coordinates. The coordinate system must then be rotated about the ecliptic by the obliquity in order to obtain the reference equitorial coordinates.

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