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Space Guidance Analysis Memo #32-64

TO: SGA Distribution
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DATE: July 14, 1964
SUBJECT: ROOTFINDER 3 - 20th Order Polynomial Factoring Program

Introduction

The ROOTFINDER 3 program was developed as a subroutine to the ROOTLOCUSPOINTS program to solve polynomials with slowly changing coefficients. In particular, a routine was needed which could extract zero and repeated roots. The program has the capability of solving polynomials ranging from second degree to twentieth degree obtaining both real and complex roots. The method of solution employed is iterated synthetic division in which successive approximations to a quadratic factor are generated. Although convergence in every case cannot be assured, the method is applicable to a very large percentage of problems, and will, in most instances, handle the troublesome situation of repeated and zero roots. The program automatically makes a linear change of variable to obtain better convergence in the more difficult cases.

Methods

A. Manner of extracting roots: The roots of the polynomial are obtained in pairs by the extraction of a quadratic factor of the form

$$x^2 + px + q .$$

If we write the polynomial in the form

$$f(x) = x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n \qquad (1)$$

and divide by

$$x^2 + px + q$$

we obtain

$$f(x) = (x^2 + px + q)(x^{n-2} + b_1 x^{n-3} + \ldots + b_{n-3}x + b_{n-2}) + Rx + S \qquad (2)$$

where R and S are functions of p and q. If the quadratic expression is to be a factor of f(x), then

$$R(p, q) = 0, \qquad S(p, q) = 0.$$

R and S may be obtained without performing the long division by equating coefficients of like powers of x in Eqs. (1) and (2). Thus, the recurrence formula

$$b_k = a_k - pb_{k-1} - qb_{k-2} \qquad (k = 1, 2, \ldots, n), \quad b_{-1} = 0, \quad b_0 = 1$$

and

$$R = a_{n-1} - pb_{n-2} - qb_{n-3}$$

$$S = a_n - qb_{n-2}$$

These two equations are solved simultaneously by Newton-Raphson iteration to obtain corrections Δp and Δq which are added to p and q respectively. The process is repeated until the change in p and q is sufficiently small.

Operation

The program ROOTFINDER 3 performs ITERATE number of iterations in an attempt to extract the quadratic factor. After each single iteration, $\text{LIMIT}_{\text{PREF}}$ is tested to see if convergence is within the preferred limit. After each ITERATE number of iterations, $\text{LIMIT}_{\text{MIN}}$ is tested. If the minimum acceptable convergence limit is met, then the current quadratic is factored. If it is not met, a change of variable is made. A variable change is made after each set of ITERATE iterations provided neither $\text{LIMIT}_{\text{PREF}}$ nor $\text{LIMIT}_{\text{MIN}}$ is met. This procedure continues for a total of NUM_{MAX} single iterations at which time the current quadratic is arbitrarily accepted and factored.

In its present form, the coefficients of the extracted quadratics (p's and q's) from the previous run are used as an initial guess in the current run. Thus a large number of polynomials with slowly changing coefficients can be solved rapidly. If one wishes to solve several unrelated polynomials, separate jobs (using * HOLD cards) must be used. In this instance,

$$p = q = 0.0$$

is the initial guess for each run.

If the coefficient of either the highest or zeroth order polynomial term is zero, the degree of the polynomial is reduced by one prior to the beginning of the quadratic iteration.

After completing its factoring, the program reconstructs the coefficients of the polynomial from the roots for comparison purposes.

Input

Card I	-	$LIMIT_{PREF}$	- preferred convergence limit (e.g. 1×10^{-8})
		$LIMIT_{MIN}$	- minimum convergence limit (e.g. 1×10^{-6})
		ITERATE	- number of trials before each variable change (e.g. 50.0)
		NUM_{MAX}	- total number of iterations (e.g. 200.0)
Card II	-	NXSAVE	- degree of polynomial
Card III to III + NX			- polynomial coefficients of decreasing powers of independent variable (one per card)