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Space Guidance Analysis Memo #56

TO: SGA Distribution  
FROM: R. D. Goss and E. S. Muller  
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SUBJECT: Deriving Random Error Vectors from Covariance Matrix.

In this memo, a method is described for generating random six dimensional error vectors from a prescribed covariance matrix of errors (E matrix). A Gaussian distribution with zero mean is assumed so that the given E matrix uniquely defines the joint probability density function which the resulting error vectors will possess.

Let:

$$\lambda_i = \text{Eigenvalues of E Matrix} \quad i = 1, 2, \dots, 6$$
$$\underline{e}_i = \text{Unit Eigenvectors of E Matrix} \quad i = 1, 2, \dots, 6$$

The  $\underline{e}_i$  may be interpreted as the "directions" in six-space along which components of the error vectors are uncorrelated. The  $\lambda_i$  may be interpreted as the respective variances of these components. Thus, the uncorrelated components of a random error vector may be generated by six independent random number generators with Gaussian distributions and zero mean. The respective variance of each generator would be  $\lambda_i$ . Actually, the six random number generators may be replaced by a single generator which: (1), produces statistically independent random numbers; and (2), has a readably adjustable variance. (e.g. the random number routine in MAC-RNDMN (sigma).) Thus, each component in the uncorrelated "directions" is given by:

$$\text{RNDMN} \left( \sqrt{\lambda_i} \right) \underline{e}_i \quad i = 1, 2, \dots, 6$$

Then, by summing up the error vector components, the desired error vector is:

$$\underline{E} = \sum_{i=1}^6 \text{RNDMN} \left( \sqrt{\lambda_i} \right) \underline{e}_i$$