

MIT Instrumentation Laboratory

DG MEMO NO. 96

TO: J. L. Nevins
FROM: A. R. Klumpp
DATE: 7 November 1963
SUBJECT: A last chance procedure for determination and correction of the LEM abort orbit.

1. General

This memo describes a procedure by which the orbit into which the LEM is initially injected can be ascertained and corrected to an accuracy at 50 thousand feet altitude of under five feet per second in vertical velocity, and fifteen feet per second in horizontal velocity (equivalent to under ten NM error in periapsis). This procedure uses only a two pound handheld sextant and tabulated values of the orbital elements and the corresponding corrections. Orbit determination requires 12 minutes, and after the first 12 minutes, a new estimate is obtained every six minutes.

The procedure is based on measuring the angle subtended by the lunar disk to an accuracy of 0.1 degrees root-sum-squared with 1000 feet elevation error of the horizon. A single fix is sufficient to determine the instantaneous altitude, the second fix yields the vertical component of velocity, and the third fix yields the horizontal component of velocity. Obtaining the horizontal component of velocity is based on the fact that the excess over circular velocity in the horizontal direction results in an apparent vertical acceleration. Obtaining the horizontal velocity component is based on an expression (see appendix) comparing the differences of three consecutive fixes, and does not involve the unreliable process of graphical differentiation. No arithmetic operations are required of the astronaut because the results are tabulated.

The procedure is based on the assumption that the spacecraft has initially been placed in an orbit with extensive deviations from circularity by another injection guidance system of which the tabulated pitch-program procedure being pursued by Mal Johnston is an example. The object of the procedure being described herein is to rapidly determine the apogee and perigee of the orbit in which the astronauts find themselves, and to yield the correction required to produce an orbit with a clear periapsis and some predetermined apoapsis, presumably that of the command module. The correction required is tabulated in the form of the pitch angle and the velocity magnitude. It is assumed that azimuth steering is accomplished by alignment of a point on the window in the direction of the pitch axis with a star known to lie normal to the desired orbital plane. Thus, this scheme produces a thrust vector in the orbital plane, but does nothing to correct any initial plane errors. Some time after the initial correction is made, the astronauts will probably make another correction, one of several options tabulated, depending upon whether the LEM reaches its apoapsis behind, in front of, or in close proximity to the command module.

2. Orbit Determination Procedure

Immediately upon injection into the initial crude orbit, the astronaut takes a fix of the angle subtended by the moon disk using the sextant. This angle determines the altitude and a page in the tabulations. The heading for this page is the subtended angle and the initial estimate of the spacecraft altitude. Six minutes later, the astronaut takes a second fix. This fix determines a row on the page. The heading for this row is the subtended angle, the new estimate of spacecraft altitude, and the initial estimate of vertical velocity. Six minutes later, the astronaut takes a third fix. This fix determines a column on the page. The heading for this column is the subtended angle. The intersection of the column and row contains a tabulation of the parameters of interest to the astronaut at that time.

These are:

1. Periapsis altitude
2. Time to periapsis
3. Apoapsis altitude
4. Time to apoapsis
5. New estimate of altitude
6. New estimate of vertical velocity
7. Estimate of excess over circular horizontal velocity
8. Pitch angle and velocity increment which will yield an immediate circular orbit.
9. Pitch angle and velocity increment which will yield an orbit with the periapsis at the present point and the apoapsis altitude equal to the orbital altitude of the command module. Assuming the present altitude is significantly less than the command module altitude, this orbit would be used for catching up with the command module.
10. Pitch angle and velocity increment which will yield an orbit with the periapsis at the present point and the apoapsis significantly higher than that of the command module. This orbit would be used for allowing the command module to catch up with the LEM. It would probably be used when the present altitude is the altitude of the command module.
11. Pitch angle and velocity increment to apply at apoapsis in order to yield a minimum periapsis. This correction would be used to provide an orbit with a clear periapsis and the maximum catch up rate with respect to the command module. For zero errors in taking fixes, the correction indicated here would be the same each time a fix is taken. Thus, the astronaut can acquire considerable confidence before making this correction by comparing the several tabulations of it.

These numbers which the astronaut obtains at 12 minutes after injection are the first estimates. Six minutes later, the astronaut can take a new fix. This new fix plus the two immediately preceding will yield a new set of estimates of these parameters. Thus, after the first twelve minutes, a new set of estimates is obtained every six minutes

of all the parameters including the critical periapsis altitude. Comparison of several successive estimates will allow the astronaut to acquire confidence in the orbit and the indicated corrections.

In all likelihood, the astronaut will have a voice link to and from earth. This link can be used to enhance the confidence of the astronaut in the correction procedures he may decide upon on the basis of the tabulations, and in fact, when the voice link is operative it may be used as the preferred mode. The astronaut will simply radio the values obtained from the fixes to earth and wait for the return of the correction procedure based on ground computations. Earth based computations can produce a least squares fit of the actual LEM trajectory which is, of course, not possible on board. The on board tabulations will serve as a check.

It is desirable to have some device which would accept each new fix as it is made and yield the estimates of the parameters based on the new fix and the two immediately preceding. However, such a device would probably be a computer or a fairly extensive calculator, particularly because of the large number of parameters required. All of the parameters can be easily tabulated prior to launch, and therefore, this appears to be the more expedient way to supply them. But a slide rule type device might be invented to perform this function well.

3. Performance Characteristics and Errors

Appendix 1 contains derivations of the equations used in the operation and error analyses, and appendix 2 contains plotted results. The results are based on approximate equations, but spot checks have been made which indicate that the results are accurate to approximately 10 percent even at 500,000 ft. altitude.

Figure 1 (appendix 2) shows the angle subtended by the lunar disk vs altitude. Assuming this procedure is to be used over the altitude range of 50 to 500 thousand feet, it will be necessary for the sextant to have a

field of view of at least 165° . This field of view is not presently available in the LEM. Possibly the easiest solution is to add a transparent bubble either in the LEM window or in the skin.

Figure 2 is intended to indicate the sensitivity of altitude determination to sextant errors as a function of altitude. Figure 3 is intended to give insight into the importance of irregularities on the lunar horizon. Note that even at 50,000 ft. altitude, the distance to the horizon is 750,000 ft. or 150 miles.

The errors are plotted in figures 4, 5, and 6 (appendix 2) based on a six minute interval between successive fixes and 0.1° sextant error. With this interval it is possible to determine the vertical and horizontal velocity components to under 5 ft/sec and 13 ft/sec respectively at 50,000 ft altitude and 10 ft/sec and 25 ft/sec at 500,000 ft altitude. With these errors in velocity, it is possible to predict the periapsis altitude to within 10 nautical miles at 50,000 ft and 19 nautical miles at 500,000 ft.

The periapsis error is four times as sensitive to horizontal velocity error than vertical velocity error, and with the horizontal velocity error over twice as large as the vertical, the periapsis error is due almost entirely to the horizontal component. It is possible to significantly reduce the horizontal velocity error if necessary by increasing the interval between successive fixes. This error is inversely proportional to the square of the time between successive fixes. Thus it would be possible to tabulate for fixes spaced at 12 minute intervals and predict the periapsis altitude to under 5 nautical miles at 500,000 ft. altitude. The six minute interval was initially chosen to provide a quick determination of the initial trajectory and to avoid covering a substantial portion of a lunar orbit during the orbit determination. It would be possible to tabulate for the six minute interval for low altitudes where a quick orbit determination is paramount, and for a 12 minute interval for high altitudes where accuracy is paramount.

PREPARED BY: KLUMPP

DATE: 6 Nov 63

APPENDIX 1. DERIVATIONS

SHEET

1

RELATIONSHIP BETWEEN H AND θ

$$\theta = \pi - 2\phi$$

$$\cos \phi = \frac{R}{R+H}$$

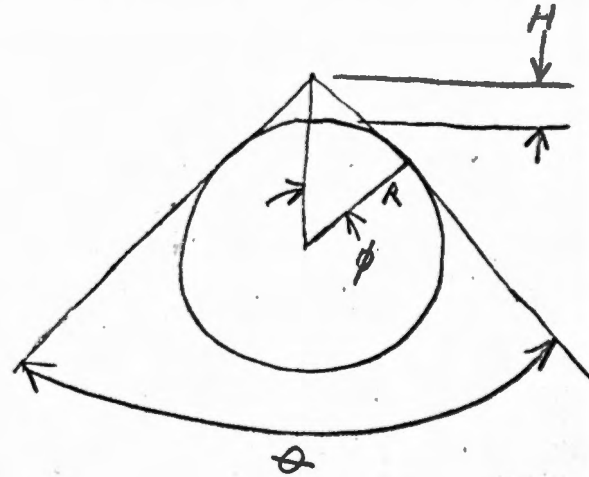
$$\sin \phi = \sqrt{1 - \frac{R^2}{(R+H)^2}}$$

FOR $H \ll 2R$, $\phi \approx \sin \phi$:

$$\phi = \sqrt{\frac{2H}{R}}$$

$$\theta = \pi - \sqrt{\frac{8H}{R}}$$

$$\frac{d\theta}{dH} = -\sqrt{\frac{2}{RH}}$$



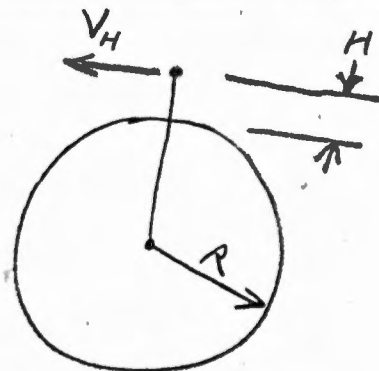
OBTAINING HORIZONTAL EXCESS VELOCITY
FROM ALTITUDE MEASUREMENTS

$$\ddot{H} = \frac{V_H^2}{R+H} - g$$

$$V_H = V_C + V_E$$

WHERE V_C = CIRCULAR ORBIT VELOCITY AT H

V_E = EXCESS OF V_H OVER V_C



$$\ddot{H} = \frac{V_C^2 + 2V_C V_E + V_E^2}{R+H} - g$$

IGNORING V_E^2 YIELDS

$$\ddot{H} = \frac{V_C^2}{R+H} + \frac{2V_C V_E}{R+H} - g$$

$$\text{BUT } \frac{V_C^2}{R+H} = g$$

$$\therefore \ddot{H} = \frac{2V_C V_E}{R+H} = 2g \frac{V_E}{V_C}$$

$$\text{AND } V_E = \frac{1}{2} V_C \frac{\ddot{H}}{g}$$

WHERE \ddot{H} IS FOUND NEXT PAGE

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APPENDIX 1 DERIVATIONS

SHEET

3

OBTAINING HORIZONTAL EXCESS VELOCITY (CONTD)
 \dot{H} FROM ALTITUDE MEASUREMENTS

$$\ddot{H}_t = \frac{\dot{H}_{t+\Delta t} - \dot{H}_{t-\Delta t}}{2\Delta t}$$

$$\dot{H}_{t+\Delta t} = \frac{H_{t+2\Delta t} - H_t}{2\Delta t}$$

$$\ddot{H}_t = \frac{\frac{H_{t+2\Delta t} - H_t}{2\Delta t} - \frac{H_t - H_{t-2\Delta t}}{2\Delta t}}{2\Delta t}$$

$$\ddot{H}_t = \frac{H_{t+2\Delta t} - 2H_t + H_{t-2\Delta t}}{4(\Delta t)^2}$$

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APPENDIX 1 DERIVATIONS

SHEET

4

ERRORS

ALTITUDE ERROR:

DUE TO ERROR IN θ :

$$\frac{dH}{d\theta} = -\sqrt{\frac{RH}{2}}$$

$$\epsilon_H = -\epsilon_\theta \sqrt{\frac{RH}{2}}$$

THIS IS ROOT-SUM-SQUARED WITH 1000 FT ERROR
DUE TO ROUGHNESS OF THE HORIZON.

VERTICAL VELOCITY ERROR:

$$\dot{H} = \frac{H_{t+\Delta t} - H_{t-\Delta t}}{2\Delta t}$$

$$\epsilon_{\dot{H}} = \frac{\sqrt{2} \epsilon_H}{2\Delta t}$$

WHERE $2\Delta t$ IS THE INTERVAL BETWEEN SUCCESSIVE FIXES

HORIZONTAL EXCESS VELOCITY ERROR:

$$V_E = \frac{1}{2} V_C \frac{\ddot{H}}{g} \quad \text{WHERE } \ddot{H} = \frac{H_{t+2\Delta t} - 2H_t + H_{t-2\Delta t}}{4(\Delta t)^2}$$

TAKING ROOT-SUM-SQUARE OF ERRORS IN H YIELDS:

$$\epsilon_{V_E} = \frac{\sqrt{6}}{8} V_C \frac{\epsilon_H}{g(\Delta t)^2}$$

WHERE $2\Delta t$ IS THE INTERVAL BETWEEN
SUCCESSIVE FIXES

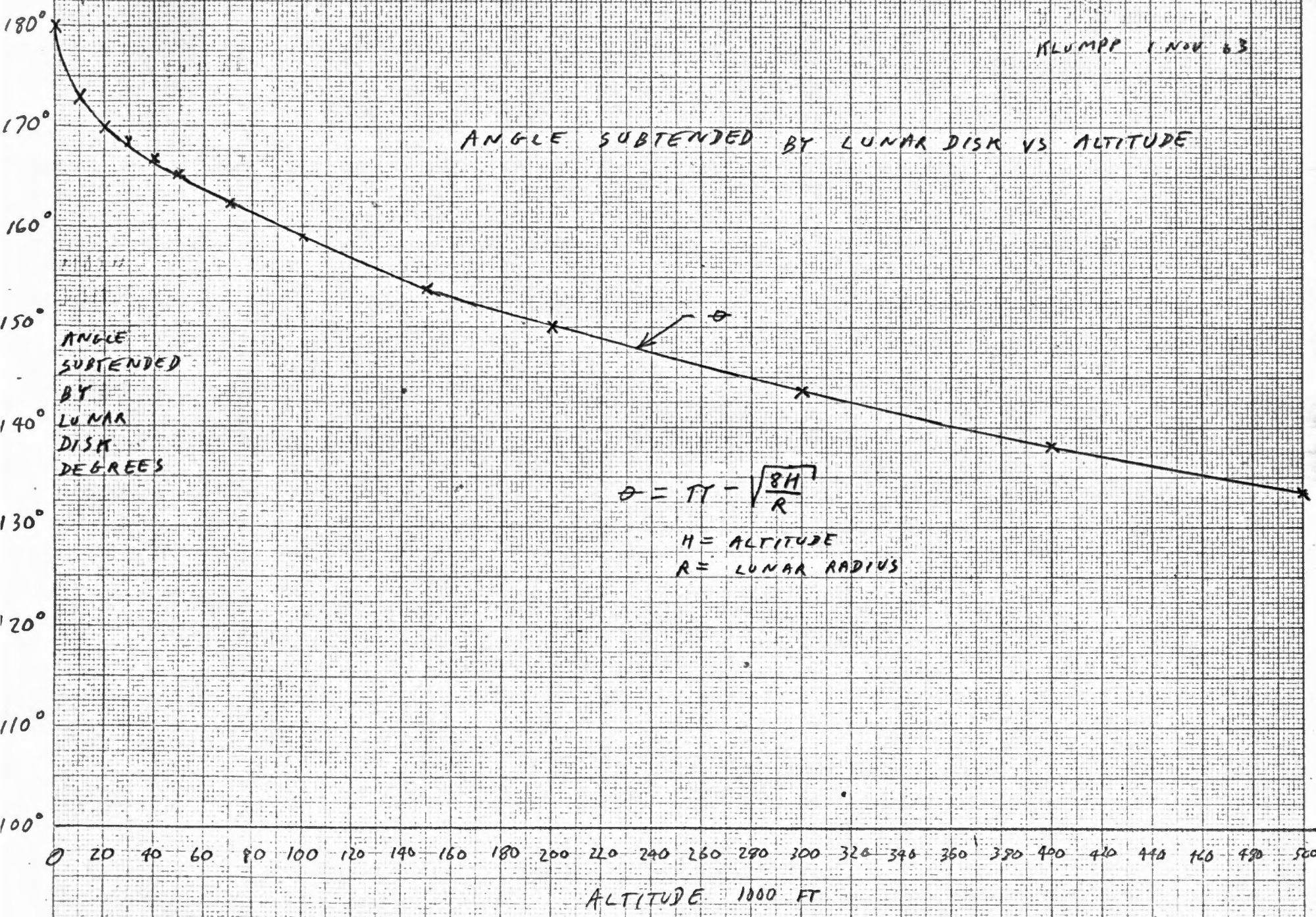
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ANGLE SUBTENDED BY LUNAR DISK VS ALTITUDE

ANGLE
SUBTENDED
BY
LUNAR
DISK
DEGREES

$$\theta = \pi - \sqrt{\frac{8H}{R}}$$

H = ALTITUDE
R = LUNAR RADIUS



APPENDIX 2 FIGURE 1

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DEGREES CHANGE IN ANGLE
SUBTENDED BY MOON'S DISK
PER THOUSAND FT CHANGE
IN ALTITUDE VS ALTITUDE

$$\frac{d\theta}{dH} \text{ VS } H$$

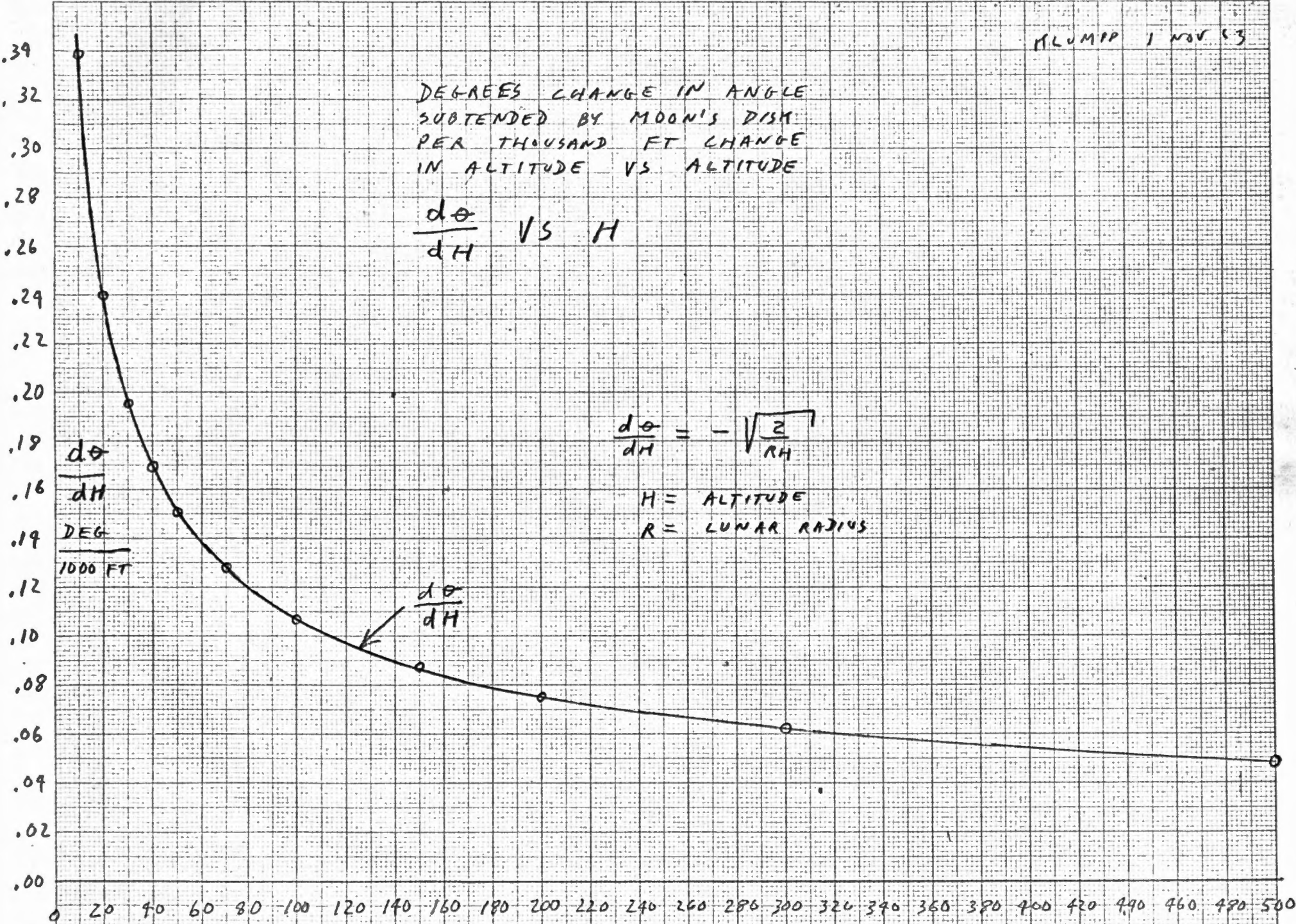
$$\frac{d\theta}{dH} = -\sqrt{\frac{2}{RH}}$$

H = ALTITUDE
R = LUNAR RADIUS

$\frac{d\theta}{dH}$
DEG
1000 FT

$\frac{d\theta}{dH}$

ALTITUDE 1000 FT



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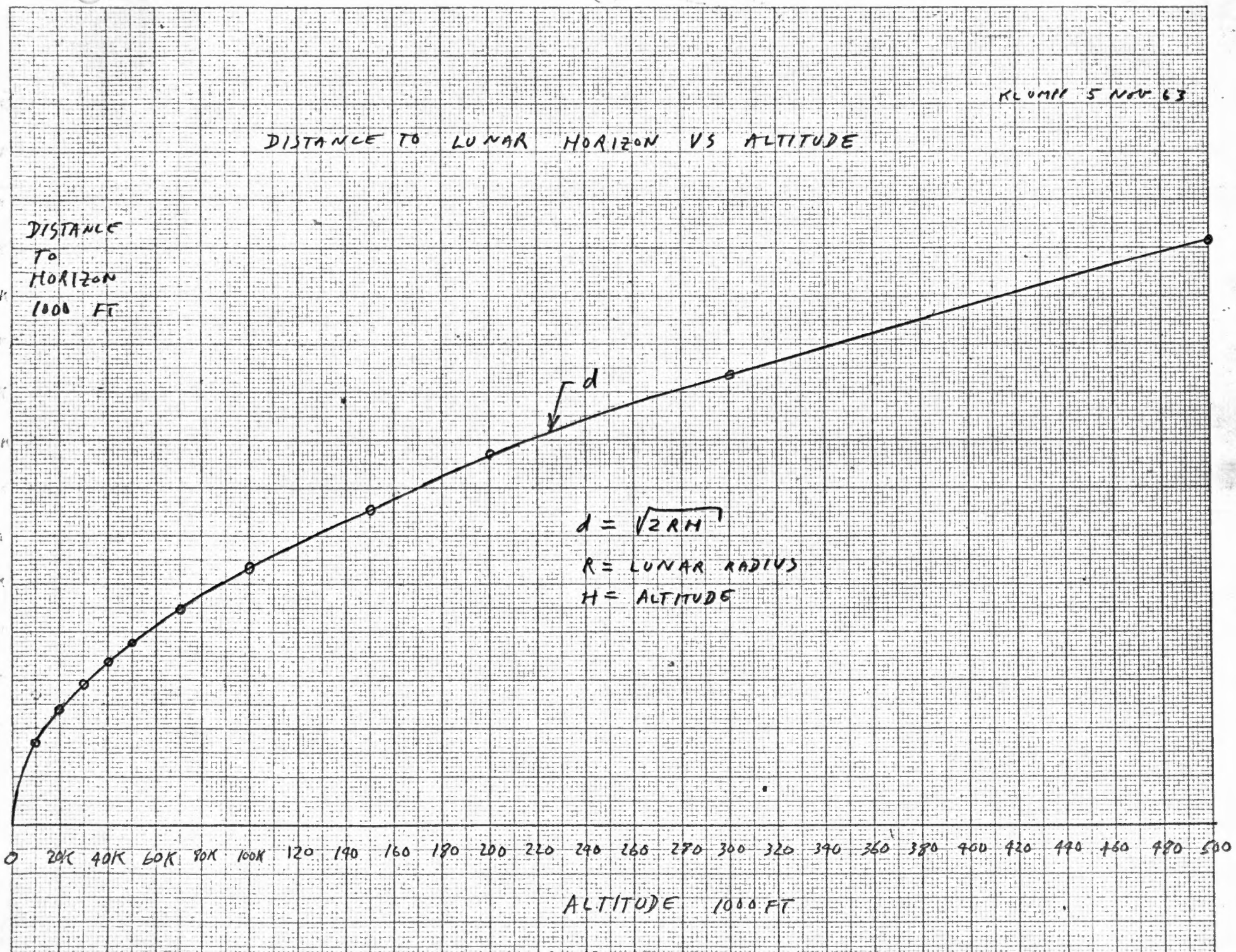
DISTANCE TO LUNAR HORIZON VS ALTITUDE

DISTANCE
TO
HORIZON
1000 FT

$$d = \sqrt{2RH}$$

R = LUNAR RADIUS

H = ALTITUDE



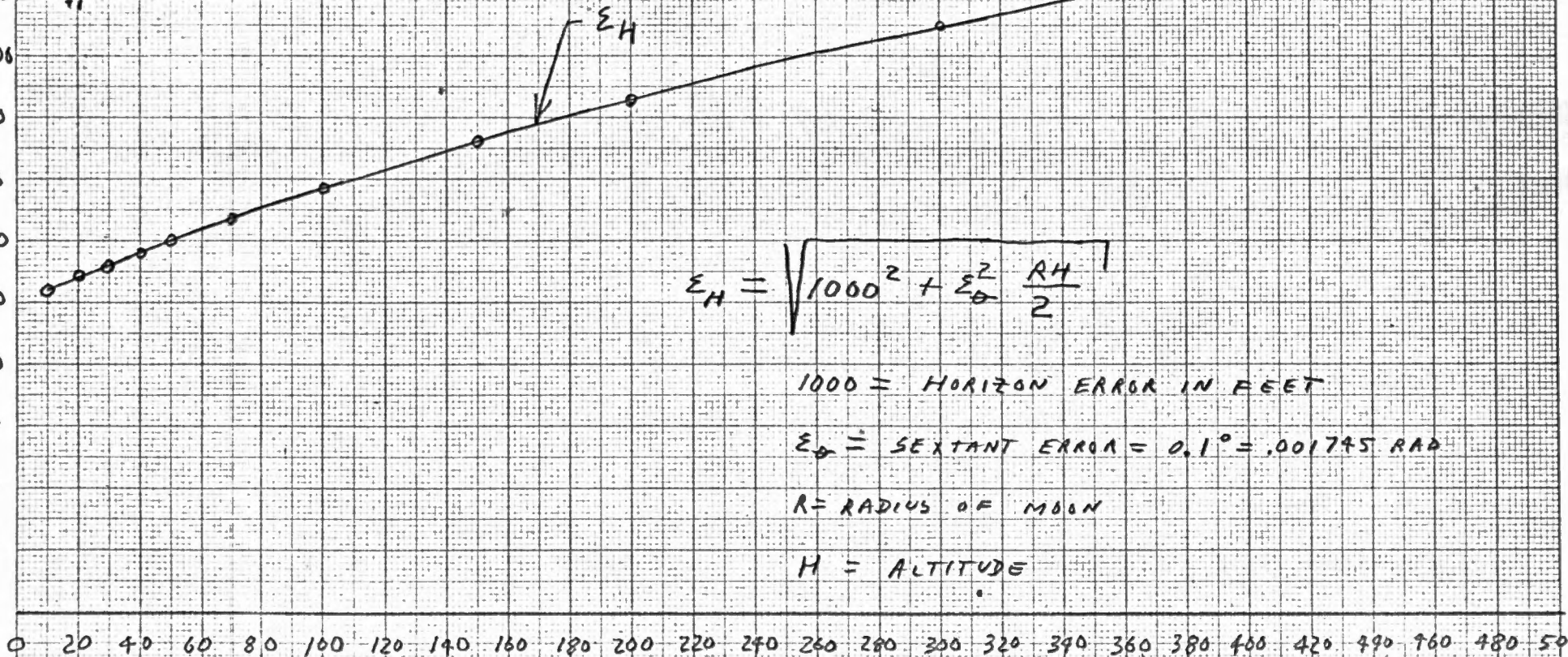
ALTITUDE 1000 FT

ERROR IN DETERMINATION OF ALTITUDE VS ALTITUDE

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2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0

ϵ_H FEET



$$\epsilon_H = \sqrt{1000^2 + \epsilon_D^2 \frac{RH}{2}}$$

1000 = HORIZON ERROR IN FEET

ϵ_D = SEXTANT ERROR = $0.1^\circ = .001745$ RAD

R = RADIUS OF MOON

H = ALTITUDE

ALTITUDE 1000 FT

APPENDIX 2 FIGURE 9

ERROR IN DETERMINATION OF VERTICAL VELOCITY
AND HORIZONTAL EXCESS VELOCITY VS ALTITUDE

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26
24
22
20
18
16
14
12
10
8
6
4
2
0

VELOCITY
ERROR
FT/SEC

HORIZONTAL
EXCESS
VELOCITY
DETERMINATION
ERROR
 ϵ_{VE}

VERTICAL
VELOCITY
DETERMINATION
ERROR ϵ_{H}

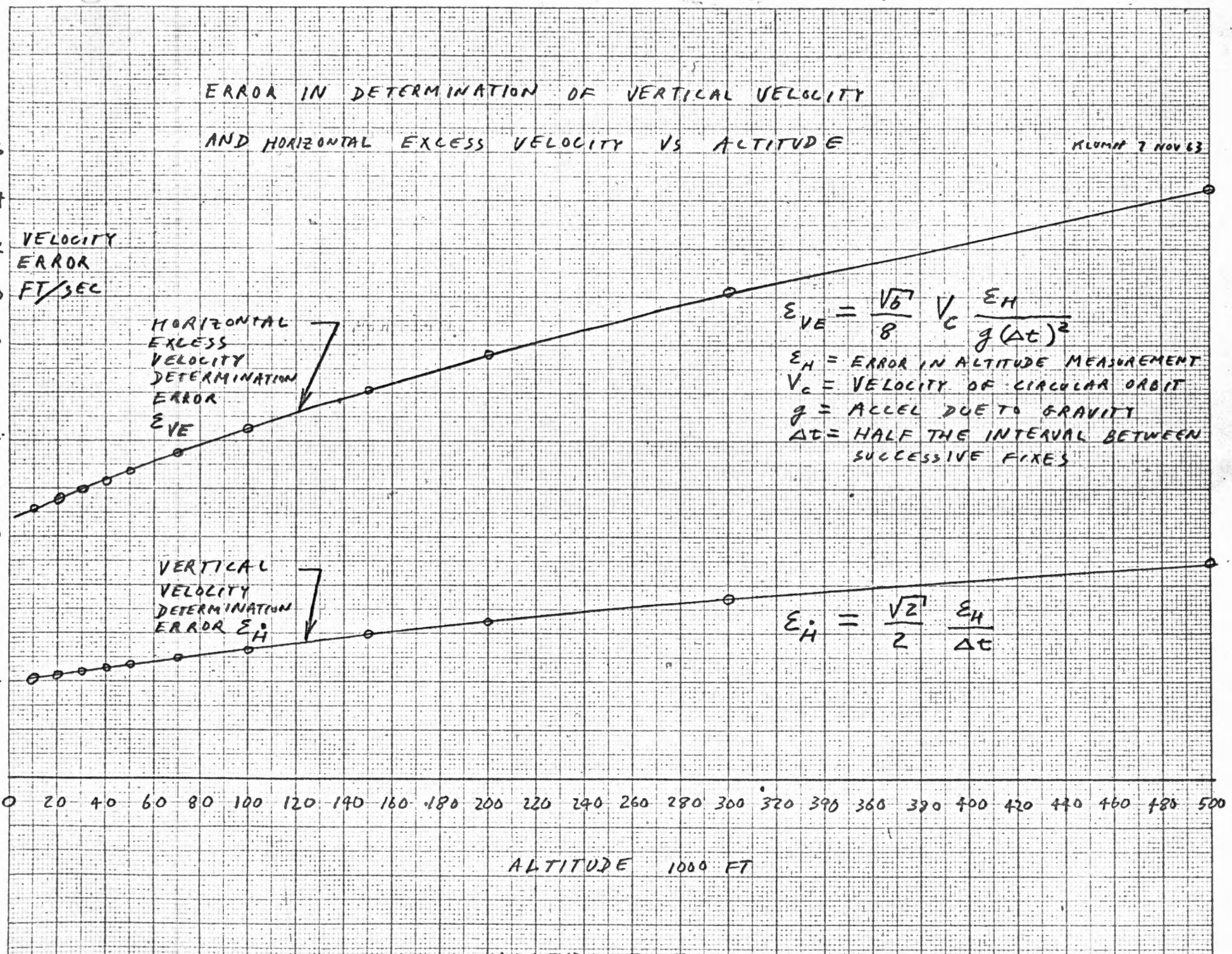
$$\epsilon_{VE} = \frac{\sqrt{b}}{8} V_c \frac{\epsilon_H}{g(\Delta t)^2}$$

ϵ_H = ERROR IN ALTITUDE MEASUREMENT
 V_c = VELOCITY OF CIRCULAR ORBIT
 g = ACCEL DUE TO GRAVITY
 Δt = HALF THE INTERVAL BETWEEN SUCCESSIVE FIXES

$$\epsilon_{H} = \frac{\sqrt{z}}{2} \frac{\epsilon_H}{\Delta t}$$

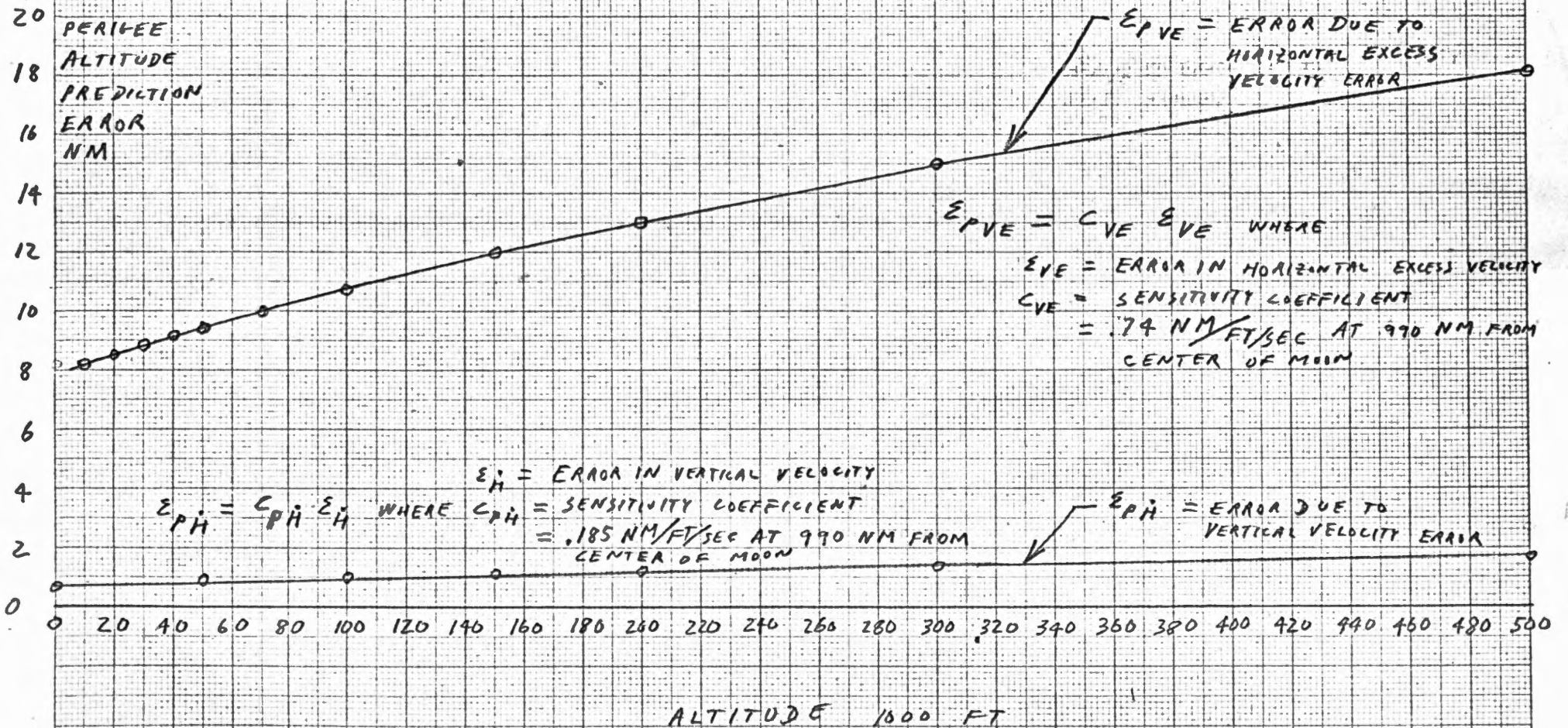
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500

ALTITUDE 1000 FT



ERROR IN PERIGEE ALTITUDE PREDICTION VS ALTITUDE

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Distribution List:

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