Space Shuttle
Astrodynamical Constants

Mission Planning and Analysis Division
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Houston, Texas 77058
SHUTTLE PROGRAM

SPACE SHUTTLE ASTRODYNAMICAL CONSTANTS

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SPACE SHUTTLE ASTRODYNAMICAL CONSTANTS

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1.0 SUMMARY

This document provides basic space Shuttle astrodynamical constants for use in mission planning and construction of ground and onboard software input loads. The data included here are provided to facilitate the use of consistent numerical values throughout the project. The document supersedes reference 1.

2.0 INTRODUCTION

The astrodynamical constants presented in this document are taken primarily from reference 2, "Natural Environment and Physical Standards for Apollo and AAP". Those data have been expanded, and other data have been included when there was an indication that additional information would be useful. The values of the constants in reference 3 have been informally adopted by the Interplanetary Trajectory Committee.

3.0 DISCUSSION OF DATA

The data included in this document are described briefly in this section. Detailed information may be found at the beginning of sections 4 and 5.

3.1 ASTRODYNAMIC CONSTANTS

The trajectory prediction and astrodynamical constants are presented in section 4. The values included are based upon (ref. 2) and the Apollo Navigation Working Group document (ref. 4), but additional data have been included.

3.2 CONVERSION FACTORS

A list of conversion factors that were used to generate the data in section 4 is given in section 5.0. These are the minimum conversions required for systems of units commonly in use at JSC.
4.0 ASTRODYNAMIC CONSTANTS AND PARAMETERS

4.1 INTRODUCTION

This section presents the values and associated uncertainties of the constants and models used in trajectory prediction. A list of conversion factors and a description of the gravitational potential equation are also included.

Uncertainties (1σ) are presented if available. For consistent conversion between units, more decimal digits are given for some quantities than are justified by the uncertainties. The values presented are in agreement with those adopted by NASA Headquarters (ref. 2).

4.2 CONSTANTS AND PARAMETERS

4.2.1 Angular Velocity of the Earth's Rotation
With Respect to the Vernal Equinox

The Earth's angular rotational velocity with respect to a precessing equinox ($\omega_p$) and inertial equinox ($\omega_I$) for OFT (calendar year 1979) based on references 6 and 16 is

$$\omega_p = .7292115854918357 \times 10^{-4} \text{ radians per second}$$

$$\omega_I = .7292115146459210 \times 10^{-4} \text{ radians per second}$$

4.2.2 Other Constants

This section defines values for other constants used in the ground and onboard software. Many of these parameters are not basic constants but are derived using some defining assumption.

a. Stellar aberration constant - the maximum aberration of a star observed from the Earth occurs when direction of motion of observer is at right angles to line of sight to star.

$$C_A = .993674 \times 10^{-4} \text{ rad}$$

b. Mean molecular weight of air - the mean molecular weight of the 10 air constituents ($N_2$, $O_2$, $Ar$, $CO_2$, $Ne$, $He$, $Kr$, $Xe$, $CH_4$, $H_2$) found by:

$$M_o = \frac{\sum (n_i \cdot M_i)}{\sum n_i} = 28.9644 \text{ Kg/Kmol}$$
where \( n_i \) is number density and \( M_i \) is molecular weight

c. The \text{gas constant} - the constant factor in the equation of state for perfect gases.

\[ R^* = 8.31432 \times 10^{-3} \text{ Newton meter (Kmol·K)} \]

d. \( g \frac{M_o}{R^*} \) - derived constant used in onboard calculations of density involving sea level gravity acceleration, mean molecular weight and the gas constant

\[ g \frac{M_o}{R^*} = 0.01041294 \text{ K}^0/\text{ft} \]

e. \text{Speed of sound constant} - derived constant used in the calculation of the speed of sound. The speed of sound is defined by:

\[ C_S = \left( \frac{\gamma R^* T_m}{M_o} \right)^{1/2} \]

where \( \gamma = 1.4 \) and is the ratio of the specific heat of air at constant pressure to that at a constant volume (dimensions).

Because \( \gamma \), \( R^* \), and \( M_o \) are constants, they are combined for computational ease and the equation is written:

\[ C_S = K \sqrt{T_m} \]

where \( K \) is the speed of sound constant

\[ K = 65.77035 \text{ ft/sec/} \sqrt{\text{K}^0} \]
4.2.3 Gravitational Potential Function

The classical expression for the gravitational potential \( V \) exerted at a point in space located at a distance \( r \) from the center of the attracting body of radius \( R_E \) and gravitational parameter \( \mu_E \) is given in equation as

\[
V(r, \phi, \lambda) = \sum_{n=0}^{\infty} \mu_E \left( \frac{R_E}{r} \right)^n \sum_{m=0}^{n} \frac{P_{nm}(\sin \phi)}{P_{nm}(\sin \phi)} \left[ C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right]
\]

\( \phi \) is latitude (geodetic)
\( \lambda \) is longitude

where \( C_{nm} \) and \( S_{nm} \) are the harmonic coefficients of the potential function and \( P_{nm}(\sin \phi) \) represents the associated Legendre functions of the first kind, of degree \( n \) and order \( m \). Because \( \sin \phi = z/r = u \), where \( u \) is a direction cosine, the associated Legendre functions may be expressed as:

\[
P_{nm}(u) = \frac{\cos^n \phi \, du}{2^n \sin^n \phi} \frac{d^{n+m}}{du^{n+m}} (u^2 - 1)^n
\]

The constants are for real time operations are:

\[
\begin{align*}
C_{2,0} &= -1082.7 \times 10^{-6} \\
C_{3,0} &= 2.56 \times 10^{-6} \\
C_{4,0} &= 1.58 \times 10^{-6} \\
C_{22} &= 1.57 \times 10^{-6} \\
S_{22} &= -.897 \times 10^{-6}
\end{align*}
\]

all others are zero.

(Higher order models are sometimes used for analysis and postmission trajectory reconstruction.)
4.2.3.1 Equatorial Earth Radius (Gravitational)

1 E. r. = .6378160000000000+007 + .50000+001 m (ref. 2).
1 E. r. = .2092572178477690+008 + .16404+002 int. ft
1 E. r. = .3443930885519158+004 + .26998-002 n. mi.
1 E. r. = .9999992160754699+000 .78392-006 E. r. (MCC)

4.2.3.2 Gravitational Parameter \( GM_e = \mu_e = \mu_{Earth} \)

\( \mu_e = .3986012000000000+015 \) + .40000+009 m\(^3\)/sec\(^2\) (ref. 2).

\( \mu_e = .14076468853278542+017 \) + .14126+001 (int. ft)\(^3\)/sec\(^2\).

\( \mu_e = .5275027808522208+005 \) + .62970-001 (n. mi)\(^3\)/sec\(^2\).

\( \mu_e = .1930931661816326+002 \) + .19979-001 (E. r.)\(^3\)/hr\(^2\) (MCC)

4.2.3.3 Mass of the Earth

\( M_e = .5973343323842350+025 \) KG
\( M_e = .1316896781981220+026 \) LBM

4.2.4 Lunar Constants

4.2.4.1 Earth-Moon Mass Ratio (ref. 2)

\( M_e/M_m = 81.3010 (\pm 0.0010) \)

4.2.4.2 Mean Lunar Radius

\( R_m = .1738090000000000+007 \) + .70000+002 m (ref. 2)
\( R_m = .5702395013123360+007 \) + .22966+003 int. ft
\( R_m = .9384935205183585+003 \) + .37797-001 n. mi.
\( R_m = .2725062772756741+000 \) + .10975-004 E. r. (MCC)

4.2.4.3 Principal Axes

\( a = .1738570000000000+007 \) + .70000+002 m (ref. 2)
\( a = .5703969816272966+007 \) + .22966+003 int. ft
\( a = .9387526997840173+003 \) + .37797-001 n. mi.
\( a = .2725815340305558+000 \) + .10975-004 E. r. (MCC)

\( b = .1738210000000000+007 \) + .70000+002 m (ref. 2)
\( b = .5702748713910761+007 \) + .22966+003 int. ft
\( b = .9385583153347732+003 \) + .37797-001 n. mi.
\( b = .2725250914643945+000 \) + .10975-004 E. r. (MCC)
where \(a\) is directed toward the center of the Earth, \(c\) is coincident with the Moon’s rotational axis, and \(b\) is perpendicular to \(a\) and \(c\).

4.2.4.4  Gravitational parameter for the Moon (\(GM_m = \mu_m = \mu\) Moon)

\[
\begin{align*}
\mu_{\text{Moon}} &= .4902780000000000 + 013 \pm .60000 + 008 \text{ \(m^3/\text{sec}^2\)} \quad \text{(ref. 2)} \\
\mu_{\text{Moon}} &= .1731400417087798 + 015 \pm .21189 + 010 \text{ \(\text{int. ft}^3/\text{sec}^2\)} \\
\mu_{\text{Moon}} &= .7718260968372028 + 003 \pm .94456 - 002 \text{ \(\text{n. mi.}^3/\text{sec}^2\)} \\
\mu_{\text{Moon}} &= .2448838571715250 + 000 \pm .29969 - 005 \text{ \(\text{E.r.}^3/\text{hr}^2\)} \quad \text{(MCC)}
\end{align*}
\]

4.2.5  General Constants

4.2.5.1  Astronomical Unit

\[
\begin{align*}
\text{AU} &= .1495978930000000 + 012 \pm .50000 + 004 \text{ \(m\)} \quad \text{(ref. 2)} \\
\text{AU} &= .4908067355643045 + 012 \pm .16404 + 005 \text{ \(\text{int. ft}\)} \\
\text{AU} &= .8077640010799136 + 008 \pm .26998 + 001 \text{ \(\text{n. mi.}\)} \\
\text{AU} &= .2345469159233102 + 005 \pm .78393 - 003 \text{ \(\text{E.r.\ (MCC)}\)}
\end{align*}
\]

4.2.5.2  Velocity of Light in a Vacuum

\[
\begin{align*}
c &= .2997925000000000 + 009 \pm .30000 + 003 \text{ \(\text{m/sec\ (ref. 2)}\)} \\
c &= .9835711942257218 + 009 \pm .98425 + 003 \text{ \(\text{int. ft/sec}\)} \\
c &= .1618750000000000 + 006 \pm .16199 + 000 \text{ \(\text{n. mi./sec}\)} \\
c &= .1692105801590269 + 006 \pm .16933 + 000 \text{ \(\text{E.r./hr\ (MCC)}\)}
\end{align*}
\]

4.2.5.3  Gravitational Parameters for the Sun

\[
\begin{align*}
\mu_{\text{Sun}} &= .1327124990000000 + 021 \pm .15000 + 014 \text{ \(m^3/\text{sec}^2\)} \quad \text{(ref. 2)} \\
\mu_{\text{Sun}} &= .4686697671960388 + 022 \pm .52972 + 015 \text{ \(\text{int. ft}^3/\text{sec}^2\)} \\
\mu_{\text{Sun}} &= .2089246235906454 + 011 \pm .23614 + 004 \text{ \(\text{n. mi.}^3/\text{sec}^2\)} \\
\mu_{\text{Sun}} &= .5623718533157138 + 007 \pm .74922 + 000 \text{ \(\text{E.r.}^3/\text{hr}^2\)} \quad \text{(MCC)}
\end{align*}
\]
4.3 EPHEMERIS TAPE SYSTEMS

4.3.1 DE19 Tape

The ephemeris tape system to be used for all missions is provided by JPL and is called the JPL Development Ephemeris Number 19 (DE19). For additional information, see references 7 and 8.

4.3.3 Tape System Conversion Factors

The following values are to be used to convert DE19 units to kilometers (refs. 7, 8, and 11).

\[ AU = 149597893 \text{ \text{km} \text{ (scale factor for planetary ephemerides)}} \]
\[ R_{em} = 6378.1492 \text{ \text{km} \text{ (scale factor for lunar ephemerides)}} \]
\[ \mu^{-1} = 81.301 \text{ \text{ (ratio of Earth mass to Moon mass)}} \]
\[ GE = 398601.2 \text{ \text{km}^3/\text{sec}^2 \text{ (gravitational parameter of the Earth)}} \]

4.4 DRAG MODEL

4.4.1 Reference Atmospheres

Seven reference atmospheres to be used are listed below.

a. Jacchia (ref. 12)
b. U. S. Standard Atmosphere, 1962 (ref. 13)
c. U. S. Standard Atmosphere Supplements, 1966 (ref. 14)
d. Cape Kennedy Reference Atmosphere (ref. 15)
e. GLOBAL Reference Atmosphere (ref. 17)
f. Edwards Atmosphere (ref. 18)
g. Vandenberg Atmosphere (ref. 19)

The MSFC Modified Jacchia Model is a computerized version of Jacchia's Static Diffusion Model (ref. 12). This model and other mechanisms may be used for density computations between 120 and 1000 km. This model is dynamic because the values of the atmospheric parameters vary widely with geomagnetic activity, season, solar activity, and latitude. If predicted values of solar
activity or geomagnetic activity are required, they may be obtained from
the Mathematical Physics Branch, Johnson Space Center, 483-A751.

The U. S. Standard Atmosphere (1962) is a static model in that the values of
density are fixed with altitude. This model is primarily used as a standard for
defining atmospheric properties and is accepted as the worldwide reference for
air data devices. Note, however, that for altitudes in the interval of 300 to
500 km, the 1962 standard atmosphere is as much as 75-percent more dense than
the Jacchia model. The standard 1962 atmosphere model is used in the MCC for
mission operations in the Encke free-flight predictor whenever the spacecraft
altitude is between 125 km and entry interface.

The U. S. Standard Atmosphere Supplements (1966) is a dynamic model very similar
to the MSFC modified Jacchia. It is currently in use for Apollo entry studies
and will be used for other entry evaluations.

The Cape Kennedy Reference Atmosphere, which is sometimes referred to as the
Patrick atmosphere, should be used for vehicle launch analysis and for launches
from the Eastern Test Range. For launches from the Western Test Range, the
Vandenberg reference should be used.

The four D global and Edwards AFB models are used for entry analysis, the
generation of operational profiles and dispersion analysis.

4.4.2 Drag Equations (for Orbit Operations)

\[ R_D = - \frac{1}{2} C_D \rho \frac{A}{M} (R - \Omega \times R) \cdot \frac{R - \Omega \times R}{R} \]

where

\[ C_D = (CDF + CDN \sin \alpha \sin^2 \beta) + CDS \sin \beta + CDA \sin \alpha \]

and

\[ \alpha = \text{angle of attack} \]
\[ \beta = \text{angle of side slip} \]

CDF, CDN, CDS, CDA, N = fit coefficients associated with vehicle
configuration

\[ C_D = 2.0 \] (above 400,000 feet for constant area drag)
\( A \) = effective cross sectional area of vehicle
\( M \) = mass of vehicle
\( \rho \) = density of atmosphere at given altitude \( (z_1) \)
\( \mathbf{R} \) = inertial velocity vector of vehicle
\( \mathbf{R} \) = position vector to vehicle from the center of Earth
\( \mathbf{R}_d \) = acceleration vector caused by drag
\( \Omega \) = Earth's rotational vector = \((0, 0, \omega)\)

\[
z_1 = r - \frac{r}{d}
\]
\[
d = \left( \frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} \right)^{1/2}
\]
\[
r = (x^2 + y^2 + z^2)^{1/2}
\]

For the ellipsoid used in the 1962 U. S. Standard Atmosphere,

\[
a = 6.378178 \times 10^6 \text{ m}
\]
\[
b = 6.356797 \times 10^6 \text{ m}
\]

4.5 FISCHER EARTH MODEL

The following constants describe the Fischer Earth model (1960), which is used for the location of radar stations and other Earth surface features (ref. 2)

4.5.1 Equatorial Earth Radius

\[
a = 6378166000000000 + 007 \text{ m}
\]
\[
a = 2092574416981527 + 008 \text{ int. ft}
\]
\[
a = 39393535125269978 + 004 \text{ n. mi.}
\]
\[
n = 1000000155784905 + 001 \text{ E.r. (MCC)}
\]
4.5.2 Flattening

\[ f = \text{flattening} = 1 - \frac{b}{a} \]

\[ f = 1/298.30 = 0.3352329869269135 \times 10^{-2} \]

4.5.3 Polar Earth Radius

\[ b = 6356784283607107 + 007 \text{ m} \]
\[ b = 208559148165061 + 008 \text{ int. ft} \]
\[ b = 3432388922034075 + 004 \text{ n. mi.} \]
\[ b = 9966478263900521 + 000 \text{ E.r. (MCC)} \]

4.5.4 Eccentricity of Ellipsoid

\[ e = \sqrt{1 - \frac{b^2}{a^2}} \]

\[ e = 0.8181333401693114 - 001 \]
\[ e^2 = 2f - f^2 \]
\[ e^2 = 0.6693421622965943 - 002 \]

5.0 EQUIVALENTS AND CONVERSION FACTORS

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<th>Unit</th>
<th>Equivalent</th>
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<tbody>
<tr>
<td>1 int. ft</td>
<td>.3048000000000000 + 000 m (exact) (ref. 2)</td>
</tr>
<tr>
<td>1 n. mi.</td>
<td>.1852000000000000 + 001 km (exact) (ref. 2)</td>
</tr>
<tr>
<td>1 E.r.</td>
<td>.6378165000000000 + 004 km (exact for scaling) for MCC internal use</td>
</tr>
<tr>
<td>1 lbm</td>
<td>.4535923700000000 + 000 kg (exact) (ref. 2)</td>
</tr>
<tr>
<td>1 hr</td>
<td>3600.000000000000 + 000 sec (exact)</td>
</tr>
<tr>
<td>1 rad</td>
<td>( \frac{180}{\pi} ) deg</td>
</tr>
<tr>
<td>1 deg</td>
<td>3600 arc sec</td>
</tr>
<tr>
<td>1 km</td>
<td>.5399568034557235 + 000 n. mi.</td>
</tr>
</tbody>
</table>
\[1 \text{ m} = 0.3280839895013123 + 001 \text{ int. ft}\]
\[1 \text{ n. mi.} = 0.6076115485564304 + 004 \text{ int. ft}\]
\[1 \text{ rad} = 0.5729577951308233 + 002 \text{ deg}\]
\[1 \text{ deg} = 0.01745329251994329 - 001 \text{ rad}\]
\[1 \text{ kg} = 0.2204622621848776 + 001 \text{ lbm}\]
\[1 \text{ int. stat. mi.} = 5280 \text{ ft (exact)}\]
\[1 \text{ lbf} = 32.174048556 \text{ (int. ft/sec}^2)\text{lbm}\]
\[\pi = 0.3141592653589793 + 001 \text{ (ref.2)}\]
REFERENCES


