SPACE SHUTTLE GN&C EQUATION DOCUMENT

No. 19

Prelaunch Alignment and Platform Compensation

by

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FOREWORD


Gerald M. Levine, Director
APOLLO Space Guidance Analysis Division
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NOMENCLATURE

\[
\mathbf{b} = \begin{pmatrix}
  b_X \\
  b_Y \\
  b_Z
\end{pmatrix}
\]

- \( x\), \( y\), and \( z\) accelerometer biases.

\[
\mathbf{d} = \begin{pmatrix}
  \text{NBD}_X \\
  \text{NBD}_Y \\
  \text{NBD}_Z
\end{pmatrix}
\]

- \( x\)-, \( y\)-, and \( z\)-gyro drifts.

\( d_3 \)

- Intermediate accumulator for down-gyro torquing commands.

\( e_I, e_M, e_O \)

- Euler angles relating platform orientation to vehicle orientation. For three-gimbal systems, these are the inner, middle, and outer gimbal angles. For four-gimbal systems, these are the inner, outer-middle, and outer gimbal angles when the inner-middle gimbal is in its normal position. The outer-gimbal axis is parallel with \( x_B \). When \( e_I = e_M = e_O = 0 \), each platform axis has the same orientation as the corresponding vehicle axis.

\( e_1, e_2, e_3 \)

- Intermediate accumulators for East-accelerometer signal processing.

\[
\mathbf{f} = \begin{pmatrix}
  \Delta v_X \\
  \Delta v_Y \\
  \Delta v_Z
\end{pmatrix}
\]

- \( x\)-, \( y\)-, and \( z\)-accelerometer scale factor errors measured in parts per part (10^6 ppm).

\( G \)

- G-sensitive matrix which transforms specific force into its effect on gyro drifts via g-sensitive terms, for example,

\[
\begin{pmatrix}
  \text{ADIA}_X & \text{ADOA}_X & \text{ADSR}_X \\
  \text{ADSR}_Y & \text{ADIA}_Y & \text{ADOA}_Y \\
  \text{ADOA}_Z & \text{ADSR}_Z & \text{ADIA}_Z
\end{pmatrix}
\]

where the g-sensitive drifts of the \( i \)th gyro are \( \text{ADIA}_i \) along the input axis, \( \text{ADOA}_i \) along the output axis, and \( \text{ADSR}_i \) along the spin reference axis.
Gyrocompass and level loop constants.

\[ k_1 = -0.062 \quad k_3 = 0.005 \quad k_4 \]

\[ k_2 = -0.0003 \quad k_3 \quad k_5 = -0.5 k_3 \]

\[ k_3 = 0.0655 \text{ mr/cm/s}; \text{ e.g., if the accelerometers are quantized at 1 cm/s and the gyros are quantized at 1 \( \mu \text{r} \), then } k_3 = 65.5 \text{ gyro pulses/accelerometer pulse.} \]

\( M \) - Transformation matrix from navigation frame (x - North, y - East, z - down) to the platform frame.

\( n \) - Number of seconds to be spent in the leveling mode before proceeding to the gyrocompass mode; 640 seconds for the first pass and 320 seconds after an azimuth change.

\( n_P \) - Number of passes through leveling/gyrocompassing loop since the last gyro torquing sequence; counts up to 10.

\( n_1, n_2, n_3 \) - Intermediate accumulators for North-accelerometer signal processing.

\( t_C \) - Cycle time for the compensation program; during pre-launch alignment, \( t_C = 0.5 \text{ sec.} \)

\( x_B', y_B', z_B \) - Vehicle axes; at launch \( x_B \) is vertical up.

\( x_P, y_P, z_P \) - Platform axes.

\( \alpha \) - Launch azimuth; desired angle from North to \( x_P \) about vertical down, e.g., a 90° launch azimuth points \( x_P \) eastward.

\( \alpha' \) - Vehicle azimuth; angle from North to \( z_B \) about vertical down.

\( \alpha'' \) - New launch azimuth; specified for an azimuth change.

\[ \Delta v = \begin{pmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} \] - Integrated specific force measurements expressed in platform coordinates.
\[ \Delta v' = \begin{pmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} \]

- Accumulators for the x, y, and z integrating accelerometers.

\[ \Delta v'' = \begin{pmatrix} \Delta v_N \\ \Delta v_E \\ \Delta v_D \end{pmatrix} \]

- Integrated specific force measurements expressed in the navigation frame; North, East, and Down.

\[ \tau = \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} \]

- Accumulators for gyro pulse-torquing angles.

\( \phi \)

- Vehicle latitude.

\( \omega_E \)

- Earth rate; approximately 15.04 deg/hr.

\( \tau_E \)

- Earth rate vector in navigation frame integrated for 5 seconds; used for earth rate compensation.
1. INTRODUCTION

The prelaunch-alignment program aligns the stable platform to some desired orientation with respect to the local navigation reference frame. The alignment progresses in three distinct phases: coarse aligning, leveling, and gyrocompassing. While the program is in the leveling or gyrocompassing phase, a new launch azimuth may be specified without reinitializing the program. At liftoff, control is passed to a navigation monitor/control program. A description of the stable-platform-compensation program is included.

The programs and equations given in this document are similar to those used for Apollo.
2. FUNCTIONAL FLOW DIAGRAMS

Since interaction between the prelaunch alignment program and the platform compensation program is desirable, a description of the latter program is included.

In the compensation program, accelerometer measurements are corrected for known biases and scale-factor errors, and gyro torquing angles which will offset the known gyro drifts are computed. If no other program is torquing the gyros or needs the integrated specific-force measurements, then synchronized torquing is not specified and the compensation program torques the gyros. Otherwise, synchronized torquing should be specified to minimize pulse-torquing errors and control is passed to the synchronizing program which is responsible for torquing the gyros. In either case, the compensation cycle time must be specified.

After initialization and coarse-aligning the gimbals, the prelaunch alignment initialization program initiates the compensation program with a .5 second cycle time specifying the prelaunch alignment program as the synchronizing program. The leveling phase is also specified for the alignment program.

When the compensation program passes control to the alignment program, the integrated specific-force measurements are filtered to produce gyro torquing angles. After every ten passes through the program, the torquing commands are actually sent to the gyros. When liftoff occurs, gyro torquing is desynchronized and control is passed to a navigation monitor/control program.

Initially, the leveling phase lasts for 640 seconds after which the gyrocompassing phase is entered. When an azimuth change is requested, the gyros are torqued to position the platform in the new desired orientation and the leveling phase is again entered. If the azimuth change is requested before first entering the gyrocompassing phase, then the next leveling phase lasts for 640 seconds; otherwise, subsequent leveling phases last for 320 seconds.
Load and/or verify initialization data

Align gimbals such that:
- $z_p$ is down
- $x_p$ is along the launch azimuth

Switch the platform to the inertial mode

Has Liftoff Occurred

- No
  - Initialize the Prelaunch Alignment Program for leveling
  - Initialize Compensation Program for synchronized torquing with the Prelaunch Alignment Program

- Yes
  - Program Alarm: Liftoff has occurred before the platform alignment was adequate
  - Navigation Monitor Control Program

Is Compensation Program Running

- Yes
  - EXIT

- No
  - Zero Accelerometers
    - Set up logic to begin the Compensation Program after .5 seconds

EXIT

Figure 1. Prelaunch Alignment Initialization Program
ENTER

Save Accelerometer Measurements

Zero Accelerometers

Set up logic to enter the Compensation Program $t_C$ seconds after the time of this entry into it.

$\text{T}_C = 0.5 \text{ sec}$
during pre-launch alignment

Compensate accelerometer measurements for scale factor errors and bias

Compute gyro torquing needed to offset gyro drift over past $t_C$ seconds

Synchronized Torquing

Torque gyros to offset computed drift error

Go to synchronizing program

EXIT

Figure 2. Compensation Program
ENTER

Has Liftoff Occurred

Desynchronize torquing for the Compensation Program

Navigation monitor/control program

Azimuth Change

Yes

Increment z-torquing angle accordingly

Torque Gyros

Replace launch azimuth with new launch azimuth

(Figure 1)

No

Figure 3a. Prelaunch Alignment Program
Figure 3b. Prelaunch Alignment Program
3. **INPUT AND OUTPUT VARIABLES**

The gyro and accelerometer performance parameters are loaded and/or verified in the prelaunch alignment initialization program, but they are passed on to and only used by the compensation program.

3.1 **Prelaunch Alignment Initialization Program**

**Input variables:**
- $\varphi$ - latitude
- $\alpha$ - launch azimuth
- $\omega$ - vehicle azimuth
- $b$ - accelerometer biases
- $f$ - accelerometer scale-factor errors
- $d$ - non-$g$-sensitive gyro drifts
- $G$ - matrix of $g$-sensitive gyro drifts

**Output variables:**
- $M$ - transformation matrix from the navigation frame to the platform frame.
- $\omega_E$ - earth rate vector in the navigation frame integrated for 5 seconds.
- $e_t, e_M$ - gimbal angles.
- $e_O$ - compensation cycle time.

Also, the following parameters are initialized:
- $n$ - number of seconds to be spent in the leveling phase.
- $n_1, n_2, n_3, e_1, e_2, e_3, d_3$ - intermediate accumulators for the prelaunch alignment program.
\( n_p \) - number of passes through the prelaunch alignment program since the last gyro-torquing sequence.

\( \tau \) - gyro-torquing angles.

\( \Delta \gamma' \) - accelerometer accumulators.

3.2 Compensation Program

Input variables:

\( \Delta \gamma' \) - accelerometer measurements.

\( t_C \) - compensation cycle time.

\( b \) - accelerometer biases

\( f \) - accelerometer scale-factor errors.

\( d \) - non-g-sensitive gyro drifts

\( G \) - matrix of g-sensitive gyro drifts.

Before initiating the compensation program, the following should be initialized:

\( \Delta \gamma' \) - Accelerometer accumulators

\( \tau \) - Gyro torquing angles

Output variables:

\( \Delta \gamma \) - Integrated specific force measurements expressed in platform coordinates.

\( \tau \) - Gyro torquing commands adjusted to compensate for known gyro drifts.

\( \Delta \gamma' \) is reset to zero

3.3 Prelaunch Alignment Program

Input variables:

\( M \) - Transformation matrix from the navigation frame to the platform frame.

\( \Delta \gamma \) - Integrated specific force measurements expressed in platform coordinates.

\( \omega_E \) - Earth rate compensation vector.
\( a' \) - New launch azimuth.
\( \tau \) - Unexecuted gyro-torquing commands.

It is assumed that the following have been initialized:

- \( n \) - Number of seconds to be spent in the leveling phase.
- \( n_1, n_2, n_3, e_1, e_2, e_3, d_3 \) - Intermediate accumulators.
- \( n_p \) - Number of passes through the program since the last gyro-torquing sequence.

Output Variables:

- \( \tau \) - Updated gyro-torquing angles.
- \( n \) - Number of seconds to be spent in subsequent leveling phases.
4. DESCRIPTION OF EQUATIONS

4.1 Prelaunch Alignment Initialization Program

This program executes the coarse aligning phase and initializes parameters used by the compensation program and the prelaunch alignment program. Since \( x_B \) is vertical, the gimbal angles used in coarse aligning take the simple form

\[
e_I = \frac{\pi}{2}, \quad e_M = 0, \quad e_O = \alpha - \alpha'
\]

If \( x_B \) or \( z_P \) is not vertical, more complicated expressions for the gimbal angles might be required.

4.2 Compensation Program

The accelerometer measurements corrected for known biases and scale-factor errors are given by

\[
\Delta v_i = (1 + \Delta SF_i) \Delta v_i' - b_i t_C, \quad i = X, Y, Z
\]

The amount of platform drift accumulated over the previous \( t_C \) seconds is

\[
t_C \left[ \begin{array}{c}
NBD_X \\
NBD_Y \\
NBD_Z \\
\end{array} \right] = \Delta v \left[ \begin{array}{c}
\Delta v_X \\
\Delta v_Y \\
\Delta v_Z \\
\end{array} \right]
\]

This drift is removed from the gyro-torquing angles before the torquing commands are issued.

4.3 Prelaunch Alignment Program

The integrated specific force measurements are transformed to the navigation frame by \( M^{-1} \). (Since \( M \) is unitary, \( M^{-1} = M^T \)) The North and East components are filtered as follows:

\[
n_{1,i+1} = n_{1,i} + 0.1 (\Delta v_{N_i} - n_{1,i})
\]

\[
n_{2,i+1} = n_{2,i} + 0.1 n_{2,i} + n_{2,i}
\]
where \( i \) represents the \( i \)th iteration and \( n \) is replaced by \( e \) to obtain the East filter. \( n_2 \) and \( e_2 \) resemble derivatives of \( \Delta v_N \) and \( \Delta v_E \), or specific forces.

In the leveling phase, the final filters are:

\[
\begin{align*}
n_{3, i+1} &= n_{3, i} + k_4 (n_{1, i+1} + k_5 n_{2, i+1}) \\
e_{3, i+1} &= e_{3, i} + k_4 (e_{1, i+1} + k_5 e_{2, i+1})
\end{align*}
\]

which resemble

\[
k_4 \text{ (specific force)} + k_4 k_5 \int \text{(specific force)}. \]

\( d_3 \) is set to zero.

In the gyrocompassing phase, the final filters are:

\[
\begin{align*}
n_{3, i+1} &= k_1 n_{2, i+1} \\
e_{3, i+1} &= e_{3, i} + k_1 e_{1, i+1} + k_2 e_{2, i+1} \\
d_{3, i+1} &= k_3 n_{2, i+1}
\end{align*}
\]

which resemble

\[
\begin{align*}
n_3 &= k_1 \text{ (specific force)} \\
e_3 &= k_1 \text{ (specific force)} + k_2 \int \text{(specific force)} \\
d_3 &= k_3 \text{ (specific force)}
\end{align*}
\]

In either phase, \( n_3, e_3, \) and \( d_3 \) are the commands to be sent to the "East", "North", and Down gyros, respectively.

After ten iterations, the earth-rate compensations are added to these commands and the results are transformed to the platform frame by \( M \):

\[
\text{gyro-torquing angles} = M \left[ \omega_E + \begin{pmatrix} e_3 \\ n_3 \\ d_3 \end{pmatrix} \right]
\]

These are added to the gyro-torquing-angle accumulators which are then used as the commands to physically torque the gyros.

4-2
5. DETAILED FLOW DIAGRAMS

UNIVERSAL CONSTANT

INPUT VARIABLES

\[ \omega_E \]

Load and/or verify Input Variables

\[ n = 640 \text{ sec} \]
\[ t_C = 0.5 \text{ sec} \]
\[ \omega_E = 5 \omega_E (\cos \phi, 0, -\sin \phi) \]

Coarse align gimbals to
\[ e_I = \pi/2 \]
\[ e_M = 0 \]
\[ e_O = \alpha - \alpha' \]

Switch platform to inertial mode

Has liftoff occurred

Program Alarm: Liftoff has occurred before platform alignment was adequate

Navigation monitor/control program

---

Figure 4a. Prelaunch Alignment Initialization Program
Figure 4b. Prelaunch Alignment Initialization Program
INPUT VARIABLES

\( \Delta v', t_C, b_f, f, d, G \)

\[ \begin{align*}
\Delta v &= \Delta v' \\
\Delta v' &= (0, 0, 0)
\end{align*} \]

Set up logic to begin this program \( t_C \) seconds after the previous pass through this program was begun.

\[ \Delta v_i = (1 + ASF_i) \Delta v_i - b_i t_C, \quad i = X, Y, Z \]

\[ T = T - t_C \left[ \begin{array}{c}
NBD_X \\
NBD_Y \\
NBD_Z
\end{array} \right] + G \left( \begin{array}{c}
\Delta v_X \\
\Delta v_Y \\
\Delta v_Z
\end{array} \right) \]

Synchronized Compensation

Yes

Torque gyro by \( \tau \)

No

EXIT

Go to synchronizing program

Figure 5. Compensation Program
INPUT VARIABLES
\[ \alpha'', \tau, \Delta v, M, \omega_E \]

PROGRAM CONSTANTS
\[ k_1, k_2, k_3, k_4, k_5 \]

Has Liftoff Occurred

Yes

Specify unsynchronized torquing

Navigation monitor/control program

Azimuth Change

No

Yes

Load \( \alpha'' \)

\[ \tau_z = \tau_z + \alpha'' - \alpha \]

Torque gyros by \( \tau \)

\[ \alpha = \alpha'' \]

(Figure 4b)

Figure 6a. Prelaunch Alignment Program
\[ \Delta v'' = M^{-1} \Delta v \]

Note: \( \Delta v_D \) is never used

\[ n_1 \leftarrow n_1 + 0.1 (\Delta v_N - n_1) \]
\[ e_1 \leftarrow e_1 + 0.1 (\Delta v_E - e_1) \]

\[ n_2 \leftarrow n_1 + n_2 \]
\[ e_2 \leftarrow e_1 + e_2 \]

Yes \( \rightarrow \) Leveling

\[ n_3 \leftarrow n_3 + k_3 (n_1 + k_5 n_2) \]
\[ e_3 \leftarrow e_3 + k_4 (e_1 + k_5 e_2) \]

No \( \rightarrow \) Leveling

\[ n_3 \leftarrow k_1 n_2 \]
\[ e_3 \leftarrow e_3 + k_1 e_1 + k_2 e_2 \]
\[ d_3 \leftarrow k_3 n_2 \]

Yes \( \rightarrow n_P \leftarrow n_P + 1 \)

No \( \rightarrow \) EXIT

Yes \( \rightarrow n_P = 10 \)

No \( \rightarrow \) EXIT

**Figure 6b. Prelaunch Alignment Program**
Figure 6c. Prelaunch Alignment Program
6. **SUPPLEMENTARY INFORMATION**

The implementation of the programs described in this document is dependent on the characteristics of the equipment used. Consequently, minor modifications to the programs may be necessary.

If the compensation program cannot be cycled precisely at the desired rate, then the compensation cycle time, \( t_C \), should be measured for each pass. It may be necessary to compute the amount of earth-rate compensation after every ten passes if the real cycle time can vary significantly.

It has been assumed that the gyro-torquing accumulators are decremented by the amount that the gyros have been commanded. If this is not the case, then the programmer must be responsible for decrementing the accumulators. For torquing accuracy considerations, fractions of pulses should be saved. It may also be desirable to torque a gyro only if it is to receive more than a certain number of pulses; that number depends on the torquing characteristics of the gyros.

The prelaunch alignment mechanization described here has a twenty minute time constant. If disturbances such as vehicle sway are minimal, then the steady state should be reached after approximately 1 1/4 hours. Approximately ten minutes are required after an azimuth change of several degrees for the platform to stabilize to the same accuracy as immediately prior to the azimuth change. This time varies with the magnitude of the change.
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