NASA Apollo

155 pages

Apollo Astronaut's Guidance and Navigation Course Notes

Prepared by MIT Instrumentation Laboratory

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APOLLO

GUIDANCE AND NAVIGATION

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E-1250

ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:

SECTION I

FUNCTIONAL VIEW OF THE APOLLO
GUIDANCE AND NAVIGATION SYSTEM

Beginning November 1962
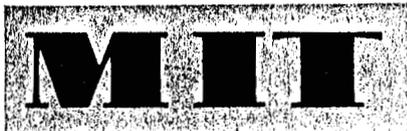
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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION I
FUNCTIONAL VIEW OF THE APOLLO
GUIDANCE AND NAVIGATION SYSTEM

ABSTRACT

This report reviews briefly the overall functions and op-
eration of the Apollo Guidance and Navigation System, defining
its major subsystems and the means by which these subsystems
accomplish the necessary guidance and navigation system functions.

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I   Introduction*

The purpose is to review briefly the overall functions and operation of the Apollo Guidance and Navigation System. First, we shall define the overall function required of the guidance and navigation (G & N) system. Next, the major subsystems of the G & N system will be identified and described. Finally, the means whereby these subsystems are used to accomplish the necessary G & N system functions will be explained for the important phases of the Apollo mission.

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*The material in Section I is adapted from information received from Mr. D. G. Hoag, Assistant Director, MIT Instrumentation Laboratory.

(OPTICAL
LOS &
INERTIAL RADAR) OPTICAL
         LOS (COMMUNICATIONS)

```
      ┌──────────┐  INITIAL CONDITIONS  ┌────────────┐
      │ GUIDANCE │◄─────────────────────│ NAVIGATION │
      │          │─────────────────────►│            │
      └──────────┘   CHANGES DUE        └────────────┘
                     TO THRUST
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CONTROL   POSITION, VELOCITY
OF ROCKETS   & TRAJECTORY
(& RE-ENTRY LIFT)   DETERMINATION
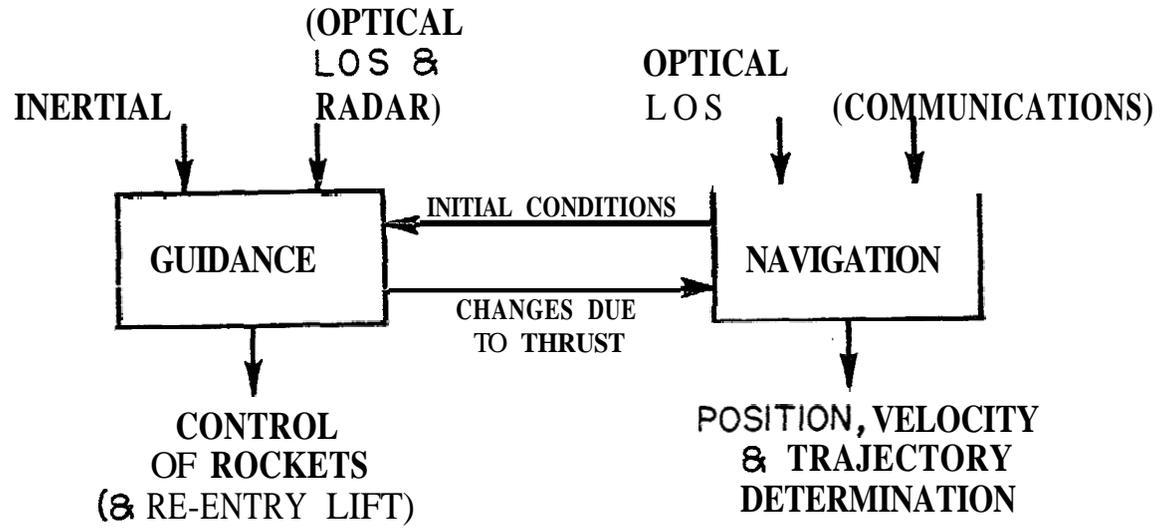
Figure 1-1

II Overall Function of the Guidance and Navigation System

Fig. 1-1 shows that the guidance and navigation system performs two basic functions in the Apollo mission:

> 1. The guidance function (sometimes referred to as the steering function) concerns control of rocket thrust during the powered or accelerated phases of a mission and control of re-entry lift during the re-entry phase.

> 2. The navigation function concerns the determination of the position and velocity of the Apollo vehicle and the determination of the required trajectories to target points.

The guidance and navigation system performs the guidance or steering function primarily on the basis of inertial measurements from gyroscopes, accelerometers, and clocks. During lunar-landing phases and rendezvous phases, optical line-of-sight and radar inputs help to perform the guidance function. With data from these sources, the system generates steering signals for the autopilot to accomplish the desired changes in the trajectory.

The navigation function of the guidance and navigation system, on the other hand, is primarily based on the use of optical line-of-sight measurements, which serve as navigation inputs. Although the primary navigation inputs are optical line-of-sight measurements, however, communications from down-tracking can serve as a backup and may be used during the mission as inputs to assist in the execution of the navigation function. With these data sources, the G & N system determines position, velocity, and trajectory parameters.

As indicated by Fig. 1-1 an interdependence of the following nature between the guidance and navigation functions exists:

> 1. An auxiliary part of the navigation function is to provide information on initial conditions, for guidance purposes during the steering phases.

> 2. An auxiliary part of the guidance function is to provide information on changes due to the thrust, for navigation purposes in updating position and velocity during the thrusting phases.

MAJOR SUBSYSTEMS

GUIDANCE AND NAVIGATION SYSTEM

SPACECRAFT

RADAR

LINE OF SIGHT → SCT — SCANNING TELESCOPE

STAR →
LAND-MARK → SXT — SEXTANT

IMU — MEASUREMENT

ASTRONAUT

D&C — DISPLAY & CONTROLS

AGC — APOLLO GUIDANCE COMPUTER

CDU — COUPLING DISPLAY UNIT

PSA — POWER SERVO ASSEMBLY
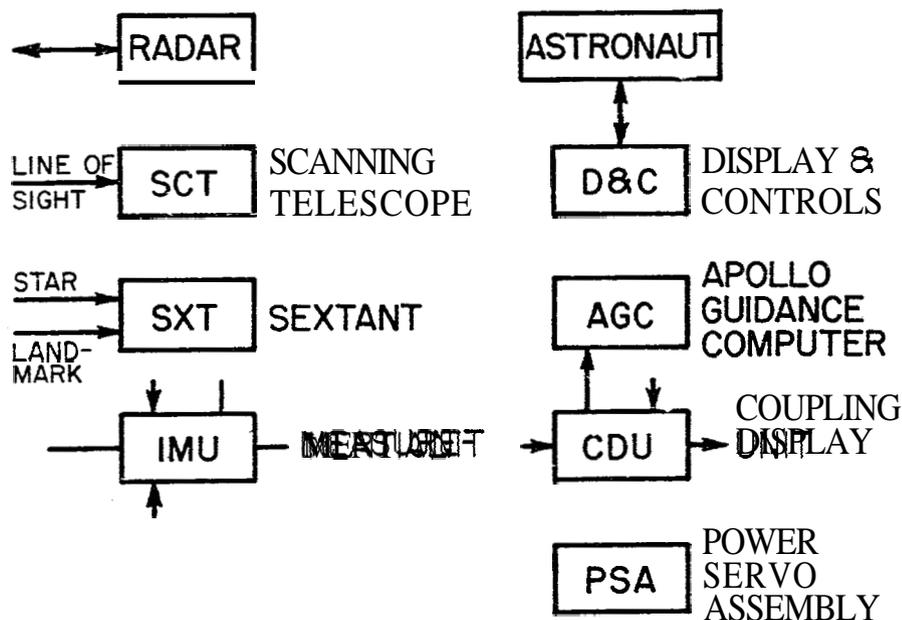
ATTITUDE CONTROL SYSTEM

ROCKET MOTOR CONTROL OF THRUST MAGNITUDE

AUTOPILOT CONTROL OF THRUST DIRECTION

Figure 1-2

III   Major Subsystems of the Guidance and Navigation System

Fig. 1-2 identifies the major subsystems of the guidance and navigation system.   The left-hand column of boxes in the figure depicts the input; sensing devices of the system.   Similarly, the center column depicts the control and data-processing devices. The right-hand column lists the other spacecraft functions of direct concern to guidance and navigation functions.

The data sensors of the G & N system are the radar, scanning telescope,, sextant, and inertial measurement unit.   The latter three are mounted on the "navigation base" in the command module of the spacecraft *so* that angle measurements can be related to a common rigid structure representing the spacecraft.

Radar

The radar employed in the guidance and navigation system is the first sensor represented in Fig. 1-2. The radar equipment. is located in the two service modules or in the lunar braking module. This equipment,, which is used for close-in sensing during the rendezvous and lunar-landing phases, consists of two components: a tracking radar and a doppler radar.   The tracking radar, which is an X-band monopulse radar, is used in conjunction with a transponder.   During earth orbital rendezvous, the transponder employed is located on the target vehicle; during a lunar-landing operation, on the other hand, the transponder is located on the lunar surface.   The transponder is needed on. the moon for landing at a precise location--near other equipment, for instance.   The doppler radar is used during the flare-out maneuver landing on the moon

Optical Instruments

The scanning telescope (SCT) is the first of the two optical instruments represented in Fig. 1-2. This device has a wide field of view for use by the astronaut in general finding, recognition, and short-range tracking.   It is a. single-line-of-sight instrument having two degrees of freedom or articulation with respect to the spacecraft.   The sextant (SXT),, the second of the two optical instruments represented, is a precise, high-magnification, narrow

field instrument. It has two lines-of-sight. It is used for making measurements such as star-to-planet angles. In addition, during phases where the inertial measurement unit (IMU) has to be aligned with precision, the sextant is used for sighting to a star for IMU orientation reference.

IMU

The IMU is the primary inertial sensing device. It measures acceleration and orientation of the spacecraft with the use of accelerometers and gyroscopes, The IMU consists of a three-degree-of-freedom gimbal system in which the outer gimbal axis is along the axis of the command module which corresponds to the roll axis at re-entry. The accelerometers are carried on the inner-most gimbal, called the stabilized member, which is held non-rotating with respect to inertial space by the action of error signals from the three gyroscopes, also mounted on this stabilized member. These gyro error signals are fed back, to stabilize the gyros and accelerometers in space, to servo motors that drive the IMU gimbals.

There are two major IMU outputs. First, the IMU produces signals from gimbal-angle transducers corresponding to the attitude of the spacecraft. Second, the IMU also produces, for the computer, velocity increments from the accelerometers. The stabilized-member gyros can be torqued from the computer to precess the stable member for initial alignment. However, the gyros are not torqued during control phases, during which they hold a fixed inertial orientation.

Control and Data Processing

The G & N system performs its control and data processing by the astronaut using display and controls, the computer, the coupling display units, and the power servo assembly.

Display and Control

The astronaut or navigator is represented in Fig 1.-2 as a major part of the guidance and navigation system. The interface between him and the rest of the guidance and navigation system

occurs at the display and controls (D & C).

Computer

The Apollo guidance computer (AGC) is the central data-processing core of the guidance and navigation system. It is a general-purpose digital computer.

Coupling Display Unit

The coupling display units (CDU) are used to couple the IMU, the computer, and the spacecraft autopilot, for the transfer of angle information, as well as to display the values of certain angles to the astronaut.

Power-Servo Assembly

The power assembly (PSA.) shown on Fig. 1-2 is a support item. It provides d-c and a-c power to the rest: of the G & N system and contains the servo control amplifiers for the TMU and optics gimbal drives.

Other Spacecraft Systems

Three spacecraft areas, outside the G & N system and nevertheless part of the spacecraft stabilization and control system, have direct bearing on the G & N system. The attitude control system determines spacecraft orientation during non-accelerated phases. It affects the ability to make optical sightings for naviga-, tion and IMU alignment purposes. The equipment for control *of* propulsion-rocket thrust magnitude, for starting and stopping the engines and modulating their thrust level. when appropriate, is regulated in its operation by the guidance system which sends signals to initiate these functions. Finally, the autopilot function of the stabilization and control system receives the guidance steering error signals during the accelerated phases to direct and control the rocket directions (or lift forces during re-entry) *so* as to achieve the desired trajectory.

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IV   Guidance and Navigation Operations

Now that the major subsystems of the guidance and naviga-
tion system have been identified and described, the use of these
subsystems in carrying out the guidance and navigation functions
during several of the important phases of the Apollo mission will
be explained.   This will be accomplished using block diagrams
having the same format as Fig. 1-2. In each diagram, cross-hatching
indicates those subsystems that are not involved in the particular
function that is under consideration.

GUIDANCE & THRUST CONTROLS

GUIDANCE AND NAVIGATION SYSTEM | SPACECRAFT
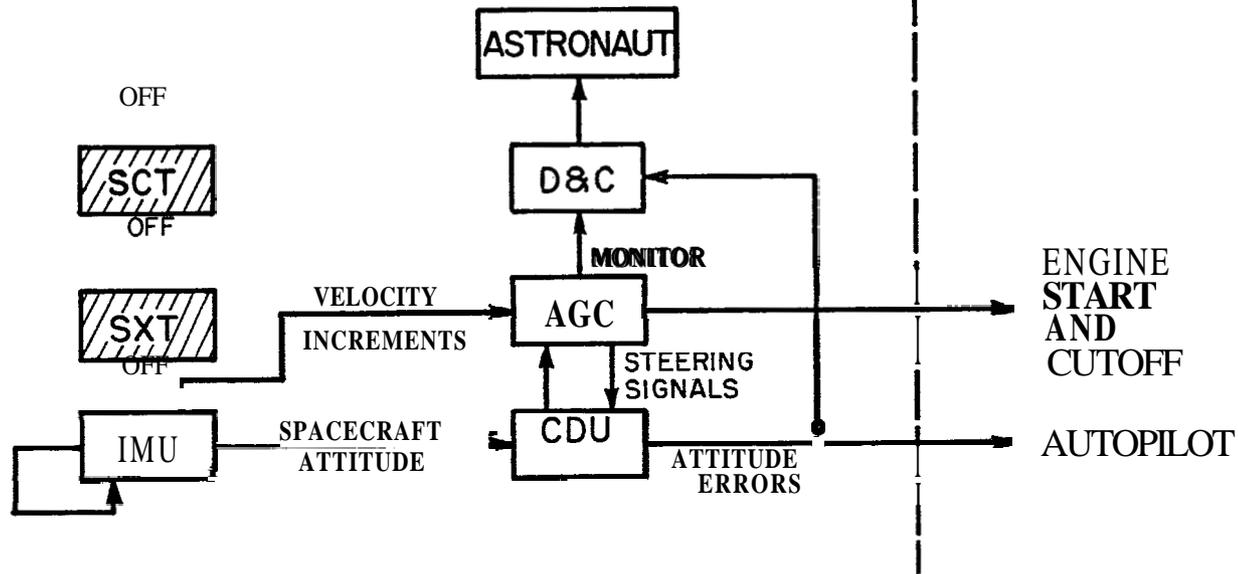
Figure 1-3

Guidance and Thrust Control

The first function considered here is the guidance function, the control of rocket thrust during the powered or accelerated phases of a mission and control of re-entry lift during the re-entry phase. The IMU is the only sensor here (see Fig. 1-3). It produces two outputs:

1. Velocity increments, which go to the computer (AGC),
2. Spacecraft attitude, which goes to the coupling display units (CDU).

The velocity increments are measured by the accelerometers in the IMU's stabilized axes. The computer determines the steering signals that it sends to the CDU in these same axes. These signals represent incremental angles, which are then compared, within the CDU, with the spacecraft attitude measured by the IMU gimbal angles. The results are the attitude error signals. The auto-pilot acts on these attitude error signals and controls so as to bring the attitude errors to zero. Meanwhile, from the same velocity measurements as those on which the steering signals are based, the computer also determines the rocket-engine cutoff and, when appropriate, modulation of the thrust. The display and controls (D & C) provide monitor functions to the astronaut. He can take control, of course, in various secondary modes to enhance mission success.

COARSE IMU ALIGNMENT

GUIDANCE AND NAVIGATION SYSTEM | SPACECRAFT

RADAR
OFF

SCT
OFF

SXT
OFF

ASTRONAUT

D&C

AGC

DESIRED
IMU GIMBAL
ANGLES

ATTITUDE
CONTROL
SYSTEM

IMU — GIMBAL ANGLES → CDU

GIMBAL ERROR
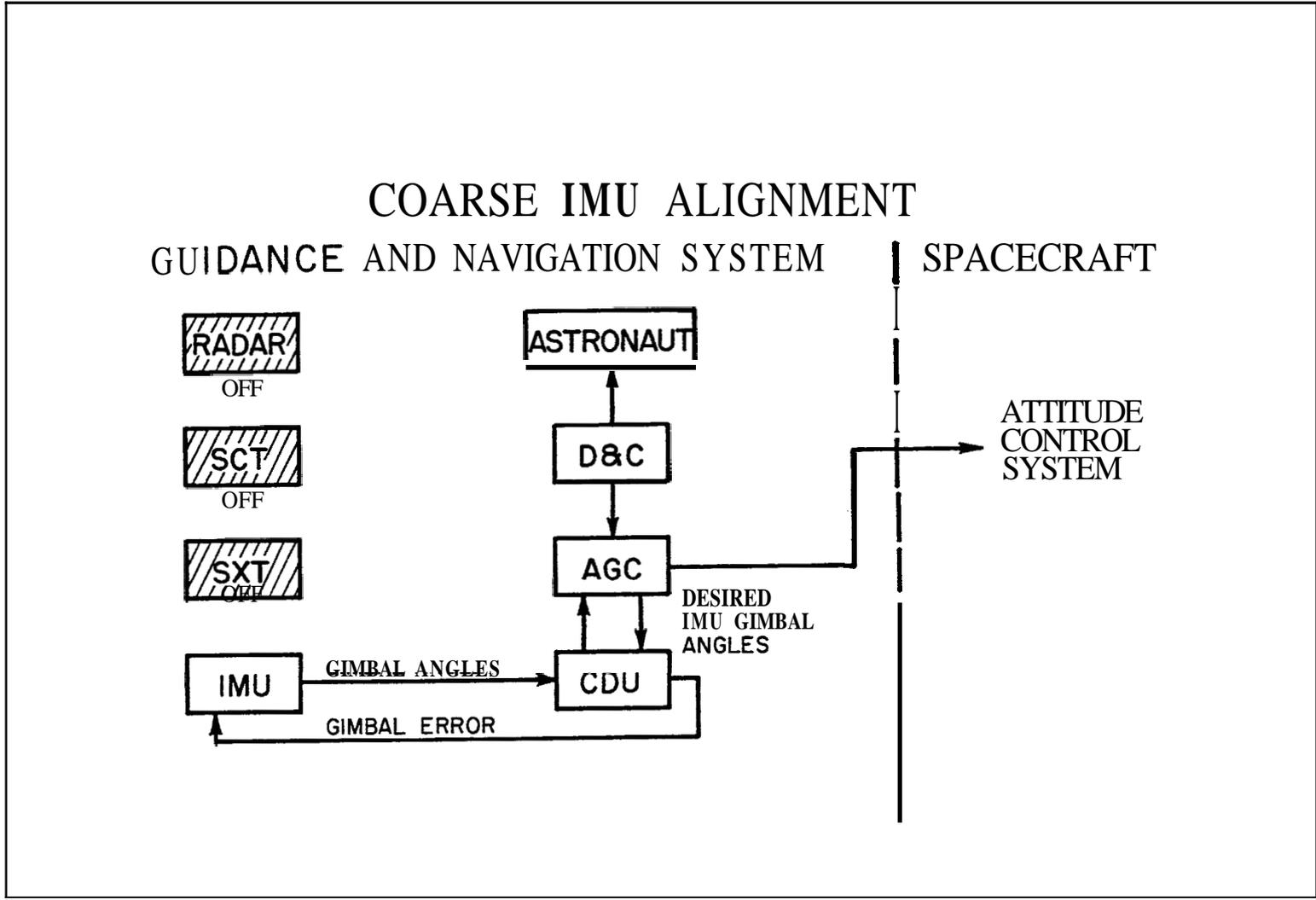
Figure 1-4

IMU Alignment

In order properly to carry out its particular functions, the stabilized member of the IMU must be prealigned with the appropriate coordinate frames. There are two phases of this alignment:

1. A. coarse alignment
2. A. fine alignment

IMU Coarse Alignment

Neither the sextant, the scanning telescope, nor the radar are involved in the coarse alignment of the IMU (see Fig. 1.-4). From the expected action of the stabilization and control system, the spacecraft has a roughly known attitude, probably one in which the spacecraft tail is toward the sun, and the spacecraft is rolled to some particular orientation with respect to the earth. Knowing this orientation, the astronaut can use the computer to determine those IMU gimbal angles which would place the IMU stabilized member in the correct orientation for its next control use, These correct angles can be fed automatically to the CDU, which compares them with actual gimbal angles and generates error signals giving the difference between actual gimbal angles and the correct gimbal angles. This error signal goes to the IMU gimbal servos, which rapidly move the stable member around to the orientation required, within an alignment accuracy of about one degree. This accuracy is limited, of course, by the accuracy of the spacecraft, attitude as determined by the spacecraft stabilization and control system.

FINE IMU ALIGNMENT

GUIDANCE AND NAVIGATION SYSTEM | SPACECRAFT
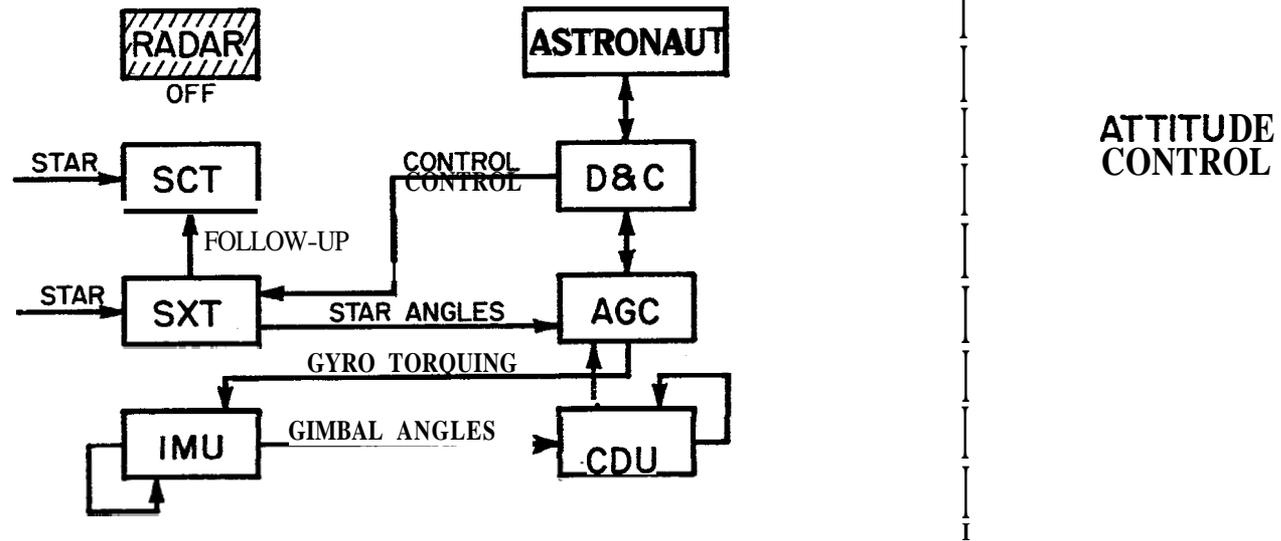
Figure 1-5

IMU Fine Alignment

The fine IMU alignment, as contrasted with the IMU coarse alignment, depends upon optical measurements (see Fig. 1-5). The sextant is the primary sensor and is used for tracking the direction to that star which is used as the orientation reference. The scanning telescope, 'with its wide field of view, is used for acquisition and to check that the correct star is being sighted. The astronaut, through the display and controls, puts the sextant on the star, thereby generating the star angle with respect to the navigation base on the spacecraft. The IMU gimbal angles with respect to the navigation base are then measured, using the CDU to feed these angles to the computer. Then a comparison between the actual and required gimbal angles is made. If the gimbal angles are not appropriate, gyro torquing signals are sent to the gyroscopes on the stabilized member of the IMU to drive the gimbals to the orientations that match up with the requirements for the fine IMU alignment. The accuracy of this fine alignment is of the order of a minute of arc. Since a single star direction can give only two degrees of freedom of orientation reference, a second star sighting is then necessary to complete the three-degree-of-freedom fine alignment of the IMU stabilized member.

MIDCOURSE NAVIGATION

GUIDANCE AND NAVIGATION SYSTEM | SPACECRAFT
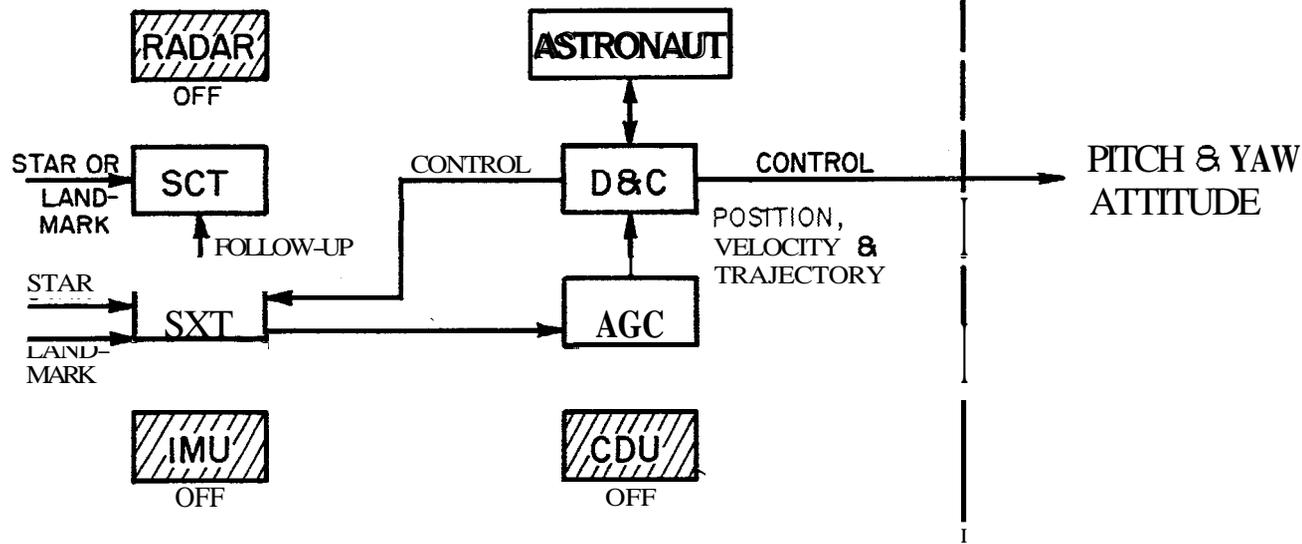
Figure 1-6

Midcourse Navigation

The next function of the guidance and navigation system to be considered is that of midcourse navigation. As indicated by Fig. 1-6, the principal sensor used is the sextant, with its two lines of sight. In its field of view, the star and the landmark are superimposed by the astronaut through the use of the controllers on the sextant. The navigator astronaut can also look through the scanning telescope for acquisition and identification as required, using its wide field of view and following either the landmark or star-line directions of the sextant. When the two targets are superimposed, the sextant feeds to the computer the angle between them. The computer uses this information to update its knowledge of free-fall trajectory, *so* that it can provide, at any time, information on position, velocity, trajectory, and trajectory extrapolation.

The sextant has only three degrees of articulation with respect to the spacecraft. Since there are two lines-of-sight, however, each requiring two degrees of freedom, an additional degree of freedom is required. This is obtained by control of the spacecraft attitude on signals from the navigator.

ORBITAL NAVIGATION

GUIDANCE AND NAVIGATION SYSTEM   SPACECRAFT
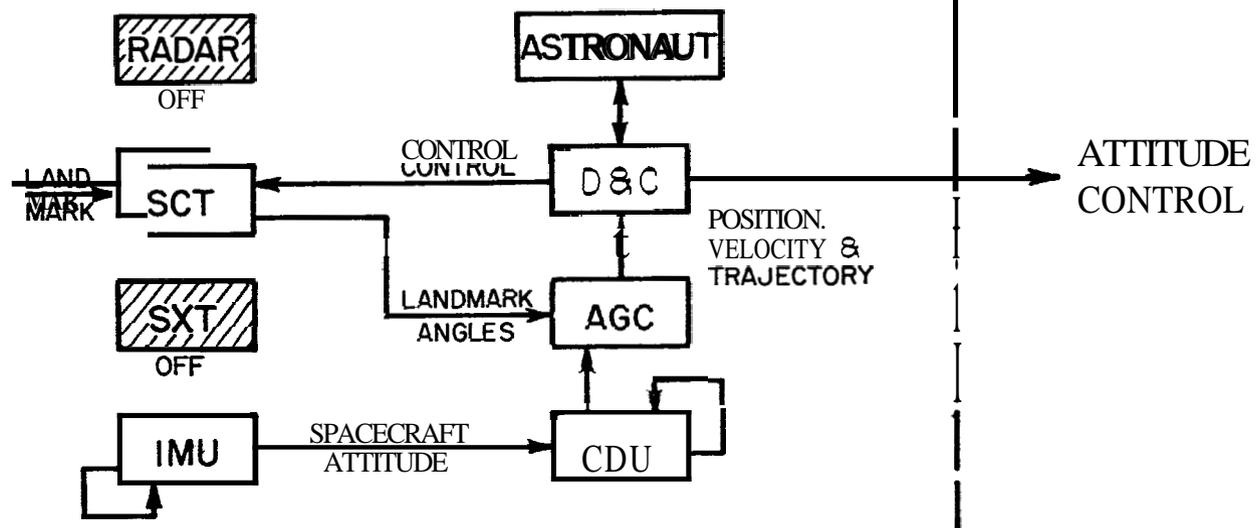
Figure 1-7

Orbital Navigation

During navigation phases in which the spacecraft is in
orbit close to the moon or the earth, angular measurements do
not have to be quite as accurate, but angular velocities are rather
extreme. In this case, the sextant is not used (see Fig. 1-7). In-
stead, the scanning telescope is used as a single-line-of-sight
instrument to track a landmark. The IMU is prealigned to a
star framework, *so* it gives spacecraft attitude with respect to
that framework. The scanning telescope on the other hand, gives
landmark angles with respect to spacecraft. From these two sub-
systems, accordingly, the landmark direction with respect to the
prealigned space direction of the IMU can be obtained. The com-
puter can absorb and compute this information for the navigator.,
to again update the trajectory parameters in this orbit, and can
supply to the navigator--by means of the display and controls--
position, velocity, and trajectory information. Attitude control
here provides stability for tracking with the scanning telescope.

RENDEZVOUS & LUNAR LANDING
GUIDANCE AND NAVIGATION SYSTEM   SPACECRAFT
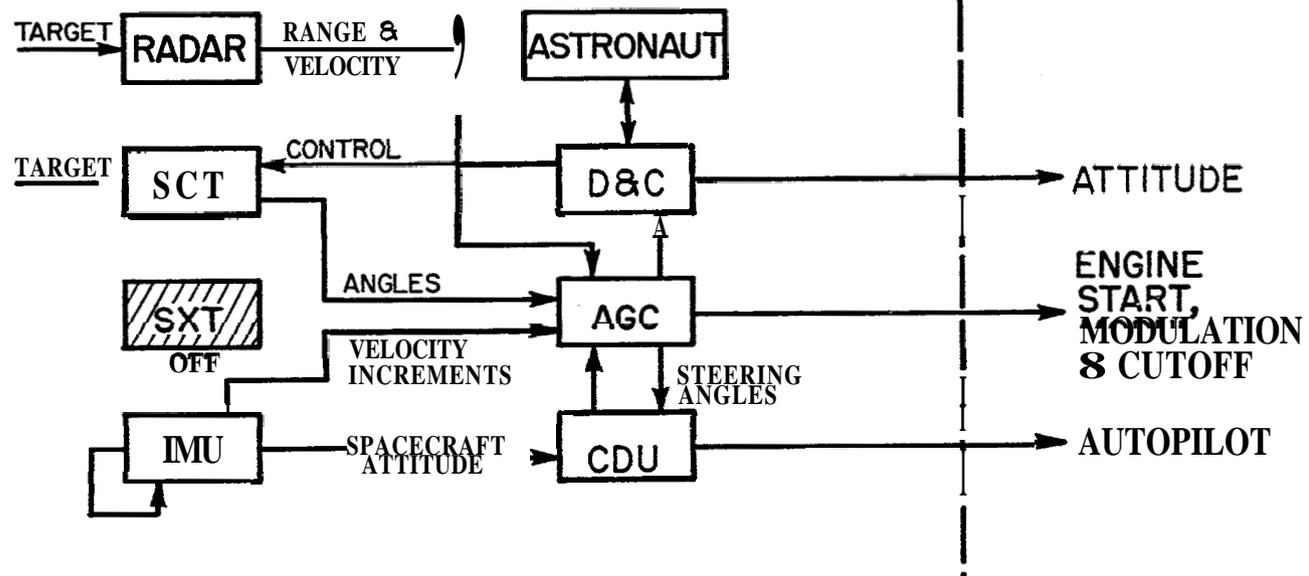
Figure 1-8

Rendezvous and Lunar Landing

The final function of the guidance and navigation system to be considered is that associated with rendezvous and lunar landing (see Fig. 1-8). Here the only subsystem not used is the sextant,. The scanning telescope gives optical tracking information, and the IMU gives inertial measurements. All of these outputs are sent to the computer. There they are processed for the pilot. Control signals go to the autopilot for steering purposes and to the rocket engines for start, modulation, and cutoff control. The pilot, of course, can take over here in any level of control he desires.

This completes the brief orientation explanation of the overall functions and operation of the Apollo Guidance and Navigation System.

AP●LLO

GUIDANCE AND NAVIGATION

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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:

SECTION II

GYRO PRINCIPLES

December 1962
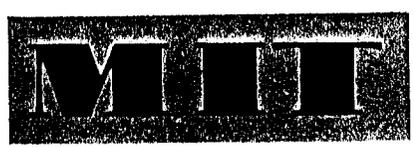
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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION II
GYRO PRINCIPLES

ABSTRACT

This section discusses the inertial guidance of space vehicles as a fundamentally geometric problem without recourse to mathematical development or engineering detail.

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TABLEOFCONTENTS

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I Properties of 'Inertial Guidance

The inertial guidance of space vehicles may be regarded as a fundamentally geometric problem. Thus it may be discussed as it is in the following sections, in terms of the actual components used, without recourse to mathematical development *or* engineering detail.

The Instrumentation of Inertial Coordinate Axes

To say that inertial guidance is geometric is to say that it deals with the location of points in certain coordinate systems. The problem is thus solvable by the instrumentation of appropriate coordinate axes, that is, by the construction of physical objects which are designed to simulate Cartesian coordinate frames. It is, of course, possible to imbed a set of body axes in any rigid object; if it is a particularly heavy object, or, say, a box containing large operating gyroscopes, *so* that it is difficult to rotate, the body may be regarded as representing coordinates which do not rotate with respect to inertial space (Fig. 2-1).
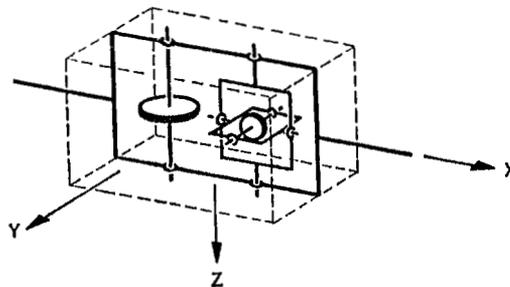
Fig. 2-1   Representation of inertial coordinates by a box containing a massive object or large gyroscopes.

These coordinates--called simply inertial coordinates--will remain non-rotating with respect to their environment if they are

2-7

coupled to it by a frictionless and massless gimbal system (Fig. 2-2).
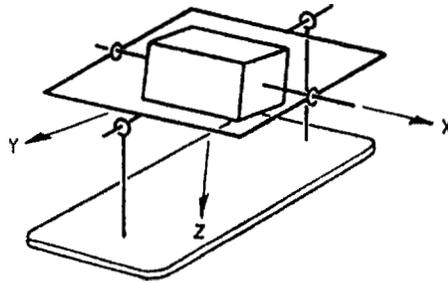
Fig. **2-2**   The coupling of mechanized inertial coordinates to the
environment by means of frictionless gimbals.

But inertial guidance systems actually contain <u>models</u> of inertial-,
coordinate axes which use neither heavy masses nor even heavy
gyros, and in which gimbal friction is, nevertheless, of secondary
importance.

<u>Mechanical Decoupling of Gyros from the Vehicle</u>

The importance attending gimbal friction stems from the
fact that it transmits into torques on the gyros at the center of the
gimbal system not only the interfering torques acting on the vehicle
(which cause it to roll, pitch, and yaw), but also any torques app-
lied directly to the gimbals.   These torques will cause the gyros
to <u>precess,</u> i.e., to rotate, and the instrumented inertial coordinates
therefore to <u>drift</u>.   Thus it will be seen that the gimbals' true func-
tion is to <u>decouple the gyros from the base</u> on which the gimbal
system is mounted.

To see how this mechanical decoupling is effected in prac-
tice, consider that even if the gyros are not massive, their re-
sponse to interfering torques which penetrate through the gimbal
system to the gyros might still be useful.   This is possible because
with suitable instrumentation, these torques can create electrical
signals connoting precession.   These signals--one from each of
the three axes--may be proportional to the rate of precession, as
in a rate gyro.   The signals may be proportional to the integral of

this rate, as in a single-degree-of-freedom integrating gyro, or in a two-degree-oi-freedom gyro. In the case of rate integration, the angle of precession is proportional to the angle through which interfering torques have turned the base about the gyro's input axis.

The gyro output signals are thus carriers of the information that the gyro package coordinates have been disturbed (Fig. **2-3**). This information is now put to use, as shown in the figure, to overcome the bearing friction and other interfering torques on the gimbals. This requires that the bearing assemblies actually be not merely shaft supports, but involve electric motors as well, *so* that, the gyro outputs, suitably processed, monitor the gimbal drive motors directly.

The result is thus a multiple closed-loop servo system. The gyros have the status of controllers of the inertial orientation of their own input axes, and the torque motors that of producing the desired orientation. Since the medium of control is a signal, the gyros need not be massive, and the gimbal drives furnish the rotational torques needed to stabilize this base motion isolation loop.

The successful mechanical decoupling of a gyro package from its environment, by the means schematically shown in Fig. **2-3**, does not set up any particular inertial coordinate system, however; its orientation with respect to, say, lines of sight to certain stars, or to the earth's polar axis, or to the vertical at some point on the earth, is still completely arbitrary. What has been done is to set up interference-proof non-rotating coordinateg; this is geometrical stabilization.

Gyro Command Signals

There is another function which the gyro package may have: rotation in inertial space, in response to command signals. These commands are presented to the gyros individually as input currents lo torque generators, which operate about the output axis of the gyro--the axis of rotation in the precession of the wheel. The

effect of these commands is to send signals from the gyros to the gimbal drives. These signals now have the function not only to mechanically decouple the gyro package from the base, but also to set the gyro package--and the instrumented coordinates--into rotation with respect to inertial space.

The base-motion-isolation loop is thus seen to provide a torque-free-environment for the operation of the gyros as angular velocity command receivers. In this connection it must be stressed that integrating gyros, used as the representative gyro example in the figure, are null-operating devices, and that they null on the commanded angular velocity (which includes, as a special case, zero angular velocity in the case of inertially-non-rotating coordinates). The reorientation of the gyro package about some line as a result of the angular velocity command will be through an angle equal to the time integral of this angular velocity relative to inertial space.

Gyro Drift

Internal to the gyro units themselves, a problem arises when unexpected torques cause drift of the wheel gimbal about the output axis. Clearly, such drift will send false signals to the base-motion-isolation servo. The means for minimizing drift are discussed presently.

Force Measurement

In inertial guidance, inertially-referred coordinates are needed for the measurement of the total force on the vehicle. When this force is considered, it is convenient to deal with it as specific force, that is, body force per unit mass. In practice, this means fastening force-measuring devices or accelerometers to the gyros, so that they measure force or acceleration with respect to a gyro-instrumented coordinate system.

The Navigation System as Dependent on Measurements

Since the Apollo G & N system is, during launch and mid-

course correction, dependent on the performance of its gyros and accelerometers, these components must be understood in terms of their operation as instruments. Although both gyros and accelerometers have had a long and useful past prior to the re-yuireruent for inertial guidance, their adaptation to inertial guid-ance has removed them a long way from the artificial horizon and the gyrocompass. The present approach abandons classical theory for certain instrumental simplifications, and has the virtue of em-phasizing function without restricting validity. Here the function to be considered concerns the application of gyros to the inertial-space-referred integrating drive system.

Gyros as Space-Stabilization Components

Any mechanism capable of indicating an orientation that remains unchanging with respect to the "fixed stars" must depend upon the inertial properties of matter. It is convenient to utilize this property as it is associated with a spinning rotor. The spin axis of this rotor will precess (that is, change its orientation with respect to inertial space) at a rate proportional to the magnitude of the applied torque; and if this torque could be reduced to zero, the rotor spin axis would hold its direction perfectly--that is, free of drift--with respect to inertial space, which, for navigation purposes, is identical with celestial space. In practice, means for supporting a spinning rotor are difficult to realize without ex-erting unwanted torques on the rotor. Experience has shown that the uncertainty torques imposed in "brute force" stabilization by mechanical systems driven directly from the rotor are intolerably great for inertial-system applications, The universally accepted remedy for this difficulty is to use servomechanism techniques for driving the mechanical members that support the spinning rotor. Any level of output torque can then be controlled by the spin-axis direction without imposing any significant reaction on the rotor, and the gyroscopic change in angular momentum of a

spinning body can be freed of externally-caused disturbances.

The elimination of outside interfering-torque effects by the use of servo-drive arrangements places the responsibility for drift uncertainties on the designers of the gyro units. These units have two related functions to perform as components of inertial-space reference systems. First, when they are forcibly displaced from preset reference orientations, they must generate output signals that represent these deviations, *so* that these signals (amplified) may be used to torque the gyros back to their reference orientations. Second, they must change these reference orientations in response to command-signal inputs, when this is required. Gyro-unit design is centered around the problem of realizing these characteristics.
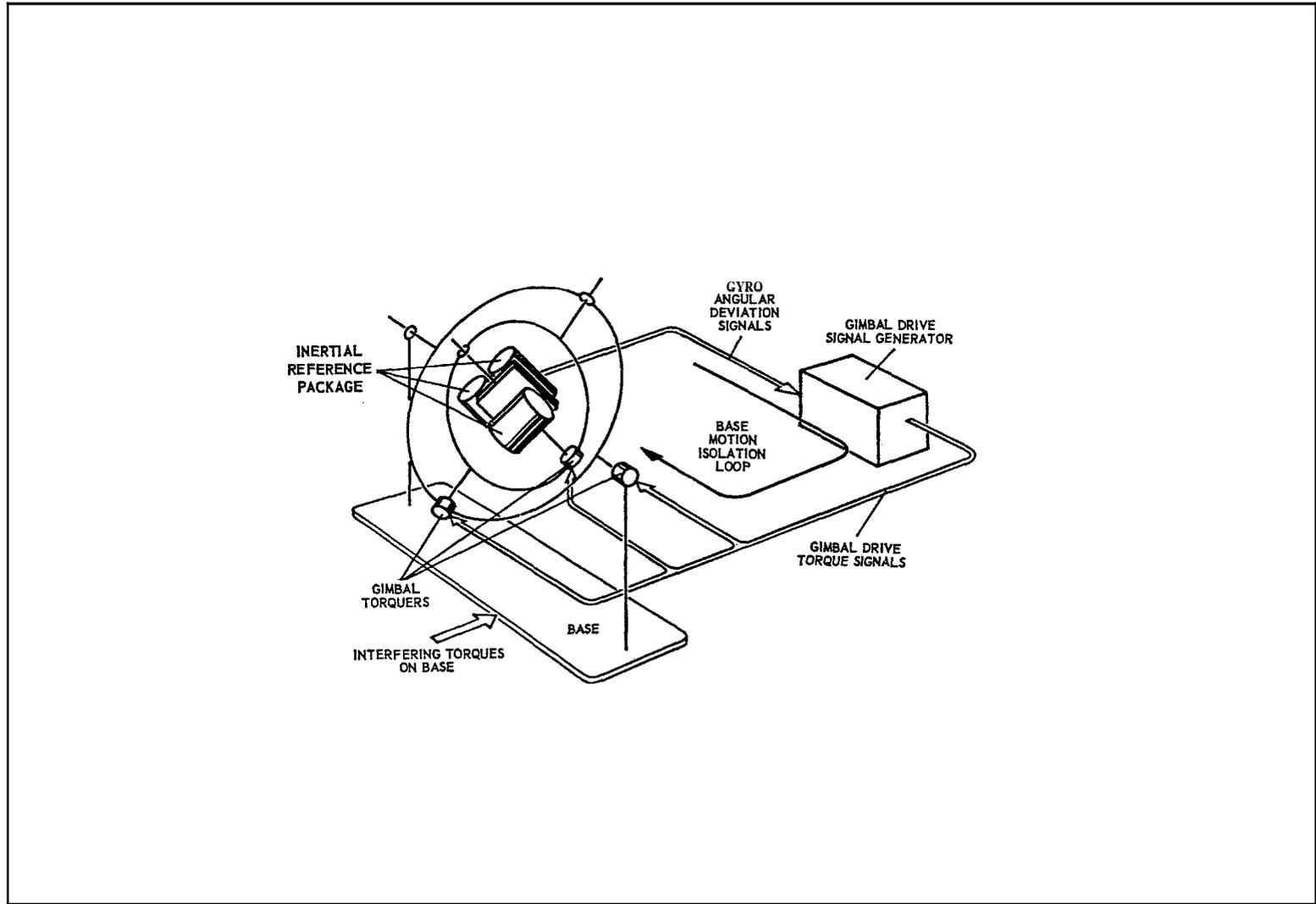
Fig. 2-3 Base motion isolation with single-degree-of-freedom
gyros as control elements and driven gimbals for decoupling the
gyros from the environment.

II   Gyroscopic Theory

        Gyroscopic theory deals with the directional aspects
of the mechanics of rotating bodies. For the description of gyro-
scopic instruments, the general theory of rotating bodies, based
on Newton's laws of motion applied to rotation, may be greatly
simplified. This simplification is made possible by the fact that,
for gyroscopic-instrument applications, a rotor must be carefully
balanced about its axis of symmetry and must be driven with a
constant angular velocity of spin relative to its mounting.   In
practice, the spin is several orders of magnitude greater than
the inertially-referred angular velocity of the instrument itself.
This fact makes it easy to deal with gyroscopic effects in terms
of simple vectors that represent rotational quantities.

        Fig. 2-4 is a summary of vector conventions for ro-
tational quantities. The gyroscopic element is most effectively
and completely represented by a "disembodied" angular momentum
spin vector. Fig. 2-5 represents a gimballed two-degree-of-
freedom gyro mechanism illustrating the vector quantities pert-
inent to precession. It is apparent that if the applied torques from
external sources and the supporting arrangement are zero, the
angular-momentum vector will have zero angular. velocity with
respect to inertial space, and the spin axis fixed to the rotor-
carrying gimbal will serve as an inertial-reference direction. The
orientation of this reference direction can be changed at will with
respect to inertial space by applying proper torque components--,
commands--to the gimbal, using the torque generators shown.
Angles between gimbals which develop as the gyro precesses are
indicated by the signal generators shown.

        Fig. 2-6 shows the essential features of the single-
degree-of-freedom gyro mechanism with (a)a damped elastic
(spring) restraint,, the rate gyro; (b)with viscous-damper integration
only, the integrating gyro; and (c)neither damped nor spring-
restrained, the unrestrained gyro.

In the integrating gyro, (b), which is used on the Apollo Inertial Measurement Unit, the rotor-carrying gimbal is directly pivoted with respect to the structure that serves as the case for attaching the gyro unit to the member whose orientation with respect to inertial space is to be indicated. For convenience in discussions, three mutually perpendicular axes fixed to the case are identified. The output axis (symbol OA) is identical with the axis about which the gimbal is pivoted with respect to the case, The spin reference axis (symbol SRA) is identical with the direction of the spin axis when the gimbal-output-angle indicator is at zero. The input axis (symbol IA) is fixed to the case *so* that it completes a right-handed set of orthogonal axes.

In operation, a torque is applied to the case about the input axis. This causes the spin axis to precess about the output axis, *so* that the spin axis turns toward the input axis. The gimbal angular velocity about the output axis sets up velocity gradients in the fluid that fill the clearance volume of the damper. For situations in which steady-state dynamic conditions exist, *so* that inertia-reaction effects are not significant, the angular velocity of the gimbal is constant and the viscous-damping torque has a magnitude equal to the output torque from the gyroscopic element.

From the standpoint of usefulness for practical applications, the essential result is that, over any given time interval during which the gimbal is free and the gimbal angle (the angle measured from the spin reference axis to the spin axis) remains small, the integral of the angular velocity of the gimbal (the angular displacement with respect to the case) is proportional to the angular displacement of the case with respect to inertial space about the input axis. Thus the signal generator angle aboard the spacecraft shows what is happening to the spacecraft in space.

It is important to note that there is no preferred natural orientation of the case from which the motion of the case is started with respect to inertial space. The reference orientation, and

2-15

initial condition, is established by the physical mechanism of the gyro unit: and its orientation at some instant that is taken as zero for integration of angular velocity. Usually it is convenient to take the reference orientation as the position of the case at an instant when the spin axis is aligned with the spin reference axis, that is, when the gimbal angle is zero; i. e. , the signal generator output is zero.

In. any practical case, the gimbal output angle is never allowed to become greater than a few seconds of arc, *so* it is valid to assume in considering the overall vehicle guidance problem, that the direction of the angular-.momentum vector is always along the spin reference axis.

GYRO
PRECESSION
VECTOR
$\overline{W}_{[i(ge)]}$

APPLIED FORCE
PRODUCING TORQUE

APPLIED
TORQUE
VECTOR
$\overline{M}$

GYRO
PRECESSION

GYRO SPIN ANGULAR
MOMENTUM VECTOR
H_S

$$\overline{W}_{[i(ge)]} \times \overline{H}_S = \overline{M}$$

The spin momentum of a gyro precesses
relative to inertial space in an attempt
to align itself with the applied torque.
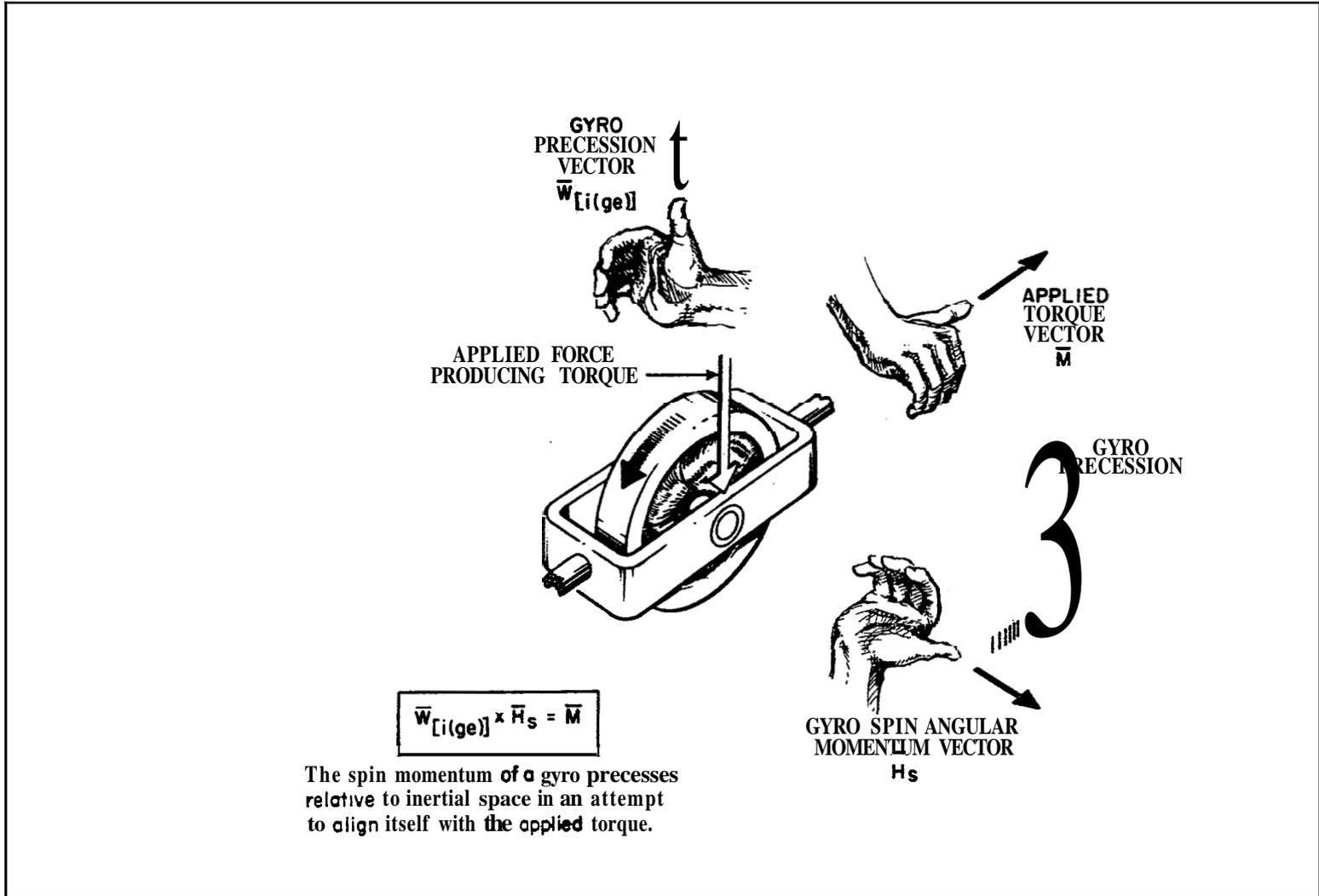
Fig. 2-4   Basic law of motion of a practical gyro.
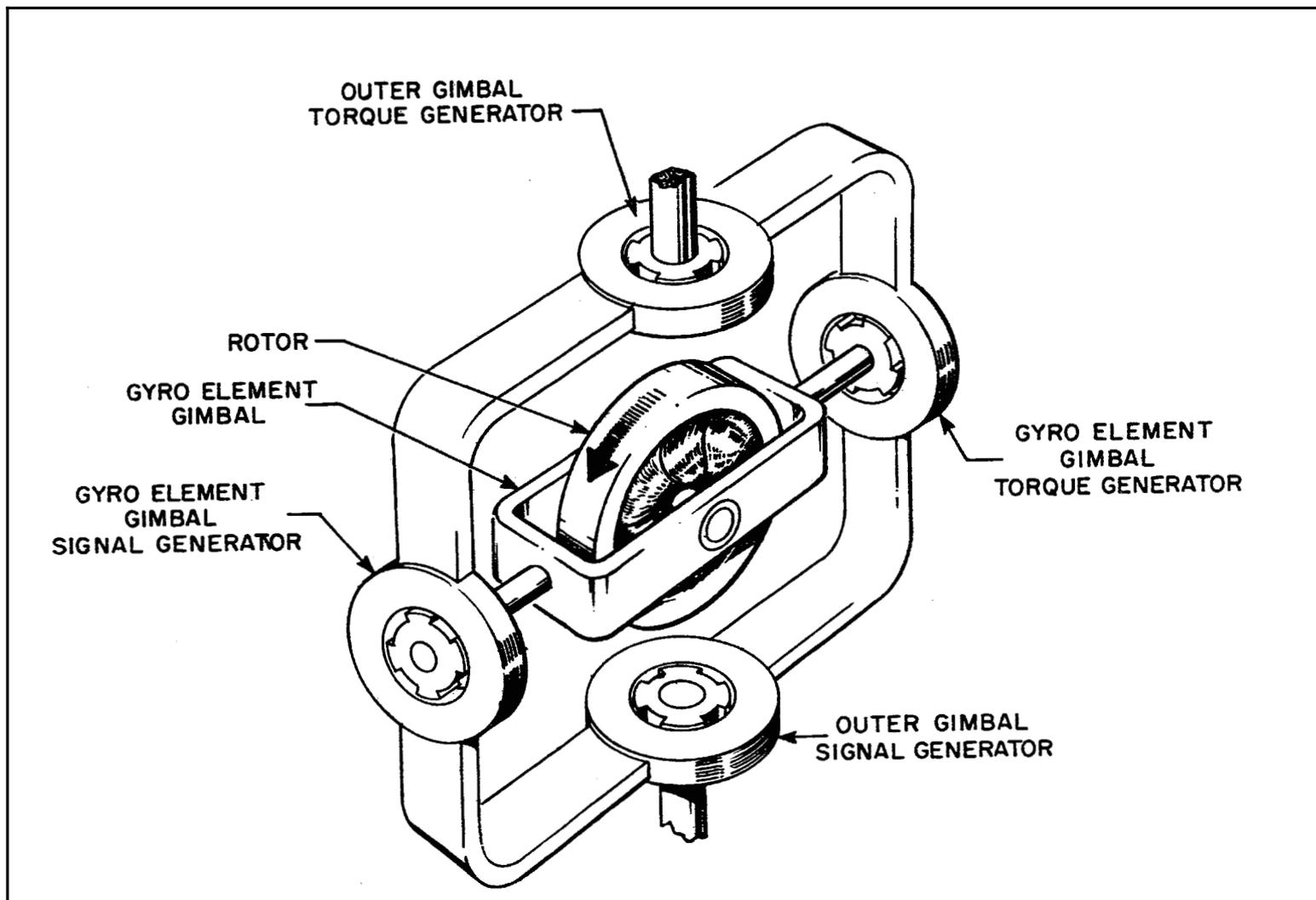
Fig. 2-5   Essential elements of a two-degree-of-freedom gyro.
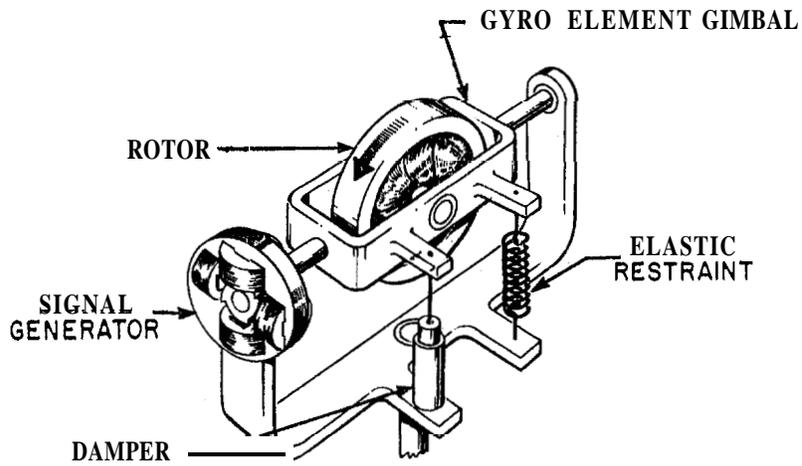
Fig. 2-6a Essential elements of a rate gyro.
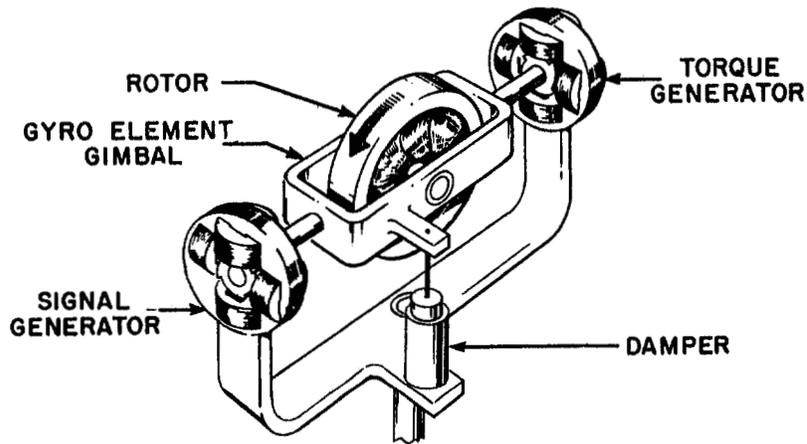
Fig. 2-6b Essential elements of an integrating gyro.
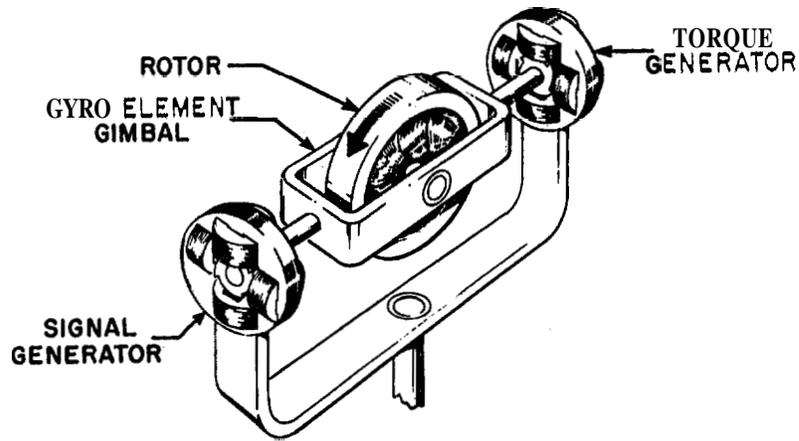
Fig. 2-6c Essential elements of an unrestrained single-degree-
of-freedom gyro.

III   Gyro Unit Applications

Inaccuracy levels permitted by the performance require-
ments of inertial systems are *so* low that calibrations (that is,
stable and accurately known input-to-output relationships) are
difficult or impossible to establish and maintain over large gim-
bal angles.   An alternate and preferable mode of operation is
servoing or nulling.   The position of the null. must be very accurate-
ly held, but the sensing units only need indicate the directions and
approximate magnitudes of input deviations from reference condi-
tions.   These input-deviation,indications are used as command sig-
nals for servo-type feedback loops that act to drive the input sensor
toward its position for null output.   In arrangements of this kind, the
gyro acts as the error-sensing means that is an essential component
of any servo system.   In order to describe the functions of gyro units
and specific-force receivers as components of inertial systems, Fig.
2-7 gives an illustrative pictorial-schematic diagram. of a single-axis
inertial-space stabilization and integration system.

In Fig. 2-7, the gyro unit is rigidly attached to a controlled
member.   This is shown as servodriven about--for illustrative pur-
poses--a single axis.   The input axis of the gyro unit is aligned with
the controlled-member axis, *so* that the output axis and the spin ref-
erence axis lie in the plane normal to the servodrive axis.   The sig-
nal-generator output of the gyro unit is connected through slip rings
(not shown) to the input of the electronic power control unit for the
servodrive motor.   When the gimbal angle is zero, the spin axis is
aligned with the spin reference axis, the signal-generator output
is at its null (minimum) level, and the gyro-unit output axis esta'b-
lishes the reference orientation for the controlled member.   When
the direction of the gyro-unit input axis is non-rotating with respect
to inertial space and the gyro-rotor gimbal is free from all applied
torques (except those stemming from power leads, friction, and the
like) the arrangement of Fig. 2-7 gives single-axis geometrical
stabilization with respect to inertial space.

Starting with the controlled member in its reference position, the direction of the controlled-member axis may be rotated in any possible way with respect to inertial space and the gyro-unit output signal will remain at its null level as long as the controlled member is not rotated about the gyro-unit input axis away from its reference orientation, although the reference orientation may itself rotate with respect to inertial space. If the controlled member does deviate from the reference orientation for any reason, the gyro rotor and gimbal rotate with respect to the case, and the output; signal changes from its null level. The slip rings and electrical connections transfer this signal change to the electronic power control unit, which in turn changes the input power to the servodrive motor in such a way that the controlled member is turned back toward the reference orientation. This action continues during any rotations of the base about the gyro-unit input axis, *so* that the controlled member hunts about the reference orientation with very small angular deviations. This entire process is called base-motion isolation or geometrical stabilization. The functional diagram for such a system is shown in Fig. **2-8.**

In practice, three single-degree-of-freedom gyro units are mounted *so* that their input axes are mutually at right angles on a controlled member, Fig. 2-9. The controlled member has three degrees of angular freedom with respect to its base, required to give complete geometrical stabilization. With this arrangement, each of the three gyro units supplies deviation signals about a controlled-member-fixed direction that changes its orientation with respect to the servodrive-motor axes, *so* that the deviation signals must be distributed by a system of resolvers to insure action by the proper motors. This is a low-accuracy resolution that serves only to maintain reasonably constant servo-loop gains. The action of each gyro protects the other two from rotations about axes other than their own input axes, *so* that it is a simple matter to achieve stabilization in the accuracy region of one second of arc. This geometrical filtering action places the engineering-design burden

on the minimizing of drift rates in the gyro units, rather than in the servo

Gyros in the Autopilot

For the purpose of steering space vehicles, three single-degree-of-freedom gyros are mounted rigidly to a vehicle structure. They generate signals that represent angular rates of the vehicle, which is then indeed the controlled member. These signals are command inputs for the vehicle steering system. The actual vehicle orientation hunts about the vehicle reference orientation with angular deviations that depend on the quality of the vehicle thrust-direction control system, which here is a servodrive in a vehicle stabilization loop.

A useful property of a servodriven gyro stabilization system is its ability to change its reference direction in response to commands. In the single-axis example under discussion, the command is an electrical signal (from the Apollo Guidance Computer or from a manual control) to the torque generator on the gyro unit. The corresponding torque-generator output torque is applied to the gyro element gimbal about the output axis of the gyro unit. The spin axis then turns away from the spin reference axis. This motion causes the signal-generator output to change from zero *so* that the servodrive motor rotates the controlled member. The gyro rotor responds to the angular velocity of the controlled member,by applying its output torque to the gimbal in the direction that tends to return the spin axis back to alignment with the spin reference axis, With suitable power-control-system design, equilibrium exists when this alignment is reached and the output signal is at its null level. This means that the angular velocity of the controlled member with respect to inertial space about the gyro input axis is directly proportional to the torque-generator output torque (if the angular momentum of the rotor is constant). In addition, when the torque-generator output torque is proportional. to the command-signal input within a negligibly small uncertainty, the controlled-

member angular velocity may be regarded as proportional to the command signal. If the base does not rotate inertially, an indication of the angular displacement of the controlled member with respect to the base represents the integral of command-signal input variations with respect to time. Conversely, an integral of the command signal with respect to time is a direct measure of the angular displacement of the controlled member with respect to inertial space about the gyro input axis.
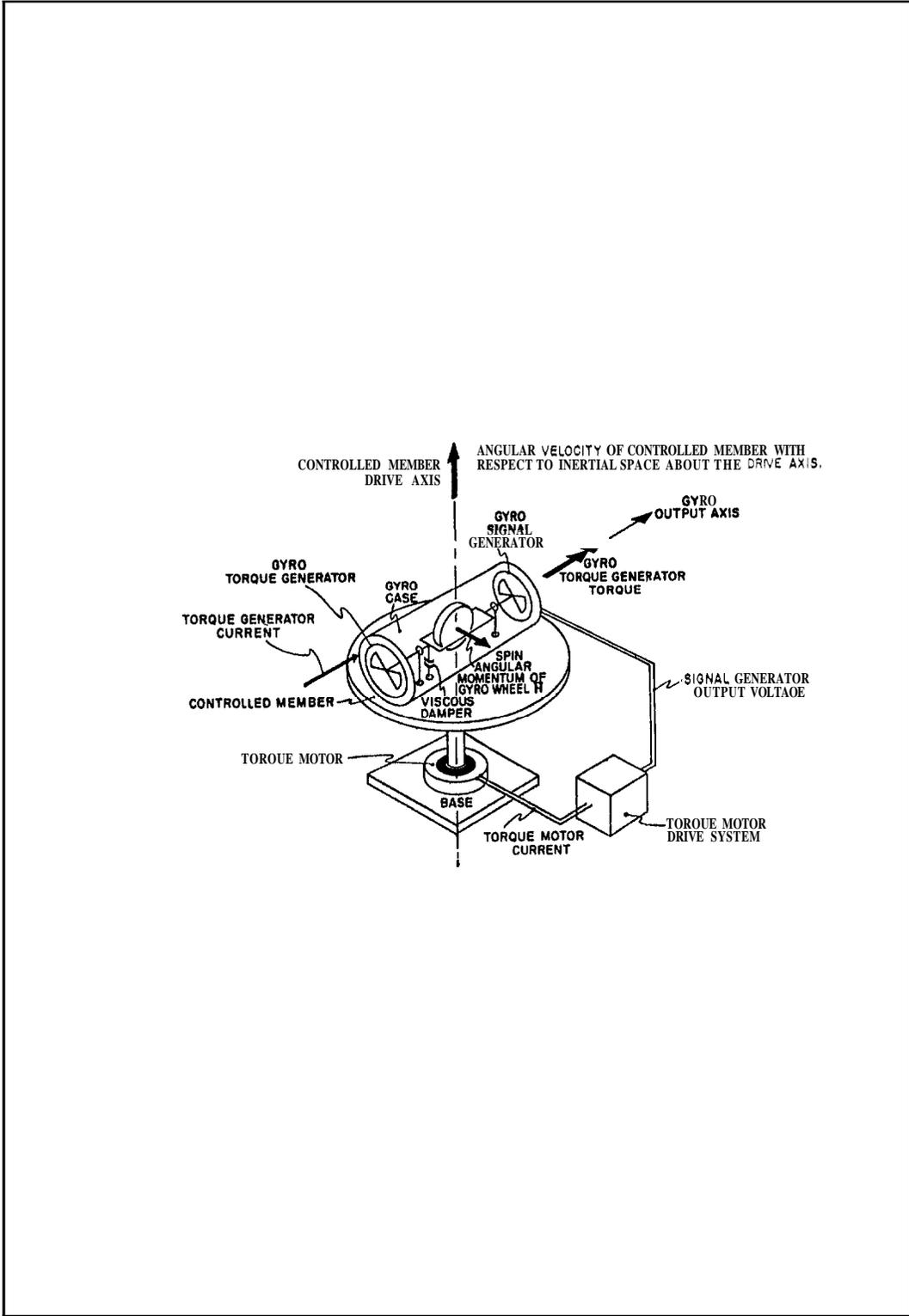
Fig. **2-7**   Schematic diagram of a single-axis gimbal drive system.
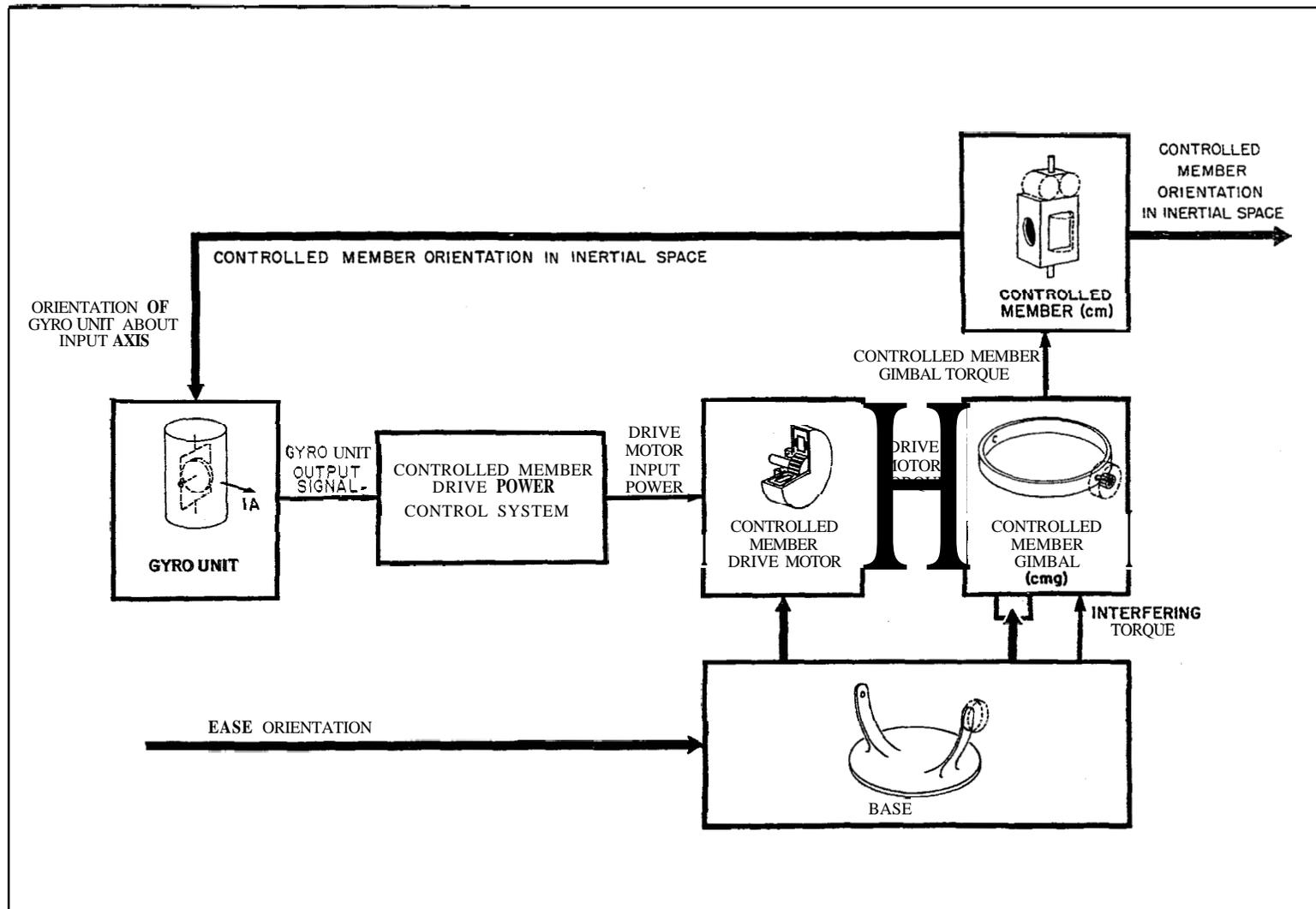
Fig. *2-8* Functional diagram for illustrative orientation **of** gimbals
in which operation maybe represented as a single-degree-of-freedom.
system.
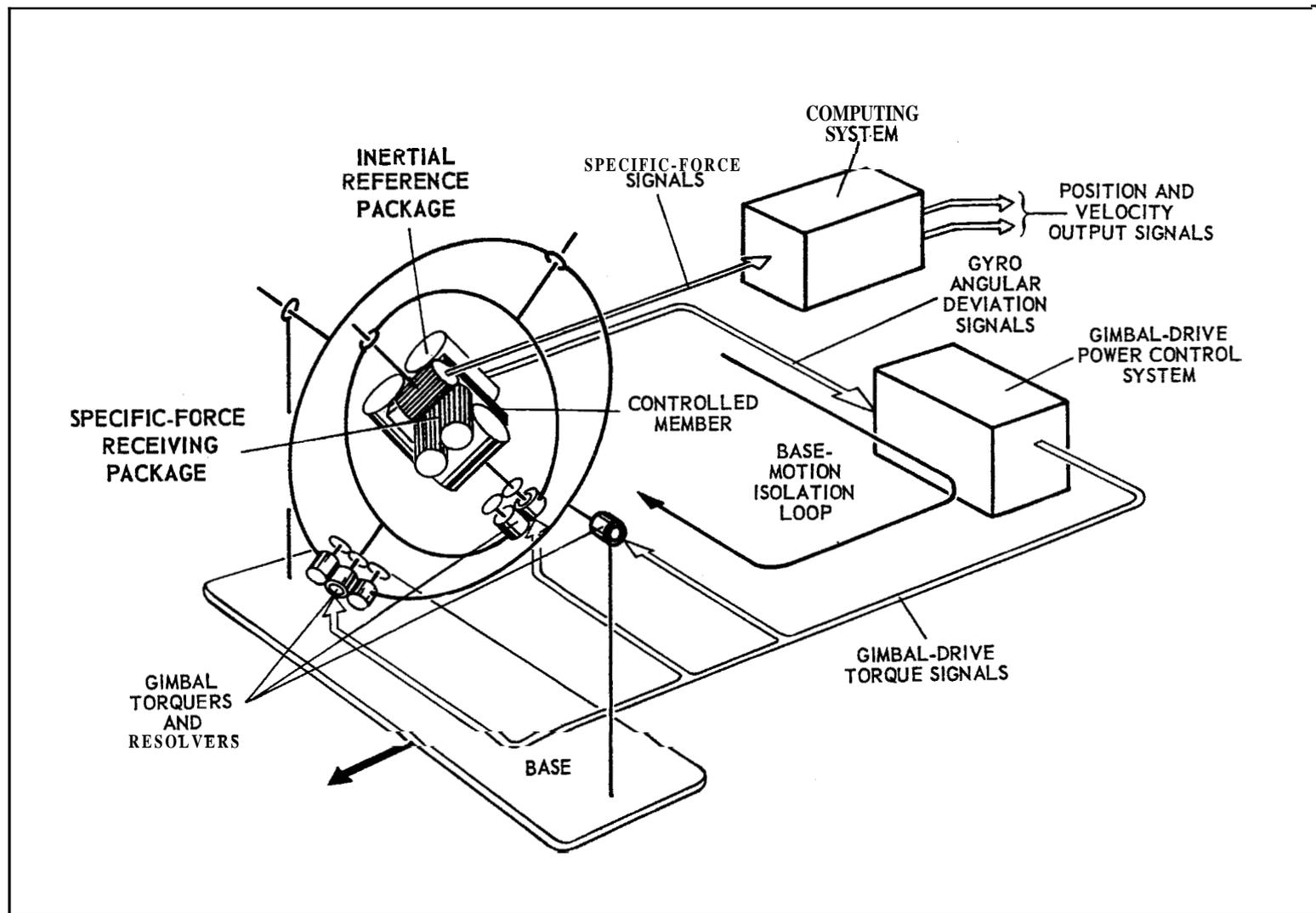
Fig. 2-9   Inertial guidance system with specific-force receiving
package fired with respect to an inertial reference package that is
stabilized with respect to inertial space.

IV   Quantitative Performance Measures

It is instructive to review the numerical values involved and the region of mechanical uncertainty that must be realized, in a general way, to meet the specifications for high-quality inertial systems. A great number of difficult problems have to be solved before satisfactory equipment is operational, but the principal limiting factor in any inertial system, given the best possible design and execution in all other aspects, is the uncertainty in center-of-mass position of rotor-carrying gimbal structures. This is because these uncertainties in gyros result in drift-rate uncertainties.

To give the reader a feeling for the magnitudes that must be considered, Table 2-1 gives names, symbols, and magnitudes in various units of the angular velocity of the earth in inertial space, in the earth's daily rotation. The drift of a gyro is commonly measured in milli-earth-rate-units (meru) and fractions thereof.

Table 2-1   Earth angular velocity units

Unit Name	Unit Symbol	Degrees Hour	Minutes of Arc Hour	Seconds of Arc Hour	Radians Hour	Radians Second	Milliradians Second
Earth Rate Unit	eru	15	900	54,000	0.26	0.73×10^{-4}	0.73×10^{-1}
Deci Earth Rate Unit	deru	1.5	90	5,400	0.26×10^{-1}	0.73×10^{-5}	0.73×10^{-2}
Centi Earth Rate Unit	ceru	0.15	9	540	0.26×10^{-2}	0.73×10^{-6}	0.73×10^{-3}
Milli Earth Rate Unit	meru	0.015	0.9	54	0.26×10^{-3}	0.73×10^{-7}	0.73×10^{-4}
Deci Milli Earth Rate Unit	d meru	0.0015	0.09	5.4	0.26×10^{-4}	0.73×10^{-8}	0.73×10^{-5}
Centi Milli Earth Rate Unit	c meru	0.00015	0.009	0.54	0.26×10^{-5}	0.73×10^{-9}	0.73×10^{-6}
Milli Milli Earth Rate Unit	m meru	0.000015	0.0009	0.054	0.26×10^{-6}	0.73×10^{-10}	0.73×10^{-7}

When a typical gyro unit is subjected to the maximum
effect of the earth's gravity acting on a mass imbalance of the
float, the drift angular velocity in radians per second is equal
to 1/2 the length of the arm between the center of symmetry and
the center of gravity in centimeters. Table 2-2 summarizes
the magnitudes of the center-of-gravity arm that corresponds to
various drift rates for a typical gyro. For example, this arm
in the case of a marginal inertial-quality gyro unit (drift rate
equal to one meru) is about one-half *of* one-tenth of a microinch,
which is about 15 angstrom units (one angstrom unit equals 10^{-8}
cm) and about five times the distance between the atoms in the
crystal lattices of steel, aluminum, and beryllium, which are
the materials commonly used for the structures of high-perform-
nnce inertial instruments.

Table 2-2 Center-of-mass positions with respect to the output axis that correspond to various drift rates;
based on the relationship developed in **Fig.** 25 for a representativt gyro unit

Drift Rate		Center-of-Mass Position With Respect to Output Axis $(arm)_{(cg)}$			
in Earth Angular Velocity Units	in $\dfrac{Radians}{Second}$	in Centimeters	in Microinches	in Angstroms	in Lattice Constants (approx) of Aluminum, Steel, or Beryllium+
1 eru	0.73×10^{-4}	1.46×10^{-4}	57. 5	1.46×10^{4}	$\cong 0.5 \times 10^{4}$ or *5000*
1 deru	0.73×10^{-5}	1.46×10^{-5}	5.75	1.46×10^{3}	0.5×10^{3} *500*
1 ceru	0.73×10^{-6}	1.46×10^{-6}	0. 575	1.46×10^{2}	0.5×10^{2} 50
1 meru	0.73×10^{-7}	1.46×10^{-7}	*0. 0575*	1.46×10	$0.5 x 10$ 5
1 d meru	0.73×10^{-8}	1.46×10^{-8}	0.00575	1.46	*0. 5* 1/2
1 *c* meru	0.73×10^{-9}	1.46×10^{-9}	0.000575	0.146	*0. 05* 1/20
1 m meru	0.73×10^{-10}	1.46×10^{-10}	0.0000575	0.0146	*0.005* 1/200

* The lattice constants of aluminum, steel and beryllium are approximately 3 angstrom units, that is, $3 \cdot 10^{-8}$
centimeter.

If the lengths of the center-of-gravity arms are considered as not fixed 'but uncertain, *so* that they contribute uncertainty to the gyro drift rate, then the data of Table 2-2 can also be considered to be that of drift-rate uncertainty versus center-of-gravity arm-length uncertainty. The numbers in the lowest line of Table 2-2 are for the case when the gyro unit of Fig. 2-10 has a drift-rate uncertainty of one millimeru, which would generally be satisfactory for inertial purposes. The small arm uncertainties that; are allowable in this typical unit are used for illustration purposes to emphasize the difficulties of gyro-unit design. It is to be noted that arm uncertainties of the same order of magnitude apply to all gyroscopic instruments, *so* that changing construction details, changing the number of degrees of freedom, or changing the method of suspension cannot solve the basic problem of gyro drift. Only careful design, good materials, and excellent techniques in manufacture and use can meet the needs of inertial guidance.

Single-degree-of-freedom gyros are symmetric about a line, (the output axis), and are thus more readily given high precision in manufacture than two-degree-of-freedom gyros, which are symmetric about a point. The required balancing to give a gyro its low drift uncertainty is most directly accomplished by single-axis assembly and calibration. Furthermore, viscosity in a supporting fluid can be used, in the single-degree-of-freedom instrument, for integration of input angular velocities.

Flotation by a liquid whose density matches that of the gyro element as nearly as possible produces a support of very low torque uncertainty and high resistance to shock or to high-g thrust. I-Iowever, a centering device must be used, such as a pivot and jewel; but in the IMU gyros the signal generator at one end of the output axis and the torque generator at the other end center the float as well as provide angle-signals or torques. With almost all the weight of the gyro element supported by the liquid, this electromagnetic support furnishes extremely accurate centering

with very low torque uncertainty. A feature of this support is that it. is entirely passive and static. The liquid flotation makes the gyro element inert to gravity and vehicle accelerations. Flotation transmits the pressure data from the case to the float to accomplish this, at the speed of sound in the liquid. The electromagnetic trim and centering is accomplished by simple tuning of external circuits around the signal and torque generators. Such support makes use of the viscosity of the liquid and produces integrating gyros.

All gyro units require precision balance to minimize torque uncertainty and all require temperature control. The objective of the gyro design engineer is to produce a unit, that will maintain its .reference direction in inertial space in the face of interferences and will rotate (precess) this direction relative to inertial space ai an angular velocity proportional to the command.

Fig. 2-10 shows the essential design features that must 'be incorporated in an Apollo IMU gyro, The rotor is driven by a multiphase, alternating current, synchronous motor. The gyro element has a spherical outer covering and is floated in its sur-rounding case with a radial clearance of about; 0..005 inch between them. The flotation fluid must be gas-free, particle-free, of appropriate density and Newtonian, i. e., the viscous torque must be proportional to the angular velocity of the float relative to the case. A spirally-wound electric heater controls the gyro unit temperature. The temperature is adjusted to make the fluid density ihe same as the average density of the gyro element:. A constant temperature distribution in the gyro unit: will tend to reduce torque uncertainties. This can be most readily obtained by control of the ambient temperature surrounding the gyro unit.

Static halance is obtained by rigid-arm compensators, which are .weighted screws that can be adjusted from outside the unit during calibration. Minute flexure of the gyro element under acceleration, or anisoelasticity, is compensated by spring-mounted weights whose shift under acceleration balances the correspond-,

ing shift of the gyro element. Power is introduced to the rotor
drive by thin flexible leads.   Such leads have a density equal to,
that of the supporting fluid and are mounted in protecting baffles
to prevent damage when the liquid solidifies during storage.   The
signal and torque generators, called ducosyns, center the float
and, as well , generate their respective tbrques or signals..
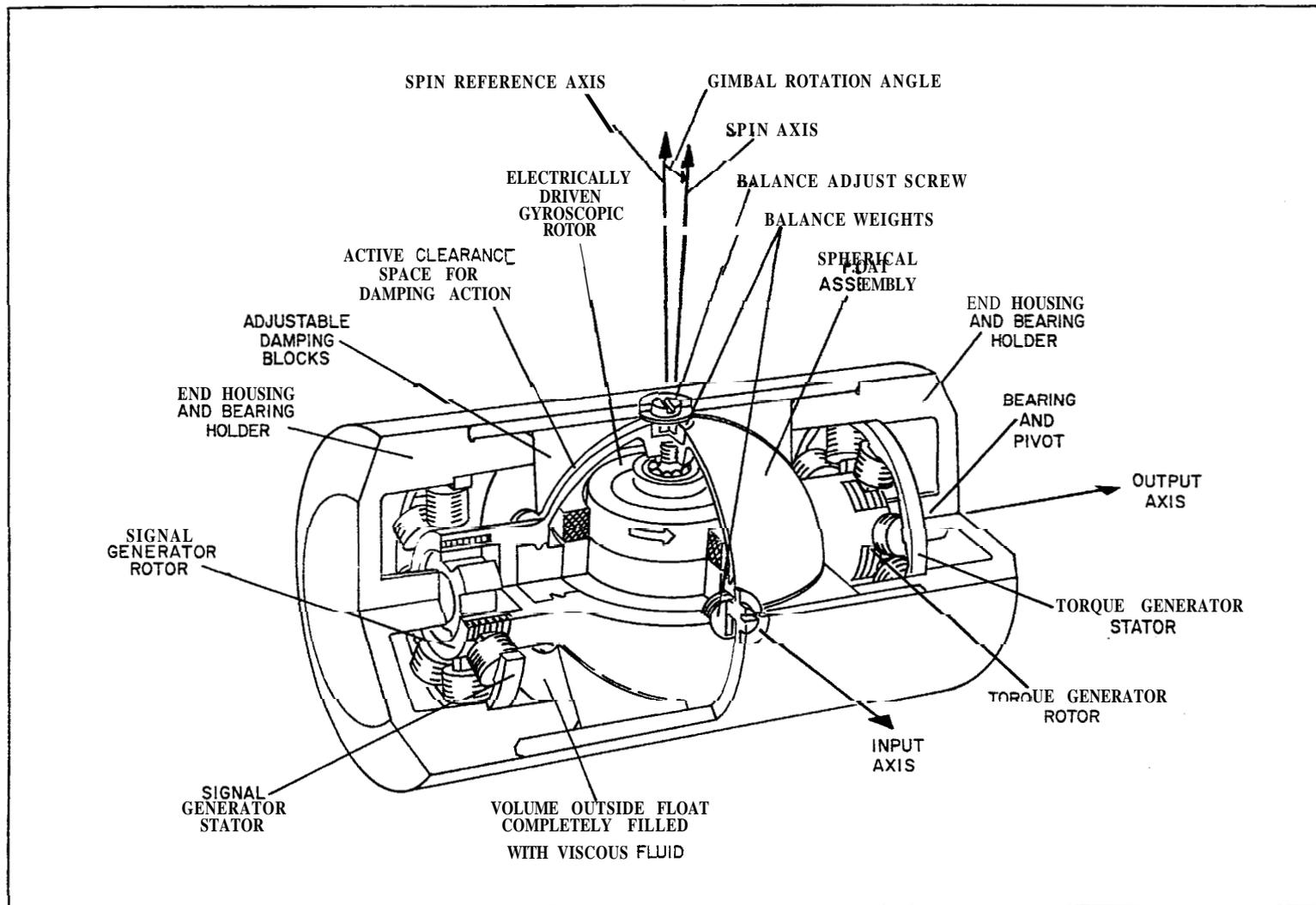
Fig. 2-10 . Cutaway view of the Inertial Reference Integrating Gyro

(IRIG) modified for use on Inertial Measurement Unit.

AP⬤LLO

GUIDANCE AND NAVIGATION

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E-1250

ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES

STABILIZATION

December 1962

MIT INSTRUMENTATION LABORATORY

CAMBRIDGE 39, MASSACHUSETTS'

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E-1250
ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION III
STABILIZATION

ABSTRACT

This section describes the design requirements of
the stabilization system.

TABLE OF CONTENTS

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I. Stabilization. System Design

Because the function of the inertial measurement. unit is
to instrument inertial coordinate axes, and because the gyros
are the sensors to be used, the gyros are selected first in de-
signing the stabilization system; and the design proceeds with the
gyros as given. A servo is designed for each gimbal drive-axis,
and each servo loop contains a gyro (Fig. 3-1).

We can specify the desired behavior of these servos super-
ficially in completely mechanical. terms. Thus, when the platform
{Fig.3-2) supporting the gyros and accelerometers is given a
steady push, we would like it to be as stiff, i. e., as resistant
to rotation with respect to inertial. space, as possible   In. me-
chanical terms, it should have a large elastic or spring constant;
in servo terms, it should have high sensitivity at low frequencies,
particularly d-c or zero frequency, the steady state. On the
other hand, an impulsive input to the platform, represented by
a sudden. push and release, should also leave it nearly undisturbed.
This means that the "spring" should be stiff at high frequencies
also; or, in servo terms, that the servo should have large high-
frequency gain.

These inputs are useful. artifices €or those which are really
encountered, but which are not so easily analyzed: the random
momentary misalignments of the platform as the spacecraft
rotates in. inertial space and tends Lo pull (through the gimbals)
on the platform. If the platform behaved like a simply-resonant
device, i. e., like a mass-spring-damper system, it would oscil-
late at a frequency near its undamped natural frequency when it
was disturbed, and this oscillation would die out at: a rate depending
on the damping. A servo can. be constructed as a simply-resonant
device, too. But this would not, in general., be what is commonly
called a "fast" servo. To see this, we can. compare the behavior
of a fast servo with a simply-resonant mechanical system.

In. a East servo there is no oscillation, in response to an impulsive input, and very little overshoot. For steady inputs, moreover, the servo loop gain may Le very high; in a mechanical device this would imply large stiffness, high resonant frequency (for given mass), and several oscillations in response to an impulse (the number of oscillations depending on the damping). In the case of the platform, where the mass-efiect on rotation, the moment of inertia, is large, the resonance could be expected to be in the region of a few cycles per second at the most, if the platform were restrained by practically-realizable springs rather than by a servo.
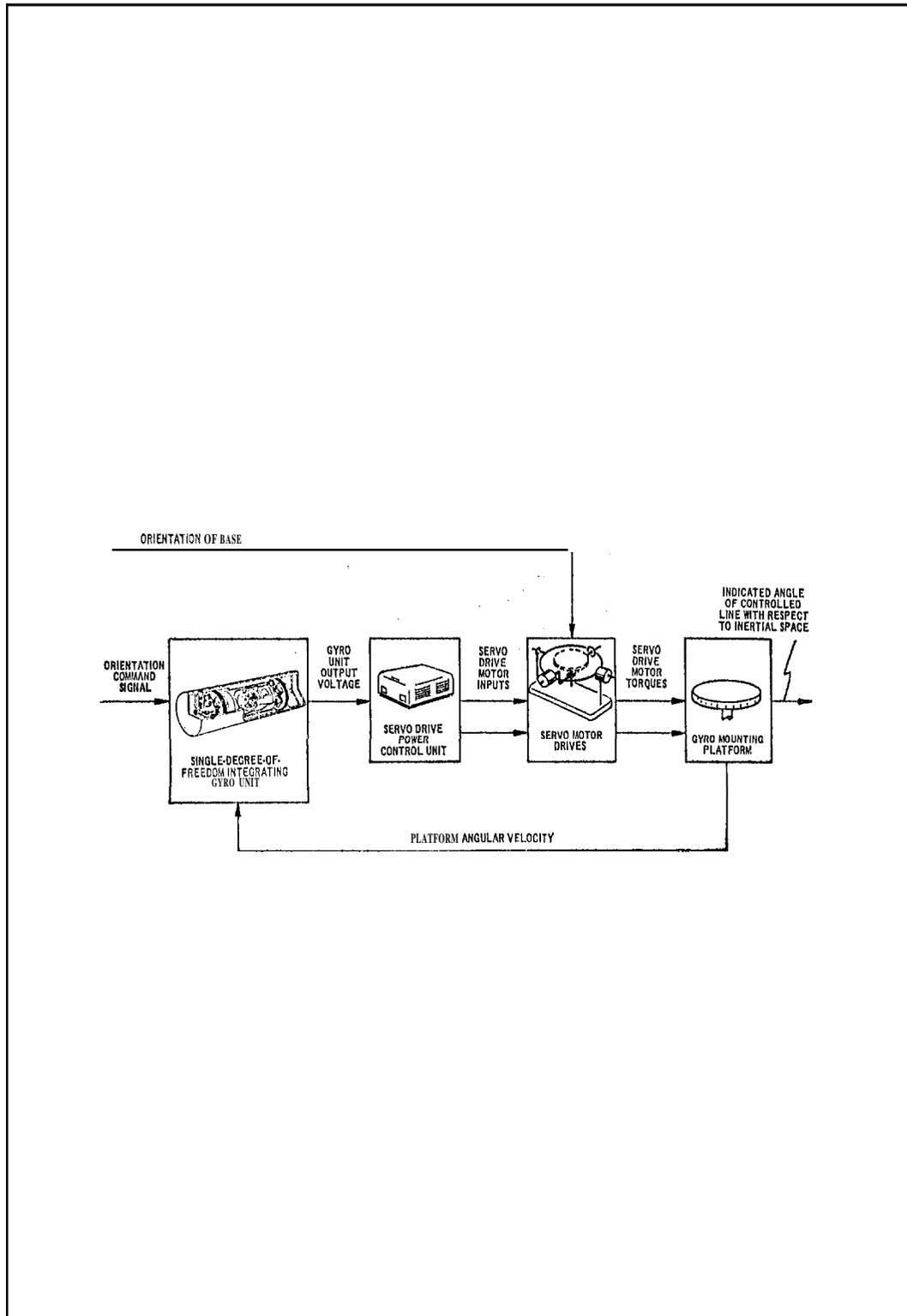
Fig. 3-1   Single-degree-of-freedom integrating gyro unit used in
a space integrator.
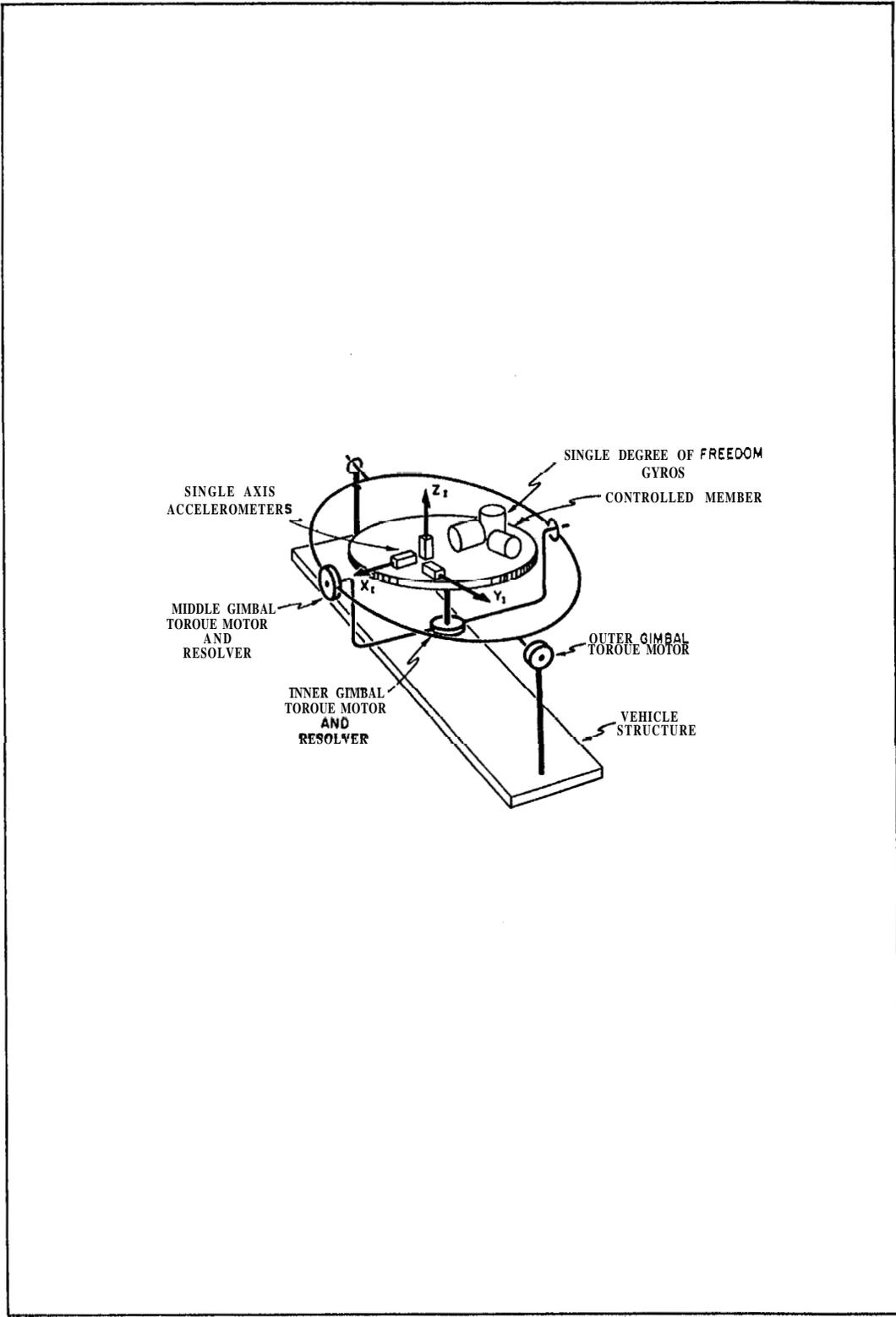
Fig. **3-2** Schematic diagram of a gimbal system for three-axis stabilization of an accelerometer package.

II   System Damping

Now, a system with only one natural frequency of oscilla-
tion can be damped in a viscous-drag manner, *so* that the energy-
dissipative force is praportional to the velocity of the mass *(or,*
in the rotational case, the energy-dissipative torque is proportional
to the angular velocity of the moment of inertia).   With this kind
of damping, a sudden change ("step") input to the system will
make it oscillate, if, as the phrase goes, it is lightly damped
(Fig. 3-3).   The oscillations die away eventually (theoretically,
after infinite time).   As the damping is increased, the oscillations
die away more rapidly, until the point is reached at which the
system just fails to oscillate--what is called critical damping.
The system simply decays back to where it was before the sudden
input change.   In a mass-spring-damper system, for example,
the mass settles back into the oil of the damper without oscillating,
despite the presence of the spring.   The damping force has super-
seded the spring force in determining the character of the motion.

Now, as the damping is increased further, the system
decays to its inertial state more and more slowly.   Actually, to
get the system back to within 95%, for example, of its initial con-
dition most rapidly, a little oscillation or overshoot is best, and
a little less than critical damping is used.   But even in-this case,
the shortest recovery time is about one-half of the undamped
natural period (Fig. 3-4).   Thus, in a massive-spring-restrained
system, with an undamped natural period of, for example, 1 sec,
damping can at best give a recovery time of about 1/2 sec.
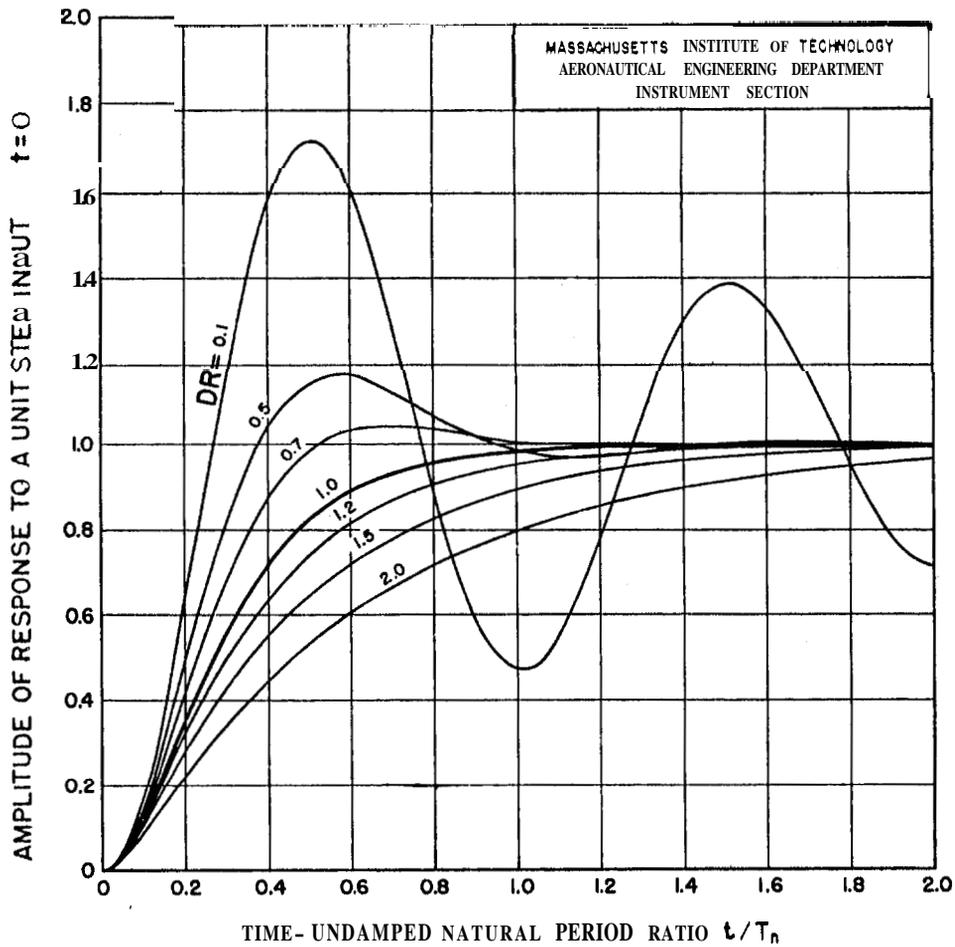
Fig. 3-3   Amplitude of response to a step input as a function of
time (in non-dimensional form) for various damping ratios of the
force per unit velocity (or the torque per unit angular velocity) to
that force *or* torque required for critical damping.
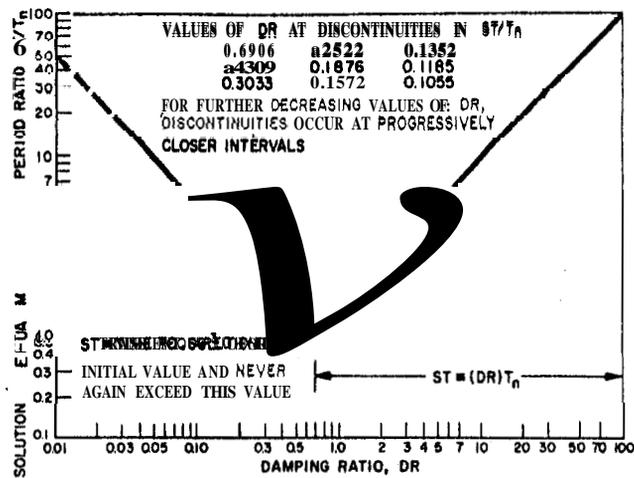
Fig. **3-4** Log-log plot of the solution time-undamped natural period ratio vs the damping ratio DR, for the transient solution of a second-order differential equation with constant coefficients.

III   Servo Stabilization

The servo, however, does not restrain the platform like a simple spring. It is true that at low and high frequencies it does; and the servo provides the stiffness of a very strong spring indeed, a stiffness that, if obtained at the resonant frequency, would cause the servo to be unstable. An unstable servo, which is one in which unforced oscillations do not die out, is obviously undesirable.

The servo is then stabilized by making it into an exponentially-decaying system for inputs with frequencies near resonance. Precisely how near is a designer's problem, and he is specifically concerned with the design of a lead-lag filter in the servo loop. Thus the "damping" is not mechanical, but is simulated at signal levels (Fig. **3-75).** The servo, therefore, is designed to provide high gain (large stiffness) at high and low frequencies, but the retarding effect on recovery from a sudden input is sidestepped by damping the servo only near its resonant frequency (in the closed-loop system), i.e., in servo terms, near the open-loop cross-over frequency, at which the logarithm of the output-input amplitude ratio is zero db. Thus an open-loop Bode plot would show a slope of (-2) from very low frequency inputs up to the first break-point of the lead-lag network, at which the slope changes to (-1), passes through cross-over at the 0 db line, and returns to (-2) at the second break-point of the lead-lag network. The next characteristic time, corresponding to a break-point at the frequency at which the slope becomes (-3), is usually that of the gyro. The other aspects of the servo are not as important for stability as those already mentioned.

We can put a tachometer on the servo and eliminate the lead-lag network; this is an advantage when such a network adversely affects signal-to-noise ratio. On the other hand, the tachometer operates by indicating the angular velocity of its

seismic element, which is viscously damped, relative to the plat-
form (Fig. 3-6). This is not exactly what we want; damping should
affect the error signal only (as it does when a lead-lag network
is used, as in Fig. **3-5**). The tachometer obviously responds to
platform motions other than those due to the error signal. Never-
theless, on a platform instrumented to be inertially nonrotating,
like the Apollo Inertial Measurement Unit,, tachometer damping
can be effective, and will probably be used.

When the platform is started up far from the null
positions of the gyros, the motion toward recovery of its correct
orientation is at first determined by the saturation of various com-
ponents in the servo loops. In this situation, we can define a
saturation torque which is the maximum that the servo can deliver;
and a saturation angle at which this torque is first reached when
the platform is displaced from its correct orientation. The re-
sulting behavior with large platform displacement is oscillatory;
the platform swings back and forth, passing through the figurative
"notch" (Fig. **3-7**) where it will eventually settle in. Each time it
passes through the notch, the system loses some energy, *so* a
kind of damping is in operation, and the system eventually settles
into the notch. This is the linear region, as opposed to the sat-
urated *or* non-linear region, of operation. The oscillations up to
this point decrease amplitude, and as in saturating nons-linear
mechanical systems, the oscillations increase in frequency
(Fig. 3-8).
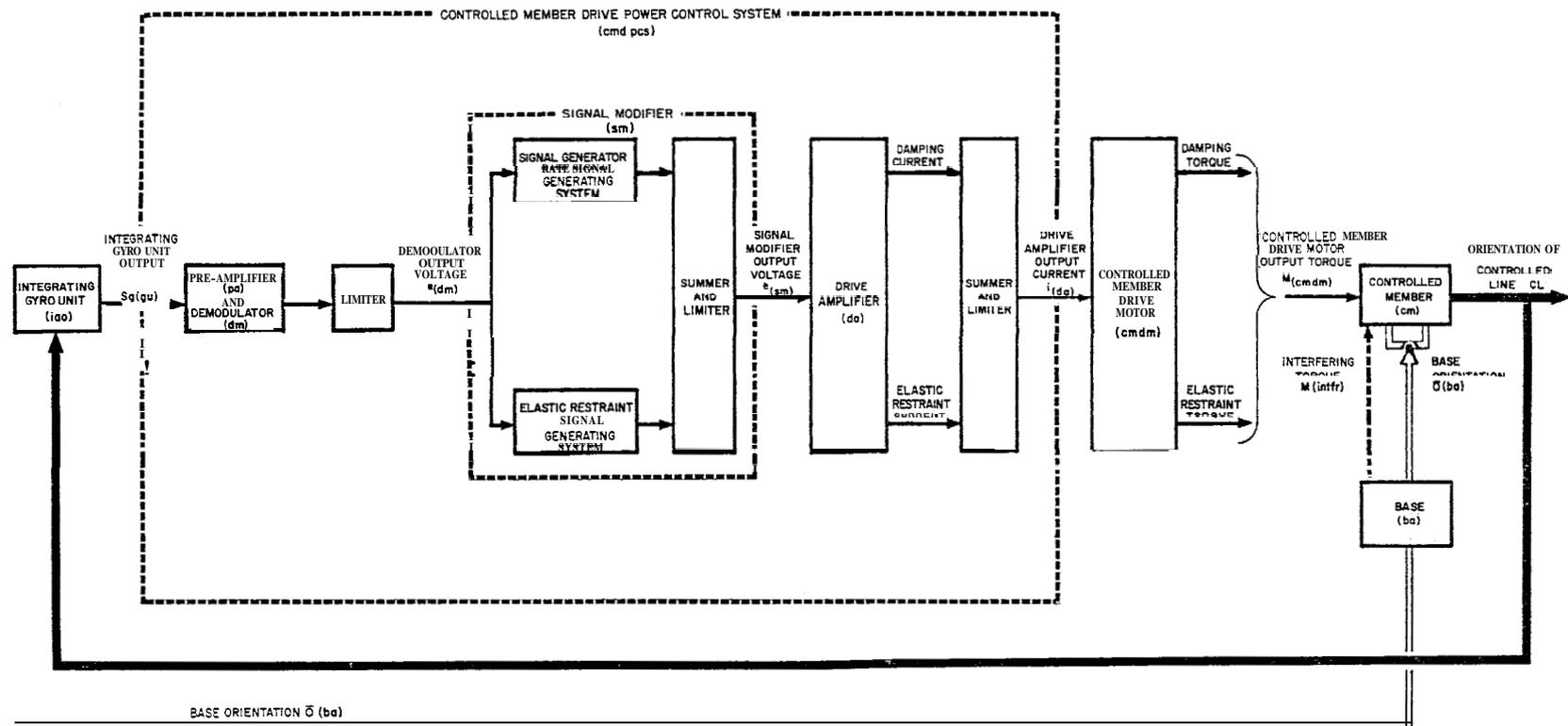
Fig. **3-5** Functional diagram for a single-axis base motion
isolation system showing subcomponents of controlled member
drive power control system leading to non-linear operation.
Signal modifier generates rate signals.
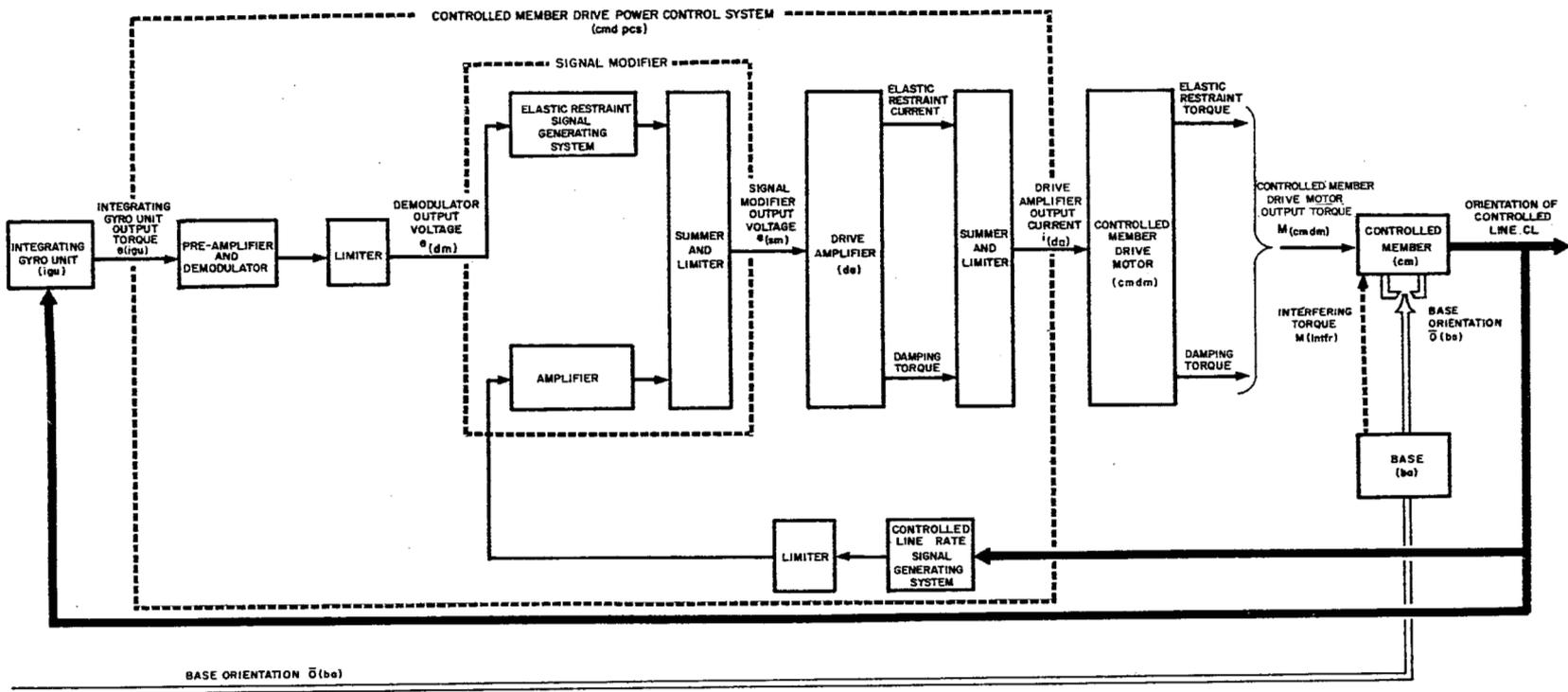
Fig. 3-6 Functional diagram for a single-axis base motion
isolation system showing subcomponents of controlled member
drive power control system leading to non-linear operation.
Device on controlled time generates rate signals.
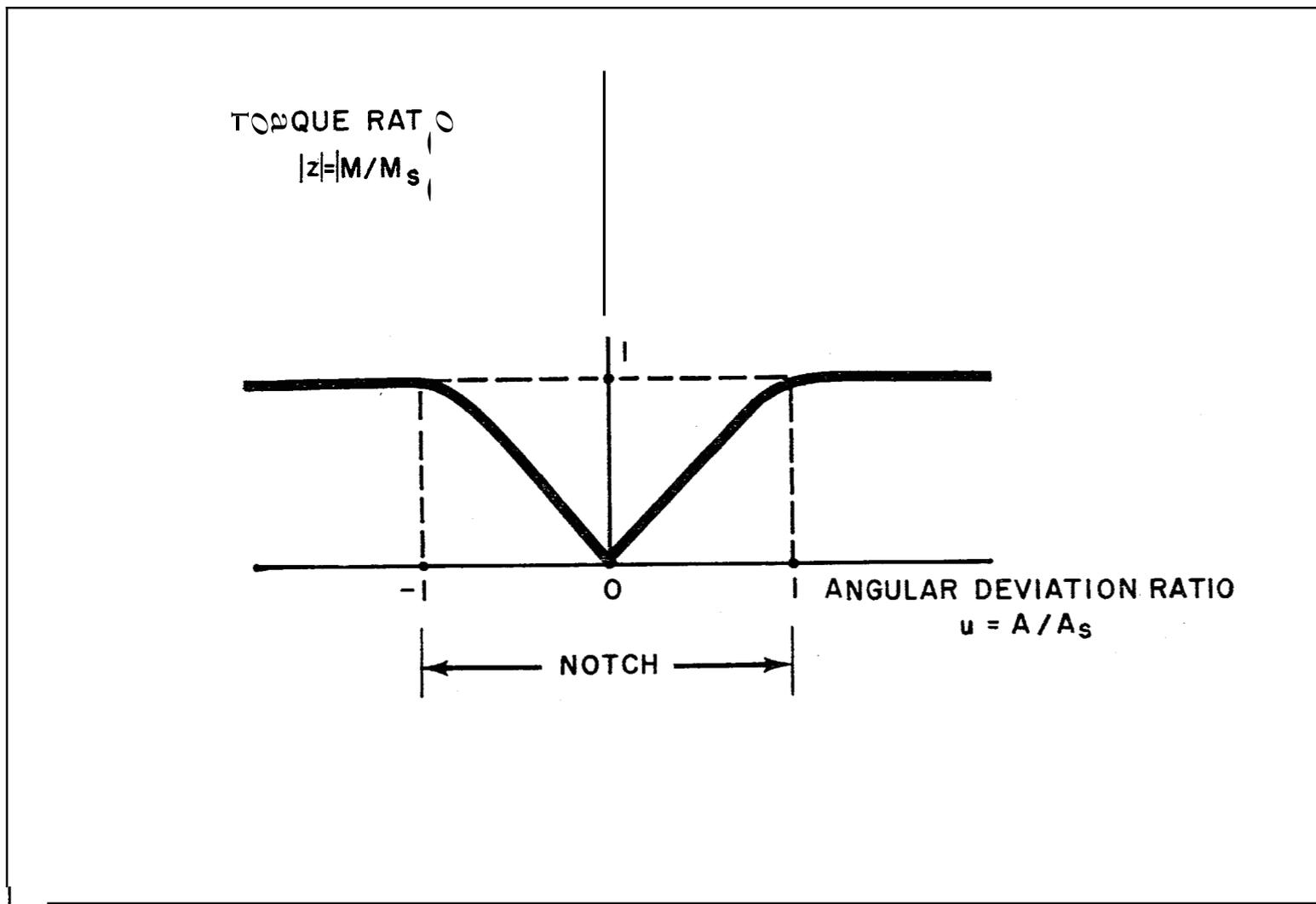
Fig. 3-7   Magnitude of torque ratio vs angular deviation ratio,
showing notch.
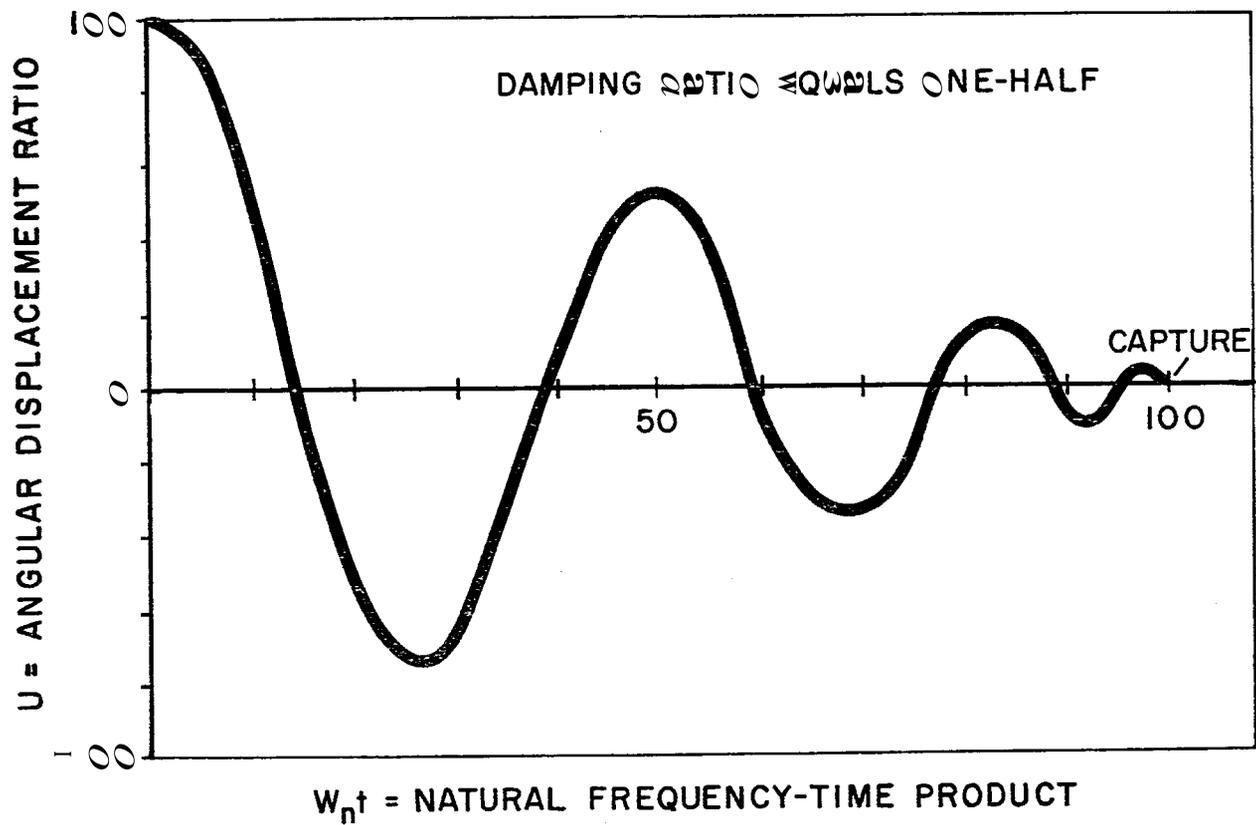
Fig. 3-8   Angular displacement ratio as a function of natural frequency-time product.

AP�LLO
GUIDANCE AND NAVIGATION

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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:

SECTION IV

ELECTROMAGNETIC NAVIGATION

by

Janusz Sciegienny

December 1962
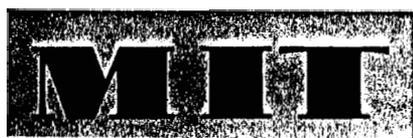
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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION IV
ELECTROMAGNETIC NAVIGATION

ABSTRACT

This section contains the slides used in a lecture on electromagnetic navigation.

by Janusz Sciegienny
December 1962

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TABLEOFCONTENTS

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Fig. 1   Operation of ERMU On LEM During LOM

Fig. 2   ERMU For LOM

```
                              TRIGGER
                                 ↓
┌──────────────────┐      ┌──────────────────┐          FLASH TUBE (TYPE
│  HIGH VOLTAGE    │ ───→ │    STORAGE       │ ───→     EGG FT-119)
│  CONVERTER       │      │   CAPACITORS     │
└──────────────────┘      └──────────────────┘

5" x 1" x 3"              2 CAP, EACH              REFLECTOR
— 0.5 LBS                 55 W-SEC                 8" DIA. 4" DEPTH
                          2" DIA. x 5" LONG        — 0.25 LBS
                          —0.7 LBS EACH
```

SPECIFICATIONS:

PULSE PEAK POWER (INPUT) **21 KW**   SIZE **260** INCH^3

PULSE DURATION **5 m** SEC   WEIGHT—3 LBS

PULSE REP. RATE I PULSE / 5 SEC   INPUT POWER **30** W

REFLECTOR BEAM WIDTH 60° (SOLID ANGLE)

THE FLASHING LIGHT HAS AN INTENSITY OF A 4th-MAGNITUDE STAR AT A
DISTANCE OF **720** Km

Fig. 3   LEM Flashing Light

EXAMPLE:

LET: R = **32** km

β = 70°

θ = I mrad

THEN:

$R\theta \tan\beta$ = 8 8 m

$R\theta / \cos\beta$ = 94 m

LUNAR AREA ILLUMINATED BY TRANSMITTED BEAM
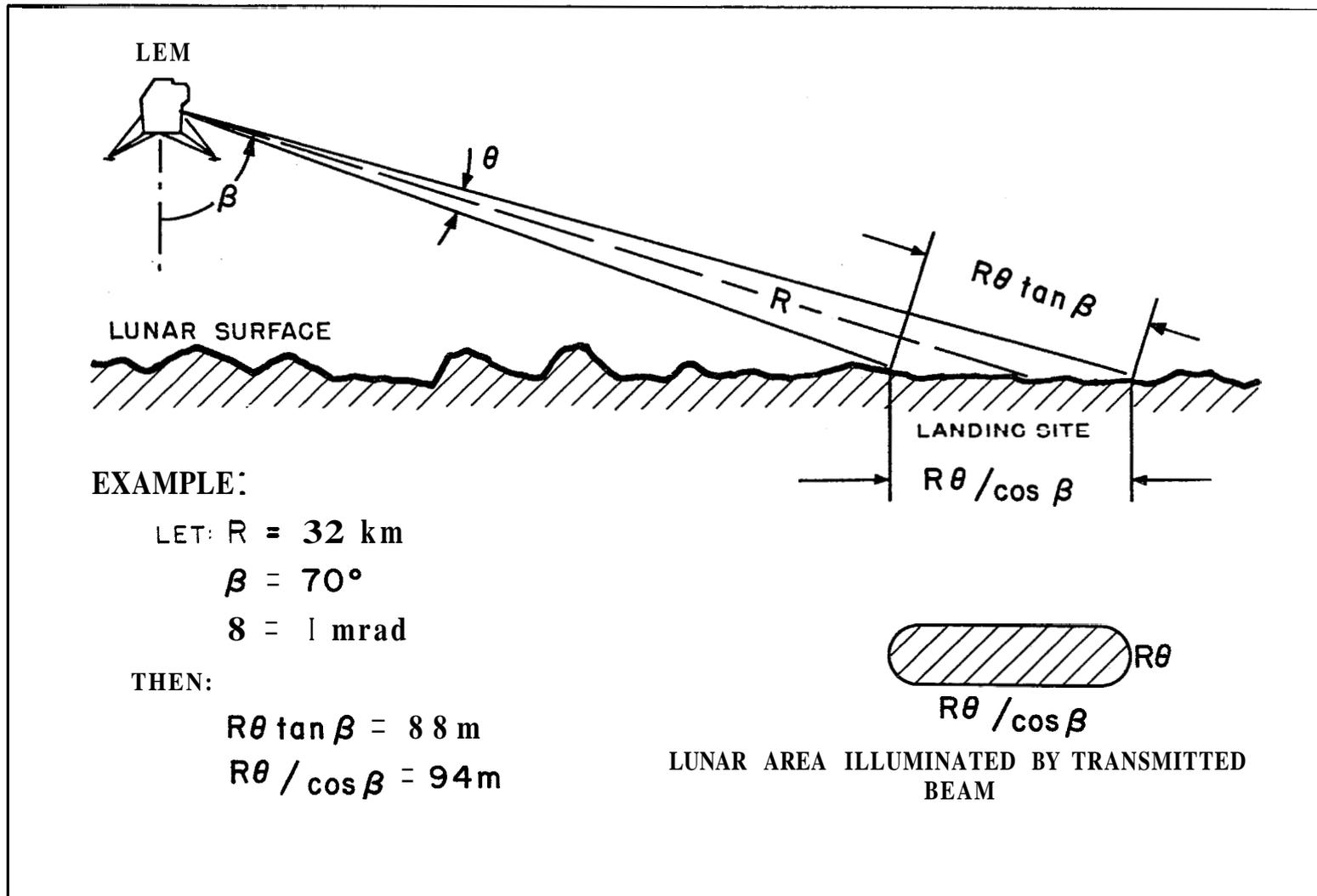
Fig. 4   Geometry Of Slant Range Measurement
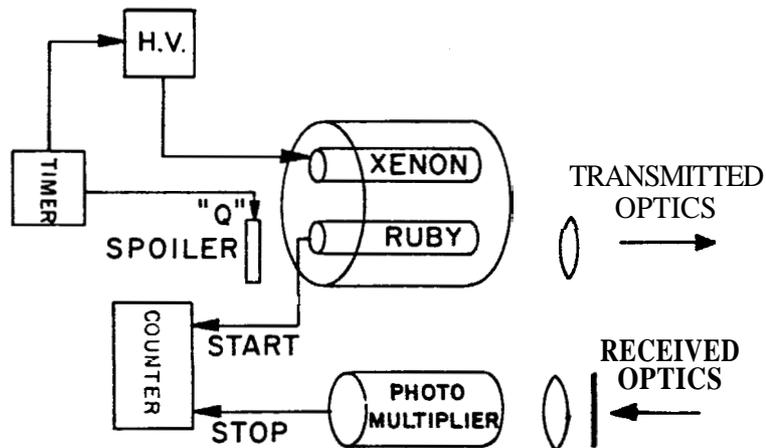
LOCATION :

NAVIGATION BASE

GIMBALLED
WITH RETICLE

MAXIMUM RANGE:
ALTITUDE -320 KM

SLANT -160 KM

ELECTRICAL CHARACTERISTICS :

WAVELENGTH $6943\mathring{A}$ (RED)
PULSE PEAK POWER   1–10 MW
PULSE DURATION 0.1 μ SEC
PULSE REPETITION RATE I PULSE/5 SEC

ACCURACY
DETERMINED BY
POINTING ACCURACY

Fig. 5   LEM LASER

TRANS. SIGNAL   DELAY $\tau = 2H/c$

Δf_T

f

Δf_T

REC. SIGNAL WITH DOPPLER

f_R

Δf
DOPPLER

REC SIGNAL NO DOPPLER

$t_m = \dfrac{1}{f_m}$

$\left| f_T - f_R \right|$

NO DOPPLER

f_H

WITH DOPPLER

f_2

f_H

f_1

$f_H = \dfrac{f_1 + f_2}{2}$

$\Delta_f = \dfrac{f_2 - f_1}{2}$

FOR NO DOPPLER $f_1 = f_2$

$H = \dfrac{C}{4 f_m \, \Delta f_T} f_H$

$\dot{H} = \dfrac{C}{2 f_T} \Delta_f$
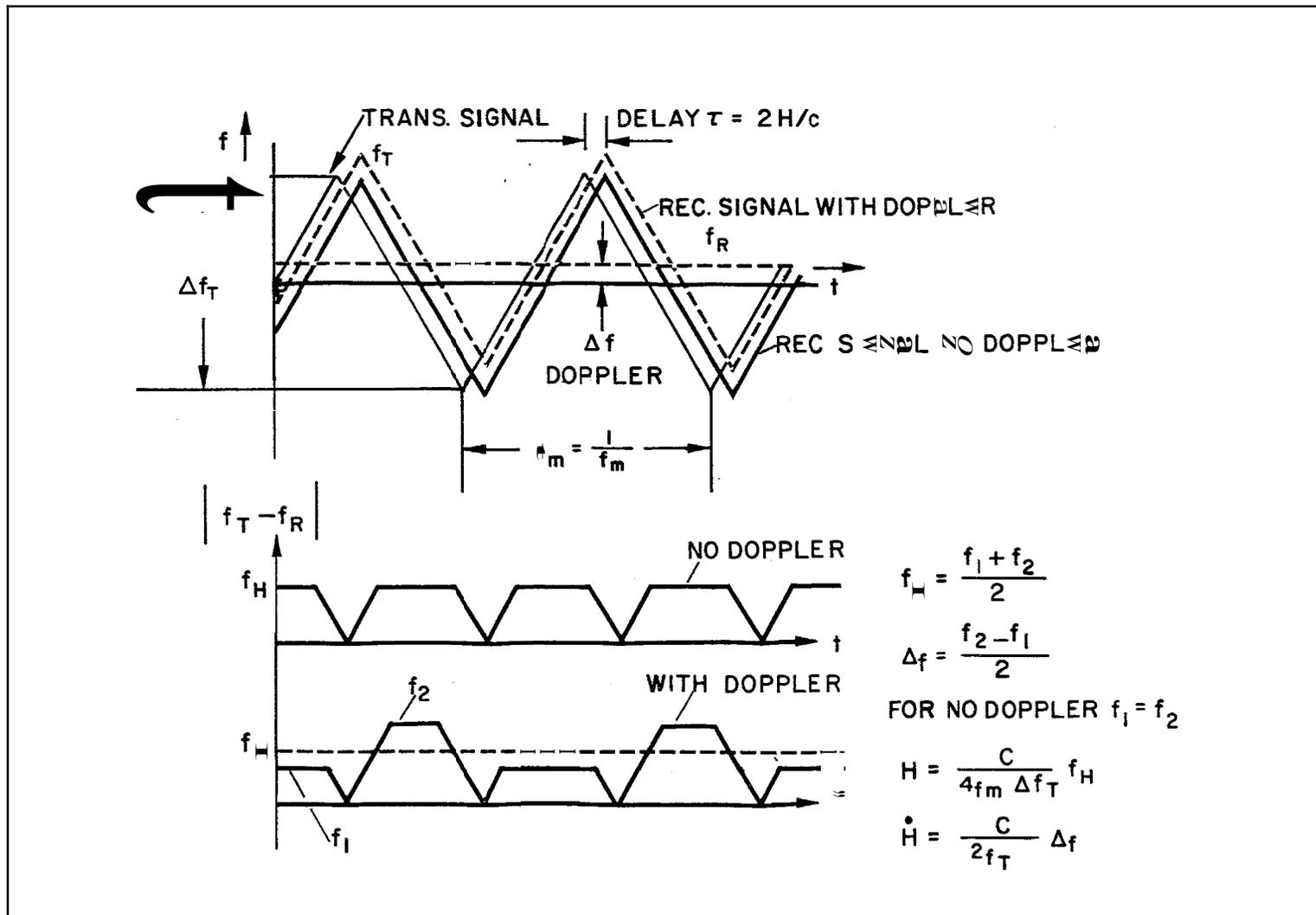
Fig. 8   Wave Forms in FM/CW Altimeter
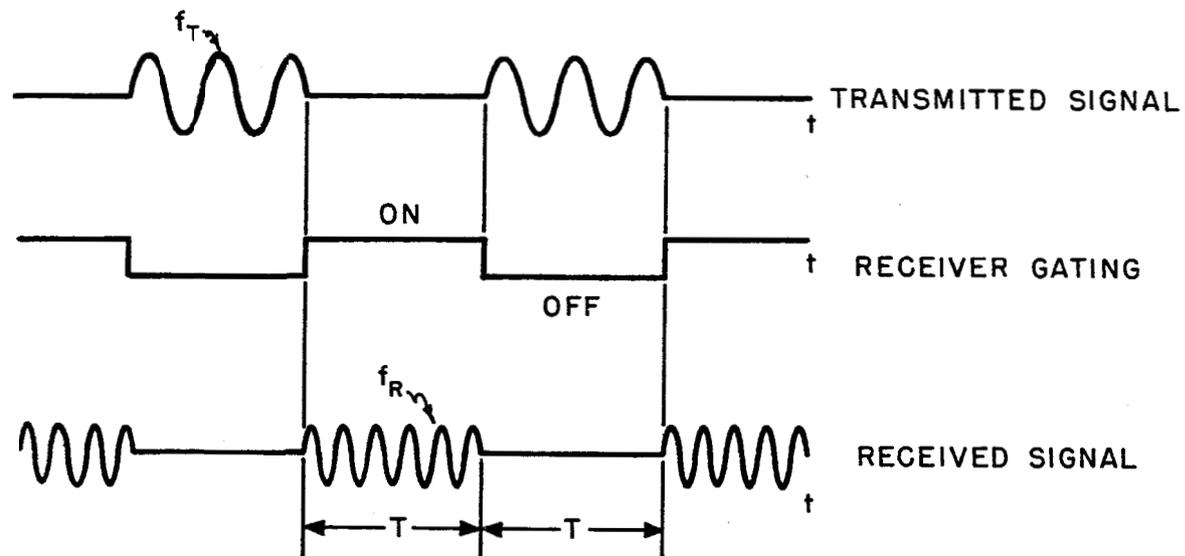
Fig. 7   ICW Wave Forms (Optimum P. R. F.)
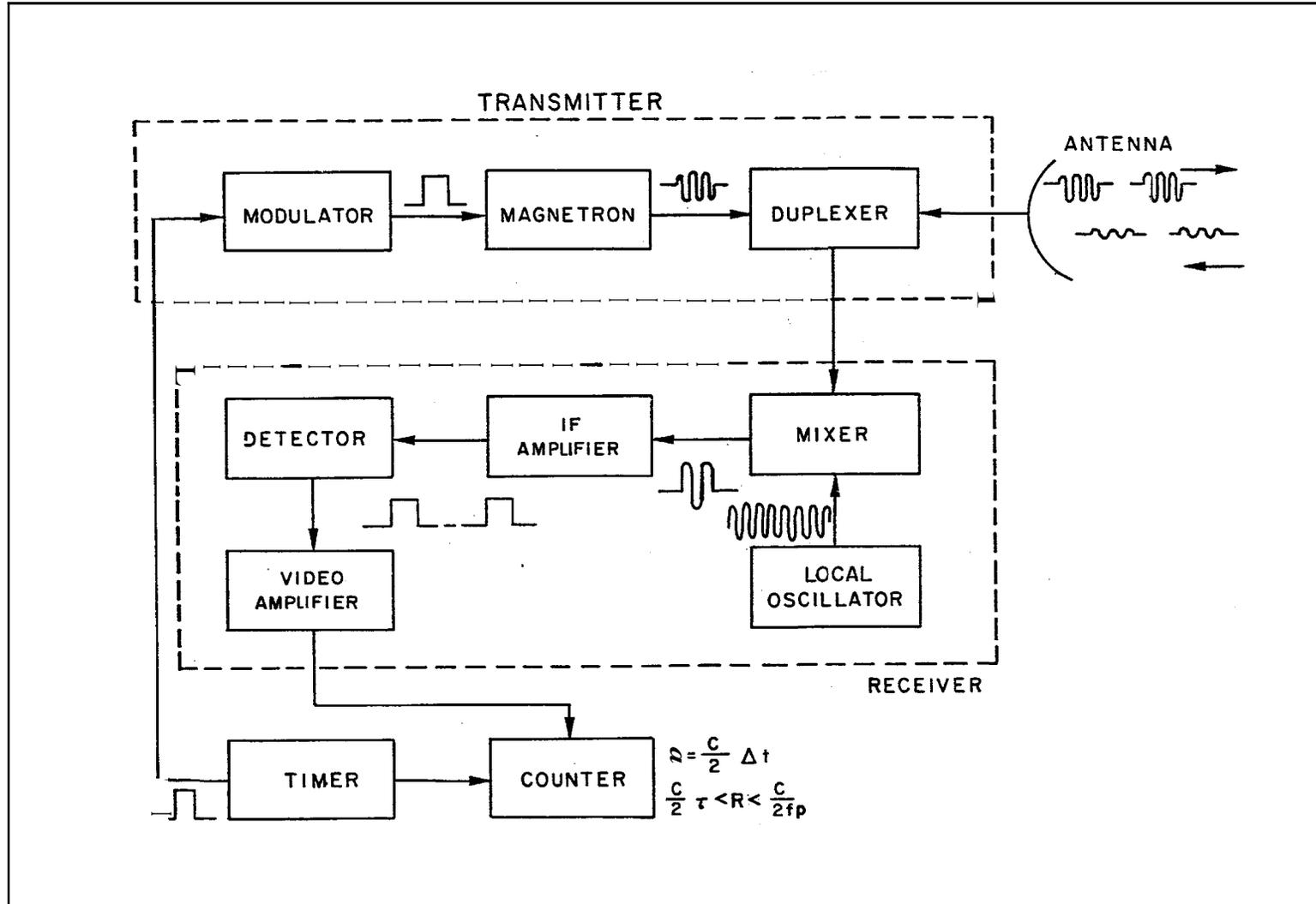
Fig. 6 Pulsed Radar Block Diagram

The figure shows a spacecraft with coordinate axes (−z, x, y) and three Doppler beams directed to points 1, 2, 3 on the ground, plus an inset defining the Doppler shift.

Inset:
$$f_d = \frac{2V}{\lambda} \cos \gamma$$

THE SPACECRAFT VELOCITY COMPONENTS:

$$V_x = \frac{X}{4 \cos a} (D_1 - D_2)$$

$$V_y = \frac{X}{4 \cos p} (D_2 - D_3)$$

$$V_z = \frac{X}{4 \cos y} (D_1 + D_3)$$

D_1, D_2, D_3 ARE THE DOPPLER SHIFTS ALONG BEAMS 1,2 &3.

X IS THE TRANSMITTER WAVELENGTH.
α, β, y ARE THE ANGLES BETWEEN
x & 1, y & 2 AND z & 3.
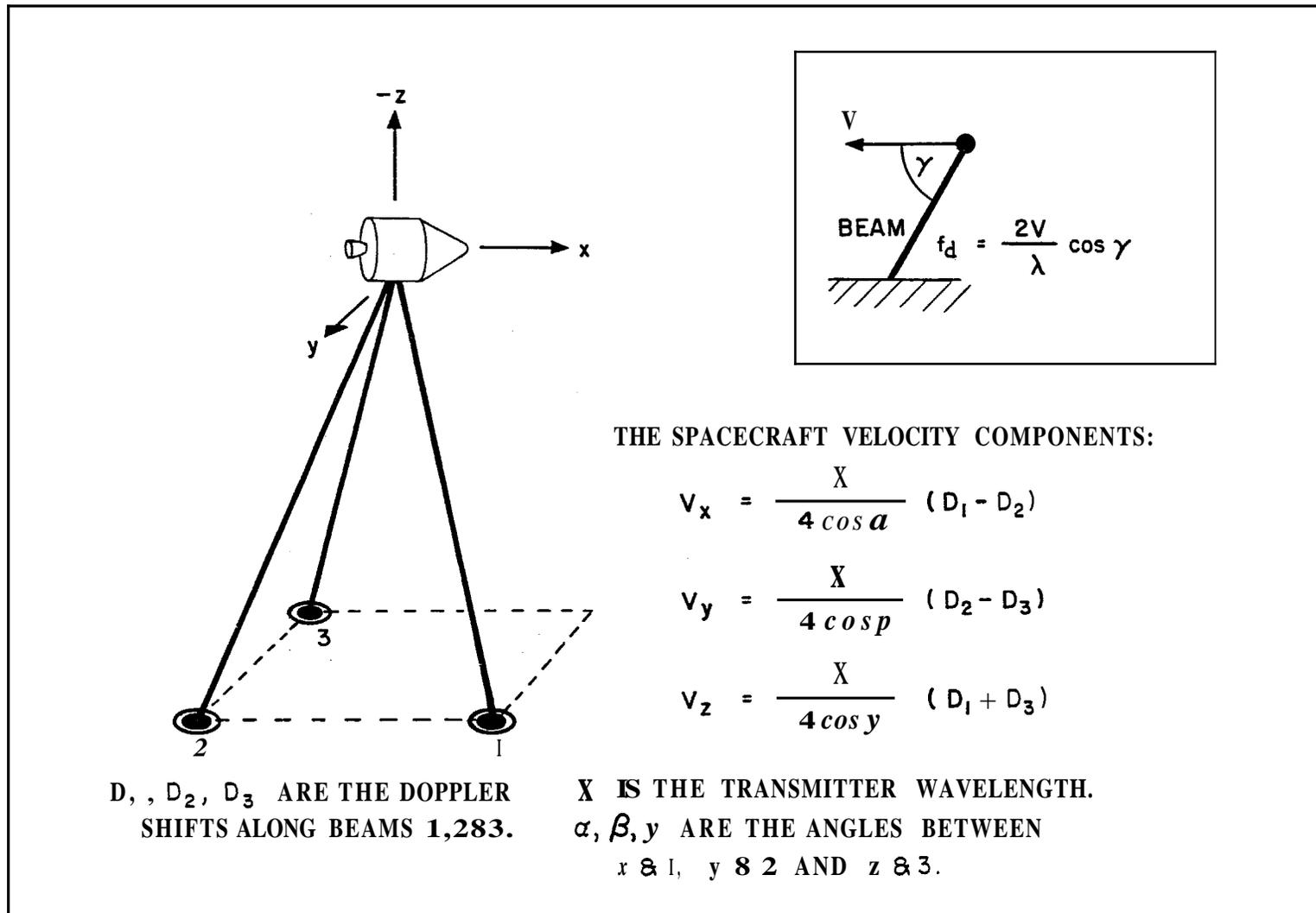
Fig. 9   Velocity Measurement By Doppler Radar
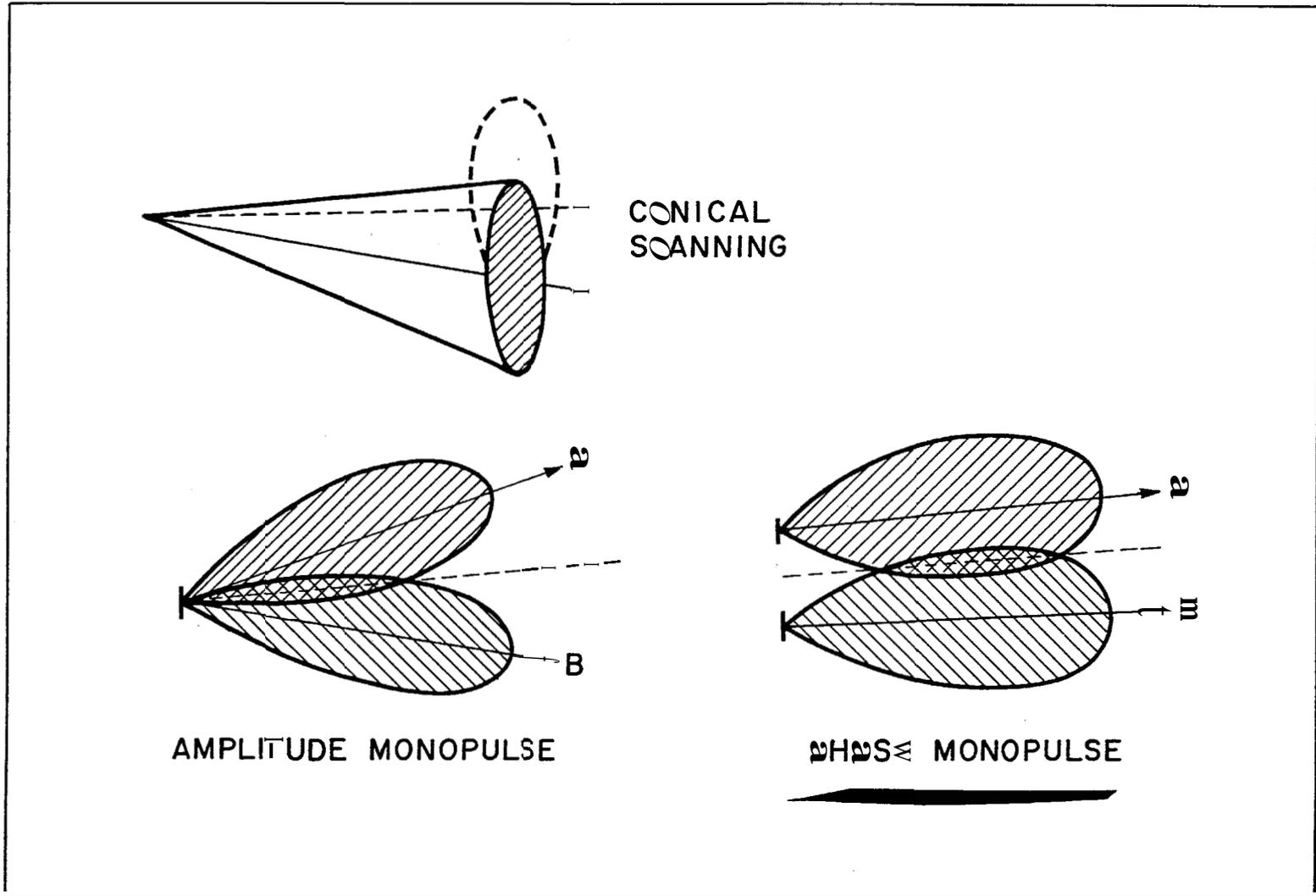
Fig. 10.   Antenna Beams in Angle Tracking Radar
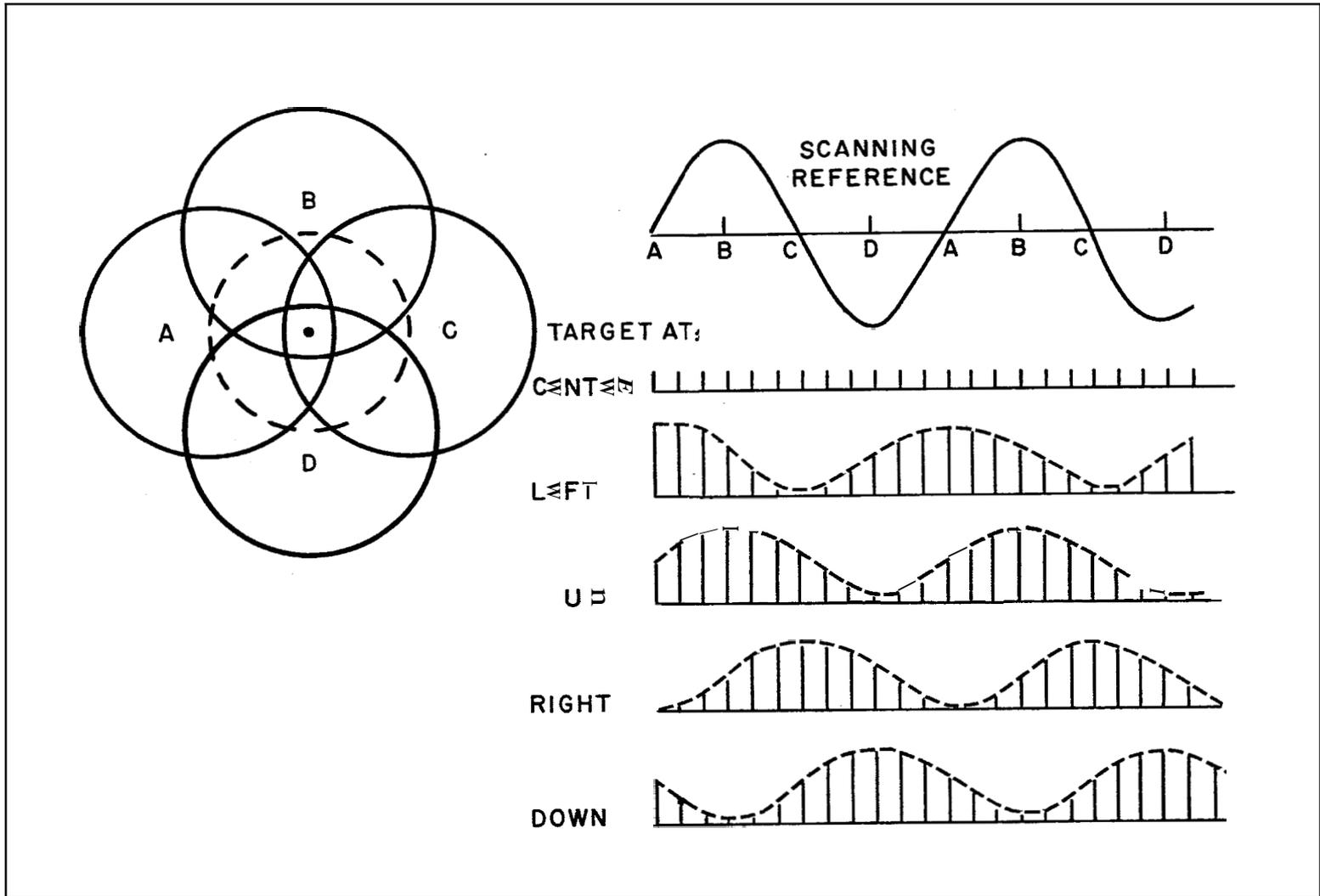
Fig. 11   Returned Pulses in Conical Scanning
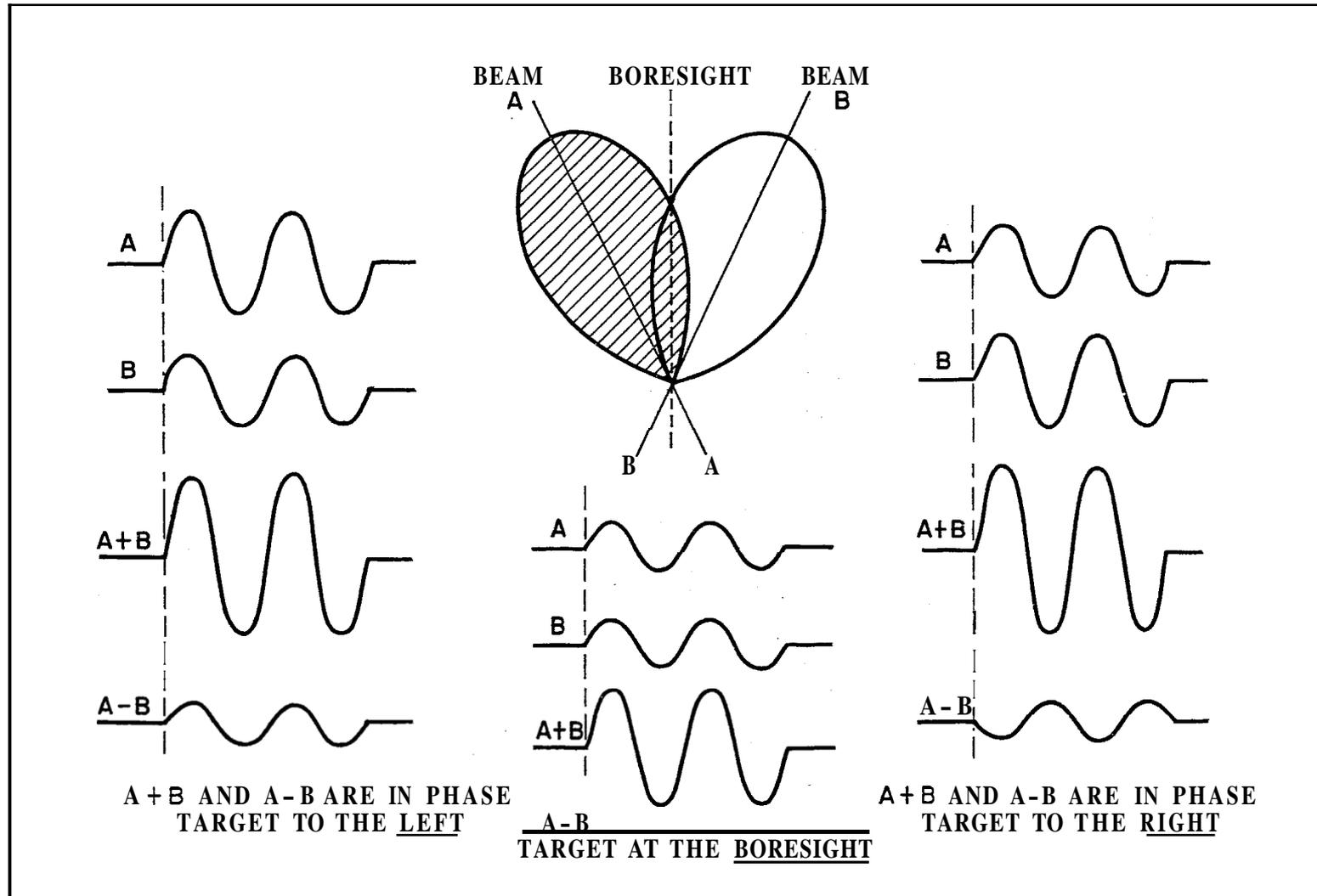
Fig. 12 Returned Pulse in Amplitude Monopulse

TRANSMITTED SIGNAL

$$c = c_0 \cos(\omega t + \phi_0)$$

RECEIVED SIGNALS

$$A = A, \cos \omega t$$

$$B = B_0 \cos(\omega t + \phi)$$

A   B

HYBRID

A + B   A - B

90°
SHIFT

| PRODUCT DETECTOR |

$\sin \phi$
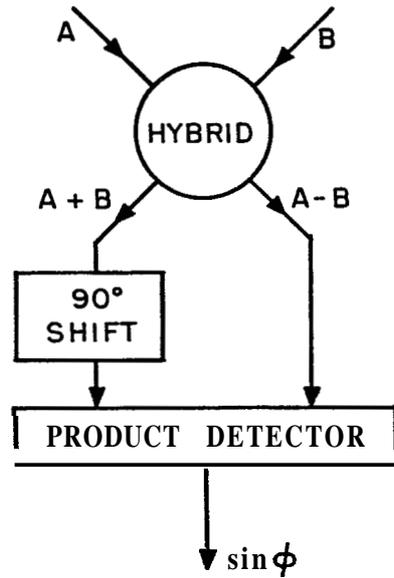
**SIGNAL PROCESSING
IN PHASE MONOPULSE**

BORE-
SIGHT   TO
TARGET

θ

X   c

c θ

A   D   B
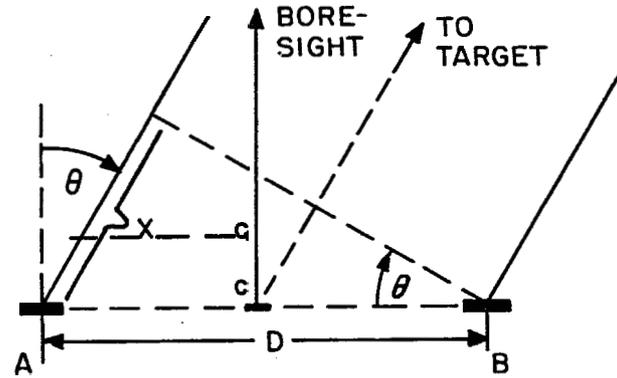
$$X = D \sin \delta$$

PHASE SHIFT

$$\phi = 2\pi \frac{X}{\lambda} = \frac{2\pi D}{\lambda} \sin \theta$$

OR

$$\sin \delta = \frac{X}{2\pi D} \quad \phi$$

KNOWN   MEASURED
CONSTANT

**PHASE MONOPULSE
BASIC EQUATION**

Fig. **13.**   Phase Monopulse

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GUIDANCE AND NAVIGATION

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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION V
MIDCOURSE NAVIGATION
AND GUIDANCE

Transcript of a Talk

by
Dr. Richard H. Battin, Jr.
February 1963
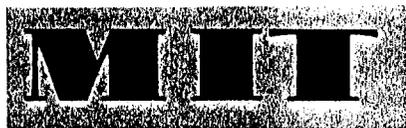
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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION V
MIDCOURSE NAVIGATION
AND GUIDANCE

ABSTRACT

This section presents an introduction to the midcourse
navigation and guidance scheme, explaining some changes in in-
tent and execution which have occurred as the scheme has developed.

Transcript of a Talk
by Dr. Richard H. Battin, Jr.
February 1963

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I   Midcourse Navigation and Guidance

Mypurpose is to set the stage for those of you who are un-
familiar with the midcourse navigation and guidance scheme.   I
think it would be appropriate to show a few slides which are pretty
well obsolete, and do not describe our current tininking; but the
ideas presented are necessary for an understanding of the method.
The midcourse,scheme originally was based on the idea of lineari-
zation around a nominal path, (see Fig. 5-1.) One can think of a
launch time and an arrival time, both fixed in inertial space with
a nominal path connecting them, and the position vector and corres-
ponding velocity vector at a particular time on the orbit being re-
presented as shown with subscript 0.   Because the vehicle was
improperly injected into orbit, it is not on the nominal path at this
particular instant of time.   The quantities that we are interested
in determining by suitable navigational measurements are the
differences between the actual position vector and the position vec-
tor at that time if we were on the nominal path.   If we know this
deviation, then by simply adding it to the nominal position vector
we would have the actual position vector.   A. similar statement
applies to the velocity vector.   For guidance it is necessary to
relate the position deviation to an appropriate velocity to take the
vehicle to the original target point in space and time.   This may
be accomplished by a matrix operation on the position deviation.
This term would be that velocity change over the nominal which
would be required to put the vehicle on a new course to the target
and the present deviation velocity experienced by being off course
to begin with.   The difference between these two is the velocity
correction.

To give an idea of the kind of information obtainable from
a single navigational measurement, consider the measurement of
the angle between the horizon of a planet and a star (see Fig. 5-2).

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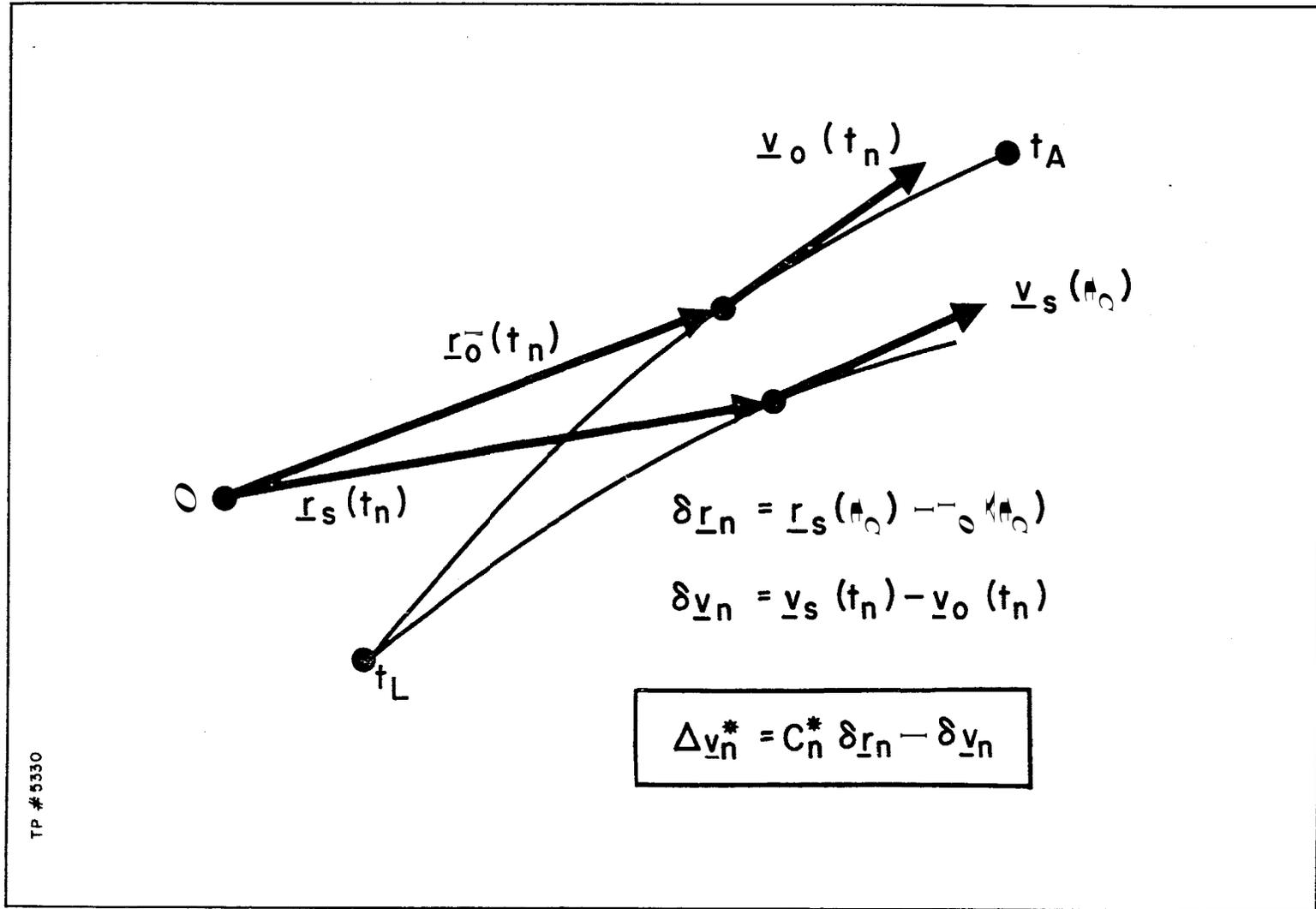
Fig. 5-1   Linearized guidance theory.

I. STAR ELEVATION

$$\delta \underline{A} = \underline{h} \cdot \delta \underline{r}$$

$$\underline{h} = \underline{u}/d$$

II. OCCULTATION

$$\delta \tau = -\frac{\underline{u}}{\underline{u} \cdot \underline{v}_r} \cdot \delta \underline{r}$$

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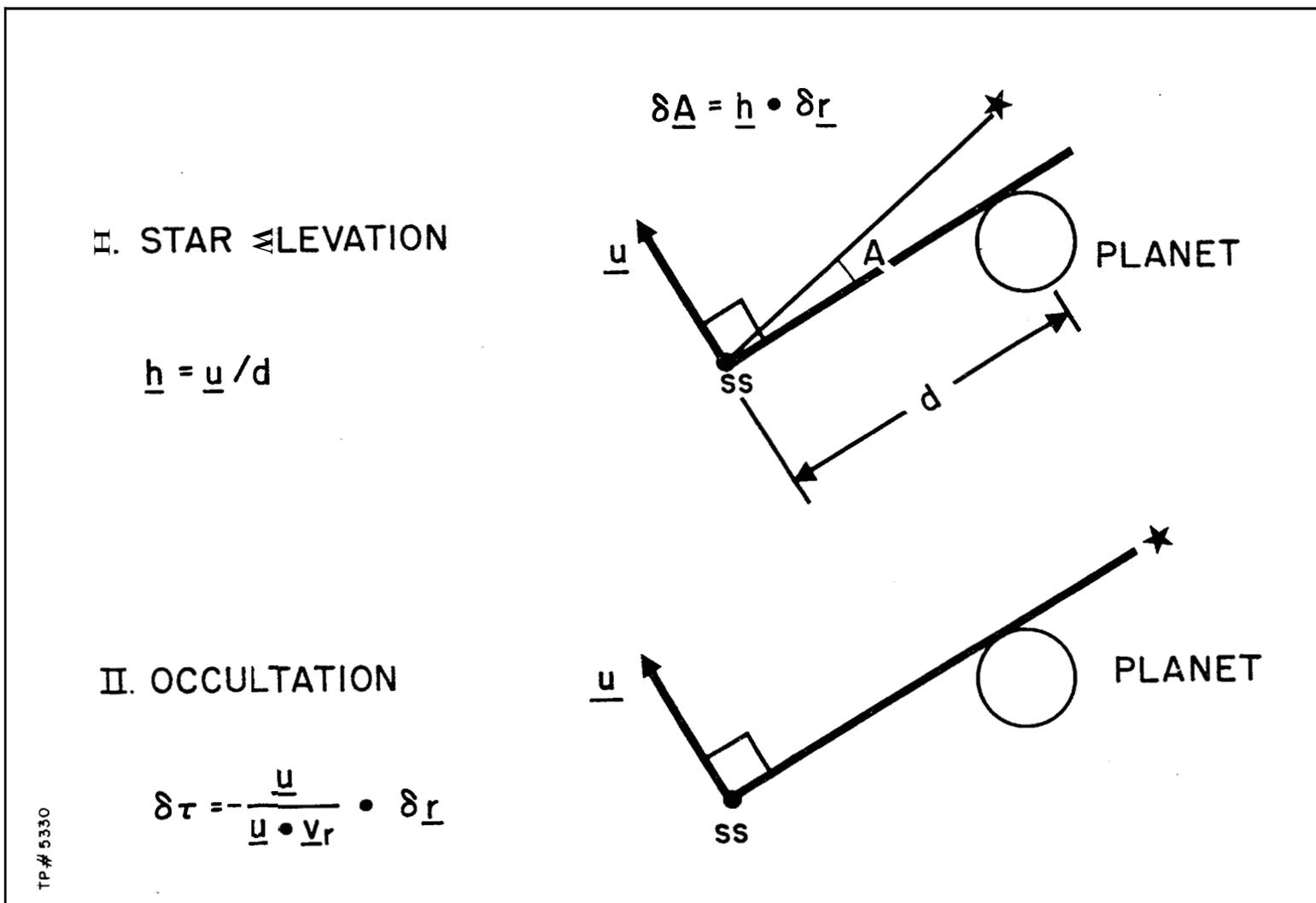
Fig. 5-2   Navigational measurements

$$\delta \underline{x}(t) = \left\| \begin{array}{c} \delta \underline{r}(t) \\ \delta \underline{v}(t) \end{array} \right\|$$

$$\delta \underline{x}_{n+1} = \Phi_{n+1,n} \, \delta \underline{x}_n$$

$$\delta A_n = \underline{h}_n \cdot \delta \underline{r}_n = \underline{h}_n^T \, K^T \, \delta \underline{x}_n$$

$$\delta A_n = \underline{h}_n^T \, K^T \, \Phi_{n+1,n}^{-1} \, \delta \underline{x}_{n+1}$$

$$\delta A_{n-1} = \underline{h}_{n-1}^T \, K^T \, \Phi_{n+1,n-1}^{-1} \, \delta \underline{x}_{n+1}$$

$$- -$$

$$\delta A_{n-5} = \underline{h}_{n-5}^T \, K^T \, \Phi_{n+1,n-5}^{-1} \, \delta \underline{x}_{n+1}$$

Fig. 5-3 Deterministic: method.

We are measuring the angle A. (The information actually obtained is only a component of the spacecraft position.). If we measure this angle and compare it with the angle that we would measure if we were on the reference path, we obtain a deviation in this angle.. This deviation is linearily related to the deviation in position. In fact, this angle is directly a component of the position deviation along a direction which is perpendicular to the line of sight to the edge of the planet, and is scaled by the reciprocal of the distance from the spacecraft to the planet edge. The h vector will 'have this particular significance.

Fig. 5-3 shows how observations made at widely different instants of time, made a single observation at a time, are processed to get a navigational fix in six dimensions. We think of a deviation vector which has six components. The first three represent position deviation and the last three represent velocity deviation. Now, because of the way in which the trajectory has been linearized, the propagation of this deviation vector from one period of time to another takes place in a linear fashion. The array of numbers controlling the propagation we call. a state transition matrix. These numbers represent the state variables of the system, and this is a six by six matrix which describes completely the propagation of the state variables from one time to another. This matrix depends only upon the reference trajectory and if we were using a reference trajectory concept could be completely predetermined. *So* by measuring a single angle, position information along one component is obtained and gives component information. about the first half of' the deviation vector. Now if you wrote down. such a relationship involving the transition matrix for six different instants of time, we could, via the transition matrix, extrapolate the measured data to a single instant. Then we would be :Faced with solving the set of six simultaneous linear algebraic equations for the com- ponents of the deviation vector. The matrix of coefficients of this

$$\delta \hat{\underline{x}}_n = \delta \hat{\underline{x}}'_n + \underline{w}_n (\delta \tilde{A}_n - \delta \hat{A}'_n)$$

$$\underline{w}_n = \frac{E'_n K \underline{h}_n}{\underline{h}_n^T K^T E'_n K \underline{h}_n + a_n^2}$$

$$E_n = E'_n - \underline{w}_n (\underline{w}'_n K \underline{h}_n)^T$$

$$\delta \hat{A}'_n = \underline{h}_n^T K^T \delta \hat{\underline{x}}'_n$$

Fig. 5-4 Navigation procedure.

system is a six-by-six coefficient matrix which would then have
to be inverted. We would like to avoid this inversion if possible.
It is not convenient to invert a large-order matrix on board the
spacecraft. It especially would be bad if some of these measure-
ments were not strongly independent. If any of the rows of this
matrix were proportional, or nearly *so,* the inversion would be
hazardous. The formulation of this navigation procedure as a
recursion law is familiar to many of you and is shown in Fig. 5-4.
Concentrate for the moment on the first equation. This is the
basic navigational equation. It tells us that the best linear esti-
mate at time T_n is obtained in two parts. One part is an extra-
polated value of the previous estimate via the transition matrix.
To this extrapolated estimate we add, in a linear fashion, the
weighted difference between what we actually measure and what
we would predict that we would measure if we really were where
we thought we were. In other words, at the latest time I had a
good estimate, I could predict what this deviation in angle would
be when I made the next measurement. I know the deviation isn't
going to be 0 because I am not on the reference path. I know I
will measure a different angle and I can predict what this difference
will be. When I compare this difference, the predicted difference
in angle deviation with what I measure it is significant if the
difference is not zero. It is then necessary to weight this difference
in some optimum. linear fashion, and add it to our previous best
estimate in order to get a better estimate. The computation of
this weighting vector is more involved than I want to discuss now.
The point is that the computation is recursive. The computation
of the weighting vector also involves a recursive evaluation of the
correlation matrix of the measurement errors. This matrix is
a six by six matrix which propagates according to the recursion
law. The weighting vector involves this correlation matrix. It
involves the h vector for the measurement, and it also involves
the variance of the error in the optical instrument. When this

- FUNCTION: ORBITAL NAVIGATION
- OBJECTIVE: TO DETERMINE POSITION AND VELOCITY OF VEHICLE USING OBSERVATIONAL DATA AND CELESTIAL DYNAMIC RELATIONS.
- PHYSICAL RESTRAINTS:
  1. LIMITED KNOWLEDGE OF PHYSICAL DATA.
  2. LIMITED ACCURACY OF NUMERICAL METHODS.
  3. LIMITED KNOWLEDGE OF INSTRUMENTATION ERRORS.
  4. ON-BOARD COMPUTATION LIMITS COMPLEXITY OF DATA PROCESSING TECHNIQUE.
- GROUND RULES:
  1. INCORPORATE MEASUREMENT DATA AS OBTAINED SEQUENTIALLY.
  2. AVOID DEPENDENCE ON REFERENCE ORBIT.
  3. UTILIZE OPTIMUM LINEAR ESTIMATION TECHNIQUES.
  4. AVOID MATRIX INVERSION.
  5. TECHNIQUE SHOULD BE APPLICABLE TO ALL PHASES FOR WHICH ONLY FIELD FORCES ARE ACTING.
  6. ADAPTABLE TO INCLUDE MEASUREMENT DATA FROM ALL SOURCES.
  7. UTILIZE GENERALIZED FORMULAE TO KEEP AGC PROGRAMS COMPACT.
  8. PROVIDE RESTART CAPABILITY IN MIDCOURSE.
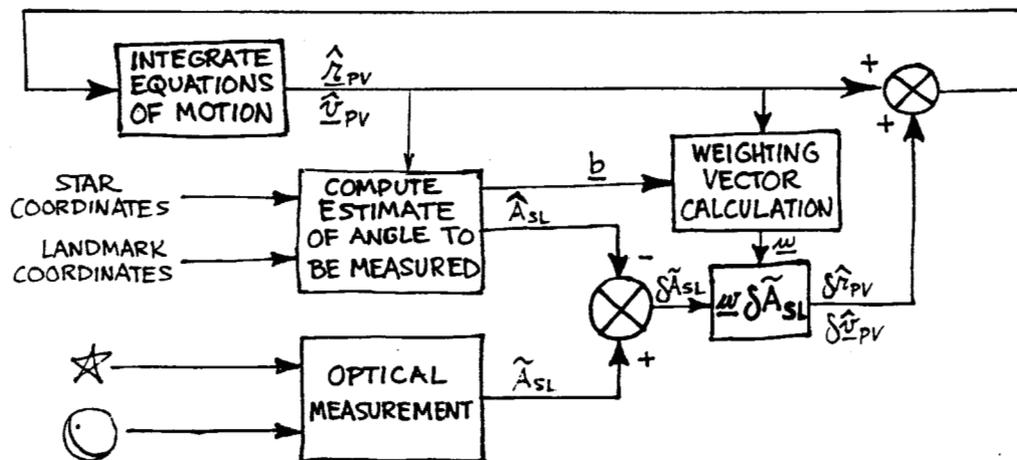
Fig. 5-5   Functional diagram

variance is known to be large, we give much less weight to this
observed difference then we would if the variance had been small.

Now, in the matrix that I was just referring to, this corre-
lation matrix of measurement errors, the 3 x 3 left hand corner
portion represents auto-correlation of the errors in position.
The lower right hand partition is the auto-correlation matrix of
the errors in velocity. The diagonal partitions are the cross-
correlations between position and velocity. It would be the intent
to have this matrix in the computer. It is needed to calculate the
optimum weight to be assigned to the incorporation of each new
piece of measurement data. *So* when we display numerical results
later, they will be the diagonal terms of this correlation matrix,
the mean-squared position uncertainties and the mean-squared
velocity uncertainties. At any instant of the time, the spacecraft
computer has and indication of these quantities, which can be used
by the astronaut to determine the uncertainties in his basic in-
formation with respect to the current estimate of positidn and
velocity.

What we have done more recently is to recognize the fact
that we need not really use a reference path for this purpose. If
we instead use the current best estimate of the position and velocity
of the spacecraft and linearize about that, we are accomplishing
the same thing. Perhaps we are even doing a better job, because
the deviation that we would experience from our estimated path
should be quite a bit smaller than a deviation from some arbitrary
reference path.

Fig. 5-5 gives a very crude indication of what the system
looks like now. First of all, we shall make a few remarks having
to do with certain physical constraints on the problem. Then we
shall mention certain ground rules which are constraints that we
imposed on ourselves to make the system as simple and flexible
as possible. The physical constraints are a result of limited

knowledge of the physical data. Actually, this limitation is really far less important than you might think from a casual inspection of the problem. The instrumentation errors will far overshadow any loss of information which results from not knowing the mass of the moon or the distance between the earth and moon. No matter what we do in the computer we can do it only approximately. We have only a finite number of digits to work with and we have series expansions which have to be truncated after a few terms. We have round-off problems because of the propagation of errors. Furthermore, the fact that we have to do these computations in a small computer, like the Apollo Guidance Computer, means that we can't even think about the problem on the same terms as we would if we were programming it for the MH 800.

Now let's consider the ground rules we have imposed on ourselves. First of all, we want to have the measurement data incorporated sequentially. I didn't make this point earlier, but on this recursion formulation of the navigation problem, there was never a time when we had to invert a matrix, *so* the calculations are simple and free of that kind of hazard. Secondly, we avoid dependence on a reference orbit by linearizing around the present best estimate in the incorporation of these measurement data. Thirdly, we want to use optimum linear estimation techniques. As a matter of fact, we are beginning to examine the possibility of relaxing this ground rule. These optimums all seem to be very, very flat and there might be a definite advantage in not doing what is mathematically optimum but in doing something a little bit simpler without really degrading the information. The technique should be applicable to all phases for which only field forces are acting. In other words, this scheme should be the basic navigation scheme for all phases of the mission in which we are not applying thrust. Thus this scheme is applicable also to the earth.-orbital phase and lunar-orbital phase. As a matter of fact, we could

also use this scheme for navigating the LEM to the surface of the moon, if we could observe the mothercraft with the radar and process radar measurements in the same overall scheme.

This leads to the next ground rule: this scheme should be capable of using all measurement data from whatever sources; radar information, optical information, star occulations, etc., a variety of data from a variety of sensors and this, without changing the overall processing scheme. Also a point which is not really easy to make in a short time the fact is that we would like to use generalized formulas, in order to keep these AGC programs compact. We would like in particular to provide a restart capability in midcourse, in case all information within the erasable part of the computer is lost. We would like to be able to have the astronaut insert as little information as possible, and manually to restart the problem.

The overall navigation scheme is as follows. We replace the reference trajectory concept with a direct integration of the equations of motion. That is, at the time of translunar injection, as soon as the engines are cut off, we have within the computer an indication of position and velocity, which we have obtained by processing the platform-accelerometer data. So initially we have an estimate of the vehicle's position with respect to P. We have position and velocity of the vehicle with respect to the earth immediately following the cut-off of the engines. We can then extrapolate the position and velocity forward by solving the equations of motion. When we desire to make a measurement, we can use this position and velocity information together with the star co-ordinates and the landmark coordinates (to measure the angle between a star and a landmark, for example) and determine or estimate the angle that we are about to measure. If our current estimate of position and velocity were exactly correct and we had no instrumentation errors, we would indeed exactly measure this

angle between the landmark and the star. At the same time we make our physical measurement, we perform the measurement of the angle between star and the landmark and obtain a measured value. The difference between the predicted angle and the measured angle is the information that we use to update our present estimate.

We have to convert this single scalar quantity into six components. That is, if we have a vector which is dependent only on the geometry of the measurement and if we have the correlation matrix of the measurement errors which we are keeping track of, we can indeed produce a six-component vector which when multiplied by this angle deviation will produce the instanteous change that should be made in our current indication of position and velocity. This deviation in angle will be small, and the step changes that are required in the position and velocity vector will be small.

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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION VI
RE-ENTRY GUIDANCE

by

Daniel J. Lickly

March. 1963

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CAMBRIDGE 39, MASSACHUSETTS

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ASTRONAUTS' GUIDANCE AND NAVIGATION COURSE NOTES: SECTION VI RE-ENTRY GUIDANCE

ABSTRACT

This section describes and briefly analyzes the problems surrounding the re-entry phase of a space mission.

by   Daniel J. Lickly

March 3.963

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OBJECTIVE: Enter the earth's atmosphere & using only the vehicle's drag and lift steer to a predetermined landing site.

Physical Constraints & Ground rules:

1) No thrust — Energy must all be dissipated by Vehicle drag.

2) Narrow corridor — to prevent excessive G loading & yet to insure capture by atmosphere allows little lee-way in entry conditions

3) Long range — Full time of month coverage coupled with low inclination angle restrictions require extended ranges.

4) Limited controllability — Low lift to drag ratio permits little tolerances in guidance.

5) Roll control Only — Lack of angle of attack modulation leaves only rolling maneuver to steer the vehicle

6) No Up-telemetry — On-board system must be self-contained

Range Prediction Equations

A_{P_1} = Equilibrium glide range angle
$$= \frac{1}{2}\left(\frac{L}{D}\right)_{ref} Ln\left(\frac{1}{1-\bar{V}^2}\right)$$

A_{P_2} = constant altitude range angle
$$= V^2 Ln(V/V_{II})/RD$$

A_{P_3} = range angle correction for transition from constant altitude to equilibrium glide
$$= 2H_s/\{(L/D)_{ref}(1-\bar{V}^2 R)\}$$

A_{P_4} = range angle correction for transition from rising flight to constant altitude
$$= (VR)V/\{R(L/D)_{max} D-(\frac{V^2}{R}-g)\}$$

A_{P_5} = flight-path angle range correction (in final phase)
$$= (VR_3-VR) V/L_{eq} R$$

A_{P_6} = potential energy range correction (in final phase)
$$= (L/D)_{ref}(H-H_f)$$

A_{PUP_1} = range angle at constant L/D
$$= 2V_2 H_s Ln\left(\frac{D}{D_{min}}\right)/(VR+VR_2)R$$

A_{PUP_2} = Kepler range angle
$$= cos^{-1}\left[(1-\bar{V}^2 cos^2\left(\frac{VR_2}{V_2}\right))\right]/(1-\bar{V}^2(\bar{V}^2-2)cos\frac{V}{V_2})$$

A_{PUP_3} = Equilibrium glide range angle
$$= \frac{1}{2}\left(\frac{L}{D}\right)_{ref} Ln\left(\frac{1}{1-\bar{V}^2}\right)$$

A_{PUP_4} = Range angle correction for transition to equilibrium glide
$$= (VR_2 - 2H_s G/V_2 (L/D)_{ref})V_2/G R(\bar{V}-1)$$

Fig. 6-1   Re-entry phase.

I   Nature of the Problem

As with almost all other phases of space travel, the problems involved in re-entering the earth's atmosphere and guiding to some landing site could be reduced to trivial ones, if only an unlimited supply of fuel were available. As a spacecraft approached the earth, thrust could be applied to reduce the vehicle's velocity. With sufficient fuel for this braking maneuver, the spacecraft could be slowed down enough to permit a safe low-speed pass through the atmosphere and power to steer to any point on the earth. However, the fuel required (on the order of 30 to 40 thousand ft/sec of velocity change) for such an operation is prohibitive for any currently proposed space venture. Fuel can be allotted only for phases in which there exists no alternative.

The re-entry phase offers an alternative to thrusting (see Fig, 6-1). The problem of terminating a space flight is to reduce the kinetic energy of the vehicle to a low enough level to land. The friction of the vehicle in the atmosphere (its drag) offers a means to dissipate the energy of the body. However, the trick in this braking type of entry technique is to dissipate the energy at just the right rate, with a control system (see Fig. 6-2).

To understand this requires some knowledge of the characteristics of our atmosphere. The density of the atmosphere as a function of altitude can be approximated as a rough guide, by an exponential curve:

$$\rho = \rho_0 e^{-h/H_s}$$

H_s is the altitude constant of air atmosphere and is called the scale height. It gives the altitude change necessary to change the density, by a factor of e. Unfortunately, the scale height of 'our atmosphere is only about 20, 000 feet at the altitudes of interest in this application; this means that for a 4 mile change in altitude,

| OPTICAL SUB-SYSTEM | INERTIAL MEASUREMENT UNIT |

ANGLE SENSORS

GYRO UNIT

SPECIFIC FORCE SENSOR

VEHICLE COORDINATE

INERTIAL REFERENCE COORDINATE

NAVIGATION BASE

SPECIFIC FORCE COMPONENT SIGNALS

VEHICLE ORIENTATION SIGNALS WITH RESPECT TO INERTIAL SPACE

VEHICLE ORIENTATION AND MOTION

COMPUTER AND CLOCK

STEERING COMMAND SIGNALS

VEHICLE SYSTEM

FLIGHT PATH DATA SIGNALS

COMPUTER REFERENCE INFORMATION

COUPLING DISPLAY UNIT

INDICATION OF FLIGHT PATH DATA

DISPLAY

CONTROL

ROLL

VEHICLE ORIENTATION AND MOTION

CONTROL SIGNAL

STEERING ERROR MONITOR

TO COMMAND MODULE JETS

INITIAL CONDITIONS AND TRAJECTORY PARAMETERS

DISPLAY AND CONTROL

Fig. 6-2   Phase 14 - earth re-entry.

the density (and the drag, since they are proportional) will change
by a factor of almost 3. This extreme variation as a function of
altitude presents a narrow "target" to hit on the return lunar trip.
The problem is to enter the atmosphere in such a manner that a
big enough "bite" is taken to get out of the nearly parabolic orbit,
and still not *so* big a bite that excessive g-loading or heating rates
will be encountered. These conflicting objectives define the limits
of the so-called re-entry corridor (see Fig. 6-3). The over-shoot
limit is defined as'the most shallow trajectory that will still en-
counter enough atmosphere to lose sufficient velocity to remain
in the atmosphere and not skip back out. The under-shoot limit
is defined as the steepest one before some arbitary limit of ac-
celeration, such as 10 g's, or of temperature, is exceeded. The
range between these limits is only about 2 degrees of flight path
angle at 400, 000 feet of altitude for the Apollo re-entry vehicle.

This corridor requirement places one of the most stringent
demands on the mid-course guidance system. To stay within this
corridor for the worst cases requires that the normal re-entry
initial conditions agree with their nominal values to within approxi-
mately 2 miles of altitude. (This is the so-called 1 sigma value.)
This is a severe test of mid-course guidance techniques and
instrumentation.
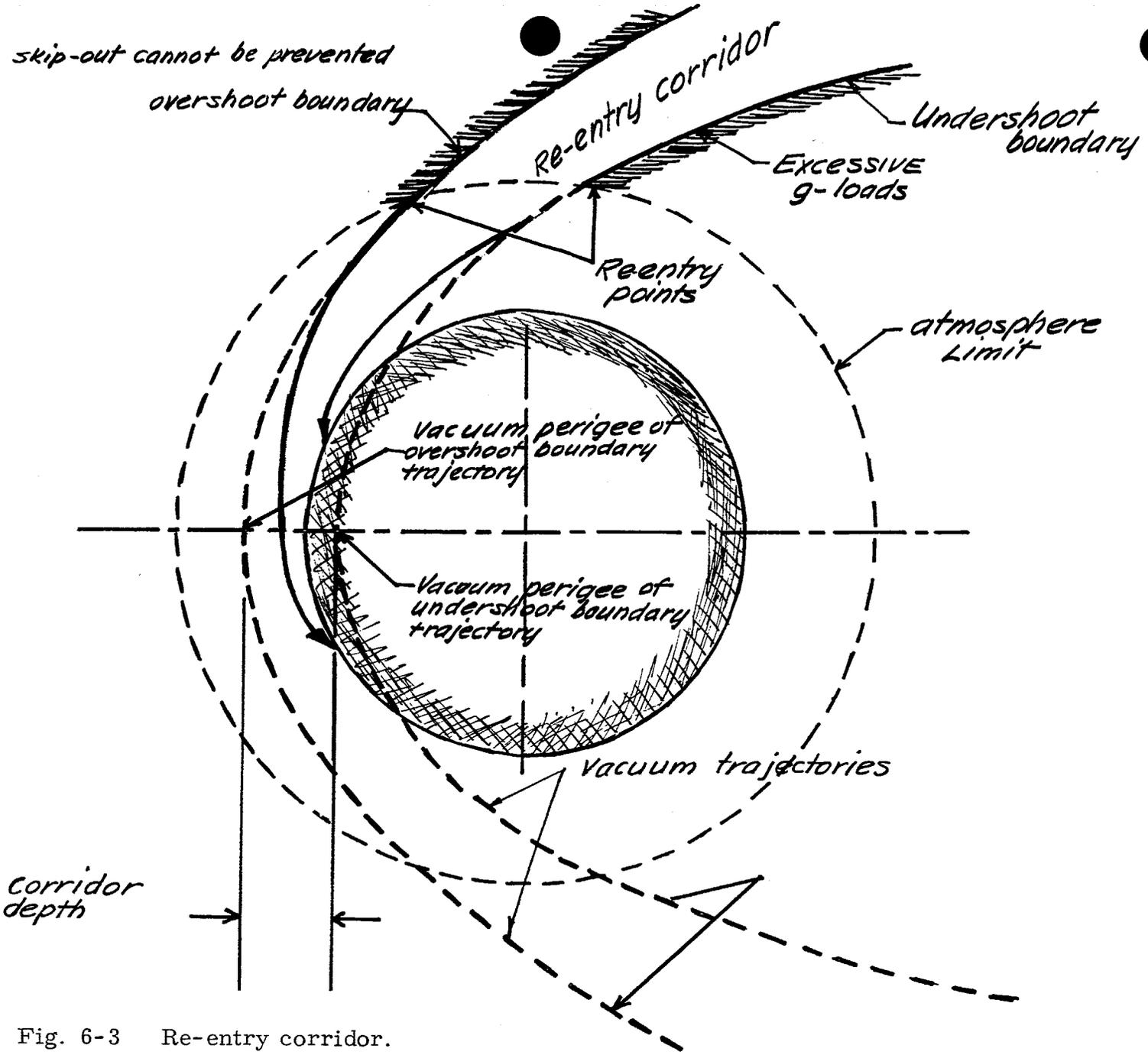
Fig. 6-3   Re-entry corridor.

II   Equations of Motion

If we make a number of simplifying assumptions, we can reduce the general equation of motion to an abbreviated form that approximates the action of the re-entry vehicle in the vertical direction. The equation is rough, of course, but it does give us an insight into the problem of controlled re-entry trajectories. The basic equation is:

$$\frac{Lift}{m} + (\frac{V^2}{R} - G) = \frac{dV_R}{dt}$$

where   Lift = vehicle lift force, positive upward

           m   = vehicle mass

           V   = velocity of the vehicle relative to the earth

           R   = radius for center of earth

           G   = acceleration of gravity

           V_R = radial velocity, altitude rate, positive upward

In words, this equation says that the lift of the craft plus the difference in centrifugal force and gravity is equal to the acceleration of the vehicle in the radial direction. ($F = ma$, or $a = F/m$).

A concept that is easy to visualize from this equation is that of "equilibrium flight". If the sum of terms on the left-hand side of the equation is zero, then the vehicle is said to be in equilibrium. That means that the lift force just balances the difference in gravity and centrifugal force.

$$\frac{Lift}{m} = G - V^2/R$$

or given the vehicle Lift-to-drag ratio, L/D:

$$L/D \, (\frac{D}{m}) = G - V^2/R$$

                            D = Drag force

On the accompanying chart labeled "Equilibrium Glide Curves" (see Fig. **6-4** in Confidential Appendix B) there is a plot of points at which the vehicle would be equilibrium. It should be emphasized that they are not trajectories, but only a locus of points at which the equilibrium conditions exists. (Indeed, they are not flyable as such, since vertical equilibrium implies constant V_R or altitude rate, and the points on the curve do not represent constant altitude rate trajectories). The apparent singularity in the plot occurs at orbital velocity and is due to the fact that no lift is needed to maintain equilibrium, since by definition, centrifugal force equals gravitational force at orbital speed.

Below orbital speed, lift upwards is needed to maintain equilibrium since G is greater than V^2/R. Above satellite speed, the reverse is true; "down-lift" is needed to hold the spacecraft in the atmosphere.

This simple approximation also allows a quick look at the, static stability of the dynamics of the vehicle. The term <u>static stability</u> is intended to describe the characteristics of the vehicle when it is disturbed slightly from its present state of motion. A system is statically stable if the forces tend to return it to its original state of motion when it is released; a system is statically unstable if the forces act in such a way *so* as to move it away from its original state of motion and in the direction of the disturbance. This is a so-called divergent system.

Let us examine a vehicle below statellite speed. Assume that it was at an equilibrium point, and we disturb it by moving it up slightly in altitude. The effect is to reduce the lift since lift is proportional to density which reduces at increased altitude. Since we had lift upwards to maintain equilibrium, a reduction in lift.will cause the vehicle .to fall down towards its original altitude. This illustrates the statically stable situation that exists below

orbital speed. However, the reverse is true above orbital speed.
Let us assume again that we disturb a hypothetical vehicle by
raising it a small amount in altitude. Again the effect is to reduce
the lift of the vehicle. At these speeds, however, downward lift
was necessary to maintain equilibrium. A. reduction is downward
lift will result in an acceleration upwards. This will bring about
a further reduction in lift and an increase in upwards acceleration.
This effect is a system divergence which is called "skip-out".

Basically, we can see that the problem of re-entry is com-
plicated by the fundamental instability of the flight at velocities
in excess of satellite speed.

AP🌑LLO

GUIDANCE AND NAVIGATION

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ASTRONAUTS' GUIDANCE AND
NAVIGATION

--S:

SECTION VII
OPTICS

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ASTRONAUTS' GUIDANCE AND
NAVIGATION COURSE NOTES:
SECTION VII
OPTICS

ABSTRACT

This section contains the slides and notes used in a lecture on optics.

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Electromagnetic Spectrum

The emphasis here will. be on the visual spectrum, a region of roughly one octave band width, from 400 mμ to 700 mμ, where

$$1\mu = 10^{-6} \text{ meter} \approx 40 \times 10^{-6} \text{ inches}$$

Optical tolerances for flatness or sphericity are in the neighborhood of a quarter wave length of visible light (approximately 5×10^{-6} in.). Optical surfaces, ball bearings, and molecular films achieve this precision. For a laser, the visual operating frequencies are thus about 5×10^{14} cps or 500 mega-mega cycles.
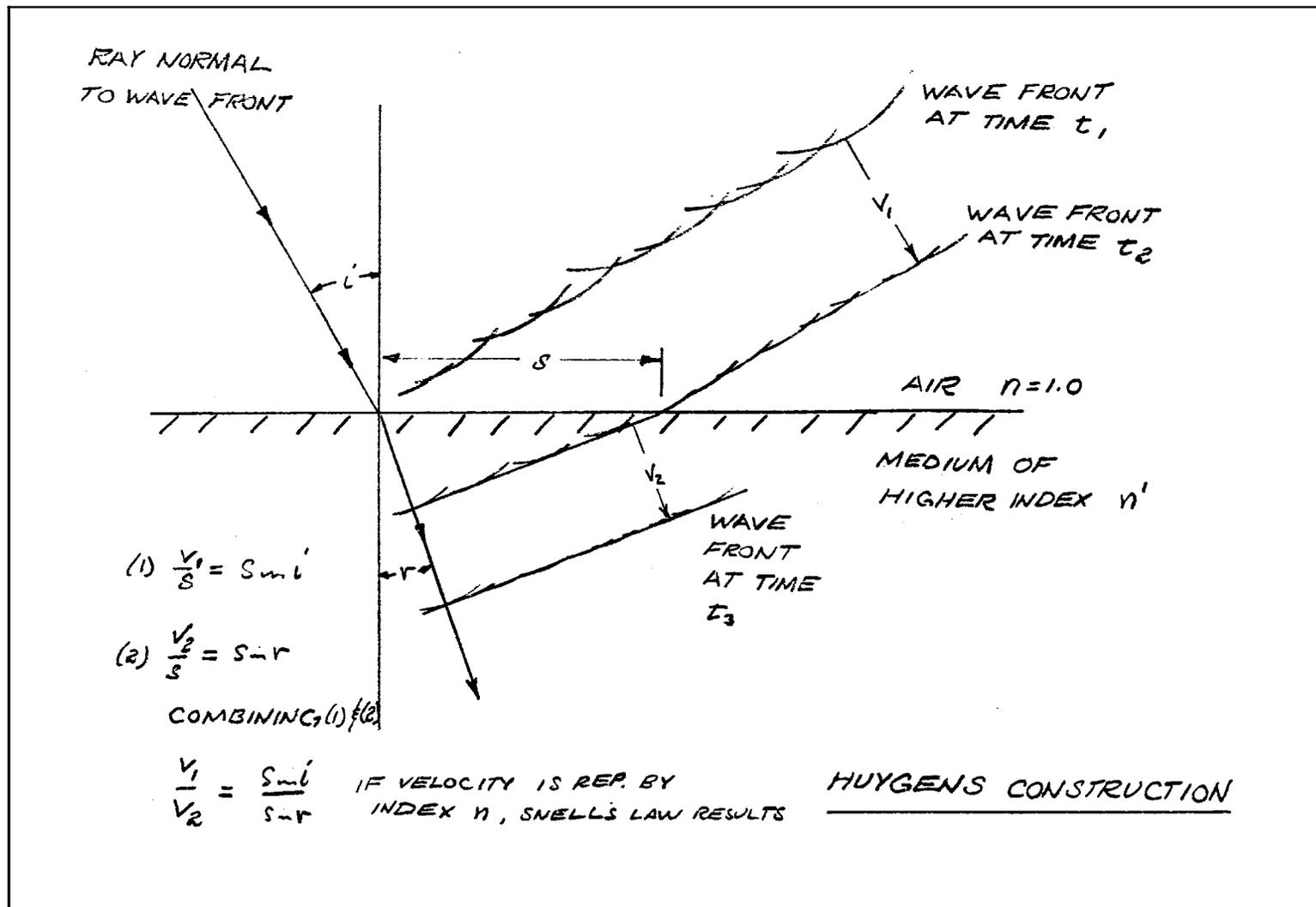
Fig. 7-1 Huygens construction.

Huygens Construction

In the Huygens Construction, radiation consists of wave-fronts in space which are spherical when observed from close aboard and plane when observed from infinity. Snell's Law can be deduced from Huygens Construction, and this is the basis for geometrical optics.

AIR OF OPTICAL INDEX $n = 1.0$

SNELL'S LAW: $\dfrac{\text{SINE } \varphi}{\text{SINE } \varphi'} = \dfrac{n'}{n}$

MATERIAL OF OPTICAL INDEX $n' = 1.5$

FOR SMALL φ THIS DISPLACE= MENT CAN BE APPROXIMATED BY $D = \varphi t\left(1 - \dfrac{1}{n'}\right)$

REFRACTION THRU GLASS
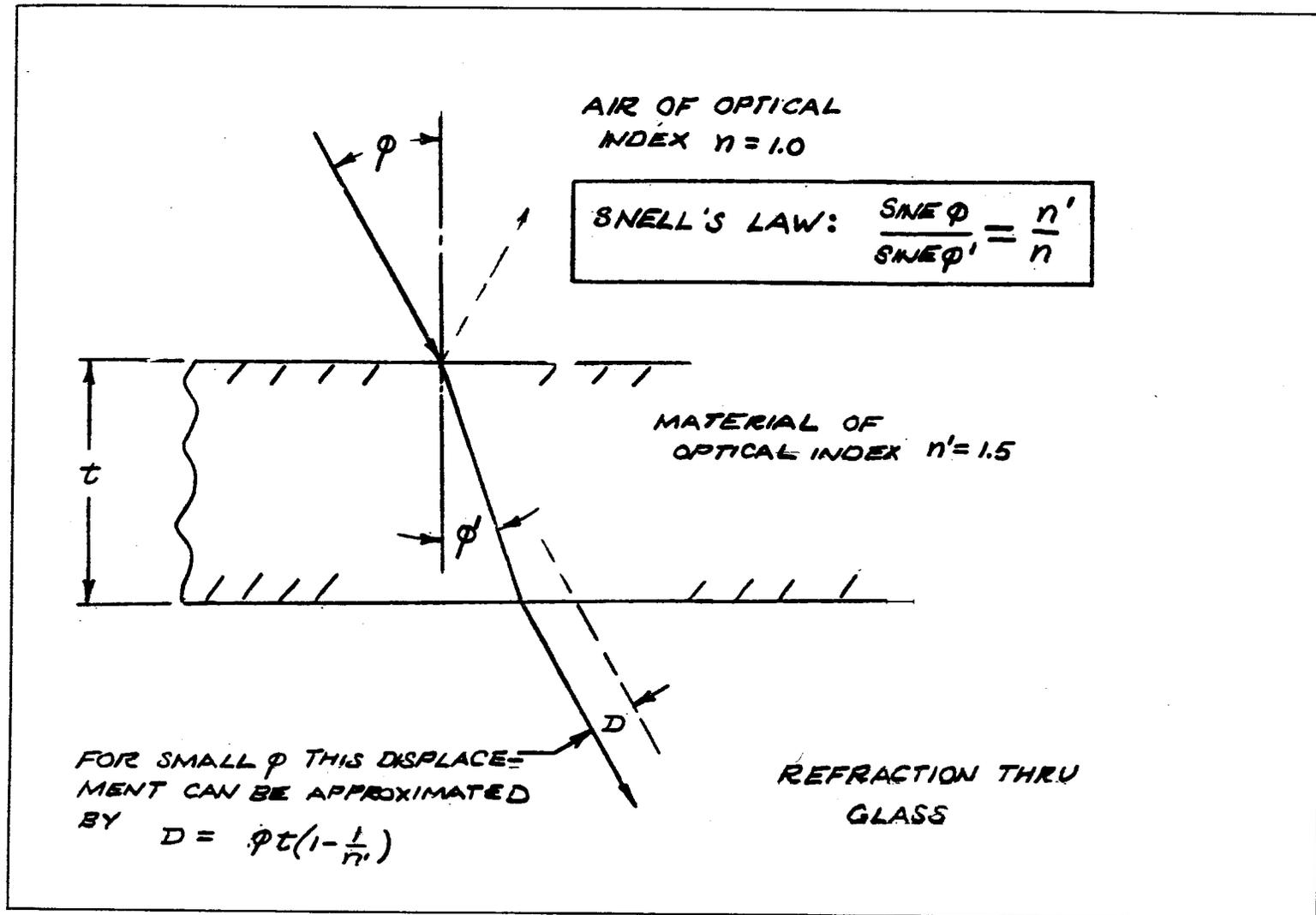
Fig. 7-2   Refraction through glass.

Refraction through Glass

The displacement of the light ray is due to a plate of thickness t. If the incident ray is considered as part of a converging set of rays, symmetrical about a normal to the plate, the converging point or focal point of this set would be displaced axially by a distance $t(1-\frac{1}{n'})$, when n' is the index of refraction of the plate material.
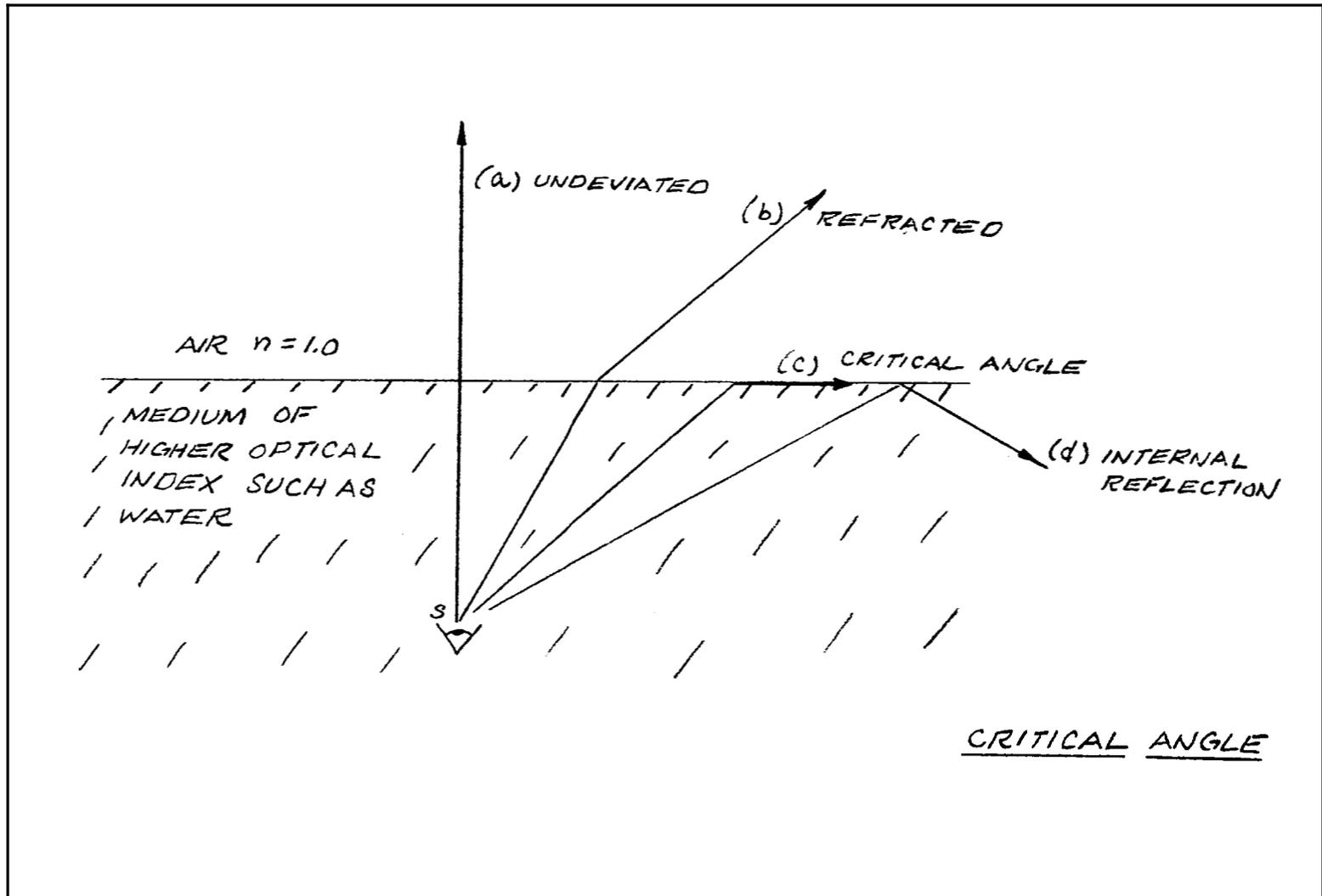
Fig. 7-3 Critical angle.

Critical Angle

Rays of light emanating from point s are refracted and c and d never emerge. The fish at point s or any point has a visual field of 180°, but a visual cone of 98°. This principle is used in the wide angle lens.

FROM SNELL'S LAW AND SMALL ANGLE APPROXIMATIONS

THE IMAGE — OBJECT RELATION

$$\frac{n}{S} + \frac{n'}{S'} = \frac{n'-1}{R}$$

NORMAL

OPTICAL INDEX n

OPTICAL INDEX n'

NORMAL

i

I

R

P θ V CTR. θ' P'

S
(OBJECT DISTANCE)

S'
(IMAGE DISTANCE)

R = RADIUS OF CURVATURE

REFRACTION AT
A SPHERICAL SURFACE
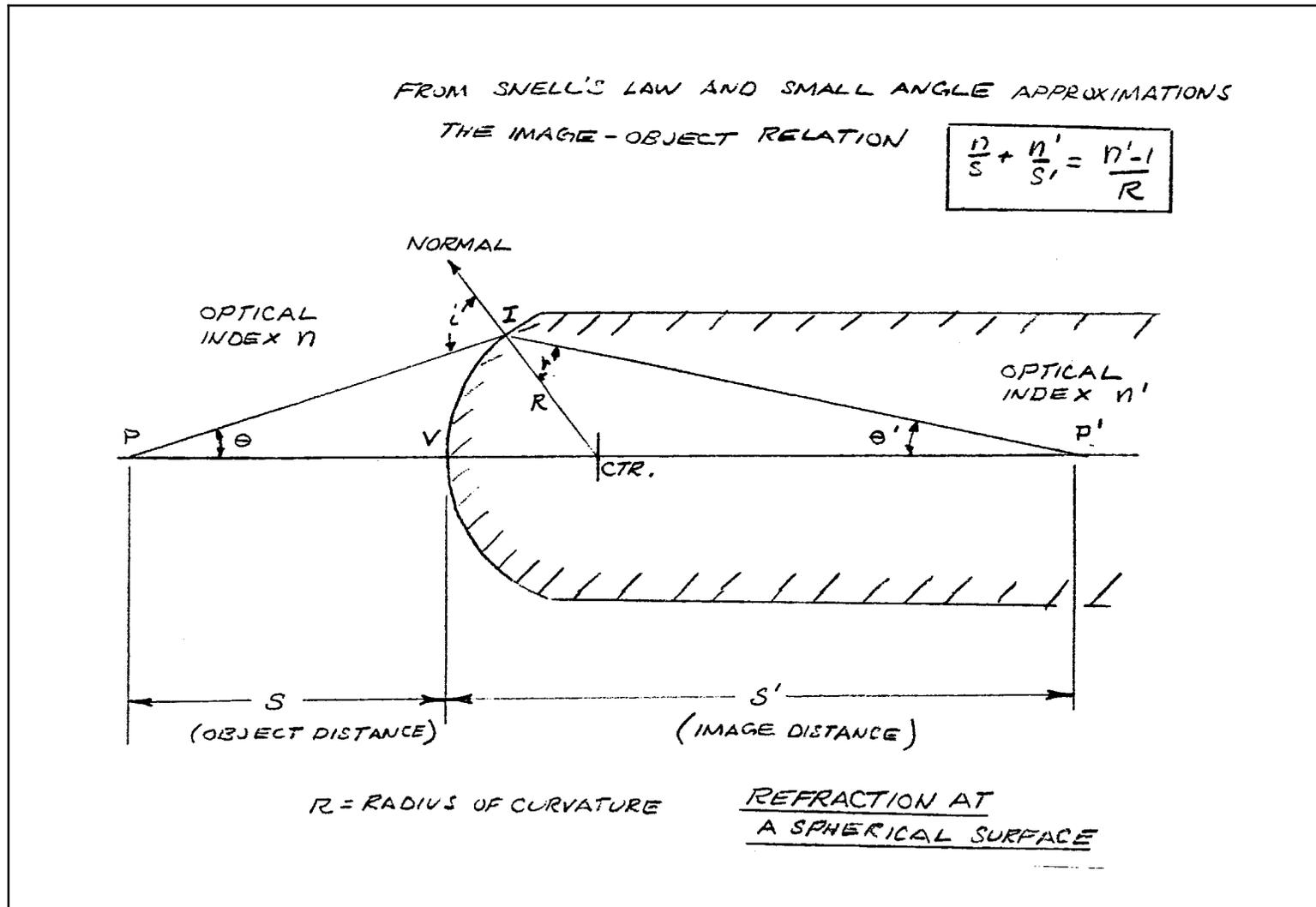
Fig. 7-4 Refraction at a spherical surface.

Refraction at a Spherical Surface

Assuming all angles are small enough to be represented by their sines, and making use of Snell's Law, the following image distance (s)—object distance (s') relation can be found (with a certain amount of work):

$$\frac{n}{s} + \frac{n'}{s'} = n' - n(\frac{1}{R})$$

where a material of index of refraction n is in the image space, of index n' is in the object space, and R is the radius of curvature of the surface. This relation can be extended from surface to surface, each image being the object for the next surface *so* that the following simple lensmaking equation can be deduced:

$$\frac{1}{s} + \frac{1}{s'} = (n - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

ACTUAL FIELD OF
VIEW IN OBJECT SPACE

EYE LENS

EXIT
PUPIL

FIELD LENS

EYE
RELIEF

OBJECTIVE LENS

APPARENT
FIELD IN
IMAGE SPACE

OPTICAL SCHEMATIC
OF TELESCOPE
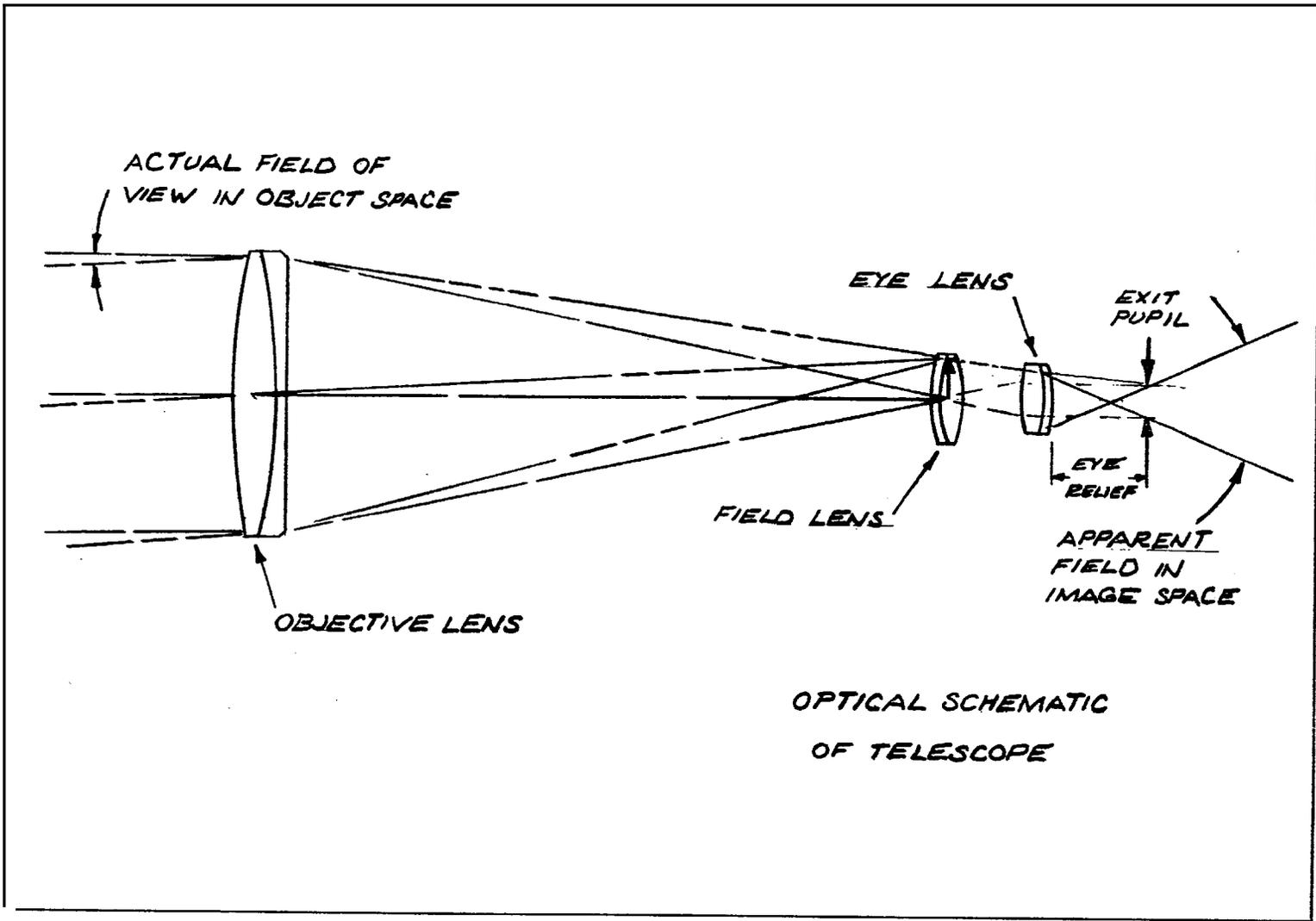
Fig. 7-5 Optical schematic of telescope.

Optical Schematic of the Telescope

A telescope is useful for:

      (1) resolution and magnification

      (2) light gathering ability.

It contains three optical components:

      (1) objective lens -- collects light and forms a real image of the field

      (2) field lens -- for light gathering efficiency only

      (3) eye lens -- magnifying glass to examine image of field formed by objective lens.

Useful relations:

      field angle $\times$ magnification = angle that image of field subtends in observer's eye

      field angle (in degrees)$\times$ magnification = 50 (as a rule of thumb)

      (for wide angle instruments, instead of 50, might be 70)

      entrance pupil - diameter D

      exit pupil - diameter d   These combine as

      magnification - M $D = Md$

Eye relief is the distance from eye lens to exit pupil and is usually comparable to the focal length of the eye piece. The exit pupil is a reduced image of an objective lens. Turning this around, since optical systems are reversible, when the observer uses the tele-. scope, his eye pupil is imaged by the telescope at the objective lens. At high light levels this pupillary image may be the effective limiting aperture of the observer - telescope system.

When the pupil diameter of the observer's eye is approximately equal to the exit pupil diameter of the telescope, the brightness of an extended object as viewed through the telescope is about the same as the brightness of the object as viewed by the unaided eye.

ONE POWER TELESCOPE USE

1. EFFECTIVELY PUTS EYE IN FRONT OF TELESCOPE

2. CAN DEFINE A LINE OF SIGHT

3. CAN MEASURE ANGLES

FIELD

ENTRANCE PUPIL

EXIT PUPIL
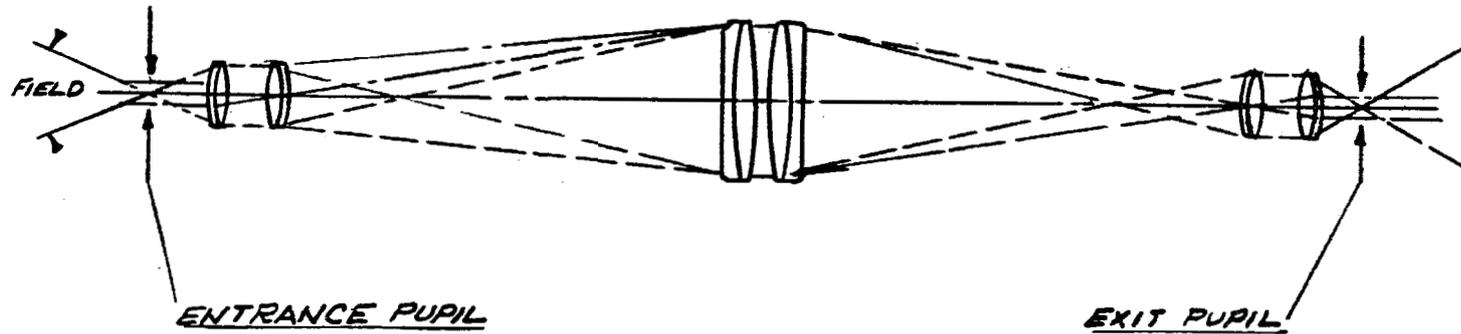
1X SYMMETRICAL TELESCOPE

Fig. 7-6   One power symmetrical telescope.

One-Power Symmetrical Telescope

Optically, this instrument consists of two similar telescopes in alignment. For this case the entrance pupil is as far forward of the telescope as the exit pupil is aft. They are the same size and the field of the one-power telescope is approximately 60°.
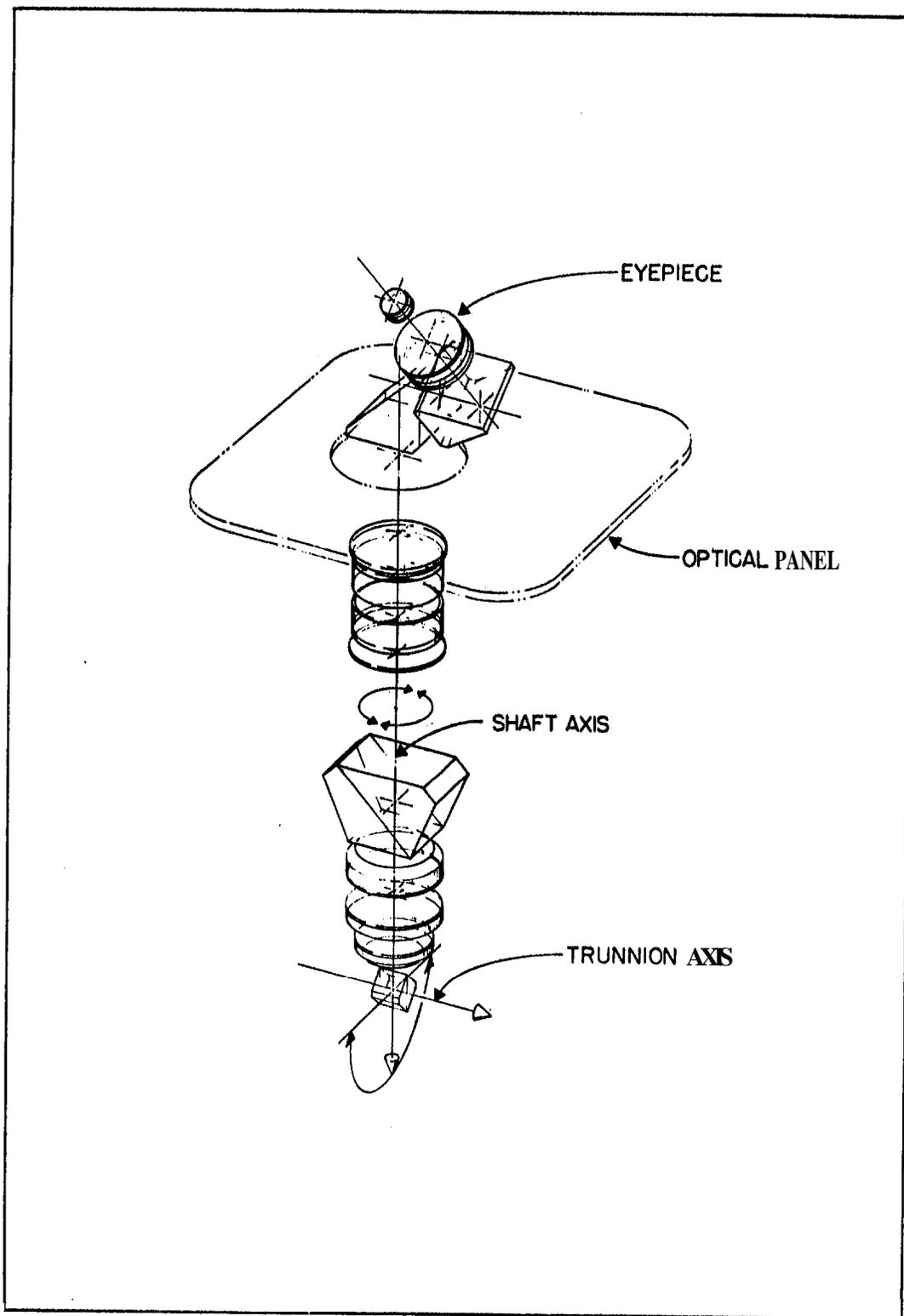
Fig. 7-7 Scanning telescope - optical. schematic.

Scanning Telescope - Optical Schematic

Light ray d in Fig. **7-3** is an example of internal reflection, the most efficient means of reflection. Prisms in binoculars and in the astronaut's scanning telescope make use of this, since for glass the critical angle is about 42°. Aluminum surfaces are about 91%-efficient reflectors.
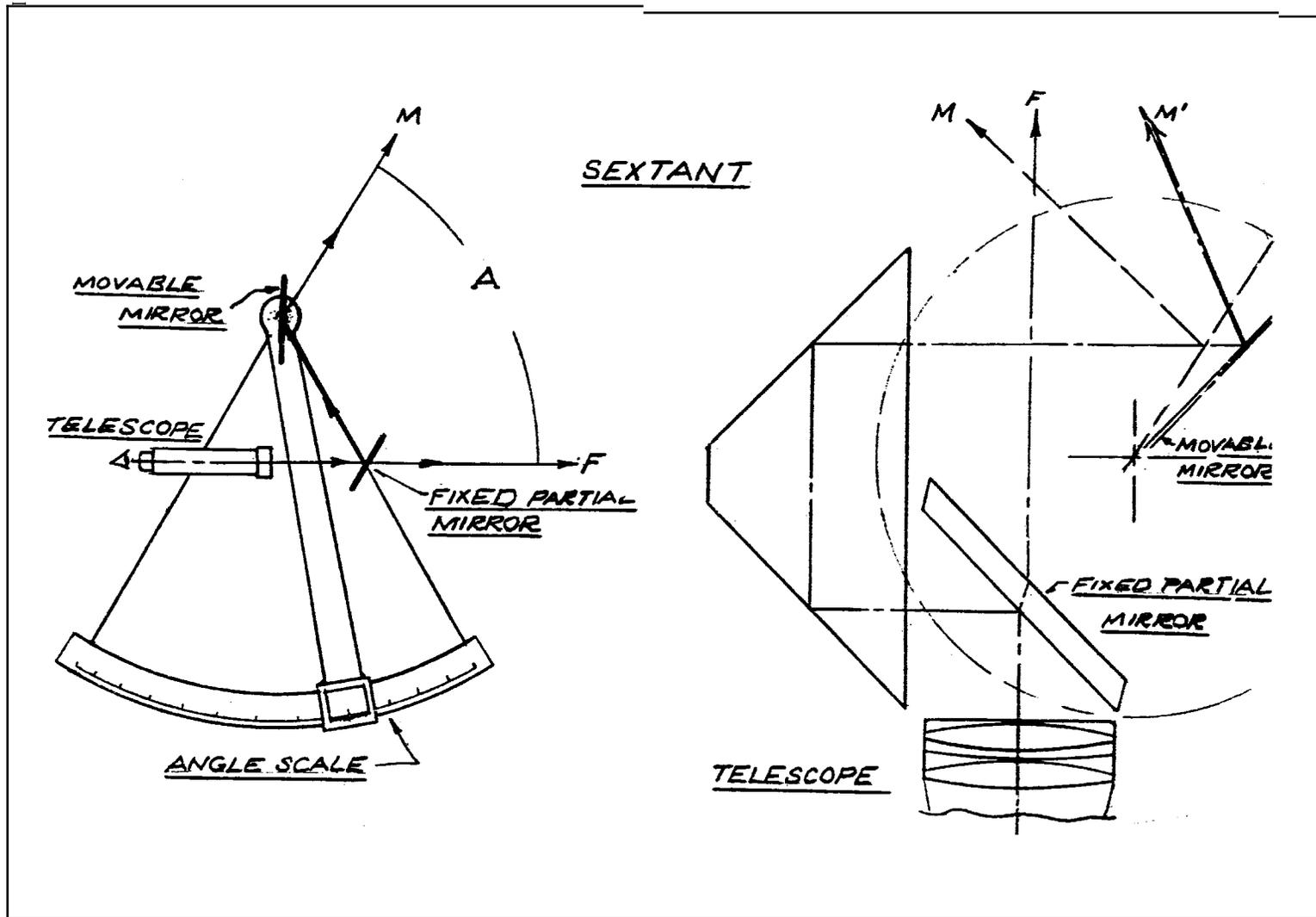
Fig. 7-8 Sextant (Apollo with nautical sextant).

Nautical Sextant and Apollo Sextant

       Chief differences between the nautical and Apollo sextants
are the 1.6 in. aperture, the **28** power magnification, and the more
accurate readout of the latter, These improvements were required
by a need for increased accuracy for navigation.

EYEPIECE

RETICLE

OBJECTIVE LENS

BEAM SPLITTER

TRUNNION MIRROR

STAR L.O.'S.
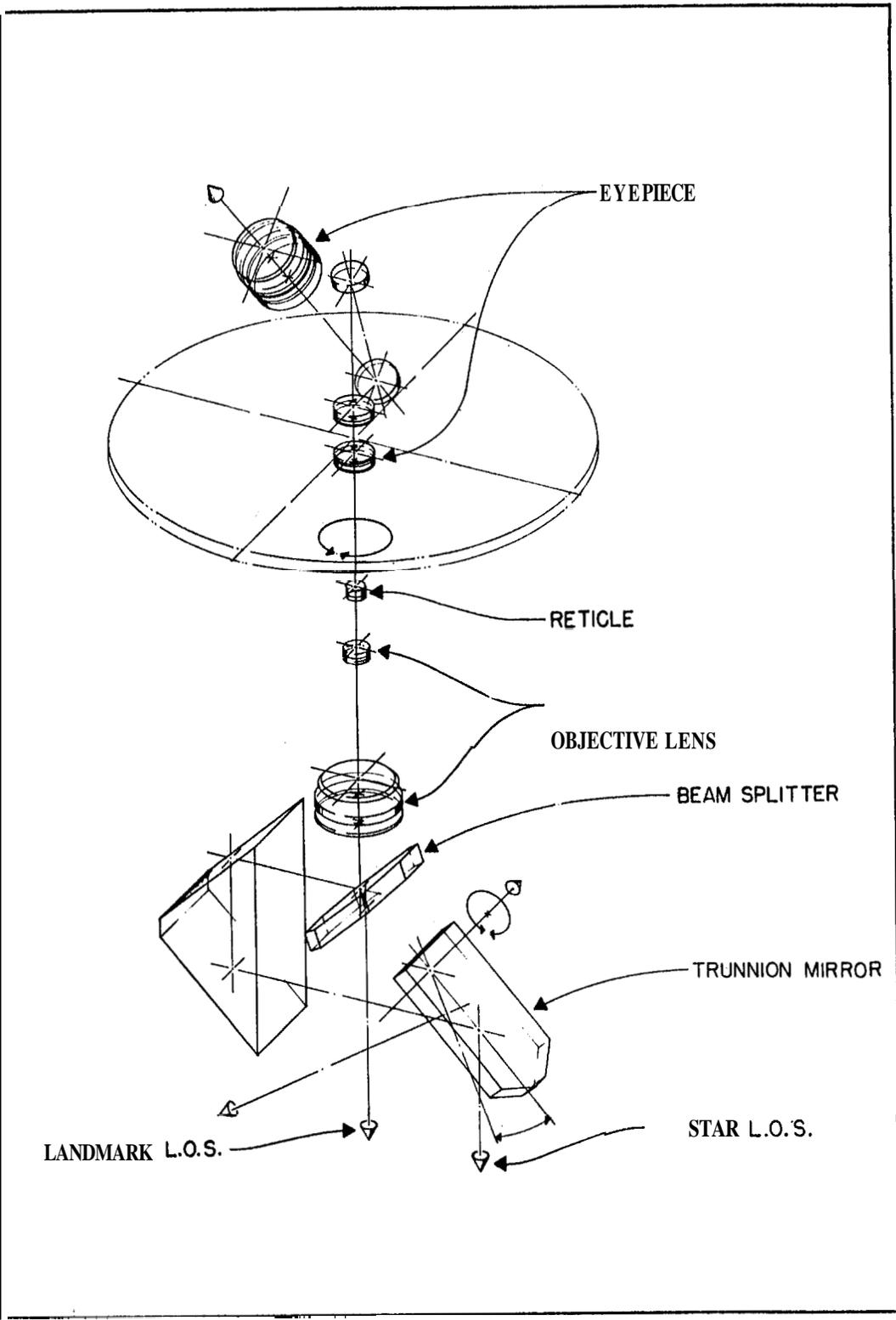
LANDMARK L.O.S.

Fig. 7-9 Sextant optical schematic

Sextant Optical Schematic

The Apollo sextant is used in the following manner: the angle between a star and a lunar or terrestrial landmark is measured by superimposing the image of that star upon the image of the landmark at the focal plane of the sextant.

Whether or not the observer can actually see the star depends upon:

(1) the magnitude of the star

(2) the brightness of the background

(3) the observer's visual adaptation level.
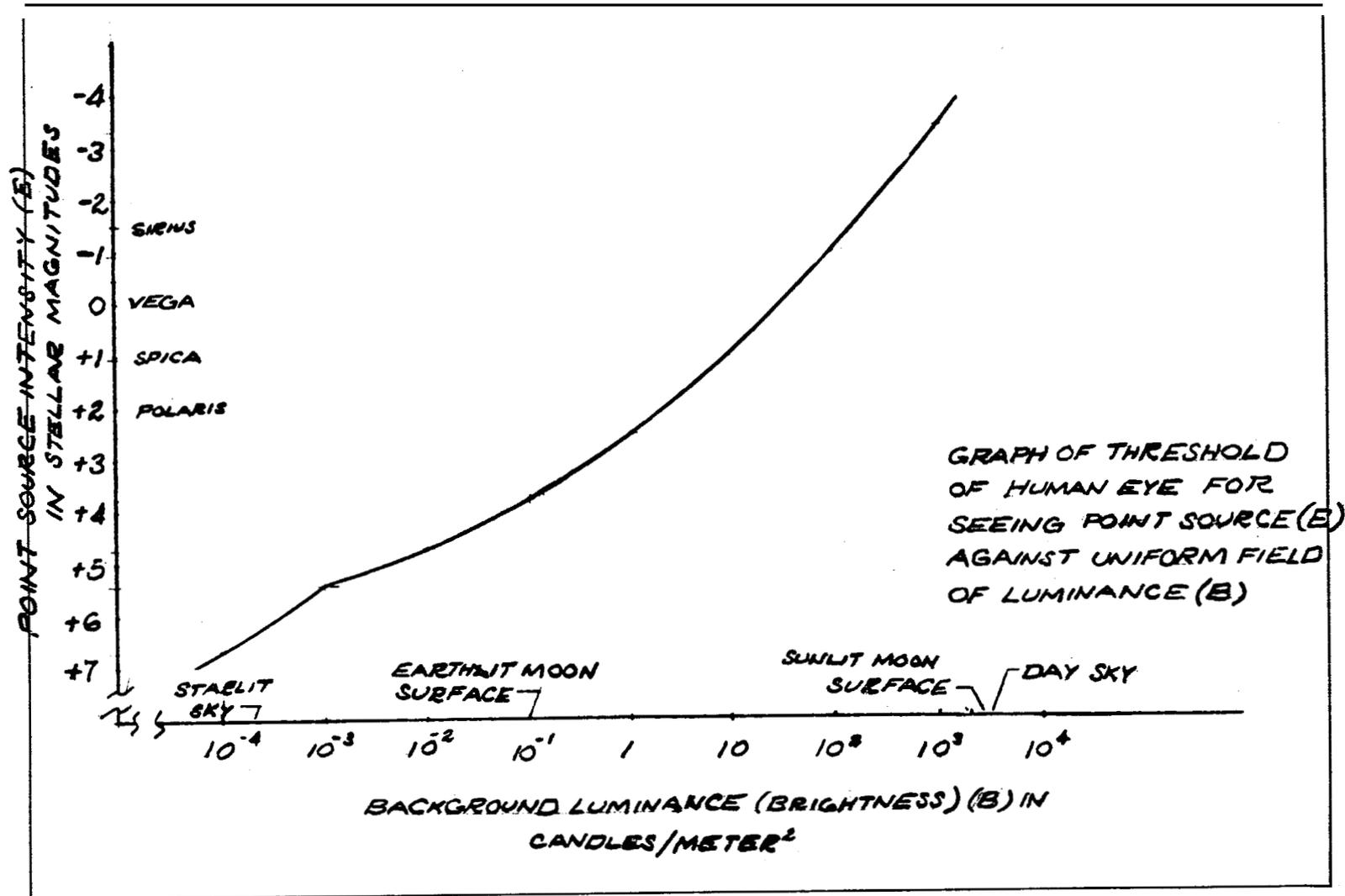
Fig. 7-10 Graph of threshold of human eye for seeing point source
(E) against uniform field of luminance (B).

Graph of Threshold of Eye

This graph, based on data obtained by A. C. Hardy, Professor Emeritus, MIT, during World War II, shows the threshold of an average eye for seeing a point source against a background of uniform luminance (brightness). All points falling above the curve should be visible; the further they are from the curve, the more easily they are seen. End points are easily confirmed and 5th and 6th magnitude stars are usually the dimmest-seen at night. Similarly, Venus (-4^m) can be seen with difficulty in the day sky,

The magnification of a telescope enables the observer to discern a star against a bright background (such as the sky or sunlit moon). This is by virtue of the unchanged brightness of the field in comparison with the increased intensity of the unresolvable star.

As additional explanation, it might be said that advantage is taken here of the smallness of the actual angle subtended by a navigational star (in the neighborhood of 0. 02 arc seconds) compared to the much larger resolution limit of the eye (close to one minute of arc and due in part to the size of the receptors at the retina).

The intensity along the sextant line of sight to the earth or moon is further attenuated by the partial mirror. The transmission ratio of the sextant lines of sight is arranged to leave the star line of sight as little affected. as is practical.

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