AMPLIFIER PERFORMANCE

© 2020-2022 by TONY R. KUPHALDT – UNDER THE TERMS AND CONDITIONS OF THE CREATIVE COMMONS ATTRIBUTION 4.0 INTERNATIONAL PUBLIC LICENSE

LAST UPDATE = 28 NOVEMBER 2022

This is a copyrighted work, but licensed under the Creative Commons Attribution 4.0 International Public License. A copy of this license is found in the last Appendix of this document. Alternatively, you may visit http://creativecommons.org/licenses/by/4.0/ or send a letter to Creative Commons: 171 Second Street, Suite 300, San Francisco, California, 94105, USA. The terms and conditions of this license allow for free copying, distribution, and/or modification of all licensed works by the general public.
Contents

1 Introduction

2 Tutorial
  2.1 Signal amplification (gain) ................................................. 6
  2.2 Amplifier energy efficiency ................................................. 9
  2.3 Classes of operation ....................................................... 10
  2.4 Rejection ........................................................................ 13
  2.5 Amplifier impedance ......................................................... 15
  2.6 Bandwidth ....................................................................... 17
  2.7 Distortion ....................................................................... 21

3 Derivations and Technical References .............................................. 27
  3.1 Decibels ........................................................................ 28

4 Questions ........................................................................ 39
  4.1 Conceptual reasoning ....................................................... 43
    4.1.1 Reading outline and reflections ....................................... 44
    4.1.2 Foundational concepts .................................................. 45
    4.1.3 Amplifier roll-off ......................................................... 47
    4.1.4 TTL logic gate as an amplifier ....................................... 48
    4.1.5 CMOS logic gate as an amplifier ..................................... 50
    4.1.6 Impulse testing ............................................................ 52
  4.2 Quantitative reasoning ....................................................... 53
    4.2.1 Miscellaneous physical constants ................................. 54
    4.2.2 Introduction to spreadsheets ......................................... 55
    4.2.3 Gain ratios into decibels ............................................... 58
    4.2.4 Decibels into gain ratios ............................................... 59
    4.2.5 Amplifier gains ............................................................ 59
    4.2.6 Gains and impedances of multi-stage amplifiers ............ 60
    4.2.7 Even versus odd harmonics ......................................... 63
  4.3 Diagnostic reasoning ....................................................... 66
    4.3.1 Audio amplifier design improvement ............................ 67
    4.3.2 Push-pull voltage follower ............................................. 68
    4.3.3 Discerning even/odd harmonics from the time domain ... 69
Chapter 1

Introduction

Amplifiers, like all electronic circuits, have limitations. This learning module focuses on common amplifier performance parameters, how to measure them, and various causes of performance limitations.

Important concepts related to amplifier performance include gain, dimensional analysis, decibels, Joule's Law, signal distortion, operational classes, BJT behavior, tank circuits, rejection ratios, Maximum Power Transfer Theorem, transmission lines, bandwidth, filtering, negative feedback, biasing, gain-bandwidth product, linearity, FET behavior, and signal decomposition (i.e. equivalent waves).

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to measure the gain of an amplifier circuit? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?

- How might an experiment be designed and conducted to measure the effective bandwidth of an amplifier circuit? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?

- What does any “decibel” value mean for a signal or for a circuit?

- Why must we multiply by 20 instead of 10 when computing voltage and current decibel figures from gain ratios?

- What factors may lead to an amplifier clipping?

- What distinguishes the different “classes” of amplifier operation from one another?

- How does a differential amplifier differ from a single-ended amplifier?

- Why is impedance-matching important in some electronic circuit applications?

- What factors limit the bandwidth of any amplifier circuit?
What is a harmonic frequency, and how does it relate to the fundamental frequency?

What does it mean to say that a waveform is “equivalent” to a harmonic series?

How may a spectrum analyzer be used to test the performance of an amplifier?
Chapter 2

Tutorial

Amplifier circuits magnify the amplitude of electronic signals by using those signals to redirect, or control, the flow of energy from some power source to a load, making that flow of energy take on the same form as the original signal:

In order to achieve this feat, amplifier circuits utilize active components such as transistors to enable electronic signals to exert control over externally-sourced electrical energy. It should be clearly understood that although amplifiers boost signal power, they do not output more total energy than they receive. The additional power necessary for the boosted signal comes from the DC power source used to energize the amplifier circuit. Energy is always conserved, and amplification is just one more example of this conservation in action.

The perfect amplifier generates a flawless reproduction of the input signal that is magnified in amplitude (i.e. greater voltage, greater current, or both). Real amplifiers, however, always fall short of this ideal. A real amplifier’s output signal is never a perfect reproduction of the input, although it may come very near to perfection. This module focuses on several practical limitations of amplifiers, and how we may quantify these circuits’ performance for the purpose of improving their design.
2.1 Signal amplification (gain)

Perhaps the most important and obvious metric of any amplifier circuit is its gain, defined simply as the ratio of output to input signal amplitude, usually symbolized by the variable A. The value of A tells us how many times larger an amplifier’s output signal is compared to its input signal. This ratio of output to input may be expressed in multiple ways. For example, we may speak of an amplifier having a power gain, a voltage gain, and a current gain, each gain calculated as the respective ratio of output to input for the specified quantity:

\[ A_P = \frac{P_{\text{out}}}{P_{\text{in}}} \quad A_V = \frac{V_{\text{out}}}{V_{\text{in}}} \quad A_I = \frac{I_{\text{out}}}{I_{\text{in}}} \]

Gain expressed as a mathematical ratio is always unitless because it is a quotient of quantities having identical units of measurement (e.g. Watts out versus Watts in, Amperes out versus Amperes in). If we approach these formulae from the perspective of dimensional analysis where units of measurement cancel out in the same way algebraic variables cancel between numerator and denominator, we see the identical units on the top and bottom of the fraction canceling so that the result has no unit of measurement at all.

Signal amplification typically results in gain values exceeding 1, but the same concept of gain (A) may be used to quantify signal attenuation as well. For example, a signal reduced in amplitude by a voltage divider network is an example of a circuit having a “gain” value less than 1.

In systems where amplification and attenuation ratios are extreme, such as radio communication systems, it is common to express gains in decibels rather than as ratios. Decibels are a logarithmic expression of power ratios, similar to how the Richter scale is used to represent the mechanical power of an earthquake: an increase or decrease of ten in the decibel scale represents a ten-fold ratio of power amplification or attenuation.

\[ \text{dB} = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \]

The following table gives examples of power gains in both ratio and dB form:

<table>
<thead>
<tr>
<th>Power gain (ratio)</th>
<th>Power gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{10000}$</td>
<td>$-40$ dB</td>
</tr>
<tr>
<td>$\frac{1}{1000}$</td>
<td>$-30$ dB</td>
</tr>
<tr>
<td>$\frac{1}{100}$</td>
<td>$-20$ dB</td>
</tr>
<tr>
<td>$\frac{1}{10}$</td>
<td>$-10$ dB</td>
</tr>
<tr>
<td>1</td>
<td>0 dB</td>
</tr>
<tr>
<td>10</td>
<td>+10 dB</td>
</tr>
<tr>
<td>100</td>
<td>+20 dB</td>
</tr>
<tr>
<td>1000</td>
<td>+30 dB</td>
</tr>
<tr>
<td>10000</td>
<td>+40 dB</td>
</tr>
</tbody>
</table>
2.1. SIGNAL AMPLIFICATION (GAIN)

Round power-of-ten gain ratios were chosen for this table simply to clarify the relationship between ratios and decibels, since decibels use the common logarithm which is based on powers of ten. However, any power gain ratio is translatable into decibels. For example, a power gain of 2 equals 3.01 dB; a power gain of 4 equals 6.02 dB; a power gain of \( \frac{1}{2} \) equals −3.01 dB, a power gain of \( \frac{1}{4} \) equals −6.02 dB, etc.

Decibels have the effect of “compressing” very large and very small ratios into relatively narrow range of numerical values. For example, a power gain of 3,566,920 is only 65.52 dB; a power “gain” (attenuation) of \( \frac{1}{3566920} \) is only −49.93 dB. Decibels also simplify the computation of multi-stage amplification systems in that total decibel gain is the sum of individual stage gains while total ratio gain is the product of individual stage gains. A multi-stage audio signal amplifier shown below illustrates how gain ratios and gain decibels work, illustrating how the total gain (as a ratio) is equal to the product of the stage gain ratios, while the total gain (as a decibel figure) is equal to the sum of the stage gain decibel figures:

\[ +V \]
\[ \text{V}_{\text{in}} \]
\[ \text{V}_{\text{out}} \]

<table>
<thead>
<tr>
<th>First amplifying stage</th>
<th>Volume control</th>
<th>Second amplifying stage</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_P = 14.7 )</td>
<td>( A_P = 0.3 )</td>
<td>( A_P = 2.6 )</td>
<td>( A_{P(\text{total})} = 11.466 )</td>
</tr>
<tr>
<td>( A_P = 11.67 \text{ dB} )</td>
<td>( A_P = -5.23 \text{ dB} )</td>
<td>( A_P = 4.15 \text{ dB} )</td>
<td>( A_{P(\text{total})} = 10.59 \text{ dB} )</td>
</tr>
</tbody>
</table>

Decibels are fundamentally an expression of power gain, which means if we wish to use dB to express either voltage or current gain we must do so in terms of how much power that voltage or current gain would imply. For any given load resistance, power is proportional to the square of voltage or current (Joule’s Law, \( P = \frac{V^2}{R} \) and \( P = I^2 R \)), and so we must incorporate this square-relationship into the decibel formulae, as follows:

\[ \text{dB} = 10 \log \left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 = 20 \log \left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) \]
\[ \text{dB} = 10 \log \left(\frac{I_{\text{out}}}{I_{\text{in}}}\right)^2 = 20 \log \left(\frac{I_{\text{out}}}{I_{\text{in}}}\right) \]
An important measure of amplifier performance related to voltage gain is clipping. This is what happens when the amplified signal attempts to rise above or below the limits of the DC power supply.

Amplifiers may also clip based on current limitations, if heavily loaded. In either case the fundamental principle at work is the very nature of electronic amplification: amplifier circuits function much like electrically-varying voltage dividers, “carving” out a reproduction of the input signal from the “raw material” provided by the DC power supply. No amplifier is able to boost its output signal to peak values beyond the “rail” voltages of the power supply, any more than an artist is able to paint a portrait larger than their canvas. Any attempt to do so results in the output signal being “clipped” as though someone cut the waveform’s shape with a knife at the limit-points.

If the clipping is symmetrical as is shown above, possible solutions include the following:

- Reduce the input signal’s amplitude
- Reduce the amplifier’s gain
- Increase the DC power supply voltage if possible
- Optimize the impedance for a better match between amplifier and load

If the clipping is asymmetrical, another possible solution exists: adjust the amplifier’s biasing.

Interestingly, some amplifier applications actually rely on clipping. For example, purposeful over-driving of an amplifier circuit is one way to convert a sine wave into a pseudo-square wave, which can be a useful technique when interfacing an analog signal with a digital (on/off) circuit. Also, intentional clipping is what gives electric guitars their characteristic “distorted” sound, where the preamplifier stage of the electric guitar amplifier is adjusted to an excessive level of gain for the express purpose of converting the string pickup coil’s AC signal into a clipped wave closely resembling a square wave. This not only alters the instrument’s sound, but it also creates a false sense of prolonged sustain because the clipped signal waveform maintains a steady peak amplitude even as the string’s oscillations gradually decay.
2.2 Amplifier energy efficiency

As defined earlier, an amplifier circuit is a collection of electronic components designed to magnify the amplitude of an electronic signal by using that signal to redirect, or control, the flow of energy from some power source to a load, such that the outgoing flow of energy reproduces the same wave-shape as the original signal. Like all energy-converting systems, some of the power source’s energy invariably diverts from the intended path (toward the load) and instead takes forms such as heat.

Show in block-diagram form, the flow of energy into and out of any electrical energy-converting circuit may be represented as arrow symbols, the width of each arrow representing the amount of power (energy per unit time) flowing into and out of the circuit:

In the case of an amplifier circuit, total power in consists of DC power supplied by the power source plus the signal source; total power out is what the load receives:

Energy efficiency for any system is the ratio between the amount of energy converted by that system into the intended form versus the amount of energy delivered to that system:

$$\text{Efficiency} = \frac{\text{Energy converted to intended form}}{\text{Energy input to system}} = \frac{P_{\text{out}}}{P_{\text{in}}}$$

100% energy efficiency is physically impossible, thanks to the Second Law of Thermodynamics. Some amplifier designs, as we shall soon see, are more energy-efficient than others.
2.3 Classes of operation

Active devices such as transistors and vacuum tubes are fundamentally DC-only devices, in that they are only able to control the flow of current in one direction. In order to use one of these devices to reproduce the entire wave-shape of an alternating-current (AC) signal, we must bias it in order to maintain a state of constant conduction throughout the AC signal’s cycle.

Consider the following example where a BJT is biased by a 3 Volt DC source in series with the AC signal source to offset that signal voltage above zero and thereby maintain the transistor in a constant state of conduction between collector and emitter:

Thanks to the transistor remaining at least partially “on” at all times, $V_{out}$ now resembles the same wave-shape as $V_{in}$ (albeit in inverted form). The signal’s biasing is clearly evident by viewing $v(1)$ in the SPICE plot: the sine-wave signal peaks at a high value of 5 Volts and at a low value of 1 Volt. The “AC portion” of this signal still has a peak-to-peak value of 4 Volts, but the biasing moves its center value to 3 Volts so that it never dips into negative values (or below the 0.7 Volt minimum required by the transistor to turn on). This “always-on” mode of amplifier operation is referred to as Class A.

A price we pay for this faithful reproduction of the entire AC signal wave-shape is significant energy dissipation in the transistor. Since the transistor never fully turns off nor fully turns on, it always dissipates energy in the form of heat. This constitutes inefficiency, which may be a problem for a high-power amplifier circuit or for an amplifier powered by some limited energy source such as a chemical battery. For this reason, other classes of operation and amplifier circuit designs exist.
If the transistor is biased such that it turns on only during one of the half-cycles of the AC signal waveform, it “rests” for half of the time and dissipates zero power when doing so. This is called *Class B* operation, and for faithful reproduction of the entire signal waveshape requires two transistors operating in a “push-pull” configuration:

A more complete example of a push-pull audio amplifier is shown here:

In this circuit proper biasing is a function of the DC supply voltage (+V) and resistor $R_1$, passing just enough current through the base terminals of each transistor to maintain them in a state of readiness to turn on at the slightest amount of input voltage, which one turning on depending on the polarity of the input signal at any given time.

Not surprisingly, class-B operation has its own unique limitation: a problem called *crossover distortion* which occurs if the transistor biasing is imperfect and causes them to conduct less than 50% of the time, resulting in a brief period during every zero-crossing of the AC signal when *neither* transistor conducts and the output signal “flat-lines” at zero when it ought to seamlessly cross zero transitioning between positive and negative. A practical solution to this problem is to slightly over-bias the push-pull transistors to achieve mutual conduction at the zero-crossing point, so that we never have a condition where both are simultaneously off. This results in slightly reduced energy
efficiency, but not nearly as poor as a class-A amplifier where the transistor never stops conducting. This compromise form of operation is called **Class AB**.

A third mode of operation is where the transistor spends significantly less than 50% of the time on. This, obviously, will fail to reproduce the AC waveshape and in fact is the very problem we sought to overcome with class-A biasing. However, in some applications this is a practical amplification technique, specifically in radio-frequency (RF) amplifiers where we may use LC “tank circuits” to sustain oscillations during the times when the transistor is off. We call this **Class-C** operation, a simple class-C amplifier shown in the following schematic:

![Class-C Amplifier Schematic](image)

The transistor in a class-C amplifier essentially “pulses” on only near the peaks of the AC input signal and remains off for the remainder of that cycle, during which time the LC tank circuit rounds out the rest of the sinusoidal signal by resonating, relying on energy stored during the transistor’s “on” time. Class-C amplifier circuits are highly efficient because the transistor spends most of its time either shut off or turned (nearly fully) on, but are limited in application to amplifying signals consisting of whole sinusoidal cycles. They find widespread application as RF (radio-frequency) power amplifiers where the signal being amplified has a very high frequency\(^1\) and where its magnitude remains mostly constant within the span of any single cycle\(^2\).

Many other amplifier classes exist, but all require substantially different circuit topologies to implement. Most of these other classes (D, F, G, I, S, T) use the transistor as a discrete on/off switch rather than as an analog “throttling” device, for the sake of reducing power dissipation at the transistor and thereby increasing the amplifier’s energy efficiency.

---

\(^1\) High frequency is practical here because it makes practical tank circuit networks using components of modest size.

\(^2\) RF amplifiers used for radio service must, of course, handle signals more complex than a simple sinusoid. RF signals are always *modulated* in some way, but these modulation schemes usually involve alterations to the magnitude and/or frequency of the “carrier” signal that are relatively slow compared to the frequency of that carrier. In other words, an RF amplifier need not be able to alter the wave-shape within the span of a single cycle, and so the tank circuit “flywheel” principle of a class-C amplifier suffices.
2.4 Rejection

Good amplification isn’t just about what an amplifier circuit should do, it’s also about what an amplifier circuit should not do. A good amplifier should magnify the amplitude of the input signal by a predictable amount, but it should not allow variations in power supply voltage or other parameters to affect the amplification of the desired signal. Expressed in different terms, an amplifier should accept only the signal presented at its input terminal(s) and reject all other influences.

Power supply rejection ratio (PSRR), often called ripple rejection ratio, refers to the effect that any variation in DC power supply voltage might have on the amplifier’s output signal. A well-designed amplifier will have a very large rejection ratio, signifying that large variations in DC power supply voltage would be necessary to significantly influence the amplifier’s output signal. An ideal amplifier has an infinite power supply rejection ratio. Since these ratio values are enormous, it is customary to express them as decibel (dB) figures, with 100 dB typically being the minimum for a modern integrated-circuit amplifier. One way to calculate power supply rejection is shown below:

\[
\text{PSR} = 20 \log \left( \frac{\Delta V_{PS}}{\Delta V_{out}} \right)
\]

Where,

- \( \text{PSR} \) = Power Supply Rejection, in decibels
- \( \Delta V_{PS} \) = Variation in DC power supply voltage, in Volts
- \( \Delta V_{out} \) = Variation in amplifier output voltage signal, in Volts

Unfortunately, there appears to be little standardization regarding how power supply rejection figures are calculated. Some manufacturers express rejection as the inverse ratio (i.e. \( \frac{\Delta V_{out}}{\Delta V_{PS}} \)) which results in a negative rather than a positive decibel figure. Another method is to calculate power supply rejection in terms of its equivalent effect on the amplifier’s input signal; i.e. how much power supply variation would be necessary to duplicate the effect of the input signal varying.

A type of rejection ratio specific to differential amplifiers is called common-mode rejection. Recall that a differential amplifier is one designed to amplify the difference in potential between two input terminals, as opposed to the more customary “single-ended input” amplifier designed to amplify the potential at one input terminal with respect to ground:

---

3This may seem like a strange way to quantify power supply rejection, because it’s really corruption of the output signal by power supply ripple that we’re worried about here. However, there is a certain rationale to it if you’re interested in comparing the effect of power supply variance against the effects of noise in the input signal. It also has the desirable marketing effect of inflating the rejection ratio, by comparing a given amount of power supply variance to the amplifier’s input signal amplitude (that input signal of course being smaller than the output signal)! As usual, caveat emptor.
With $V_{in}$ defined as a potential difference between two input terminals on a differential-input amplifier, such an amplifier should only amplify that potential difference and reject any amount of voltage common to both of those input terminals and ground. The following diagrams illustrates an ideal differential amplifier with a voltage gain of five rejecting a change in “common-mode” voltage from 6 Volts to 1 Volt:

Note how this ideal differential-input amplifier only accepts the differential input voltage and rejects any common-mode voltage: its output remains unchanged at 10 Volts despite a six-fold change in common-mode voltage.

Real differential amplifiers, of course, are actually affected by common-mode voltage just as they are affected by DC power supply fluctuations. The quantification of this rejection follows a formula similar to that of power supply rejection:

$$CMR = 20 \log \left( \frac{\Delta V_{CM}}{\Delta V_{diff}} \right)$$

Where,

- $CMR =$ Common-Mode Rejection, in decibels
- $\Delta V_{CM} =$ Variation in common-mode input voltage, in Volts, to produce a certain amount of $\Delta V_{out}$
- $\Delta V_{diff} =$ Variation in differential input voltage, in Volts, to produce the same amount of $\Delta V_{out}$

As with power supply rejection, modern integrated-circuit amplifiers often boast common-mode rejection figures exceeding 100 dB.
2.5 Amplifier impedance

The amount of impedance an amplifier presents as a load to its input signal source, as well as the amount of impedance an amplifier presents to a driven load, are both important amplifier specifications. An amplifier with input and/or output impedance values poorly matched to its external device(s) will not perform optimally.

On the input side, amplifier impedance is defined by the biasing resistor network as well as the transistor itself, and to a lesser degree the load it powers. The Maximum Power Transfer Theorem tells us a network will accept the most power from a source when its impedance matches the internal impedance of that source. Power transfer optimization is important when the signal source is weak (i.e. high-impedance, incapable of delivering much current), because extracting as much power from that source as possible helps ensure the strongest amplified signal.

On the output side, amplifier impedance simply limits power delivery to the external load. In terms of raw power delivery, the less internal impedance at an amplifier’s output terminal, the more effectively it may deliver energy to the load. However, this does not necessarily mean zero output impedance is ideal: one beneficial use of output impedance in an amplifier is self-protection against a short-circuited load.

---

4A common misconception about the Maximum Power Transfer Theorem is that source impedance needs to be adjusted to match load impedance for maximum energy transfer. This is not true. It is the load which must be adjusted to match the source, not the other way around. The ideal source impedance for maximum energy transfer is always zero!
Another factor dictating ideal impedances for an amplifier is *transmission line termination*. This is relevant for high-frequency amplifiers sending and/or receiving signals through relatively long conductors. Whenever a pulse or otherwise alternating electrical signal is sent through a long cable (called a *transmission line*), that pulse energy travels along the cable’s length and will “reflect” off the far end of that cable unless it encounters an impedance value matching the cable’s own *surge impedance* or *characteristic impedance*.

Ideally an amplifier receiving a pulse signal from a transmission line will be designed to have an input impedance matching that of the line itself, to prevent reflected signals returning to the source. Likewise, an amplifier sending a pulse signal into a transmission line should be designed to have an output impedance matching the line’s, to absorb any reflected energy that might return from the other end of the line.

If an amplifier’s input and/or output impedance happens to be mis-matched to the transmission line(s) it connects to, external resistors may be added to make the necessary corrections. In the example shown below, two transmission lines rated at 50 Ω surge impedance each connect to a fully differential amplifier having 1 MΩ input impedance and 20 Ω output impedance:

Here, we have augmented the amplifier’s mis-matched internal impedances with external resistors so that both cables “see” 50 Ω when “looking into” the amplifier.

---

5. Here, “relatively” refers to conductor length as compared to the AC signal’s wavelength $\lambda$.

6. Yes, technically the resistor paralleled with the amplifier’s input terminals should be *slightly* greater than 50 Ohms so that in parallel with the 1 million Ohms of the amplifier it will make exactly 50 Ohms for perfect cable termination. However, cable termination need not be *perfect* to adequately dissipate reflected signal energy, and no cable has an impedance of exactly 50 Ohms anyway. Besides, where would you find a 50.0025 Ohm resistor? The reason we use two 15 Ohm resistors on the output rather than a single 30 Ohm resistor is to ensure both of the amplifier’s differential output terminals are *balanced* with respect to ground because we are assuming a balanced (i.e. ungrounded) transmission line.
2.6 Bandwidth

The ideal electronic signal amplifier is capable of amplifying signals of all frequencies equally well, but this is an unattainable goal in practice. Instead, real amplifiers behave somewhat like filter networks with practical frequency-cutoff limits.

Consider the multi-stage audio amplifier shown earlier, as an example of an amplifier circuit with definite frequency limitations:

![Amplifier Circuit Diagram]

The existence of capacitors throughout this circuit creates a frequency-dependent response. Clearly, we could not amplify a DC voltage signal using this amplifier because the three coupling capacitors (drawn horizontally) would all block that signal’s passage through. The two bypass capacitors (drawn vertically) boost gain by shunting around the emitter resistor of each amplification stage, furthermore limiting the amplifier’s practicality at low signal frequencies.

Conversely, this amplifier cannot amplify radio-frequency signals either due to the parasitic capacitance existing between each transistor’s base and collector terminals. This parasitic capacitance acts to “feed back” signal from the collector (output) to the base (input) at each amplifying stage, and because those two signal voltages are 180 degrees out of phase with each other (measured with respect to any other point in the circuit), this feedback is negative or degenerative, the effect being to cancel voltage gain.

As a result of these dual limitations, this amplifier’s frequency-response function looks something like a band-pass filter with a fairly flat pass-band:
Direct-coupled multi-stage amplifiers have frequency responses similar to low-pass filter networks, their usable bandwidths extending all the way down to 0 Hz (DC), but still cutting off at high frequencies as shown in the following graph:

![Graph showing bandwidth and gain relationship](image)

Of course, direct-coupled multi-stage transistor amplifiers have other limitations too, namely the tendency for each stage’s biasing to affect the biasing of all the other stages. AC-coupled multi-stage amplifiers avoid this problem because their inter-stage coupling blocks all DC signals, of which biasing could be considered a DC signal. The price we pay in avoiding the “bias drift” problem of direct-coupled amplifiers is the inability to convey DC and other low-frequency signals.

One technique useful for extending the low end of an amplifier’s bandwidth without abandoning AC coupling is to incorporate “compensating” resistance and capacitance on the collector side of the transistor:

![Circuit diagram](image)

The idea of this is to increase the effective collector-side resistance of each common-emitter amplifier stage as frequency decreases. Voltage gain for a common-emitter amplifier is approximately $\frac{R_C}{R_E}$, and so increasing $R_C$ will boost gain. The $R_{\text{comp}} / C_{\text{comp}}$ network effectively does this by “un-bypassing” $R_{\text{comp}}$ as frequency decreases.
2.6. BANDWIDTH

A similar technique used to help compensate for the degradation of common-emitter amplifier gain at high frequencies is to install a peaking coil in the collector pathway of the circuit. Inductive reactance increases with increasing frequency \((X_L = 2\pi f L)\), and so by placing an inductor on the collector-side of a common-emitter amplifier the total impedance on that side will grow with frequency and thereby provided a high-frequency gain boost:

![Diagram of a peaking coil amplifier](image)

When constructing amplifiers intended for extremely high (i.e. “radio”) frequencies even peaking coils are not enough to maintain a usable voltage gain, and so instead of using resistance on the collector side of a common-emitter amplifier to develop an output voltage we commonly see an LC “tank” circuit network used instead:

![Diagram of an LC tank circuit](image)
Of course, the resonant LC network gives the amplifier a fairly narrow bandwidth, but this is generally a desirable feature in an amplifier intended for radio applications where the communication channel is defined by a specific frequency.

A common performance parameter specified for electronic amplifiers is called the *gain-bandwidth product*, abbreviated *GBW*. This figure is calculated by multiplying the measured gain of an amplifier by the frequency of the test signal at that measured gain value, and for many amplifiers this figure is approximately constant over a wide range of frequencies. For example, if an amplifier circuit has a GBW of 1.2 million, it ideally means it can offer a gain of 1.2 million at a signal frequency of 1 Hz, a gain of one at a signal frequency of 1.2 MHz, or any combination in between where the product of gain and frequency is 1.2 million.

Approximate though this parameter may be, it holds an important lesson for us in understanding the frequency response of an amplifier: it is impossible for any amplifier to exhibit both an arbitrarily high gain and an arbitrarily high bandwidth. Instead, in order to achieve one of these goals we must sacrifice the other.
2.7 Distortion

To provide flawless reproduction of an input signal, an amplifier must be linear. A good way to visually understand the role that an amplifier’s linearity plays in the faithful reproduction of an AC signal is to graph the amplifier’s response as a line against which points along the AC input signal’s waveform will be projected and reflected to form new points on the AC output signal’s waveform. In the following illustration, a straight red line represents the linear response of the amplifier while blue curves outline the AC signal waveforms:

So long as the amplifier response is a straight line (i.e. linear), the wave-shape of $V_{\text{out}}$ must be the same as the wave-shape of $V_{\text{in}}$. A truly linear amplifier may magnify the wave, but it cannot distort the wave.\(^7\)

\(^7\)The slope of this line represents the amplifier’s voltage gain. A gain of 1 (0 dB) would be a line at 45° (i.e. a slope of 1).

\(^8\)This basic principle applies to any linear network, not just linear amplifiers. Voltage dividers and other attenuator networks are also fundamentally linear, and as such they will never change the shape of the wave even though they may diminish its amplitude. The linear response line of a resistive attenuator network, for example, is merely a line with a slope less than 1 so that $V_{\text{out}}$ is less than $V_{\text{in}}$.\(^8\)
The same cannot be said of an amplifier having a non-linear response. Such an amplifier is guaranteed to distort the wave-shape because points along the output waveform’s amplitude will not be proportionate in amplitude to those corresponding points along the input waveform. Below is an illustration showing an example of this phenomenon, where the amplifier’s response is a J-shaped curve rather than a straight line:

Note how one-half of the output waveform appears “fatter” than the other: clearly a distorted reproduction of the original (symmetrical) wave-shape.
2.7. DISTORTION

Field-effect transistors, being square-law devices, exhibit a similar J-shaped response curve, which explains why single-transistor FET amplifier circuits tend to exhibit the same distortion characteristic\(^9\) as seen in the following SPICE simulation:

Gross levels of distortion such as what we see with this single-stage JFET amplifier is easy enough to detect by visual inspection of the oscillograph, but more subtle degrees of distortion require other means of detection. One such diagnostic technique is to view an amplifier’s output using a spectrum analyzer rather than an oscilloscope. Spectrum analyzers represent signals in the frequency domain rather than in the time domain.

Fourier’s Theorem states that any wave-shape – no matter how simple or complex – may be expressed in terms of pure sinusoids (i.e. sine-wave or cosine-wave shapes). A simple sine or cosine wave, for example, is by definition sinusoidal. What is not so obvious is the fact that even non-sinusoidal wave-shapes are actually equivalent to a series (summation) of multiple sinusoids\(^10\) at different frequencies and amplitudes. Jean Baptiste Joseph Fourier made this mathematical discovery, and for this reason we often refer to the “decomposition” of non-sinusoidal waves into sinusoidal series Fourier decomposition. A Fourier series consists of a summation of multiple sine or cosine terms, each with its own amplitude, frequency, and phase angle. For periodic waveforms the frequencies are all integer-multiples of the lowest frequency (called the fundamental), with each of the higher frequencies called harmonics.

\(^9\)The reason the distorted output waveform in the FET simulation appears inverted compared to our sketch is because the common-source FET amplifier is inverting in the same manner as a common-emitter amplifier.

\(^10\)Interestingly, it is also possible to decompose any wave-shape into a series of other function types, so long as those function types are mathematically orthogonal. For example, it is possible to express any waveform as a series of impulse functions, or as a series of exponential functions, or as a series of step functions. Sinusoids just happen to be convenient from both mathematical and engineering perspectives, and so this is by far the most popular form of decomposition.
Sinusoidal decomposition is not just an interesting theoretical principle, but a very practical tool for analyzing amplifier performance. The basic idea is quite simple: a pure sine wave output by a signal generator should consist of just one peak on a spectrum analyzer’s display (the “fundamental”). After amplifying that sine-wave signal, the amplifier’s output spectrum should also consist of just one peak representing that same fundamental frequency, albeit at a greater amplitude than the input signal, and likely shifted in phase from the input signal as well. However, if the amplifier is not perfectly linear, any distortion it contributes to the output signal make that signal’s wave-shape non-sinusoidal, and this will translate into multiple peaks on the spectrum. The number and amplitude of these spectral peaks indicates how much harmonic distortion the amplifier has.

In the illustration shown below, a 1 kHz sinusoidal signal is sent to the input of an electronic amplifier circuit, with spectrum displays shown of input and output signal voltages:

On the input spectrum we see a single peak at 1 kHz, representing the fundamental frequency of the pure sine-wave signal generated by the AC signal source. On the output spectrum we see a fundamental peak at 1 kHz as well as smaller peaks at 3 kHz and 5 kHz, the 3rd and 5th harmonics of the fundamental. In this particular example the 3rd harmonic amplitude is −100 dB less than the fundamental, and the 5th harmonic is −110 dB below the fundamental. These figures represent very low levels of distortion: a 1 kHz sinusoid added to a 3 kHz sinusoid of $\frac{1}{10000}$ the fundamental’s amplitude.

11Note how the fundamental peak on both input and output spectra measure 0 dB. This may seem strange if we expect the amplifier to boost the amplitude of the signal. However, this is typical for two separate signal measurements, where the fundamental peak is usually adjusted to be the reference amplitude against which all other peak heights are compared. This is called a normalized measurement. The input and output spectra would have to be simultaneously displayed by a spectrum analyzer, normalized to the output signal fundamental, in order to compare the amplitudes of output versus input and thereby perceive the amplifier’s gain.
amplitude, added to a 5 kHz sinusoid of only \( \frac{1}{\sqrt{2}} \) the fundamental’s amplitude. Such low levels of distortion would be utterly imperceptible on an oscillograph display of the output waveshape – i.e. it would appear to the unaided eye as a perfect sine wave – but on a spectrum display the presence of distortion is obvious.

That said, a real amplifier test will not be quite as simple as this example. No signal generator outputs perfectly pure sine waves, and so what we would typically see is an input spectrum with some low-level harmonics, and an output spectrum with those same harmonics (and perhaps others as well!) at greater amplitudes. The comparison of normalized harmonic amplitudes between output and input would tell us how much distortion the amplifier contributes above and beyond the inherent distortion of the signal generator. Also, real spectra always show a noise floor consisting of a jagged\(^\text{12}\) rather than a flat base-line at the bottom of the display. The following screenshot of a digital oscilloscope’s FFT (spectrum) display shows a 2 kHz sine-wave signal generator output with substantial noise:

![Digital oscilloscope's FFT display](image)

A common metric of amplifier distortion is Total Harmonic Distortion, or THD. This is mathematically defined as the ratio of total harmonic signal power versus fundamental signal power, usually expressed as a percentage:

\[
\text{THD} = \frac{P_H}{P_F} \times 100\%
\]

Where,

- \( P_F \) = Signal power of the fundamental frequency (i.e. the 1st harmonic), in Watts
- \( P_H \) = Signal power of all higher-order harmonic frequencies, in Watts

Single-purpose test instruments called distortion analyzers exist to measure THD. They work on the principle of notch filtering: using a narrow-band band-stop filter to filter out the fundamental frequency from a signal, leaving nothing but the higher-order harmonics which are then measured.

\(^{12}\)While periodic waveforms consist of a fundamental sinusoid added to one or more harmonics of that fundamental, non-periodic waveforms contain non-harmonic frequencies, and in most cases solid bands of frequencies spanning certain ranges. Noise is usually random in nature, and so a spectrum’s noise floor appears as a jagged “band” of frequencies rather than a set of clearly-defined peaks.
Chapter 3

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.
3.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the common logarithm, expressed as a measurement unit called the decibel. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of “decibel” you can think: this is an expression of how much greater (or how much smaller) one power is to another. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a gain. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:

\[
\text{Gain} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{18.4 \text{ W}}{40 \text{ mW}} = 460
\]

An alternative way to express the gain of this amplifier is to do so using the unit of the Bel, defined as the common logarithm of the gain ratio:

\[
\log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) = \log \left( \frac{18.4 \text{ W}}{40 \text{ mW}} \right) = 2.66276 \text{ B}
\]

When you see an amplifier gain expressed in the unit of “Bel”, it’s really just a way of saying “The output signal coming from this amplifier is \(x\) powers of ten greater than the input signal.” An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the “Bel” (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the deciBel (1 dB = \(\frac{1}{10}\) of a Bel). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

\[
\text{dB} = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)
\]

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid\(^1\).

\(^1\)It is interesting to note that although the “Bel” is a metric unit, it is seldom if ever used without the metric prefix
An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

\[
\text{dB} = 10 \log(\text{Ratio})
\]

If we wish to solve for the ratio, we must “undo” all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

\[
\frac{\text{dB}}{10} = 10 \log(\text{Ratio})
\]

\[
\frac{\text{dB}}{10} = \log(\text{Ratio})
\]

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

\[
10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}
\]

\[
10^{\frac{\text{dB}}{10}} = \text{Ratio}
\]

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

\[
\text{Ratio} = 10^{26.6276 \text{ dB}}
\]

\[
\text{Ratio} = 10^{2.66276 \text{ B}}
\]

\[
\text{Ratio} = 460
\]

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

\[\text{“deci” (} \frac{1}{10} \text{). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.}\]
We may also use decibels to express power losses in addition to power gains. There are many practical applications of this in signaling systems, both electronic and optical. One such application is filtering, where a “filter” circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is attenuation, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: cable loss. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal’s energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length\(^2\), such that the power out is less than the power in:

\[
\text{Gain} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{37 \text{ mW}}{40 \text{ mW}} = 0.925
\]

\[
10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) = 10 \log \left( \frac{37 \text{ mW}}{40 \text{ mW}} \right) = -0.3386 \text{ dB}
\]

Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power gain is expressed as a positive decibel value, while any power loss is expressed as a negative decibel value. Any component outputting the exact same power as it takes in will exhibit a “gain” value of 0 dB (equivalent to a gain ratio of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of “greater than” and “less than”. Positive values represent powers that are greater while negative values represent powers that are lesser. Zero Bel or decibel values represent no change (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of \(10 \times 10 = 100\), whereas -30 dB represents a power ratio reduction of \(\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}\). Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division or power ratio: e.g. +6 dB is approximately equal to a power ratio gain of \(2 \times 2 = 4\), whereas -12 dB is approximately equal to a power ratio reduction of \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}\). We may combine \(\pm 10\) dB and \(\pm 3\) dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of \(10 \times 10 \times 2 \times 2 = 400\).

\(^2\)For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is dielectric heating, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.
Observe what happens if we combine a “gain” component with a “loss” component and calculate the overall power out versus power in:

The overall gain of this amplifier and cable system expressed as a ratio is equal to the product of the individual component gain/loss ratios. That is, the gain ratio of the amplifier multiplied by the loss ratio of the cable yields the overall power ratio for the system:

\[
\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5
\]

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the sum of the individual component decibel values. That is, the decibel gain of the amplifier added to the decibel loss of the cable yields the overall decibel figure for the system:

\[
\text{Overall gain} = 10 \log \left( \frac{17.02 \text{ W}}{40 \text{ mW}} \right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}
\]

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

\[
425.5 \approx 400 = (10) \times (10) \times (2) \times (2)
\]

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

\[
(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}
\]

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.
Decibels always represent comparisons of power, but that comparison need not always be $P_{\text{out}}/P_{\text{in}}$ for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard “reference” power of exactly 1 milliWatt. The resulting decibel figure would be written as “dBm” in honor of the 1 milliWatt reference:

$$P_{\text{in}} = 10 \log \left( \frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}$$

The unit of “dBm” literally means the amount of dB “greater than” 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

$$P_{\text{out}} = 10 \log \left( \frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}$$

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.

Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply adding the amplifier’s gain factor yields the amplifier’s output power in dBm:

$$P_{\text{in}}(\text{dB}) + P_{\text{gain}}(\text{dB}) = P_{\text{out}}(\text{dB})$$

$$16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}$$

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.
We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of “dBW” (decibels greater than 1 Watt):

\[ P_{in} = 10 \log \left( \frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW} \]

\[ P_{out} = 10 \log \left( \frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW} \]

Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is less than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is more than the 1 Watt reference. A positive dB figure means “more than” while a negative dB figure means “less than.”

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply adding the amplifier’s gain factor yields the amplifier’s output power in dBW:

\[ P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB}) \]

\[ -13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW} \]

An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.
This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of ratios involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they “transform” one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of $-0.15$ decibels per foot at a signal frequency of $2.4$ GHz will suffer $-15$ dB over 100 feet, and $-150$ dB over 1000 feet. To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:

A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, multiply to make the over-all amplifier’s power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, add to make the over-all amplifier’s power gain (dB):

---

In fact, logarithms are one of the simplest examples of a transform function, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the Fourier transform (converting a time-domain function into a frequency-domain function) and the Laplace transform (converting a differential equation into an algebraic equation).
3.1. DECIBELS

Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a power ratio, we cannot simply substitute $V$ or $I$ for $P$ in any of the formulæ we’ve seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. $V_{out}$ is twice as large as $V_{in}$), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load’s power dissipation will be more than double when energized by a voltage twice as large. Joule’s Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power quadrupling because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule’s Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2 R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left( \frac{P_{out}}{P_{in}} \right) = 10 \log \left( \frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left( \frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

$$\log x^y = y \log x$$

Therefore, we may simplify the decibel formula for voltage gain by removing the “2” power and making it a multiplier:

$$10 \log \left( \frac{V_{out}}{V_{in}} \right)^2 = (2)(10) \log \left( \frac{V_{out}}{V_{in}} \right) = 20 \log \left( \frac{V_{out}}{V_{in}} \right)$$

$$10 \log \left( \frac{I_{out}}{I_{in}} \right)^2 = (2)(10) \log \left( \frac{I_{out}}{I_{in}} \right) = 20 \log \left( \frac{I_{out}}{I_{in}} \right)$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.
We can see the practicality of using decibels to represent something other than electrical power by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter’s face, calibrated in decibels (DB):

Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB\(^4\). A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC voltage. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically *undefined*\(^5\). Practically, it means \(-\infty\) dB, which is why the needle at the 0 Volt position “falls off” the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the *nonlinear* nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the *logarithm* function which is nonlinear.

\(^4\)Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

\(^5\)Your electronic calculator will complain if you attempt to take the logarithm of zero!
Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

\[ V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts} \]

Setting the VOM to the 2.5 V AC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):

In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range \textit{four times} as great), the meter’s needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

\[ 20 \log \left( \frac{2.5}{10} \right) = -12.04 \text{ dB} \]

Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

\[ 20 \log \left( \frac{2.5}{50} \right) = -26.02 \text{ dB} \]

\[ 20 \log \left( \frac{2.5}{250} \right) = -40.0 \text{ dB} \]
Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read\(^1\) the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture\(^2\), the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

---

1. Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

2. Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 4. QUESTIONS

General challenges following tutorial reading

- **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.

- **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

- Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

- Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

- Identify any **new concept(s)** presented in the text, and explain in your own words.

- Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

- Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

- Devise an experiment to **disprove** a plausible misconception.

- Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

- Describe any useful **problem-solving strategies** applied in the text.

- Devise a question of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

• Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.

• Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.

• Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?

• Is there more than one way to solve this problem? Which method seems best to you?

• Show the work you did in solving this problem, even if the solution is incomplete or incorrect.

• What would you say was the most challenging part of this problem, and why was it so?

• Was any important information missing from the problem which you had to research or recall?

• Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?

• Examine someone else’s solution to identify where they applied fundamental laws or principles.

• Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).

• For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.

• For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.

• For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.

• Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?

• Identify where it would be easy for someone to go astray in attempting to solve this problem.

• Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

• In what way(s) was this experiment or project easy to complete?

• Identify some of the challenges you faced in completing this experiment or project.
CHAPTER 4. QUESTIONS

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system’s function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component \(X\)) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?
4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

---

3Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
4.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

√ Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

√ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

√ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

√ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

√ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

√ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Energy
- Conservation of Energy
- Gain
- Decibels
- Dimensional analysis
- Common logarithm
- Joule’s Law
- Rejection
- Ripple
- Common-mode voltage
- Maximum Power Transfer Theorem
- Impedance
Transmission line

Filter

Negative feedback

Parasitic effect

Bandwidth

Resonance

Linearity

Square-law characteristic

Time domain

Frequency domain

Sinusoidal decomposition (i.e. Fourier’s Theorem)

Fundamental frequency

Harmonic frequency

Fourier series
4.1.3 Amplifier roll-off

Most of the simple amplifiers you will be initially studying tend to lose gain as the frequency of the amplified signal increases. This loss of gain is sometimes quantified in terms of roll-off, usually expressed in units of decibels per octave (dB/octave).

What, exactly, is “roll-off”? Furthermore, explain the meaning of “octave” in the context of the units of measurement used to specify roll-off.

If we were to plot the response of a typical amplifier in the form of a Bode plot, what type of filter circuit characteristic (band-pass, band-stop, etc.) would it best resemble?

Challenges

- Identify some reasons why amplifiers behave as filters.
4.1.4 TTL logic gate as an amplifier

Integrated-circuit (IC) devices known as logic gates are essentially transistor amplifiers, designed to operate in discrete (on/off) mode rather than amplify analog signals. An example of a logic gate is shown below, this one a transistor-transistor-logic (TTL) gate known as an inverter:

Start by determining the state of each transistor when the input signal is “high”:

• $Q_1 =$ on or off?
• $Q_2 =$ on or off?
• $Q_3 =$ on or off?

Next, determine the state of each transistor when the input signal is “low”:

• $Q_1 =$ on or off?
• $Q_2 =$ on or off?
• $Q_3 =$ on or off?

Lastly, apply the performance criteria you’ve been studying for amplifiers in general to this logic gate. If you are already familiar with digital logic circuitry, relate some of the terms and concepts from analog amplifier performance measurement to the equivalent terms and concepts in digital.
4.1. CONCEPTUAL REASONING

- Logic gates are intended to function strictly with discrete (high/low) signals. Comment on how this perspective may be gainfully used to help you analyze the basic behavior of any amplifier circuit.

- From the perspective of $Q_2$’s base terminal, is $Q_1$ functioning as a common-collector, common-emitter, or common-base amplifier?

- From the perspective of $Q_3$’s base terminal, is $Q_1$ functioning as a common-collector, common-emitter, or common-base amplifier?

- A DC logic signal is either high or low (+V or zero), but an AC signal could be thought of as having three states: high (+V), zero, and low (−V). How would this logic circuit respond to a “low” AC signal (−V)?

- Identify distinct amplification stages within this logic gate circuit.
4.1.5 CMOS logic gate as an amplifier

Integrated-circuit (IC) devices known as logic gates are essentially transistor amplifiers, designed to operate in discrete (on/off) mode rather than amplify analog signals. An example of a logic gate is shown below, this one a complementary-metal-oxide-semiconductor (CMOS) gate known as an inverter:

Start by determining the state of each transistor when the input signal is “high”:

- \( Q_1 \) = on or off?
- \( Q_2 \) = on or off?
- \( Q_3 \) = on or off?
- \( Q_4 \) = on or off?
- \( Q_5 \) = on or off?
- \( Q_6 \) = on or off?

Next, determine the state of each transistor when the input signal is “low”:

- \( Q_1 \) = on or off?
- \( Q_2 \) = on or off?
- \( Q_3 \) = on or off?
- \( Q_4 \) = on or off?
- \( Q_5 \) = on or off?
- \( Q_6 \) = on or off?
Lastly, apply the performance criteria you’ve been studying for amplifiers in general to this logic gate. If you are already familiar with digital logic circuitry, relate some of the terms and concepts from analog amplifier performance measurement to the equivalent terms and concepts in digital.

**Challenges**

- Logic gates are intended to function strictly with discrete (high/low) signals. Comment on how this perspective may be gainfully used to help you analyze the basic behavior of any amplifier circuit.

- Are transistors $Q_5$ and $Q_6$ operating as common-source, common-drain, or common-gate amplifiers?

- A DC logic signal is either high or low (+V or zero), but an AC signal could be thought of as having three states: high (+V), zero, and low (−V). How would this logic circuit respond to a “low” AC signal (−V)?

- Identify distinct amplification stages within this logic gate circuit.
4.1.6 Impulse testing

The Hewlett-Packard Application Note 17 ("Square wave and pulse testing of linear systems") written by B.M. Oliver in 1966 explains very clearly how square-wave signals and brief pulses ("impulse" signals) may be used to test the response of electronic circuits such as amplifiers. In this document the author explains how linear systems may be tested using pure sinusoidal signals of various frequency values (called "steady-state" testing), and then goes on to explain how testing with short-duration "impulse" signals and/or square-wave signals often provides the same useful information in less testing time.

His rationale for using non-sinusoidal test signals to "probe" a linear electronic network is explained in the first two paragraphs of a section of this document entitled The Response of Linear Systems to Impulses:

While steady state measurements are very useful for the reasons pointed out in the last section, they are quite time consuming and for many purposes observing the transient response of the system certain particular types of input waves may provide all the information necessary . . .

Consider, for example, the case where the input, \( f(t) \), is an impulse of negligible duration and, say, unit area. The spectrum of such an impulse contains all frequencies at equal amplitude, and in phase in the sense that they all add at \( t = 0 \). In other words the spectrum is a constant \( F(\omega) = 1 \). Now by superposition, it obviously makes no difference whether all frequencies are introduced one after the other as in steady state testing, or simultaneously by applying an impulse. The frequencies will all be modified in the same way by the linear system. You might say that an impulse test is equivalent to an instantaneous steady state test. They both give the same information. The problem is merely one of interpreting the results in either case.

Explain what the author of this Application Note means when he says “an impulse test is equivalent to an instantaneous steady state test”.

**Challenges**

- Propose a pulse-generating circuit capable of outputting extremely short-duration impulse signals.
4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

---

4In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
4.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation ($\sigma$) of uncertainty in the final digits: for example, Avogadro’s number given as $6.02214179(30) \times 10^{23}$ means the center value $(6.02214179 \times 10^{23})$ plus or minus $0.00000030 \times 10^{23}$.

Avogadro’s number ($N_A$) = $6.02214179(30) \times 10^{23}$ per mole (mol$^{-1}$)

Boltzmann’s constant ($k$) = $1.3806504(24) \times 10^{-23}$ Joules per Kelvin (J/K)

Electronic charge ($e$) = $1.602176487(40) \times 10^{-19}$ Coulomb (C)

Faraday constant ($F$) = $9.64853399(24) \times 10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space ($\mu_0$) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space ($\epsilon_0$) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space ($Z_0$) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant ($G$) = $6.67428(67) \times 10^{-11}$ cubic meters per kilogram-seconds squared (m$^3$/kg-s$^2$)

Molar gas constant ($R$) = $8.314472(15)$ Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant ($h$) = $6.62606896(33) \times 10^{-34}$ joule-seconds (J-s)

Stefan-Boltzmann constant ($\sigma$) = $5.670400(40) \times 10^{-8}$ Watts per square meter-Kelvin$^4$ (W/m$^2$-K$^4$)

Speed of light in a vacuum ($c$) = **299792458 meters per second** (m/s) = $186282.4$ miles per second (mi/s)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from [http://physics.nist.gov/constants](http://physics.nist.gov/constants), National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s 2018 CODATA recommended values database.
4.2. QUANTITATIVE REASONING

4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g., Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g., cell B3; cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>= B1 / B2</td>
<td>km/h</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula \(= B1 / B2\), because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln(), log10())

Parentheses may be used to ensure proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of \( ax^2 + bx + c \):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>(-B4 + sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>(-B4 - sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
</tr>
<tr>
<td>3</td>
<td>a =</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>b =</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>c =</td>
<td>-2</td>
</tr>
</tbody>
</table>

This example is configured to compute roots of the polynomial \( 9x^2 + 5x - 2 \) because the values of 9, 5, and −2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new \( a \), \( b \), and \( c \) coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

---

7 Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

8 Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

9 Reviewing some algebra here, a root is a value for \( x \) that yields an overall value of zero for the polynomial. For this polynomial \( (9x^2 + 5x - 2) \) the two roots happen to be \( x = 0.269381 \) and \( x = -0.82494 \), with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[
y = \sqrt{b^2 - 4ac} \quad z = 2a
\]

\[
x = \frac{-b \pm y}{z}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1 = \frac{(-B4 + C1)}{C2}</td>
<td>= \sqrt{(B4^2) - (4<em>B3</em>B5))</td>
</tr>
<tr>
<td>2</td>
<td>x_2 = \frac{(-B4 - C1)}{C2}</td>
<td>= 2*B3</td>
</tr>
<tr>
<td>3</td>
<td>a = 9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b = 5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c = -2</td>
<td></td>
</tr>
</tbody>
</table>

Note how the square-root term \(y\) is calculated in cell C1, and the denominator term \(z\) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
4.2.3 Gain ratios into decibels

Convert the following amplifier gains (either power, voltage, or current gain ratios) into gains expressed in the unit of decibels (dB):

- $A_P = 25$ ; $A_P(dB) =$
- $A_V = 10$ ; $A_V(dB) =$
- $A_I = 37$ ; $A_I(dB) =$
- $A_P = 150$ ; $A_P(dB) =$
- $A_I = 41$ ; $A_I(dB) =$
- $A_V = 3.4$ ; $A_V(dB) =$
- $A_P = 18$ ; $A_P(dB) =$
- $A_V = 100$ ; $A_V(dB) =$

**Challenges**

- Suppose a student mistakenly computes 10 dB for $A_V = 10$ (ratio). Explain what went wrong, and why the correct answer is different.
4.2.4 Decibels into gain ratios

Convert the following amplifier gains expressed in the unit of decibels (dB), to gain figures expressed as unitless ratios:

- $A_P = 5 \text{ dB} : A_P^{(\text{ratio})} =$
- $A_V = 23 \text{ dB} : A_V^{(\text{ratio})} =$
- $A_I = 20 \text{ dB} : A_I^{(\text{ratio})} =$
- $A_P = 2.5 \text{ dB} : A_P^{(\text{ratio})} =$
- $A_I = 7.4 \text{ dB} : A_I^{(\text{ratio})} =$
- $A_V = 45 \text{ dB} : A_V^{(\text{ratio})} =$
- $A_P = 12.8 \text{ dB} : A_P^{(\text{ratio})} =$
- $A_V = 30 \text{ dB} : A_V^{(\text{ratio})} =$

Challenges

- Suppose a student mistakenly computes 100 (ratio) for $A_I = 20 \text{ dB}$. Explain what went wrong, and why the correct answer is different.

4.2.5 Amplifier gains

An amplifier has a voltage gain of 5 and a current gain of 75, both figures being ratios. Calculate the following gains:

- Power gain (as a ratio) =
- Power gain (dB) =
- Voltage gain (dB) =
- Current gain (dB) =

Challenges

- Suppose a student calculates 6.9897 dB for the voltage gain of this amplifier. Explain where they made a mistake in this calculation.
4.2.6 Gains and impedances of multi-stage amplifiers

Write a formula estimating the over-all AC voltage gain (ratio) for each of the following multi-stage amplifier circuits, as well as other formulae for each circuit’s input and output impedance values:
4.2. QUANTITATIVE REASONING
Challenges

- Identify components that do not substantially influence $A_V$, and explain what their role is in each circuit.
4.2.7 Even versus odd harmonics

Note the effect of adding the second harmonic of a waveform to the fundamental, and compare that effect with adding the third harmonic of a waveform to the fundamental:
Now compare the sums of a fundamental with its fourth harmonic, versus with its fifth harmonic:
And again for the 1st + 6th, versus the 1st + 7th harmonics:

Examine these sets of harmonic sums, and indicate the trend you see with regard to harmonic number and symmetry of the final (Sum) waveforms. Specifically, how does the addition of an even harmonic compare to the addition of an odd harmonic, in terms of final waveshape?

Challenges

- Identify a practical application of this knowledge.
4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.
4.3.1 Audio amplifier design improvement

A student wishes to build a variable-gain audio amplifier circuit using an operational amplifier and a potentiometer, to drive a small loudspeaker so he can listen to the output of a digital audio player without having to use headphones:

Before building the project in a finalized form, the student prototypes it on a solderless breadboard to make sure it functions as intended. And it is a good thing he decided to do this before wasting time on a final version, because it sounds terrible!

When playing a song, the student can hear sound through the headphones, but it is terribly distorted. Taking the circuit to his instructor for help, the instructor suggests the following additions:

After adding these components, the circuit works great. Now, music may be heard through the speaker with no noticeable distortion.

Explain what functions the extra components perform, and why the circuit did not work as originally built.
• Another problem this student notes while prototyping his audio amplifier is that he can never turn the volume all the way off. Even at the lowest volume setting, you can still hear some sound from the loudspeaker. Identify the problem, and modify the circuit accordingly to fix it.

4.3.2 Push-pull voltage follower

A complementary push-pull transistor amplifier built exactly as shown would perform rather poorly, exhibiting crossover distortion:

Explain how such distortion would present on the display of a spectrum analyzer.

• Propose a design change to this circuit that will eliminate crossover distortion.
4.3.3 Discerning even/odd harmonics from the time domain

By visual inspection, determine which of the following periodic waveforms contain even-numbered harmonics:

A  B  C
D  E  F
G  H  I
J  K  L
M  N  O
P  Q  R

Challenges
• Identify a practical application of being able to discern even/odd harmonics from time-domain plots.

4.3.4 Single-ended versus push-pull amplifier distortion

A criticism of single-ended power amplifier stages versus push-pull power amplifier stages is that the single-ended designs tend to produce more even-numbered harmonic distortion:

![Single-ended vs Push-pull Diagram]

Explain why this is.

Challenges

• What other disadvantage(s) do single-ended power amplifiers have compared to push-pull?
4.3.5 Improving low-end amplifier performance

Suppose this amplifier circuit is tested and found to have insufficient voltage gain at low frequencies. Its mid-range and high-end gains are perfectly acceptable, but its bandwidth “rolls off” too much on the low end:

Identify any modifications that may help.

**Challenges**

- How could we measure roll-off at each stage of this circuit, rather than just at the very end.
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify** what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. **Annotate** all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.
- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

- Articulate communication is fundamental to work that is complex and interdisciplinary.

- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
APPENDIX B. INSTRUCTIONAL PHILOSOPHY

These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

---

1In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

---

4 As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

5 Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s TeX typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TeX is a programmer’s approach to word processing. Since TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TeX makes it relatively easy to learn how other people have created their own TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft *Word* suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s LaTeX extensions to TeX

Like all true programming languages, TeX is inherently extensible. So, years after the release of TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LaTeX, which is the markup language used to create all ModEL module documents. You could say that TeX is to LaTeX as C is to C++. This means it is permissible to use any and all TeX commands within LaTeX source code, and it all still works. Some of the features offered by LaTeX that would be challenging to implement in TeX include automatic index and table-of-content creation.

Tim Edwards’ Xcircuit drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
**Gimp** graphic image manipulation program

Essentially an open-source clone of Adobe’s **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

**SPICE** circuit simulation program

**SPICE** is to circuit analysis as **TeX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my “go to” application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang’s **ePiX** mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is **not** a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical visualization tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.
APPENDIX C. TOOLS USED

**gnuplot** mathematical visualization software

Another open-source tool for mathematical visualization is **gnuplot**. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my **gnuplot** output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of Gimp to do rudimentary image editing, my use of **gnuplot** only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

**Python** programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

Creative Commons License

Creative Commons Attribution 4.0 International Public License

By exercising the Licensed Rights (defined below), You accept and agree to be bound by the terms and conditions of this Creative Commons Attribution 4.0 International Public License (“Public License”). To the extent this Public License may be interpreted as a contract, You are granted the Licensed Rights in consideration of Your acceptance of these terms and conditions, and the Licensor grants You such rights in consideration of benefits the Licensor receives from making the Licensed Material available under these terms and conditions.

Section 1 – Definitions.

a. Adapted Material means material subject to Copyright and Similar Rights that is derived from or based upon the Licensed Material and in which the Licensed Material is translated, altered, arranged, transformed, or otherwise modified in a manner requiring permission under the Copyright and Similar Rights held by the Licensor. For purposes of this Public License, where the Licensed Material is a musical work, performance, or sound recording, Adapted Material is always produced where the Licensed Material is synched in timed relation with a moving image.

b. Adapter’s License means the license You apply to Your Copyright and Similar Rights in Your contributions to Adapted Material in accordance with the terms and conditions of this Public License.

c. Copyright and Similar Rights means copyright and/or similar rights closely related to copyright including, without limitation, performance, broadcast, sound recording, and Sui Generis Database Rights, without regard to how the rights are labeled or categorized. For purposes of this Public License, the rights specified in Section 2(b)(1)-(2) are not Copyright and Similar Rights.

d. Effective Technological Measures means those measures that, in the absence of proper authority, may not be circumvented under laws fulfilling obligations under Article 11 of the WIPO Copyright Treaty adopted on December 20, 1996, and/or similar international agreements.

e. Exceptions and Limitations means fair use, fair dealing, and/or any other exception or
APPENDIX D. CREATIVE COMMONS LICENSE

limitation to Copyright and Similar Rights that applies to Your use of the Licensed Material.

f. **Licensed Material** means the artistic or literary work, database, or other material to which the Licensor applied this Public License.

  g. **Licensed Rights** means the rights granted to You subject to the terms and conditions of this Public License, which are limited to all Copyright and Similar Rights that apply to Your use of the Licensed Material and that the Licensor has authority to license.

  h. **Licensor** means the individual(s) or entity(ies) granting rights under this Public License.

  i. **Share** means to provide material to the public by any means or process that requires permission under the Licensed Rights, such as reproduction, public display, public performance, distribution, dissemination, communication, or importation, and to make material available to the public including in ways that members of the public may access the material from a place and at a time individually chosen by them.

  j. **Sui Generis Database Rights** means rights other than copyright resulting from Directive 96/9/EC of the European Parliament and of the Council of 11 March 1996 on the legal protection of databases, as amended and/or succeeded, as well as other essentially equivalent rights anywhere in the world.

  k. **You** means the individual or entity exercising the Licensed Rights under this Public License. **Your** has a corresponding meaning.

Section 2 – Scope.

a. License grant.

1. Subject to the terms and conditions of this Public License, the Licensor hereby grants You a worldwide, royalty-free, non-sublicensable, non-exclusive, irrevocable license to exercise the Licensed Rights in the Licensed Material to:

   A. reproduce and Share the Licensed Material, in whole or in part; and
   B. produce, reproduce, and Share Adapted Material.

2. Exceptions and Limitations. For the avoidance of doubt, where Exceptions and Limitations apply to Your use, this Public License does not apply, and You do not need to comply with its terms and conditions.

3. Term. The term of this Public License is specified in Section 6(a).

4. Media and formats; technical modifications allowed. The Licensor authorizes You to exercise the Licensed Rights in all media and formats whether now known or hereafter created, and to make technical modifications necessary to do so. The Licensor waives and/or agrees not to assert any right or authority to forbid You from making technical modifications necessary to exercise the Licensed Rights, including technical modifications necessary to circumvent Effective Technological Measures.
For purposes of this Public License, simply making modifications authorized by this Section 2(a)(4) never produces Adapted Material.

5. Downstream recipients.

A. Offer from the Licensor – Licensed Material. Every recipient of the Licensed Material automatically receives an offer from the Licensor to exercise the Licensed Rights under the terms and conditions of this Public License.

B. No downstream restrictions. You may not offer or impose any additional or different terms or conditions on, or apply any Effective Technological Measures to, the Licensed Material if doing so restricts exercise of the Licensed Rights by any recipient of the Licensed Material.

6. No endorsement. Nothing in this Public License constitutes or may be construed as permission to assert or imply that You are, or that Your use of the Licensed Material is, connected with, or sponsored, endorsed, or granted official status by, the Licensor or others designated to receive attribution as provided in Section 3(a)(1)(A)(i).

b. Other rights.

1. Moral rights, such as the right of integrity, are not licensed under this Public License, nor are publicity, privacy, and/or other similar personality rights; however, to the extent possible, the Licensor waives and/or agrees not to assert any such rights held by the Licensor to the limited extent necessary to allow You to exercise the Licensed Rights, but not otherwise.

2. Patent and trademark rights are not licensed under this Public License.

3. To the extent possible, the Licensor waives any right to collect royalties from You for the exercise of the Licensed Rights, whether directly or through a collecting society under any voluntary or waivable statutory or compulsory licensing scheme. In all other cases the Licensor expressly reserves any right to collect such royalties.

Section 3 – License Conditions.

Your exercise of the Licensed Rights is expressly made subject to the following conditions.

a. Attribution.

1. If You Share the Licensed Material (including in modified form), You must:

A. retain the following if it is supplied by the Licensor with the Licensed Material:

i. identification of the creator(s) of the Licensed Material and any others designated to receive attribution, in any reasonable manner requested by the Licensor (including by pseudonym if designated);

ii. a copyright notice;
iii. a notice that refers to this Public License;

iv. a notice that refers to the disclaimer of warranties;

v. a URI or hyperlink to the Licensed Material to the extent reasonably practicable;

B. indicate if You modified the Licensed Material and retain an indication of any previous modifications; and

C. indicate the Licensed Material is licensed under this Public License, and include the text of, or the URI or hyperlink to, this Public License.

2. You may satisfy the conditions in Section 3(a)(1) in any reasonable manner based on the medium, means, and context in which You Share the Licensed Material. For example, it may be reasonable to satisfy the conditions by providing a URI or hyperlink to a resource that includes the required information.

3. If requested by the Licensor, You must remove any of the information required by Section 3(a)(1)(A) to the extent reasonably practicable.

4. If You Share Adapted Material You produce, the Adapter’s License You apply must not prevent recipients of the Adapted Material from complying with this Public License.

Section 4 – Sui Generis Database Rights.

Where the Licensed Rights include Sui Generis Database Rights that apply to Your use of theLicensed Material:

a. for the avoidance of doubt, Section 2(a)(1) grants You the right to extract, reuse, reproduce, and Share all or a substantial portion of the contents of the database;

b. if You include all or a substantial portion of the database contents in a database in which You have Sui Generis Database Rights, then the database in which You have Sui Generis Database Rights (but not its individual contents) is Adapted Material; and

c. You must comply with the conditions in Section 3(a) if You Share all or a substantial portion of the contents of the database.

For the avoidance of doubt, this Section 4 supplements and does not replace Your obligations under this Public License where the Licensed Rights include other Copyright and Similar Rights.

Section 5 – Disclaimer of Warranties and Limitation of Liability.

a. Unless otherwise separately undertaken by the Licensor, to the extent possible, the Licensor offers the Licensed Material as-is and as-available, and makes no representations or warranties of any kind concerning the Licensed Material, whether express, implied, statutory, or other. This includes, without limitation, warranties of title, merchantability, fitness for a particular purpose, non-infringement, absence of latent or other defects, accuracy, or the presence or absence of errors,
whether or not known or discoverable. Where disclaimers of warranties are not allowed in full or in part, this disclaimer may not apply to You.

b. To the extent possible, in no event will the Licensor be liable to You on any legal theory (including, without limitation, negligence) or otherwise for any direct, special, indirect, incidental, consequential, punitive, exemplary, or other losses, costs, expenses, or damages arising out of this Public License or use of the Licensed Material, even if the Licensor has been advised of the possibility of such losses, costs, expenses, or damages. Where a limitation of liability is not allowed in full or in part, this limitation may not apply to You.

c. The disclaimer of warranties and limitation of liability provided above shall be interpreted in a manner that, to the extent possible, most closely approximates an absolute disclaimer and waiver of all liability.

Section 6 – Term and Termination.

a. This Public License applies for the term of the Copyright and Similar Rights licensed here. However, if You fail to comply with this Public License, then Your rights under this Public License terminate automatically.

b. Where Your right to use the Licensed Material has terminated under Section 6(a), it reinstates:

1. automatically as of the date the violation is cured, provided it is cured within 30 days of Your discovery of the violation; or

2. upon express reinstatement by the Licensor.

For the avoidance of doubt, this Section 6(b) does not affect any right the Licensor may have to seek remedies for Your violations of this Public License.

c. For the avoidance of doubt, the Licensor may also offer the Licensed Material under separate terms or conditions or stop distributing the Licensed Material at any time; however, doing so will not terminate this Public License.

d. Sections 1, 5, 6, 7, and 8 survive termination of this Public License.

Section 7 – Other Terms and Conditions.

a. The Licensor shall not be bound by any additional or different terms or conditions communicated by You unless expressly agreed.

b. Any arrangements, understandings, or agreements regarding the Licensed Material not stated herein are separate from and independent of the terms and conditions of this Public License.

Section 8 – Interpretation.

a. For the avoidance of doubt, this Public License does not, and shall not be interpreted to, reduce, limit, restrict, or impose conditions on any use of the Licensed Material that could lawfully
APPENDIX D. CREATIVE COMMONS LICENSE

be made without permission under this Public License.

b. To the extent possible, if any provision of this Public License is deemed unenforceable, it shall be automatically reformed to the minimum extent necessary to make it enforceable. If the provision cannot be reformed, it shall be severed from this Public License without affecting the enforceability of the remaining terms and conditions.

c. No term or condition of this Public License will be waived and no failure to comply consented to unless expressly agreed to by the Licensor.

d. Nothing in this Public License constitutes or may be interpreted as a limitation upon, or waiver of, any privileges and immunities that apply to the Licensor or You, including from the legal processes of any jurisdiction or authority.
Creative Commons is not a party to its public licenses. Notwithstanding, Creative Commons may elect to apply one of its public licenses to material it publishes and in those instances will be considered the “Licensor.” Except for the limited purpose of indicating that material is shared under a Creative Commons public license or as otherwise permitted by the Creative Commons policies published at creativecommons.org/policies, Creative Commons does not authorize the use of the trademark “Creative Commons” or any other trademark or logo of Creative Commons without its prior written consent including, without limitation, in connection with any unauthorized modifications to any of its public licenses or any other arrangements, understandings, or agreements concerning use of licensed material. For the avoidance of doubt, this paragraph does not form part of the public licenses.

Creative Commons may be contacted at creativecommons.org.
Appendix E

References


Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

3 November 2021 – explicitly stated Fourier’s Theorem.

29 September 2021 – corrected a typographical error in the Tutorial, identified by Jacob Stormes.

13 September 2021 – added a one-page section reviewing energy efficiency as it applies to amplifier circuits.

10 May 2021 – commented out or deleted empty chapters.

2 December 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

25 November 2020 – added Tutorial section on class of operation.

10 August 2020 – added Tutorial text on gain-bandwidth product (GBW).

11 May 2020 – added a new Quantitative problems showing a variety of different multi-stage amplifiers and asking for formulae estimating the voltage gain and impedances of each.

29 April 2020 – added new Conceptual problems on logic gates, as well as a new Diagnostic problem on improving low-end performance for a transistor-coupled amplifier.


26 March 2020 – document first created.
Index

Adding quantities to a qualitative problem, 74
Annotating diagrams, 73
Bel, 28
Bias voltage, 10
Characteristic impedance, 16
Checking for exceptions, 74
Checking your work, 74
Class A, 10
Class AB, 12
Class B, 11
Class C, 12
Code, computer, 81
Common logarithm, 7, 28
Conservation of Energy, 5
dB, 28
dBm, 32
dBW, 33
Decibel, 6, 28
Degenerative feedback, 17
Dimensional analysis, 73
Edwards, Tim, 82
Efficiency, 9, 12
Feedback, 17
FET, 23
Field-effect transistor, 23
Fourier transform, 34
Fourier's Theorem, 23
Fourier, Jean Baptiste Joseph, 23
Fundamental frequency, 23, 25
Gain, 6
Gain, amplifier, 28
Gain-bandwidth product, 20
Graph values to solve a problem, 74
Greenleaf, Cynthia, 39
Harmonic frequency, 23, 25
How to teach with these modules, 76
Hwang, Andrew D., 83
Identify given data, 73
Identify relevant principles, 73
Inefficiency, 9, 10
Instructions for projects and experiments, 77
Intermediate results, 73
Inverted instruction, 76
Joule's Law, 7, 35
Knuth, Donald, 82
Lamport, Leslie, 82
Laplace transform, 34
Limiting cases, 74
Linearity, 21
Logarithm, common, 7, 28
Maximum Power Transfer Theorem, 15
Metacognition, 44
Moolenaar, Bram, 81
Murphy, Lynn, 39
Negative feedback, 17
Noise floor, 25
Normalized measurement, 25
Open-source, 81
Parasitic capacitance, 17
Power supply rejection, 13
Problem-solving: annotate diagrams, 73
97
Problem-solving: check for exceptions, 74
Problem-solving: checking work, 74
Problem-solving: dimensional analysis, 73
Problem-solving: graph values, 74
Problem-solving: identify given data, 73
Problem-solving: identify relevant principles, 73
Problem-solving: interpret intermediate results, 73
Problem-solving: limiting cases, 74
Problem-solving: qualitative to quantitative, 74
Problem-solving: quantitative to qualitative, 74
Problem-solving: reductio ad absurdum, 74
Problem-solving: simplify the system, 73
Problem-solving: thought experiment, 73
Problem-solving: track units of measurement, 73
Problem-solving: visually represent the system, 73
Problem-solving: work in reverse, 74

Qualitatively approaching a quantitative problem, 74

Rail, power supply, 8
Reading Apprenticeship, 39
Reductio ad absurdum, 74–76
Rejection, 13
Richter scale, 6
Ripple rejection, 13

Schoenbach, Ruth, 39
Scientific method, 44
Second Law of Thermodynamics, 9
Simplifying a system, 73
Sinusoid, 23
Socrates, 75
Socratic dialogue, 76
Spectrum analyzer, 23
SPICE, 39
Square-law device, 23
Stallman, Richard, 81
Surge impedance, 16

Tank circuit, 12
Thought experiment, 73
Torvalds, Linus, 81
Transform function, 34

Transmission line, 16
Units of measurement, 73
Visualizing a system, 73
Waste, 9
Wavelength, 16
Work in reverse to solve a problem, 74
WYSIWYG, 81, 82

Waste, 9
Wavelength, 16
Work in reverse to solve a problem, 74
WYSIWYG, 81, 82