Radio Antennas

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Chapter 1

Introduction

Antennas are deceptively complex components in an RF system, because at first inspection they resemble nothing more than some pieces of wire (or even a single piece of wire). However, the parasitic capacitance and inductance inherent to those pieces of wire make antennas complex indeed.

This module is a very shallow introduction to the very deep world of antenna function and design. The fact that RF engineers may easily specialize in their careers as antenna designers gives you some indication of just how complex this subject is. In fact, antenna design is complex enough that computers running genetic algorithms have been used to evolve novel antenna designs by simulation-testing random shapes until an optimum shape emerges from the trials!

Important concepts related to the topic of antennas include frequency versus wavelength, electromagnetic radiation, resonance, fundamental versus harmonic frequencies, polarization, reciprocity, directionality, antenna gain, dipole antennas, monopole antennas, loop antennas, transmission lines, standing wave ratio, signal reflections, radiation patterns, SCADA systems, RTU and MTU units, radiation resistance, balanced versus unbalanced signals, decibels, and the Law of Energy Conservation.

Here are some good questions to ask of yourself while studying this subject:

- How does an antenna work to radiate (i.e. transmit) electromagnetic waves?
- How does an antenna work to intercept (i.e. receive) electromagnetic waves?
- How does the physical size of an antenna relate to its ideal operating frequency?
- Why is the physical orientation of an antenna important to its function?
- How can an antenna have a gain as a purely passive device, without violating the Law of Energy Conservation?
- What purpose is served by the “reflector” and “director” elements of a Yagi antenna?
- What useful information may be gleaned from an antenna’s radiation pattern diagram?
- Must transmitting and receiving antennas be differently designed, or will the same design work equally well in both modes?
• What is an isotropic antenna, and why is the concept an important one?
• Is an antenna an electrical load, source, or something else?
• How can you tell if an antenna’s length has been properly trimmed for the transmitter it connects to?
• Where might a directional antenna be best applied?
• What is a balanced signal, and how does this differ from an unbalanced signal?
• What do the following terms mean: $dB$, $dBd$, and $dBi$?
Chapter 2

Case Tutorial

The idea behind a Case Tutorial is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain why the circuits behave as they do?
2.1 Example: making a 900 MHz antenna

A great many industrial and consumer grade electronic devices use the 902 MHz to 928 MHz frequency band which is one of several “ISM” (Industrial, Scientific, Medical) bands requiring no FCC license within the United States if operating below specified maximum EIRP power levels. Making your own monopole ("whip") antenna for one of these devices is easy to do using an RF connector and a set of solid metal wires for the driven monopole element and the radial counterpoise elements (used to form a "ground plane") for the driven element, as we see in the following photograph:

When the radio transceiver uses a coaxial connector for the antenna connection as is the case in our application, the signal is necessarily unbalanced. Since a monopole antenna is also unbalanced, there will be no need for a balun between the two as would be the case if we used a balanced antenna design such as a dipole.

The wavelength of a 902 MHz signal is approximately 0.3324 meters, with one-quarter of that being 83.1 millimeters. This will be the (ideal) length of the driven monopole element. Radial lengths are less critical, and in this case were 72 millimeters each. By using a suitable test instrument such as a vector network analyzer (VNA) or impedance analyzer, we will be able to measure the antenna to determine its fundamental resonant frequency, and to trim the driven element wire length to obtain the best results.
2.1. *EXAMPLE: MAKING A 900 MHZ ANTENNA*

When tested on a VNA sweeping between 800 MHz and 1 GHz the following result was obtained and plotted in the form of a Smith Chart showing the antenna’s impedance as a complex-numbered value:

Markers 2 and 3 represent the bounds of our ISM band, 902 MHz and 928 MHz respectively. We achieve a non-reactive (i.e. purely resistive) impedance where the trace crosses the horizontal line inside the Smith chart, and this appears to happen at a frequency *below* 902 MHz. This means our 83.1 millimeter driven element length is in reality slightly too long for the 902-928 MHz frequency range. The reason for this is a phenomenon known as *end effect* where the effective capacitance at the tip of the driven element is greater than what it would be if the wire were infinite in length (i.e. no end). This extra capacitance results in any antenna’s driven element resonating at a frequency that is *lower* than what we ideally predict based on signal wavelength.
CHAPTER 2. CASE TUTORIAL

Another useful plot of antenna performance is its *Standing Wave Ratio*, or SWR. The ideal and lowest-possible SWR value for any antenna is unity (1), with any value greater than that being less than ideal:

Here we see the SWR achieving its lowest value at a frequency that is one again below 902 MHz. This also tells us the antenna's driven element is physically too long.
2.1. EXAMPLE: MAKING A 900 MHZ ANTENNA

Perhaps the most dramatic plot showing the antenna’s resonant frequency is the VNA’s *phase* measurement, showing the polar phase angle of the antenna’s measured impedance. At resonance that phase angle should be zero. Immediately above or below resonance the impedance phase angle will jump to a large positive value (inductive) or negative value (capacitive), respectively. The sharp vertical “jump” we see in this phase plot represents the narrow frequency over which the antenna transitions from predominantly capacitive to predominantly inductive:

Once again this transition occurs below 902 MHz (to the left of marker #2), telling us the antenna’s driven monopole element is physically too long.
Lastly, we see the VNA’s plot showing reflected signal magnitude in decibels. Ideally an antenna radiates all energy received from the wired signal source into space, reflecting none of that energy back to the signal source. So, ideally the reflected signal value at the antenna’s operating frequency should be zero in magnitude, or infinitely less than the “forward” energy sent to the antenna from the source. A large negative decibel value, therefore, is what we should expect to see at the operating frequency for the reflected signal magnitude:

As with the other measurements, we see this ideal point occurring at a frequency below marker #2 (902 MHz) which tells us the driven element is physically too long and needs to be trimmed shorter.
In these next three screenshots, we see SWR measurements taken of this antenna with the original length (top), trimmed by 3 millimeters (middle), and trimmed an additional 2.25 millimeters (bottom):

After the first 3 millimeter trimming the SWR reaches its lowest value at a frequency of 911 MHz (marker #1). After the additional 2.25 millimeter trimming the SWR minimum point is at 927 MHz, again shown by marker #1. As you can see, the process of trimming an antenna at this frequency range is a delicate one indeed!
With the SWR minimum at 927 MHz, the Smith chart showed an antenna impedance of $37.7 + j1$ Ohms at that frequency which is nearly perfectly resistive:
Chapter 3

Tutorial

3.1 Basic antenna types

A radio wave is a form of electromagnetic radiation, comprised of oscillating electric and magnetic fields, with the wavelength being the physical distance traveled by the wave in the time of one cycle (period). An antenna is nothing more than a conductive structure designed to emit radio waves when energized by a high-frequency electrical power source, and/or generate high-frequency electrical signals when intercepting radio waves. Three common antenna designs appear here:

The Yagi antenna, with its “director” and “reflector” elements fore and aft of the dipole element, exhibits a high degree of directionality, whereas the dipole and monopole (“whip”) antennas tend to emit and receive electromagnetic waves equally well in all directions perpendicular to their axes. Directional antennas are ideal for applications such as radar, and also in point-to-point communication applications. Omnidirectional antennas such as the dipole and whip are better suited for applications requiring equal sensitivity in multiple directions.
A photograph of an actual Yagi antenna used in a SCADA system appears here:

The wavelength ($\lambda$) of any wave is its propagation velocity divided by its frequency. For radio waves, the propagation velocity is the speed of light ($2.99792 \times 10^8$ meters per second, commonly represented as $c$), and the frequency is expressed in Hertz:

$$\lambda = \frac{c}{f}$$

Antenna dimensions are related to signal wavelength because antennas work most effectively in a condition of electrical resonance, and an antenna’s geometry dictates the amount of parasitic capacitance and inductance its elements possess. In other words, the physical size of the antenna is such that it will electrically resonate at certain frequencies: a fundamental frequency as well as certain harmonics (integer-multiples) of that fundamental frequency. For this reason, antenna size is inversely proportional to signal frequency: low-frequency antennas must be large, while high-frequency antennas may be small.
3.2. **ANTENNA SIZE**

For example, a quarter-wave “whip” antenna designed for a 900 MHz industrial transceiver application will be approximately\(^1\) 8.3 centimeters in length. The same antenna design applied to an AM broadcast radio transmitter operating at 550 kHz would be approximately *136 meters* in length!

The following photograph shows a *half-wave* “whip” antenna, located at the roofline of a building. The additional length of this design makes it more efficient than its quarter-wave cousin. This particular antenna stands approximately one meter in length from connector to tip, yielding a full wavelength value \((\lambda)\) of 2 meters, equivalent to 150 MHz:

3.2 **Antenna size**

Antennas are fundamentally resonant components: their parasitic capacitance and inductance able to store and release energy in the forms of electric and magnetic fields, respectively, and this enables antennas to exchange energy back and forth between these complementary forms. An antenna will function most efficiently — whether transmitting or receiving — when operated in a condition of resonance where the signal frequency matches that of the antenna’s natural oscillations. The relationship between ideal frequency and antenna length is inverse: the longer the antenna, the lower its fundamental frequency, and vice-versa.

\(^1\)Due to the “end effect” of lumped capacitance at the tip of the antenna, an actual quarter-wave antenna needs to be slightly shorter than an actual quarter of the wavelength. This holds true for dipoles and other antenna designs as well.
Dipole antennas operate best at a wavelength twice as long as the total length of wire:

\[ \lambda = \frac{c}{f} \]

"Half-wave" dipole antenna operating in transmission mode

A “half-wave” dipole antenna with a total length of 5 meters\(^2\) will radiate optimally at a frequency of 30 MHz, given that the velocity of electromagnetic waves in open space is approximately \(3 \times 10^8\) meters per second. This same antenna will also effectively resonate at any harmonic (integer multiple) of 30 MHz (e.g. 60 MHz, 90 MHz, 120 MHz, etc.) just as a transmission line is able to resonate at a fundamental frequency as well as any harmonic thereof.

A popular variation on the theme of the half-wave dipole antenna is the so-called monopole, quarter-wave, or “whip” antenna, which is exactly what you might think it is: one-half of a half-wave antenna. Instead of two wires pointed away from each other, we substitute an earth-ground connection for one of the wires:

"Quarter-wave" monopole antenna operating in transmission mode

Quarter-wave antennas tend to be less effective than half-wave antennas, but are usually much more convenient to construct for real applications.

\(^2\)In practice, the ideal length of a dipole antenna turns out to be just a bit shorter than theoretical, due to lumped-capacitive effects at the wire ends. Thus, a resonant 30 MHz half-wave dipole antenna should actually be about 4.75 meters in length rather than exactly 5 meters in length.
3.2. ANTENNA SIZE

For applications where the ideal length of an antenna is impractically long, resonance may be achieved using a shorter antenna by adding a loading coil, usually located in the center of the antenna. The loading coil adds inductance to the antenna structure, reducing the resonant frequency just as increased inductance will decrease resonant frequency for any LC “tank circuit” network. The photograph on the right-hand side shows a Citizen’s Band (CB) antenna magnetically mounted to the roof of a vehicle, enabling the antenna to resonate while being much shorter than the 2.73-2.78 meters normally required for a quarter-wave antenna operating between 26.965 MHz and 27.405 MHz (the “11 meter” band permitted for CB communications in the United States):

It is important to understand that quarter-wave (or “monopole”) antennas absolutely require a conductive surface perpendicular to the antenna in order to properly function. In other words, this conductive plane is really the other half of the antenna, serving the same purpose as the second element of a dipole. In many installations the Earth itself serves as this conductive surface, and good antenna performance requires an electrically sound connection with the Earth. If the soil beneath an antenna is too dry and therefore not very conductive, it may be augmented by an array of horizontal or angles wires extending radially from the base of the antenna functioning as a ground plane, or counterpoise.

Locating a quarter-wave antenna above a metal-roofed building is another way to achieve a
good ground plane for effective operation. On boats and ships the water itself acts as an excellent ground plane for the antenna, but once again a good “solid” electrical connection between the radio equipment and the water must be established for good performance. For metal-hulled vessels this task is easy: simply use the metal hull itself as the connection point with the water. On fiberglass-hulled vessels some metallic conductor must be inserted into the water to make this connection.

While proper antenna length and grounding benefits any radio receiver’s reception, these factors are especially important for safe and effective radio transmitter operation because the antenna must act as a resistive load to receive the radio transmitter’s output power and convert as much of that electrical power into electromagnetic waves as possible. A poorly-configured antenna system will not only reduce the effectiveness of a radio transmitter, but may actually cause damage to the transmitter if too much of the RF energy is “reflected” back to the transmitter from the mismatched antenna.

Aside from ensuring secure grounding for a quarter-wave antenna, the length of that antenna should be carefully adjusted for resonance. A good tool to use for antenna “trimming” is a Standing Wave Ratio or SWR meter which shows the magnitude of “standing waves” along the transmission line connecting transmitter to antenna, with the ideal SWR being unity (1) and increasing as standing waves grow stronger. An antenna operating in a condition of resonance is purely resistive, which means its inductive and capacitive reactances ($X_L$ and $X_C$) cancel out at that frequency. Lengthening an antenna increases both its inductance and capacitance, resulting in a lower resonant frequency; shortening an antenna raises its resonant frequency. Proper antenna length may be checked by measuring its SWR while adjusting the transmitter’s frequency up and down. If an antenna is too long, it will resonate at too low of a frequency, which means the SWR will improve (i.e. SWR decreases toward 1) as transmitter frequency is decreased. If an antenna is too short, its resonant frequency will be too high, which means the SWR will improve as transmitter frequency increases. A properly trimmed antenna will exhibit its best SWR at the transmitter’s center frequency. Some radio transmitters have an SWR meter built in as a feature for antenna adjustment, as seen below in the close-up view of a Citizen’s Band transceiver faceplate, where the upper-most scale of the analog meter face reads in SWR (unitless) as the unit outputs power to the antenna with the microphone’s “talk” button depressed:

3Recall how pulse energy reflects off the end of any transmission line that is imperfectly terminated (i.e. terminated by any impedance not precisely matching the transmission line’s characteristic impedance). An ideal transmitter/line/antenna system is one where the antenna functions as a purely resistive load matching the line’s impedance, which in turn matches the transmitter’s output impedance. An antenna of improper length will exhibit reactance as well as resistance, causing reflected signals back to the transmitter, manifesting as standing waves along the transmission line that may be measured with an SWR meter. Ideally the SWR of any properly-terminated transmission line will be unity (1), with SWR increasing as standing waves grow in intensity.

4On a Citizen’s Band transmitter, the “channel” number is proportional to frequency, with channel 1 being 26.965 MHz and channel 40 being 27.405 MHz.
3.3 Antenna orientation and directionality

Another factor affecting antenna performance is the orientation of the receiving antenna with respect to the transmitting antenna. Antenna orientation matters because electromagnetic waves are polarized: their electric and magnetic fields oscillating in specific spatial planes (at right angles to each other). The plane of a dipole antenna’s wires, for example, defines the plane of electric field oscillation, while the magnetic field will be in a plane perpendicular to the wires. Antenna conductors should therefore be parallel to each other in order to maximize reception, in order that the electric and magnetic fields emanating from the wires of the transmitting antenna will “link” properly with the wires of the receiving antenna(s). If the goal is optimum communication in any direction (omnidirectionality), dipole and whip antennas should be arranged vertically so that all antenna conductors will be parallel to each other regardless of their geographic location.

Omnidirectionality may seem like a universally good trait for any antenna: to be able to transmit and receive electromagnetic waves equally well in any direction. However, there are good reasons for wanting directionality in an antenna. One reason is for greater security: perhaps you have an application where you do not wish to broadcast information in all directions, where anyone at all could receive that information. In that case, the best antennas to use would be those that work best in one direction and one direction only, with transmitting and receiving antenna pairs pointed directly at each other.

Another reason for antenna directionality is improved reception. As noted before, the AC signal received at an antenna is very small, typically on the order of microVolts. Since electromagnetic radiation tends to “spread” as it travels, becoming weaker with distance from the transmitting antenna, long-range radio communication benefits from increased sensitivity. A transmitting antenna that is directional will focus more of its power in one direction than in others, resulting in less “wasted” power radiated in unwanted directions. Likewise, a receiving antenna that is directional does a better job of “collecting” the electromagnetic energy it receives from that direction (as well as ignoring electromagnetic waves coming from other directions).
A simple yet highly effective antenna design for directional transmission and reception is the Yagi, named after its inventor Hidetsugu Yagi. A Yagi is based on a half-wave dipole element surrounded by one or more wires longer than $\lambda/2$ to the rear (called “reflectors”) and one or more wires shorter than $\lambda/2$ to the front (called “directors”). The terms “reflector” and “director” are quite apt, describing their interaction with electromagnetic waves from the perspective of the dipole: reflectors reflect the waves, while directors direct the waves. The result is an antenna array that is much more directional than a simple dipole:
3.3. **ANTENNA ORIENTATION AND DIRECTIONALITY**

An example of a Yagi antenna used as part of a municipal wastewater SCADA system is shown in this photograph, the antenna of course being the multi-element array in the upper-right corner:

This particular antenna is part of a radio communication network between “pumping stations” in a city, these stations acting to pump wastewater from gravity-fed collection sumps to the centralized treatment facility.
Another example of a highly directional antenna design is the parabolic dish, often used in microwave and satellite communications. This photograph shows two “dish” antennas, one open to the weather (right) and the other covered with a weather-proof diaphragm to keep the antenna elements protected (left):

Some dish antennas use mesh or metal-tube reflectors rather than a solid parabolic reflector, as is the case in this next photograph:
3.4 Antenna characteristics

Different antenna designs vary in several important performance metrics, including:

- Radiation pattern
- Input impedance
- Feed signal type (balanced or unbalanced)

**Radiation pattern** is a geometric expression of how well an antenna sends and receives electromagnetic radiation at different directions. No real antenna is perfectly symmetrical in this regard, and in fact it can be a decided advantage to use an antenna with a “preferred” direction of operation.

We generally denote the radiation pattern for any antenna by sketching a shaded area surrounding the antenna in all directions. One should not think that the outer edges of these shaded areas represents some “hard” boundary beyond which no radiation is emitted (or detected). In reality, the radiation patterns extend out to infinity (assuming otherwise empty space surrounding the antenna). Instead, the size of each shaded area simply represents how effective the antenna is in that direction compared to other directions – just like a “polar” style mathematical plot, where the distance from the graph’s center (origin) point to the periphery of the drawn function represents magnitude at that particular angle or direction. In the case of the vertical whip and dipole antennas, for instance, the radiation patterns show us that these antennas have zero effectiveness along the vertical (Y) axis centerline. To express this in anthropomorphic terms, these antenna designs are “deaf and mute” in those directions where the radiation pattern is sketched having zero radius from the antenna center.

**Input impedance** for any antenna is the amount of electrical impedance it presents when part of a functioning radio system. This is important because the antenna must connect to a transmitter or receiver circuit, generally through a cable of some sort, and in order to avoid “reflected” signals and other inefficiencies the antenna’s impedance must match (or be transformed to match) with other components in the system.

Regarding **feed signal type**, voltage signals may be broadly grouped into two categories: *balanced* and *unbalanced*. All voltages are measured between two points, and if one of those points is Earth ground then the signal is known as “single-ended” or “unbalanced”. When the voltage in question is measured between two points, neither one of which is equipotential to Earth ground, then the signal is known as “differential” or “balanced”. If an antenna is connected to the wrong type of signal, its radiation resistance, input impedance, and general performance may significantly degrade.

Antenna positioning is an extremely important factor for its performance. The height above the Earth, distance from electrically conductive surfaces, and orientation all affect an antenna’s characteristic measurements and performance. Proximity to any conductive mass in particular (including the Earth) distorts an antenna’s radiation pattern, which in turn affects how well it can transmit and receive in certain directions. If there is any consistent rule to follow when setting up antennas for maximum performance, it is this: position them as high as possible and as far away from interfering objects as possible!
One final note on these antenna types and characteristics is that they are cited for antennas operating at their fundamental (resonant) frequency. If an antenna is operated at some frequency substantially different than this, its radiation pattern and input impedance will substantially change!

### 3.4.1 Vertical monopole antenna

Often called a *whip* antenna, these are quite popular on mobile radio equipment.

### 3.4.2 Vertical dipole antenna
3.4. ANTENNA CHARACTERISTICS

3.4.3 Vertical loop antenna

So far all the antenna designs discussed utilize open-ended conductors for the radiating elements. However, conductive loops\(^5\) are also capable of radiating and receiving electromagnetic energy:

The radiation pattern for a vertical loop antenna with a perimeter length equal to \(\lambda/2\) is orthogonal to that of a half-wave dipole, with maximum gain in the plane of the loop rather than in the plane of Earth ground (i.e. the Z plane). Loop antennas are seldom used because their loss resistance to radiation resistance ratio is fairly high, which means a greater amount of transmitter power becomes “wasted” in the form of heat than with “open” types of antennas.

One particular variation of the loop design is called the folded dipole, where the loop is rectangular in shape rather than square. If the shorter dimension of this loop is a small fraction of one wavelength, the antenna will have the same basic radiation pattern as a normal (open-ended) dipole:

Folded dipole antennas typically exhibit input impedance values approximately four times that of an open-ended dipole, usually around 280 to 300 Ohms.

\(^5\)The actual shape of the loop is less important than one might suspect. Square, circular, and rhombus shapes all function well, with the primary consideration being physical support of the wire loop.
3.4.4 Vertical Yagi antenna

A popular directional antenna design is the Yagi, using passive “reflector” and “director” elements to focus the dipole’s sensitivity in one direction:

Yagi antenna pairs may be horizontally or vertically oriented, so long as the transmitting and receiving Yagis are both mounted with the same polarization and face each other. In industrial SCADA radio applications, Yagi antennas are generally oriented with the dipole wires vertical, so that they may be used in conjunction with omnidirectional whip or dipole antennas. An illustration of such use is shown here, with multiple “Remote Terminal Unit” (RTU) transceivers communicating with a central “Master Terminal Unit” (MTU) transceiver:

Here, all the Yagi antennas on the RTUs are vertically oriented, so that they will match the polarization of the MTU’s whip antenna. The Yagi antennas all face in the direction of the MTU for
optimum sensitivity. The MTU – which must broadcast to and receive from all the RTUs – really needs an omnidirectional antenna. The RTUs – which need only communicate with the one MTU and not with each other – work best with highly directional antennas.

If the MTU were equipped with a Yagi antenna instead of a whip, it would only communicate well with one of the RTUs, and perhaps not at all with some of the others. If all RTUs were equipped with whip antennas instead of Yagis, they would not be as receptive to the MTU’s broadcasts (lower gain), and each RTU would require greater transmitter power to transmit effectively to the MTU.

The proximity of passive “reflector” and “director” elements near the “driven” dipole element have the effect of reducing the dipole’s radiation resistance. Some Yagi antennas therefore employ a folded dipole driven element rather than a standard dipole because the folded dipole’s impedance is naturally greater than that of a standard dipole, thus compensating for the “loading” effect of the passive elements, and bringing the Yagi antenna’s impedance closer to standard coaxial cable impedance values. A photograph of a small Yagi antenna with a folded dipole driven element is shown here:
3.4.5 Vertical corner-reflector antenna

Another directional variant of the dipole antenna is the so-called *corner-reflector* antenna, where the driven dipole element is placed in front of a V-shaped reflector array or surface. The following photograph shows a corner-reflector antenna with a folded dipole as the driven element:

![Corner-reflector antenna](image)

The radiation pattern for a corner reflector antenna is quite similar to that of a Yagi. If a solid metal reflector element is used, the rearward attenuation is quite substantial and typically better (i.e. more directive) than a Yagi.
3.4.6 Vertical log-periodic dipole antenna

An antenna design similar to the Yagi in appearance but different in purpose is the log-periodic dipole, with multiple pairs of driven dipole pairs having different lengths and opposing polarities. Unlike the Yagi design which achieves greater antenna gain with more director elements, the log-periodic antenna’s purpose is to function well over a broad range of frequencies, that “bandwidth” becoming broader with additional dipole elements.

Log-periodic antennas find application where one antenna must handle the output of a transmitter covering a range of different frequencies, and/or where a receiver unit must function well over a broad range of frequencies using the same antenna. An example of the latter is this log-periodic antenna used inside of an RF test chamber to monitor emitted radio signals from various types of electronic devices. The range of possible emitted signals from an arbitrary selection of electronic devices is quite wide, which is why the log-periodic antenna is a good choice in this application:

If you closely examine this photograph, you will note that the aluminum antenna elements attach alternately to two aluminum “rails” which are the balanced signal conductors, so that each pair of matched-length elements has a polarity opposite its neighboring pairs.
3.5 Antenna reciprocity

A basic principle in antenna theory called reciprocity states that the efficiency of an antenna as a radiator of electromagnetic waves mirrors its efficiency as a collector of electromagnetic waves. In other words, a good transmitting antenna will be a good receiving antenna, and an antenna having a preferred direction of radiation will likewise be maximally sensitive to electromagnetic waves approaching from that same direction. To use a Yagi as an example:

Related to reciprocity is the concept of equivalent orientation between transmitting and receiving antennas for maximum effectiveness. The electromagnetic waves emitted by a transmitting antenna are polarized in a particular orientation, with the electric and magnetic fields perpendicular to each other. The same design of antenna will be maximally receptive to those waves if its elements are similarly oriented. A simple rule to follow is that antenna pairs should always be parallel to each other in order to maximize reception, in order that the electric and magnetic fields emanating from the wires of the transmitting antenna will “link” properly with the wires of the receiving antenna(s). If the goal is optimum communication in any direction (omnidirectionality), dipole and whip antennas should be arranged vertically so that all antenna conductors will be parallel to each other regardless of their geographic location.
3.6 Antenna gain calculations

A common way to express the maximal effectiveness of any antenna design is as a ratio compared to some idealized form of antenna with a more uniform radiation pattern. As with most ratio measurements in radio technology, the standard unit for antenna gain is the decibel (dB), related to a ratio of powers as follows:

\[
\text{Gain in dB} = 10 \log \left( \frac{P}{P_{\text{ref}}} \right)
\]

The most common reference standard used to calculate antenna gain is a purely theoretical device called an isotropic antenna. This is an ideally omnidirectional antenna having a perfectly spherical radiation pattern:

If a directional antenna such as a Yagi radiates (and/or receives) 20 times as much power in its most sensitive direction as an isotropic antenna, it is said to have a power gain of 13.01 dBi (13.01 decibels more than an isotropic). An alternative “reference” for comparison is a half-wave dipole antenna. Decibel comparisons against a dipole are abbreviated dBd. The assumption of an isotropic antenna as the reference is so common in radio engineering, though, you often see antenna gains expressed simply in units of dB. The assumption of isotropic reference (dBi) for antenna gain expressions is analogous to the assumption of “RMS” measurements in AC circuits rather than “peak” or “peak-to-peak”: if you see an AC voltage expressed without any qualifier (e.g. “117 Volts AC”), it is generally assumed to be an RMS measurement.

Whip antennas typically exhibit an optimal gain of 6 dBi (6 dB being approximately equal to a 4-fold magnification compared to an isotropic antenna), while Yagis may achieve up to 15 dBi (approximately equal to a 32-fold magnification). Parabolic “dish” antenna designs such as those used in microwave communication systems may achieve gains as high as 30 dBi (≈ 1000-fold magnification). Since antenna gain is not a real amplification of power – this being impossible according to the Law of Energy Conservation – greater antenna gain is achieved only by greater focus of the radiation pattern in a particular direction.
The concept of antenna gain is very easy to mis-comprehend, since it is tempting to think of any type of gain as being a true increase in power. Antenna gain is really nothing more than a way to express how concentrated the RF\textsuperscript{6} energy of a radiating\textsuperscript{7} antenna is in one direction compared to a truly omnidirectional antenna. An analogy to antenna gain is how a horn-style loudspeaker focuses its audio energy more than a loudspeaker lacking a horn. The horn-shaped speaker sounds louder than the horn-less speaker (in one direction only) because its audio energy is more focused. The two speakers may be receiving the exact same amount of electrical energy to produce sound, but the more directional of the two speakers will be more efficient transmitting sound in one direction than the other. Likewise, a horn-shaped microphone will have greater sensitivity in one direction than a comparable “omni” microphone designed to receive sound equally well from all directions. Connected to a recording device, the directional microphone seems to present a “gain” by sending a stronger signal to the recorder than the omnidirectional microphone is able to send, in that one direction.

The flip-side of high-gain antennas, loudspeakers, and microphones is how poorly they perform in directions other than their preferred direction. Any transmitting or receiving structure exhibiting a “gain” due to its focused radiation pattern must likewise exhibit a “loss” in performance when tested in directions other than its direction of focus. Referring back to directional radio antennas again, a Yagi with an advertised gain of 15 dBi (in its “preferred” direction) will exhibit a strong negative gain in the rearward direction where its ability to radiate and receive is almost non-existent.

If even more signal gain is necessary than what may be achieved by narrower radiation focus, an actual electronic amplifier may be added to an antenna assembly to boost the RF power sent to or received from the antenna. This is common for satellite antenna arrays, where the RF amplifier is often located right at the focal point of the parabolic dish. Satellite communication requires very high transmitter and receiver gains, due to inevitable signal weakening over the extremely long distances between a ground-based antenna and a satellite antenna in geosynchronous orbit around the Earth.

\textsuperscript{6}RF simply stands for Radio Frequency.
\textsuperscript{7}Or – applying the principle of reciprocity – antenna gain is really nothing more than a way to express how sensitive a receiving antenna is compared to a truly omnidirectional antenna.
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.
4.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the common logarithm, expressed as a measurement unit called the **decibel**. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of “decibel” you can think: *this is an expression of how much greater (or how much smaller) one power is to another*. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a **gain**. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:

![Diagram of an electronic amplifier with signal input and output](https://via.placeholder.com/150)

\[
\text{Gain} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{18.4 \text{ W}}{40 \text{ mW}} = 460
\]

An alternative way to express the gain of this amplifier is to do so using the unit of the **Bel**, defined as the common logarithm of the gain ratio:

\[
\log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) = \log \left( \frac{18.4 \text{ W}}{40 \text{ mW}} \right) = 2.66276 \text{ B}
\]

When you see an amplifier gain expressed in the unit of “Bel”, it’s really just a way of saying “The output signal coming from this amplifier is \(x\) powers of ten greater than the input signal.” An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the “Bel” (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the **deciBel** (\(1 \text{ dB} = \frac{1}{10} \text{ of a Bel}\)). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

\[
\text{dB} = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)
\]

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid\(^1\).

\(^1\)It is interesting to note that although the “Bel” is a metric unit, it is seldom if ever used without the metric prefix
An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

\[
\text{dB} = 10 \log(\text{Ratio})
\]

If we wish to solve for the ratio, we must “undo” all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

\[
\frac{\text{dB}}{10} = \frac{10 \log(\text{Ratio})}{10}
\]

\[
\frac{\text{dB}}{10} = \log(\text{Ratio})
\]

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

\[
10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}
\]

\[
10^{\frac{\text{dB}}{10}} = \text{Ratio}
\]

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

\[
\text{Ratio} = 10^{\frac{26.6276 \text{ dB}}{10}}
\]

\[
\text{Ratio} = 10^{2.66276 \text{ B}}
\]

\[
\text{Ratio} = 460
\]

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

---

“deci” (\(\frac{1}{10}\)). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.
We may also use decibels to express power losses in addition to power gains. There are many practical applications of this in signaling systems, both electronic and optical. One such application is filtering, where a “filter” circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is attenuation, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: cable loss. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal’s energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length\(^2\), such that the power out is less than the power in:

\[
\text{Signal } P_{\text{in}} \quad \text{Cable} \quad \text{Signal } P_{\text{out}}
\]

\[
\text{Gain} = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{37 \text{ mW}}{40 \text{ mW}} = 0.925
\]

\[
10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) = 10 \log \left( \frac{37 \text{ mW}}{40 \text{ mW}} \right) = -0.3386 \text{ dB}
\]

Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power gain is expressed as a positive decibel value, while any power loss is expressed as a negative decibel value. Any component outputting the exact same power as it takes in will exhibit a “gain” value of 0 dB (equivalent to a gain ratio of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of “greater than” and “less than”. Positive values represent powers that are greater while negative values represent powers that are lesser. Zero Bel or decibel values represent no change (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of \(10 \times 10 = 100\), whereas \(-30\) dB represents a power ratio reduction of \(\frac{1}{10} \times \frac{1}{10}\) \(\times \frac{1}{10} = \frac{1}{1000}\). Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division or power ratio: e.g. +6 dB is approximately equal to a power ratio gain of \(2 \times 2 = 4\), whereas \(-12\) dB is approximately equal to a power ratio reduction of \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}\). We may combine \pm 10 dB and \pm 3 dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of \(10 \times 10 \times 2 \times 2 = 400\).

\(^2\)For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is dielectric heating, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.
4.1. **DECIBELS**

Observe what happens if we combine a “gain” component with a “loss” component and calculate the overall power out versus power in:

![Diagram of power supply, amplifier, and cable with power values and decibel calculations.]

The overall gain of this amplifier and cable system expressed as a ratio is equal to the *product* of the individual component gain/loss ratios. That is, the gain ratio of the amplifier *multiplied* by the loss ratio of the cable yields the overall power ratio for the system:

\[
\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5
\]

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the *sum* of the individual component decibel values. That is, the decibel gain of the amplifier *added* to the decibel loss of the cable yields the overall decibel figure for the system:

\[
\text{Overall gain} = 10 \log \left(\frac{17.02 \text{ W}}{40 \text{ mW}}\right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}
\]

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

\[425.5 \approx 400 = (10) \times (10) \times (2) \times (2)\]

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

\[(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}\]

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.
Decibels always represent comparisons of power, but that comparison need not always be $P_{\text{out}}/P_{\text{in}}$ for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard “reference” power of exactly 1 milliWatt. The resulting decibel figure would be written as “dBm” in honor of the 1 milliWatt reference:

\[
P_{\text{in}} = 10 \log \left( \frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}
\]

The unit of “dBm” literally means the amount of dB “greater than” 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

\[
P_{\text{out}} = 10 \log \left( \frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}
\]

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.

Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply adding the amplifier’s gain factor yields the amplifier’s output power in dBm:

\[
P_{\text{in}}(\text{dB}) + P_{\text{gain}}(\text{dB}) = P_{\text{out}}(\text{dB})
\]

\[
16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}
\]

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.
We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of “dBW” (decibels greater than 1 Watt):

\[ P_{in} = 10 \log \left( \frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW} \]

\[ P_{out} = 10 \log \left( \frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW} \]

Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is less than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is more than the 1 Watt reference. A positive dB figure means “more than” while a negative dB figure means “less than.”

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply adding the amplifier’s gain factor yields the amplifier’s output power in dBW:

\[ P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB}) \]

\[ -13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW} \]

An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.
This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of ratios involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they “transform” one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of 
\[\begin{align*}
-0.15 \text{ decibels per foot} & \quad \text{at a signal frequency of 2.4 GHz}
\end{align*}\]
will suffer
\[\begin{align*}
-15 \text{ dB over 100 feet},

\text{and -150 dB over 1000 feet.}
\end{align*}\]
To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:

\[
\begin{align*}
\text{Power output} &= 21.8 \text{ dBm} \\
\text{Cable loss} &= -0.17 \text{ dB/ft} \
\text{Length} &= 6 \text{ feet} \\
\text{Gain} &= 45 \text{ dB} \\
\text{AC line power} &= \text{Power delivered to the load} = 62.38 \text{ dBm} \\
\end{align*}
\]

\[
\begin{align*}
21.8 \text{ dBm} + (-0.17 \text{ dB/ft}(6 \text{ ft})) + 45 \text{ dB} + (-0.17 \text{ dB/ft}(20 \text{ ft})) \\
21.8 \text{ dBm} - 1.02 \text{ dB} + 45 \text{ dB} - 3.4 \text{ dB}
\end{align*}
\]

A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, multiply to make the over-all amplifier’s power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, add to make the over-all amplifier’s power gain (dB):

\[
\begin{align*}
\text{Stage 1} \quad \text{Stage 2} \quad \text{Stage 3}
\end{align*}
\]

\[\text{In fact, logarithms are one of the simplest examples of a transform function, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the Fourier transform (converting a time-domain function into a frequency-domain function) and the Laplace transform (converting a differential equation into an algebraic equation).}\]
Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a power ratio, we cannot simply substitute $V$ or $I$ for $P$ in any of the formulae we’ve seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. $V_{out}$ is twice as large as $V_{in}$), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load’s power dissipation will be more than double when energized by a voltage twice as large. Joule’s Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power \textit{quadrupling} because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule’s Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left( \frac{P_{out}}{P_{in}} \right) = 10 \log \left( \frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left( \frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

$$\log x^y = y \log x$$

Therefore, we may simplify the decibel formula for voltage gain by removing the “2” power and making it a multiplier:

$$10 \log \left( \frac{V_{out}}{V_{in}} \right)^2 = (2)(10) \log \left( \frac{V_{out}}{V_{in}} \right) = 20 \log \left( \frac{V_{out}}{V_{in}} \right)$$

$$10 \log \left( \frac{I_{out}}{I_{in}} \right)^2 = (2)(10) \log \left( \frac{I_{out}}{I_{in}} \right) = 20 \log \left( \frac{I_{out}}{I_{in}} \right)$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.
We can see the practicality of using decibels to represent something other than electrical power by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter’s face, calibrated in decibels (DB):

Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB. A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC voltage. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically undefined. Practically, it means $-\infty$ dB, which is why the needle at the 0 Volt position “falls off” the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the nonlinear nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the logarithm function which is nonlinear.

---

4 Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

5 Your electronic calculator will complain if you attempt to take the logarithm of zero!
4.1. DECIBELS

Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

\[ V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts} \]

Setting the VOM to the 2.5 V AC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):

In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range four times as great), the meter’s needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

\[ 20 \log \left( \frac{2.5}{10} \right) = -12.04 \text{ dB} \]

Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

\[ 20 \log \left( \frac{2.5}{50} \right) = -26.02 \text{ dB} \]

\[ 20 \log \left( \frac{2.5}{250} \right) = -40.0 \text{ dB} \]
Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read\(^1\) the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture\(^2\), the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

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\(^1\)Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

\(^2\)Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 5. QUESTIONS

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

• **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an **intelligent child**: as simple as you can without compromising too much accuracy.

• **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

• Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

• Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

• Identify any **new concept(s)** presented in the text, and explain in your own words.

• Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

• Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

• Devise an experiment to **disprove** a plausible misconception.

• Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

• Describe any useful **problem-solving strategies** applied in the text.

• Devise a question of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.

- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.

- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?

- Is there more than one way to solve this problem? Which method seems best to you?

- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.

- What would you say was the most challenging part of this problem, and why was it so?

- Was any important information missing from the problem which you had to research or recall?

- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?

- Examine someone else’s solution to identify where they applied fundamental laws or principles.

- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).

- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.

- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.

- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.

- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?

- Identify where it would be easy for someone to go astray in attempting to solve this problem.

- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?

- Identify some of the challenges you faced in completing this experiment or project.
• Show how thorough documentation assisted in the completion of this experiment or project.

• Which fundamental laws or principles are key to this system’s function?

• Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.

• What will happen if (component X) fails (open/shorted/etc.)?

• What would have to occur to make this system unsafe?
5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

---

3Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
5.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

- Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

- Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

- Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

- Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

- Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

- Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
5.1. CONCEPTUAL REASONING

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Energy
- Conservation of Energy
- Parasitic effect
- Capacitance
- Inductance
- Electric field
- Magnetic field
- Electromagnetic wave
- Fundamental frequency
- Harmonic frequency
- Antenna
- Period
Frequency

Wavelength

Directionality

Resonance

Dipole

Reciprocity

SCADA

Decibels

Parabola

Radiation pattern

Isotropic

Gain

Radiation resistance

Joule’s Law
5.1.3 Parabolic dish feed

A close inspection of the following photograph reveals a small *feed antenna* element located at the focal point of the dish:

Identify the type of antenna structure used for “feeding” this dish, and explain how it functions.

**Challenges**

- Why is this dish antenna made of metal bars rather than being a solid reflector?
5.1.4 Antenna blind spots

Identify the “blind spots” of various antenna types – that is, physical locations near the antenna where radiation strength will be at a minimum (if the antenna is transmitting) and where the antenna will be least sensitive to any incoming signals.

- Whip =
- Dipole =
- Yagi =

Challenges
- Where is the blind spot for an isotropic antenna?

5.1.5 Dummy antenna

When radio technicians need to test the operation of a transmitter at significant power levels, but without actually broadcasting any signal through space, they disconnect the antenna from the transmitter’s RF output and in its place connect a dummy load consisting of a high-power resistor with a value equal to the “radiation resistance” of the normal antenna.

Explain why a resistive “dummy load” simulates an antenna realistically to the transmitter, since real antennas are simply made of metal struts and therefore contain negligible resistance.

Challenges
- What practical considerations should we consider if building a dummy load for a radio transmitter of significant power, such as a 50 kW AM broadcast transmitter?
5.1. CONCEPTUAL REASONING

5.1.6 Microwave detector

The model RFD102A-DET microwave energy detector consists of a small dipole antenna soldered to an RF-to-DC converter circuit, which in turn is soldered to a light-emitting diode. Together, these components act as a microwave field detector efficient enough to energize the LED by the radiated power from a standard home Wi-Fi network router.

In the following photographs we see one of these detectors held near a Wi-Fi router in two orientations, one of which results in the LED illuminating and the other resulting in no light:

![Microwave detector orientations](image)

Explain why the orientation matters to the LED’s illumination.

**Challenges**

- Identify one additional orientation we should test the detector in, to make this experiment thorough.
5.1.7 Transmitter testing

The Federal Communication Commission (FCC) bulletin number 63 (“Understanding the FCC Regulations for Low-Power, Non-Licensed Transmitters”) contains the following paragraph:

A particular transmitter that generates a constant level of power (Watts) can produce electric fields of different strengths ($\mu$V/m) depending on, among other things, the type of transmission line and antenna connected to it. Because it is the electric field that causes interference to authorized radio communications, and since a particular electric field strength does not directly correspond to a particular level of transmitter power, most of the Part 15 emission limits are specified in field strength.

Explain why measured electric field strength doesn’t necessarily correspond to transmitter output power. What are some of the reasons why these two may not be proportional? Be as specific as you can in your answers.

<table>
<thead>
<tr>
<th>Challenges</th>
</tr>
</thead>
<tbody>
<tr>
<td>• What shape of antenna might be best for intercepting the electric field component of an electromagnetic wave, versus intercepting the magnetic field component?</td>
</tr>
</tbody>
</table>
These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

---

4In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation ($\sigma$) of uncertainty in the final digits: for example, Avogadro’s number given as $6.02214179(30) \times 10^{23}$ means the center value ($6.02214179 \times 10^{23}$) plus or minus $0.00000030 \times 10^{23}$.

Avogadro’s number ($N_A$) = $6.02214179(30) \times 10^{23}$ per mole (mol$^{-1}$)

Boltzmann’s constant ($k$) = $1.3806504(24) \times 10^{-23}$ Joules per Kelvin (J/K)

Electronic charge ($e$) = $1.602176487(40) \times 10^{-19}$ Coulomb (C)

Faraday constant ($F$) = $9.64853399(24) \times 10^{4}$ Coulombs per mole (C/mol)

Magnetic permeability of free space ($\mu_0$) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space ($\varepsilon_0$) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space ($Z_0$) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant ($G$) = $6.67428(67) \times 10^{-11}$ cubic meters per kilogram-seconds squared (m$^3$/kg-s$^2$)

Molar gas constant ($R$) = $8.314472(15)$ Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant ($h$) = $6.62606896(33) \times 10^{-34}$ joule-seconds (J-s)

Stefan-Boltzmann constant ($\sigma$) = $5.670400(40) \times 10^{-8}$ Watts per square meter-Kelvin$^4$ (W/m$^2$.K$^4$)

Speed of light in a vacuum ($c$) = **299792458 meters per second** (m/s) = $186282.4$ miles per second (mi/s)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from [http://physics.nist.gov/constants](http://physics.nist.gov/constants), National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s *2018 CODATA recommended values* database.
5.2. QUANTITATIVE REASONING

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g., Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g., cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>( \frac{B1}{B2} )</td>
<td>( \text{km/h} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula \( \frac{B1}{B2} \), because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common\textsuperscript{7} arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure\textsuperscript{8} proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>(-B4 + sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>(-B4 - sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
</tr>
<tr>
<td>3</td>
<td>a =</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>b =</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>c =</td>
<td>-2</td>
</tr>
</tbody>
</table>

This example is configured to compute roots\textsuperscript{9} of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and $-2$ have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new $a$, $b$, and $c$ coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

\textsuperscript{7}Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

\textsuperscript{8}Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

\textsuperscript{9}Reviewing some algebra here, a root is a value for $x$ that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>= (-B4 + C1) / C2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= sqrt((B4^2) - (4<em>B3</em>B5))</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>= (-B4 - C1) / C2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 2*B3</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>b</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>c</td>
<td>-2</td>
</tr>
</tbody>
</table>

Note how the square-root term \((y)\) is calculated in cell C1, and the denominator term \((z)\) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
5.2.3 Antenna dimensions

A dipole antenna designed to resonate at a fundamental frequency of 1.8 GHz will have a total length of approximately 0.083275 meters, based on the fact that dipole antennas operate at half the wavelength ($\frac{\lambda}{2}$) of the RF energy transmitted and/or received.

If we wished to calculate this same length in a different unit of measurement such as centimeters, we could do so using a simple fraction-based technique whereby we set up the original quantity as a fraction and then multiply by other fractions consisting of physically equal numerator and denominator quantities such that the old unit is replaced by a new unit:

$$\left( \frac{0.083275 \text{ m}}{1} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) = 8.3275 \text{ cm}$$

We may continue with this same unit-conversion strategy to compute an answer in inches, knowing there are exactly 2.54 centimeters to one inch:

$$\left( \frac{0.083275 \text{ m}}{1} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) \left( \frac{1 \text{ in}}{2.54 \text{ cm}} \right) = 3.2785 \text{ in}$$

. . . and again, to compute an answer in feet:

$$\left( \frac{0.083275 \text{ m}}{1} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) \left( \frac{1 \text{ in}}{2.54 \text{ cm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 0.27321 \text{ ft}$$

This method of unit-conversion offers several advantages over other methods. First, one need only commit a few conversion ratios to memory in order to convert back and forth between many combinations of units rather than memorize (or research) conversion ratios for every combination of units encountered. Second, the accuracy of the answer is assured by careful inspection of the canceled units, which is important because this tends to be the most common way in which unit-conversions go astray: by accidently dividing when multiplication is necessary, or vice-versa.

Knowing this, determine the dimensions of a “whip” antenna designed to resonate at the fundamental frequency of 840 MHz, expressing your answer in units of meters, centimeters, feet, and inches.

Both whip and dipole antennas operate effectively at odd harmonics as well as the fundamental. Based on this fact, calculate some other operating frequencies that would be a good match for this same antenna.

**Challenges**

- Convert 40 miles into kilometers using nothing but the following ratios: 2.54 cm to 1 inch, 5280 feet to 1 mile, 12 inches to 1 foot, and of course metric prefixes (centi = 100, kilo = 1000, etc.).
5.2.4 Transmitter power and field strength

A simple pair of formulae relating approximate transmitted power to measured electric field and magnetic field strengths, respectively, useful when estimating the RF power output by a radio transmitter at a certain distance away based on measurements of the $E$ field, is as follows:

\[ S = \frac{PG}{4\pi D^2} = \frac{E^2}{Z_0} = \frac{H^2}{Z_0} \]

Where,
- $S =$ Power density of field (the Poynting vector), in Watts per square meter
- $P =$ Transmitter output power, in Watts
- $G =$ Antenna gain (1 = theoretical “isotropic” antenna)
- $D =$ Distance between transmitter antenna and receiving instrument
- $E =$ Electric field strength detected by receiving instrument, in Volts per meter
- $H =$ Magnetic field strength detected by receiving instrument, in Amperes per meter
- $Z_0 =$ Characteristic impedance of free space, approximately 376.73 Ohms

The $4\pi D^2$ term is a calculation of spherical surface area, used to estimate how much the emitted RF energy “spreads out” from the antenna based on the assumption that it radiates away equally in all directions.

First, algebraically simplify this formula as much as possible to eliminate any unnecessary constant values.

Next, algebraically manipulate this formula to solve for $P$ given a measured value $E$.

Next, replace $G$ in that manipulated formula with the antenna’s gain figure in $dBi$. 
Lastly, estimate the following transmitter powers given these measured electric field strength ($E$) values and distances:

- $E = 170$ microVolts per meter; $D = 40$ meters; Antenna gain = +2 dBi; $P =$
- $E = 800$ microVolts per meter; $D = 15$ meters; Antenna gain = +6 dBi; $P =$
- $E = 19$ milliVolts per meter; $D = 55$ meters; Antenna gain = +3 dBi; $P =$
- $E = 0.25$ Volts per meter; $D = 8$ meters; Antenna gain = +4 dBi; $P =$
- $E = 1.3$ Volts per meter; $D = 22$ meters; Antenna gain = +10 dBi; $P =$

**Challenges**

- If you double the distance between the transmitting antenna and the receiving instrument, what will this do to the measured field strength?
- These formula assume certain conditions, namely that the receiver is in the far-field region of the transmitting antenna (i.e., not too close), that there are no impedance mismatches in the receiver’s antenna system, and that the receiving antenna is properly oriented. Explain why each of these caveats matter.
5.2.5 Monopole radiation strength

A particular monopole, or “whip”, antenna has a radiation pattern as shown below:

Identify the approximate amount of signal attenuation experienced as a result of receiving this antenna’s transmitted radiation at different elevations (i.e. different angles with respect to Earth ground and the antenna’s location):

- Receiver A =
- Receiver B =
- Receiver C =
- Receiver D =

Challenges
Antenna radiation pattern plots are an example of a polar plot. Explain how this type of mathematical plot differs from the more conventional Cartesian style.
5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Whip antenna orientations

An RF consultant is summoned onto an industrial facility to diagnose a problem the technicians there are experiencing with some newly-installed wireless sensors. Each sensor has a “whip” style antenna on it with a swivel base allowing it to be positioned at any angle. When the consultant arrives, she finds each and every one of these antennas pointed straight at the base station (MTU) for the sensor network.

Identify the problem, and explain how to remedy it.

Challenges

• How well would the system work if all the whip antennas were angled so as to be parallel with the ground.
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution.** Annotate all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.
- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

• The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

• Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

• Articulate communication is fundamental to work that is complex and interdisciplinary.

• Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

• Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

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1In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

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4 As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

5 Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s \TeX \ typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, \TeX *is a programmer’s approach to word processing.* Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft *Word* suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s \LaTeX \ extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX, which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as C is to C++. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards’ Xcircuit drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
Gimp graphic image manipulation program

Essentially an open-source clone of Adobe’s PhotoShop, I use Gimp to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although Gimp does offer its own scripting language (called Script-Fu), I have never had occasion to use it. Thus, my utilization of Gimp to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as \TeX{} is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer SPICE for the following reasons: it is free, it is fast, it is reliable, and it is a fantastic tool for teaching students of electricity and electronics how to write simple code. I happen to use rather old versions of SPICE, version 2g6 being my “go to” application when I only require text-based output. NGSPICE (version 26), which is based on Berkeley SPICE version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all SPICE example netlists I strive to use coding conventions compatible with all SPICE versions.

Andrew D. Hwang’s ePiX mathematical visualization programming library

This amazing project is a C++ library you may link to any C/C++ code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as Mathematica or Maple to do. It should be said that ePiX is not a Computer Algebra System like Mathematica or Maple, but merely a mathematical visualization tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own C/C++ code!), but it can graph the results, and it does so beautifully. What I really admire about ePiX is that it is a C++ programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a C++ library to do the same thing he accomplished something much greater.
Another open-source tool for mathematical visualization is gnuplot. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my gnuplot output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of Gimp to do rudimentary image editing, my use of gnuplot only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. phasor) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

References


Young, Michael F., “Planning a Microwave Radio Link”, YDI Wireless, Falls Church, VA, 2002.

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

7-9 May 2022 – added a Case Tutorial chapter, with a section showing a home-made 900 MHz antenna.

23 April 2022 – clarified some details on radiation patterns of loop antennas, and moved this section ahead of the Yagi antenna section so that I could discuss folded dipoles as Yagi driven elements. Edited image 4242 and image 4244 for Yagi antennas. Added a subsection on corner reflector antennas. Added more index entries for antenna types.

20 April 2022 – added some content to the Tutorial on log-periodic antennas.

9 March 2022 – added some content to the Introduction chapter, and also made minor edits to some of the antenna.

12 January 2022 – added folded dipole antennas to the “Loop antenna” subsection.

7 January 2022 – changed the name of the “Antenna radiation patterns” section to “antenna characteristics” and then divided it into subsections for each antenna type. Also included comments on radiation resistance and balanced/unbalanced for each antenna type (text within the images).

10-13 October 2021 – added a dimension showing half-wave ($\frac{\lambda}{2}$) length of a Yagi antenna’s active element, in image 4237. Also, made minor edits to illustrations.

19 September 2021 – added “unity fraction” unit-conversion example to the “Antenna dimensions” Quantitative Reasoning question.

29 May 2021 – added photograph of a monopole antenna with counterpoise.

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13 May 2021 – added graphic images and text about loading coils to the Tutorial, as well as explanations of “trimming” an antenna for resonance.

10 May 2021 – commented out or deleted empty chapters.

4 May 2021 – added comments to the Tutorial about the importance of monopole antenna grounding and counterpoises.

14-15 April 2021 – added a Quantitative Reasoning question on electric field strength versus estimated transmitter power, and also corrected some capitalization errors (“volts” should have been “Volts”). Added more content to some questions, and made minor edits to the Tutorial including “elevation plane” and “azimuth plane” labels to antenna radiation pattern illustrations. Corrected error on loop antenna radiation pattern (I had azimuth and elevation views swapped).

18 November 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions. Also made some minor edits to the Tutorial.

3 September 2020 – added a Conceptual Reasoning question regarding a passive microwave detector’s orientation versus a Wi-Fi router.

27 July 2020 – document first created.
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