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Comparators

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Chapter 1

Introduction

Comparators bridge the worlds of analog and digital: analog inputs accepting continuously-variable voltage signals, followed by an output state producing a discrete (on or off) voltage signal. A comparator is very similar in function to its more sophisticated cousin the operational amplifier, but designed for discrete output states rather than an analog output as is the case for an opamp.

Important concepts related to comparators include single-ended versus differential voltage signals, amplification, gain, integrated circuits, bridge networks, feedback, Kirchhoff’s Voltage Law, voltmeter usage, pulse-width modulation, differential pairs, active loading, BJT behavior, FET behavior, and electrical noise.

Here are some good questions to ask of yourself while studying this subject:

• How might an experiment be designed and conducted to explore the operation of comparator ICs? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?

• How might an experiment be designed and conducted to explore the phenomenon of positive feedback in a comparator circuit? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?

• How may we reliably predict the output signal polarity of a comparator circuit?

• Why are the power supply “rail” voltages for an amplifier important to its performance?

• What is feedback in any system?

• Do we typically find negative feedback or positive feedback used with comparators? Why?

• How do single-ended and differential amplifiers differ from one another?

• What role does the differential pair network play inside of a comparator?

• Why is active loading often used in the internal design of comparators?
• Why might an open-collector output be preferable to a push-pull output stage for a comparator in certain applications?

• What are some common comparator features?
Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?
2.1 Example: Kirchhoff’s Voltage Law in action
2.2  Example: comparator response to various inputs

In these illustrations, I have likened the comparator’s action to that of a single-pole, double-throw switch, showing the “connection” made between power supply terminals and the output terminal:
2.3 Example: comparator demonstration circuit

This circuit uses a model LM339 comparator (just one of the four comparators contained in the LM339 IC) to demonstrate basic comparator operation:

The purpose of the potentiometer is to serve as an adjustable voltage divider, to take the 6 Volt battery’s voltage and divide it down to some adjustable value between 0 Volts and 6 Volts, inclusive. The two 10 kilo-Ohm fixed resistors also form a voltage divider, providing a fixed (non-variable) voltage equal to one-half of the battery’s voltage.

Below is a pictorial representation of the LM339 IC and other components inserted into a solderless breadboard:

Note the unusual location of the LM339’s power supply terminals: (+) on pin 3 and (−) on pin 12. Also note that the model LM339 cannot source output current, but can only sink. This is why the LED must be connected the way it is, sending current into the comparator’s output terminal when energized.
While individual LEDs will suffice, a ten-segment LED module looks much nicer:

Note the unusual location of the LM339's power supply terminals: (+) on pin 3 and (−) on pin 12. Also note that the model LM339 cannot source output current, but can only sink. This is why the LEDs must be connected the way they are, sending current into the respective comparator output terminals when energized.
2.5 Example: simple dot-graph display circuit

This circuit illuminates a single LED in the multi-LED array based on the voltage level of a signal (set by the position of the potentiometer):

The purpose of each Exclusive-OR logic gate is to activate its respective LED whenever the two input lines are different logic states. Thus, only the LED at the output of the highest-level comparator with a low output state will be lit.
2.6 Example: the effects of noise

The following oscilloscope screenshots were captured with channels A (blue) and B (red) connected to the inverting input and output terminals of a comparator, respectively, while the noninverting terminal connected to a constant DC reference voltage (not shown on the display). As you can see, instead of the output (red) being a clean “square” wave as the inverting input signal\(^1\) (blue) varies above and below the constant reference, there is some “chattering” due to noise on the channel A (blue) signal voltage:

\(^1\)This signal happened to be a low-frequency sine wave, only a small fraction of a Volt in magnitude.
This “chattering” problem was corrected by adding some positive feedback to the comparator circuit. All the screenshots you see here were taken when the inverter circuit had no positive feedback in it at all.
2.7 Example: multi-comparator voting array

In applications where high accuracy voltage signal comparison is required, one way to minimize the error-inducing effects of comparator offset voltage is by using multiple comparators to compare the same two voltage signals and use digital logic gates to “vote” based on a majority determination from the comparators.

For example, here we use three comparators to compare two ground-referenced voltage signals, their outputs going to a two-out-of-three (2oo3) voting network:

The output should go to a “high” state whenever \( V_2 > V_1 \) and go to a “low” state whenever \( V_1 > V_2 \), but the 2oo3 logic allows any single comparator to be “out-voted” by the other two if there is disagreement. This same 2oo3 voting logic also provides a greater level of reliability for the circuit as a whole in the event that one of the comparators fails with its output stuck either high or low (i.e. if it becomes unresponsive to the input signals).
2.8 Example: simple opamp experiments

A very simple arrangement for exploring fundamental comparator and operational amplifier circuit behavior may be fashioned using a pair of 9-Volt batteries, a potentiometer, and a single operational amplifier as such:

The “split” DC power supply provides a center connection (called Ground) and two “rail” connections between which we may measure voltages spanning +9 Volts (from +V to Ground) to −9 Volts (from -V to ground). The potentiometer divides the total 18 Volt differential into proportions adjustable by its wiper position, so that a voltmeter measuring between the wiper terminal ($V_{\text{signal}}$) and Ground may experience anything between +9 Volts and −9 Volts inclusive.

With this split DC power supply and adjustable signal source, we may build a range of different comparator and opamp circuits useful for self-instruction. In all cases I recommend choosing resistor values between 1 kΩ and 100 kΩ, and using a TL082 or comparable operational amplifier.
2.8. EXAMPLE: SIMPLE OPAMP EXPERIMENTS

Comparator demonstration (with hysteresis)

Positive-feedback resistor should be much larger in value than the potentiometer

Voltage follower demonstration

\( V_{out} \) should equal \( V_{in} \)
Non-inverting amplifier demonstration

\[ V_{out} \text{ should be equal to } V_{in}(\frac{R_2}{R_1} + 1) \]

Inverting amplifier demonstration

\[ V_{out} \text{ should be equal to } V_{in}(\frac{-R_2}{R_1}) \]
2.8. EXAMPLE: SIMPLE OPAMP EXPERIMENTS

Oscillator demonstration

Sample-and-hold voltage follower demonstration

$V_{\text{out}}$ should equal last $V_{\text{in}}$ value when button was pressed
Chapter 3

Tutorial
3.1 Differential amplification

It is often useful to be able to compare the magnitudes of two voltage signals and take action based on which one of them is larger. An electronic device designed for this purpose is called a comparator, because its purpose is to compare the two signals against one another. In this Tutorial we will explore how to construct comparators, as well as some of their practical uses.

Comparators are amplifiers because the output signal often needs to be stronger than the input signals being compared. The generic symbol for a simple electronic amplifier appears below, the complex internal circuitry of the amplifier represented in total by a simple triangle shape with the input terminal on the wide end and the output terminal at the far point. The amplifier’s output voltage will be determined by the input voltage multiplied by a factor \( A_V \) called voltage gain:

\[
V_{out} = A_V V_{in}
\]

Other types of input and output exist for electronic amplifiers, though. Shown below are three other variations, each one sporting at least one differential signal where the voltage in question is not referenced to ground but rather between two dedicated terminals:

In the study of electronic circuits we often encounter these two different ways of expressing signals represented by electrical voltages: differential signals where the voltage in question is measured between two un-grounded conductors (also called balanced signals), and single-ended signals where the voltage in question is measured between one un-grounded conductor and ground (also called ground-referenced or unbalanced signals). The fact that voltage by its very nature is a relative quantity – meaning it is always measured between two points and never at a single point – give us multiple ways to define how a signal voltage is expressed, conveyed, and in this case amplified.
3.1. DIFFERENTIAL AMPLIFICATION

The differential-input type of amplifier lends itself well to signal comparison. If we take two ground-referenced signals and connect them to the two inputs of a differential amplifier, the amplifier will amplify the difference between those two voltage signals and present that amplified difference at its output terminal:

\[
\begin{align*}
+V_{\text{supply}} & \quad \text{"Red lead"} \\
- V_{\text{supply}} & \quad \text{"Black lead"} \\
2 \text{ V} & \quad 5 \text{ V} \\
\end{align*}
\]

\[V_{\text{out}} = -12 \text{ V}\]

In the example shown above, \(V_{\text{out}}\) is equal to \(-12\) Volts because the differential between the two input terminals is \(-3\) Volts (i.e. the + input terminal is at a potential \(-3\) Volts with respect to the \(-\) input terminal) and the amplifier’s voltage gain is four.

Determining input voltage for a differential amplifier is often confusing for students new to this topic. A true differential amplifier responds only to the potential difference between its two input terminals. It is helpful to think of the + and − input terminals of any differential amplifier as analogous to the red and black leads, respectively, of a digital DC meter; also, to consider the mathematical sign of that digital meter’s indication as the polarity of the amplifier’s output signal.

In the following diagrams we see this “voltmeter analogy” applied to multiple scenarios for a differential amplifier with a voltage gain of one:
If we imagine building a differential amplifier with an extremely large voltage gain, then even very small differential voltages will be enough to cause the amplifier’s output to fully saturate, which means $V_{out}$ will attain (or very nearly attain) the DC power supply “rail” voltage of $+V$ or $-V$ depending on the polarity of the differential.

In the following diagrams we see the same input conditions shown previously, but this time applied to a differential amplifier with a voltage gain greater than 100,000. As you can see, the only results we get are full-saturation to the $+V$ rail or to the $-V$ rail. The extremely high gain forces the amplifier’s output to become a practically discrete (i.e. on-or-off) indication of differential input voltage polarity – i.e. the differential amplifier now behaves as a comparator:

Another way of thinking about any differential amplifier’s response is simply to compare the actual polarity of its differential input voltage against the + and – labels of its input terminals. If the real polarity matches the terminal markings, the output swings in the positive direction; if opposite, it swings negative.

The + and – input terminals of a differential amplifier are also known as the noninverting and inverting inputs, respectively. These labels also make sense if we consider the effect each of those inputs has on the output. A voltage applied to the noninverting input (with respect to ground) tends to drive the output voltage in the same direction, whereas a voltage applied to the inverting input tends to drive the output voltage in the opposite direction.
3.1. DIFFERENTIAL AMPLIFICATION

Comparators exist as integrated circuits (ICs) with all internal components on a single wafer of silicon semiconductor material, often with more than one amplifier packaged in one IC “chip”. Examples of single, dual, and quad comparators contained within DIP (Dual Inline Package) integrated circuits appear in the following illustration:

A photograph of an LM339 quad comparator IC manufactured by Motorola appears in this photograph:

Note the notch on the left-hand end of this DIP device, as well as the dot in the lower-left corner, both of which serve to “index” the integrated circuit for easy identification of pin 1 (lower-left). For integrated circuits lacking such dots and/or notches, one sure way to properly orient your view of the device is to position it so that the markings are right-side-up (as shown here). When the printed text on the device’s cover is oriented to be readable, the lower-left pin of the DIP will be pin number 1.
3.2 Practical applications

Practical applications of comparators abound. One, shown in the schematic diagram below, signals the direction of imbalance for a bridge circuit with two active sensors:

![Schematic Diagram]

The LED will energize any time the ratio $\frac{R_{\text{sensor1}}}{R_1}$ exceeds $\frac{R_{\text{sensor2}}}{R_2}$, regardless of variations in DC supply voltage. This basic circuit structure is useful for any type of rheometric (i.e. resistance-based) sensors where a comparison of two physical measurements must be determined. For example, if we were to use two cadmium-sulfide (CdS) photocells as the sensors, this circuit would indicate when sensor 1 became darker than sensor 2. If we swap out the photocells for strain gauges, this circuit will indicate when one gauge 1 stretched farther than gauge 2. If we use thermistors, this circuit will tell us when one becomes warmer than the other.\(^1\) We need not use an LED as the indicating device, either: it is just as practical to have the comparator’s output signal drive a transistor to control power to a load (e.g. heating element, electric motor, solenoid).

As you can see, the simple comparison of two DC voltage signals to energize or de-energize a load is filled with possibilities.

Another practical application is the generation of what is called a pulse-width modulation or PWM waveform from a DC voltage signal. Simply defined, a PWM waveform is a periodic pulse signal where the “width” of the “on” portion of the pulse varies but the period of its oscillation remains fixed. PWM waveforms are useful for varying power to loads by means of fast pulsing, with many practical applications including heat control for ovens and fuel injection control for internal combustion engines. A flip-book animation of a simple PWM circuit in action may be found in section 5.1 beginning on page 60.

\(^1\)Just which sensor must become warmer depends on whether we use thermistors with positive or negative temperature coefficients.
3.3 Comparator internal circuitry

The internal circuitry of a comparator is, as one might expect, fairly complex. A good representative example of a comparator is the popular model LM339 shown in the following diagram:

Most of this circuitry exists to give the comparator an extremely large differential voltage gain (typically 200,000!). Again, the purpose of having such a high voltage gain is to ensure the output of the comparator will saturate with just the slightest difference in potential between the two input terminals – i.e. that the output’s state becomes a discrete (“high” or “low”) indication of which input signal is strongest. With this being the design goal for a comparator, there is no such thing as having too much differential voltage gain!

This extremely high voltage gain comes by way of active loading of the amplifier stages. Note the absence of resistors in the differential pair stage (Q1 through Q4) and in the final output stage (Q11 and Q12). Instead, these stages have transistors as loads, with these “active” loads wired to “mirror” the same amount of current passing through other transistors (i.e. Q7 mirroring Q6’s current; Q5, Q8, and Q9 all mirroring Q10’s current.). With these active-load transistors behaving as current regulators, they exhibit extremely high dynamic resistance while passing normal amounts of quiescent (bias) current. That high dynamic resistance gives each amplifier stage enormous levels of voltage gain.
If we simplify this circuit to a bare-minimum design, it will be easier to understand:

Transistors $Q_1$ and $Q_2$ along with resistors $R_1$, $R_2$, and $R_3$ form a differential pair amplifier. Transistor $Q_3$ and resistor $R_4$ form the comparator’s gain stage: simply a common-emitter amplifier boosting the differential pair’s voltage gain. Transistor $Q_4$ forms the comparator’s output stage.

Note how in this simplified schematic as well as the full schematic that the model LM339 comparator cannot source current, but can only sink current through its output stage. Those readers familiar with TTL digital logic circuitry will recognize the similarity between this and an open-collector logic gate output. While this may seem limiting, it offers the advantage of allowing load devices to be powered by voltage levels beyond that of the comparator’s own power supply rails. Consider the following example circuit where a model LM339 comparator operating on a 5 Volt DC supply is able to send a discrete signal to a logic gate operating on a 15 Volt DC supply:

$R_{\text{pullup}}$ is necessary to provide a default “high” (+15 Volt) logic state to the inverter gate whenever the comparator is not sinking to ground.

---

2Technical Reference section 4.1 beginning on page 34 reviews the operation of differential pair amplifiers.
A comparator equipped with a *push-pull* or “totem-pole” output stage is able to both source and sink load current:

![Simplified internal schematic of a push-pull comparator](image)

The ability to source load current comes at a cost, of course, and that cost is only being able to source load current at the comparator’s +V rail voltage and no higher\(^3\). A good example of a push-pull comparator IC is the model TLC3704 which has the same “pinout” (pin assignments) as the venerable LM339, but with output current sinking and sourcing capability unlike the LM339 which can only sink current at its output:

![Pinout diagram for LM339 and TLC3704 quad comparator integrated circuits](image)

---

\(^3\)In fact, with the simple design shown, the comparator’s output voltage cannot even go that high! \(Q_4\)’s \(V_{CE}\) voltage drop of approximately 0.3 Volts when saturated “on” added to the diode’s 0.7 Volt drop means the comparator’s high-state output voltage will be approximately 1 Volt less than +V.
3.4 Comparator features

You will find that a common variation among comparator models is output type. Open-collector (also, open-drain for FET-output comparators) versus push-pull is just one pair of options. Other output options include:

- **Rail-to-Rail output** – if the comparator’s output voltage is able to “swing” all the way from one power supply “rail” voltage to the other, it is known as a rail-to-rail output. Some comparators cannot achieve this, but can only come close to the rail voltage limits at their output terminals.

- **Tri-state** – an auxiliary input terminal called Enable forces the comparator’s output terminal to become “floating” when inactivated. In this “high-impedance” mode, the comparator cannot sink or source current but is essentially “disconnected” from its load.

- **Shutdown** – similar to the tri-state output option, when the “shutdown” terminal state changes, the comparator’s output switches to “high-impedance” mode and the rest of the comparator enters an idle state where power supply current dramatically decreases. This is an energy-saving feature for battery-powered circuits.

- **Strobe** – an auxiliary input terminal by this same name forces the comparator’s output to assume a particular state regardless of noninverting/inverting input voltages.
3.5 Schematic diagram simplifications

Comparator circuits, like semiconductor logic gate circuits, are often quite complex. For this reason, those who draw schematic diagrams for these circuits often intentionally simplify the represented wiring in order to make their schematics easier to follow. We will explore common methods for doing so, beginning with this fairly simple comparator circuit monitoring the voltage produced by a resistive bridge network:

![Comparator Circuit Diagram]

In this circuit, the LED will energize whenever $R_{\text{sensor1}}$ drops more voltage than $R_{\text{sensor2}}$. Each and every connecting wire necessary to make this circuit functional is represented as a line within this schematic. It is a full-detail schematic with nothing simplified, except of course for the transistors and other devices internal to the comparator IC that are collectively represented by the comparator’s triangle symbol.

One way to simplify this schematic is to show all DC power connections using “rail” and “ground” symbols rather than connecting wires:

![Simplified Comparator Circuit Diagram]

Benefits of this simplification are immediately evident: no longer is there a need to cross any wires in the schematic, and our eyes no longer have to track (or be distracted by) power wiring. Instead, each network or device requiring a connection to the DC voltage source simply has $+V$ and/or Ground symbols showing those global connections.
However, we may take this process of graphic simplification one step further by completely omitting the DC power rail connections of the comparator:

![Comparator Circuit Diagram]

This is an acceptable practice because any competent electronics professional already knows that all comparators, like the vast majority of all integrated circuits, require DC power in order to operate. With this being common knowledge within the profession, we need not pedantically represent it in every schematic we draw.

Much the same is true for digital logic gate circuits, where gate symbols need not have their DC power terminals shown. Consider this collection of gates used to make a two-out-of-three (2oo3) redundant “voting” logic circuit:

![Logic Gate Diagram]

All four of the AND gates, the two OR gates, and the one Exclusive-OR gate in this voting logic circuit all require DC power in order to properly function. Every electronics professional knows this, and so we allow ourselves to omit these connections on schematic diagrams in order to enjoy the rewards of simpler diagrams.
3.6 Adding hysteresis

A perfect comparator saturates its output with just the slightest difference in voltage applied between its two input terminals, the transition from “high” to “low” (or vice-versa) output states occurring exactly as the differential input voltage crosses zero. However, this “perfect” comparator behavior is often less than ideal for real-world applications where input signal voltages exhibit small amounts of AC “noise”. Imagine, for example, the following circuit designed to activate an alarm LED if ever the speed sensed by a tachogenerator\(^4\) becomes excessive:

![Comparator circuit diagram](image)

If the tachogenerator’s signal is perfectly noise-free, the LED will make a clean transition from “off” to “on” whenever the shaft speed exceeds the threshold value set by the potentiometer. However, analog signals are never completely free of noise, with tachogenerators in particular guaranteed to produce noisy voltage signals due to their brushes making and breaking contact with commutator segments as the shaft spins. What will happen as the tachogenerator’s voltage approaches the threshold value is the comparator’s output will cycle between “high” and “low” states at the noise frequency, causing the LED to “chatter” on and off rather than just turn on once. The illustration below shows this phenomenon graphically:

![Graph showing comparator behavior](image)

\(^4\)A “tachogenerator” or tach is a permanent-magnet DC generator designed to output a voltage directly proportional to shaft speed, intended to function as an analog machine speed sensor.
A rather clever solution to this practical problem is to add some positive feedback to the comparator circuit in the form of a large resistance connecting the output terminal to the noninverting (+) input terminal:

The inclusion of $R_{\text{feedback}}$ causes the threshold value (i.e. the voltage signal between the noninverting input terminal and ground) to slightly adjust in the same direction as the comparator’s output signal. When the LED is off (i.e. output “high”), the threshold voltage increases a small amount. When the comparator switches state to turn on the LED (i.e. output “low”), the threshold voltage suddenly shifts down slightly. The practical effect is that there are now two threshold voltages centered about the potentiometer’s wiper setting, the comparator now tending to remain in its last state such that it will ignore noise on the signal:

This is an example of a general phenomenon called hysteresis, when a system fails to return to its original state after some stimulus returns to its original state. Hysteresis is often undesirable (e.g. looseness in a mechanism making it difficult to achieve a desired position, magnetic remanence in a transformer core resulting in energy dissipation with every reversal of the alternating current), but here it serves the very practical purpose of eliminating the “chatter” effect.

Some integrated-circuit comparator models are designed with some internal positive feedback for this very purpose.
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.
4.1 Differential pair amplifiers

An amplifier circuit known as a *differential pair* receives two input signals and is able to generate either a single-ended or a differential output voltage signal. A differential pair using two NPN transistors is shown here:

![Differential pair amplifier](image)

This circuit is really nothing more than two common-emitter amplifiers with a shared emitter resistor. In fact, if one input is left floating (i.e. not connected to anything) the other input and its respective output terminal is just that: a simple common-emitter amplifier with a voltage gain approximately equal to \( \frac{R_C}{R_E} \):

![Common-emitter amplifier](image)

As \( V_1 \) increases, transistor \( Q_1 \) turns on to a greater degree, passing more current. This increases the voltage dropped across both \( R_E \) and \( R_C \), and with \( R_C \) dropping more voltage the output terminal experiences less voltage with respect to ground (applying Kirchhoff’s Voltage Law with \( V_{\text{supply}} \) assumed to be constant). In other words, \( V_1 \) has an *inverting* effect on the voltage at terminal \( \text{Out}_1 \),
just as we would expect for a common-emitter amplifier. With $I_n_2$ floating, the other transistor is fully “off” and passes no current, leaving $O_u_t_2$ at full supply potential and unaffected by $V_1$.

If, however, we connect a fixed-voltage signal source to the second input terminal (of sufficient magnitude to partially turn on transistor $Q_2$) and re-run our “thought experiment”, we see something more interesting behavior from this differential pair circuit:

As $V_1$ increases, transistor $Q_1$ again turns on to a greater degree, increasing voltage drops across $R_E$ and $R_{C1}$ and consequently causing $O_u_t_1$’s terminal voltage to decrease as before. The increased voltage drop across the shared emitter resistor, however, now affects the other transistor ($Q_2$) because it has been biased by $V_2$ to be partially on. With $V_2$ fixed, an increased voltage drop across $R_E$ leaves less voltage to forward-bias $Q_2$’s base-emitter junction (once again applying Kirchhoff’s Voltage Law). This causes transistor $Q_2$’s to restrict collector current, which in turn causes $R_{C2}$ to drop less current and makes the potential at $O_u_t_2$ increase. In summary, an increase at $V_1$ causes $O_u_t_1$’s voltage to decrease and $O_u_t_2$’s voltage to increase, all voltages measured with respect to ground).

Examining the interaction between the two halves of the differential pair circuit with greater scrutiny, we will find it helpful to characterize transistors $Q_1$ and $Q_2$ in terms of “common” amplifier configurations. We have already noted $Q_1$ functions as a common-emitter amplifier from the perspective of terminal $O_u_t_1$. However, from the perspective of terminal $O_u_t_2$ it is a much more complex relationship. In our thought experiment we left input voltage $V_2$ at some fixed magnitude sufficient to make $Q_2$ partially conduct, which makes $V_2$ a bias voltage for $Q_2$. We also noted that changes in voltage dropped across $R_E$ caused by changes in input signal $V_1$ affected $Q_2$. From the perspective of $R_E$, transistor $Q_1$ is a common-collector amplifier for signal $V_1$, also known as a voltage follower: as $V_1$ increases, $V_{RE}$ increases by an almost identical amount. This increased voltage drop across $R_E$ as a result of $V_1$ increasing acts as an input signal to transistor $Q_2$. With $V_2$ acting as a constant “bias” and $V_{RE}$ acting as an input signal for $Q_2$, transistor $Q_2$ functions as a common-base amplifier from the perspective of output terminal $O_u_t_2$. Recall that common-base amplifiers are noninverting and have voltage gains approximately equal to $R_{C2}$/R_E (the same as common-emitter amplifiers). Therefore, we would expect $O_u_t_2$’s voltage to rise as much as $O_u_t_1$’s signal falls for any increase in $V_1$. Since the two halves of the differential pair are symmetrical (i.e. identical transistors and collector resistors), fixing $V_1$ and varying $V_2$ should manifest the exact same effects, except on opposite output terminals.
CHAPTER 4. DERIVATIONS AND TECHNICAL REFERENCES

Mapping these conclusions on to a standard comparator symbol with differential inputs and outputs, we have the following:

If we desire a single-ended output from our differential pair amplifier, we may simply omit Out\(_1\) and use Out\(_2\) as the only output terminal:
Plain differential pair circuits such as this exhibit poor common-mode rejection, and have voltage gain values too low to be immediately useful as either comparators or as operational amplifiers. The circuit’s performance is usually augmented by current mirrors as shown in the next schematic, and to facilitate good matching between all the transistor pairs the circuit is usually fashioned on an integrated circuit (IC) on a single wafer of silicon:

Current passing through the “programming” resistor \( R_{pgm} \) creates a voltage drop across \( Q_4 \)’s base-emitter PN junction which biases \( Q_3 \), causing the latter transistor to function as a current regulator passing approximately the same amount of current as through \( R_{pgm} \). With a nearly constant current assured through \( Q_3 \), any variations in current through \( Q_1 \) and \( Q_2 \) are forced to be complementary by Kirchhoff’s Current Law. If a rise in voltage at \( I_{n(+) \text{a}} \) causes current to rise through \( Q_1 \), \( Q_3 \)’s tendency to regulate current means \( Q_2 \) will be robbed of current by the same degree, forcing a more pronounced effect at the Out terminal.

The current mirror formed by \( Q_5 \) and \( Q_6 \) enhance differential voltage gain even more by accentuating the effects of any imbalance in voltage signals between \( I_{n(+) \text{a}} \) and \( I_{n(-) \text{a}} \). Once again imagining a rise in voltage at \( I_{n(+) \text{a}} \) with respect to ground causing \( Q_1 \)’s current to increase, \( Q_6 \) will try to match that current increase through \( Q_2 \) by becoming more conductive. This, on its own, will force the output voltage to rise even without any response from \( Q_2 \). Putting all these effects together results in a differential pair circuit with far greater differential voltage gain than before.

\[^{1}\text{An ideal differential-input amplifier responds only to the differential voltage between its two input terminals, and not to the common-mode voltage between either terminal and ground. However, the simple differential pair is affected by both.}\]
In an effort to decrease input bias current we may replace transistors $Q_1$ and $Q_2$ with Darlington pairs, which is often how we find the input stages of IC comparators and operational amplifiers constructed:
Additionally, single-ended amplifier stages may be added to this circuit to increase voltage gain and output current-drive ability. When one examines the internal circuitry of comparators and operational amplifiers alike, we always find additional stages of amplification beyond the augmented differential pair. Consider for example the model LM339 comparator:

Here, the augmented differential pair embodies most of the comparator ($Q_1$ through $Q_7$). The gain-boosting and output drive stages are found in transistors $Q_9$, $Q_{11}$, and $Q_{12}$.
Examining a more sophisticated differential amplifier circuit, the venerable 741 operational amplifier:

![Internal schematic of a model 741 operational amplifier](image)

The long horizontal wire connecting to the base of $Q_{16}$ is what bridges the differential input stage to the gain-boosting stage. The final output stage consists of the four transistors to the far-right of the diagram.
4.2 Split DC power supplies

A popular way to allow electronic amplifier circuits to input and output truly AC (alternating current) signals, as well as handle “bipolar” DC signals (i.e. signals having either positive or negative potential with respect to ground) is to power them using a split DC source. This is a DC power supply configuration providing three points of connection and three different voltage levels as shown below:

From +V to Ground we measure the voltage of the upper source; from −V to ground we measure the negative voltage of the lower source; between +V and −V we measure those sources’ voltage sum. Typical split DC supplies are symmetrical, meaning that +V and −V have the same absolute values.

If we consider a common-emitter BJT amplifier energized by such a supply, we see it becomes possible to achieve Class-A amplification of an AC signal source without the need for biasing resistors anywhere. Moreover, the output voltage is capable of achieving true AC status as well, alternating positive and negative with respect to ground, all because of the split power supply:
In the following diagrams we show the simplified internal schematic diagram of a comparator powered by a split supply, in two “limiting case” conditions. In keeping with the problem-solving strategy of “limiting cases” we will assume all transistor states to be either fully on (saturated) or fully off, marking those saturated-on transistors in red:

In the upper scenario we see the comparator subjected to a negative differential voltage input (i.e. inverting terminal more positive than noninverting), and this causes its output to saturate low by turning on transistor $Q_5$. With $Q_5$ on, the output terminal is nearly equipotential with the negative supply rail ($-V$), and so is negative with respect to ground.

In the lower scenario the comparator experiences a positive differential input (noninverting terminal more positive than inverting), causing $Q_4$ to saturate on. With $Q_4$ fully on, the output terminal is nearly equipotential with the positive rail ($+V$), and so is positive with respect to ground.
Comparators and operational amplifiers are incredibly useful and practical as electronic circuit building-blocks, but they are not perfect. For a great many applications these imperfections are insignificant, but for others they may be debilitating. Therefore, it is important to understand the ways in which operational amplifiers deviate from ideal behavior.
4.3.1 Output voltage limits

The output voltage of any amplifier circuit cannot exceed its DC supply “rail” voltages, because all an amplifier does is modulate its supply voltage. A useful model of a differential amplifier is that of a potentiometer with its wiper driven by the pointer of an analog voltmeter driven by the differential voltage applied to the input terminals:

No matter how much voltage is applied between the + and − input terminals, the potentiometer can only “swing” as far as the +V and −V “rails” of the DC power supply. This behavior is often referred to as rail-to-rail output, where the \( V_{\text{out}} \) limits are exactly (or very nearly) equal to the DC rail potentials.

However, many practical amplifier models cannot even swing that far. A more accurate model for many is one where the potentiometer cannot quite reach the +V and −V rails:

With the diodes’ voltage drops in effect, \( V_{\text{out}} \) is only able to “swing” to within approximately 0.7 Volts of each rail. Such internal voltage drops are typical of comparators and operational amplifiers using BJTs in the final output stage. Rail-to-rail capability demands FET output transistors which are capable of operating with mere milliVolts of drain-source voltage drop.
4.3. COMPARE AND OPERATIONAL AMPLIFIER IMPERFECTIONS

4.3.2 Input voltage limits and latch-up

As a general rule, input voltages for any integrated circuit should remain within the limits of its DC power supply “rail” potentials. Many integrated circuits including comparators and operational amplifiers include “protection” diodes at the input terminals effectively clamping input terminal potentials to the power supply rails (to within fractions of a Volt). Expanding upon our previous circuit model, we may show these protection diodes (two per input terminal) connected such that one will begin conducting if the potential at any input exceeds either power supply rail, essentially making that input terminal the new “rail” to power the amplifier:

![Diagram showing protection diodes at input terminals of an operational amplifier.]

If the common-mode$^2$ voltage applied to the inputs of a differential amplifier exceeds the point at which both transistors in the differential pair subcircuit turn off, the amplifier’s output may saturate rather than respond properly to the differential input voltage. In some designs this may even lead to a condition where the output remains “latched” in this saturated mode even after the offending input condition ceases – a phenomenon known as latch-up$^3$.

Amplifiers may fail to function properly even with input voltage values falling well within their supplied rail voltages. For example, the popular TL08x series of operational amplifiers specify common-mode voltage limits between $+4$ Volts above the negative rail voltage and $−4$ Volts below the positive rail voltage: if we happened to power one of these opamps on $±12$ Volt rails, the permissible input signal voltage range for either of its inputs would be $−8$ Volts to $+8$ Volts. In

$^2$For a differential amplifier, the common-mode voltage is the amount of voltage with respect to ground that is shared among both input terminals, typically calculated as the average of the two inputs’ ground-referenced voltages. In other words, if $V_{diff} = V_{in(+)} - V_{in(-)}$ then $V_{CM} = \frac{V_{in(+)} + V_{in(-)}}{2}$. However, for the sake of determining safe limits for amplifier input voltage we may more simply define “common-mode voltage” as being any voltage measured with respect to ground for either of a differential amplifier’s two inputs that may be either high or low enough to cause a problem for that amplifier.

$^3$Interestingly, the term “latch-up” more properly describes a condition in an integrated circuit where the particularly layering of semiconductor materials forms a PNPN (thyristor) structure accidentally triggered by certain abnormal input conditions. Like the “latched” state just described, this triggered-thyristor state can only be reset by cycling power to the circuit.
contrast, other opamp models such as the MCP600x series offer *rail-to-rail inputs* capable of receiving any common-mode voltage up to and including the two DC supply rail voltages.

### 4.3.3 Input bias current

A generally safe assumption for most circuits is that the input terminals of a comparator or operational amplifier carry no current. Like the input of an ideal voltmeter, we assume these inputs have infinite input impedance and therefore pose no load on the sources they sense.

However, this is not entirely true. The input transistor stage of a differential amplifier represents a finite – albeit very large – impedance, and bipolar junction transistors do require *some* amount of base current to function. An inspection of the internal circuitry for the model 741 opamp reveals the source of these currents as well as their directions:

![Internal schematic of a model 741 operational amplifier](image)

The two input terminals for the 741 connect to the bases of NPN transistors $Q_1$ and $Q_2$. In order for these transistors to turn on, current *must* enter the base terminals from outside the opamp. These *bias currents* are quite small – on the order of tens of *nano*Amperes, but they do indeed exist and must find a path through the external circuit.
4.3. COMPARATOR AND OPERATIONAL AMPLIFIER IMPERFECTIONS

Another legacy opamp model is the 324. Its internal circuitry is shown below:

Note the two input PNP transistors $Q_1$ and $Q_2$. In order for these transistors to turn on, current must exit their base terminals and find its way to the negative pole of the power supply through external circuitry.

Some comparators and opamps have FET inputs rather than BJT inputs, usually in an attempt to increase input impedance (i.e. decrease bias current). The TL08x opamp series is an example of this design:

Knowing that JFETs are designed to have their gate-channel PN junctions reverse-biased, we may conclude the two P-channel JFETs $Q_1$ and $Q_2$ will exhibit extremely small bias currents entering their gate terminals from outside the opamp.
When we connect components to the input terminals of an opamp, we must be sure some pathway exists for these bias currents to flow in the proper directions. Consider the following examples, where a voltage divider provides a voltage signal which each opamp then “buffers” as a voltage follower:

Again, these currents are extremely small in magnitude, and their effects are usually negligible. However, if they must pass through a high impedance, they will cause an undesired voltage drop to develop. A common strategy to mitigate the effects of this voltage drop is to insert an equivalent amount of resistance within the feedback bias current path, assuming the two bias currents will be approximately equal in magnitude, and therefore the two unwanted voltage drops should be approximately equal in magnitude but opposite in polarity and so should cancel each other out as common-mode (not differential) voltage at the opamp inputs:
If we regard the voltage divider as a voltage source with its own internal Thévenin resistance, we see the bias current must pass through that resistance:

The value of the "source's" Thévenin resistance is equal to the parallel equivalent resistance of $R_1$ and $R_2$, and so our feedback resistance needs to be equal to the same:

$$R_{feedback} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$
4.3.4 Input bias voltage

If the input terminals of a differential amplifier are shorted together to ensure zero differential input, the amplifier’s output should ideally assume a state of zero Volts as well. However, this rarely happens in practice due to asymmetries in the internal circuitry of the amplifier. In practice, a very small amount of differential input voltage is necessary to achieve exactly zero Volts at the output, and this small differential voltage is called the bias voltage of the amplifier.

Some operational amplifier models provide external terminals for the user to connect a potentiometer, which may be used to “trim” the offset of the amplifier. The legacy 741 opamp is one of those designs, with offset null terminals labeled:

Externally, the “nulling” potentiometer connects to these two offset terminals and to the negative power supply terminal:

Essentially, the two portions of the potentiometer become connected in parallel with $R_1$ and $R_2$, respectively, inside the opamp. By externally skewing the equivalent resistances of $R_1$ and $R_2$, the bias currents through the two halves of the opamp’s differential pair input stage may be likewise skewed to achieve the desired negation of bias voltage.
4.3.5 Frequency compensation

A problem common to high-gain amplifier circuits is undesired oscillation, and the root of this problem is positive feedback: where some of the amplifier’s output signal becomes “fed back” to its input with a 360° phase shift, so that it continually drives itself into oscillation. This principle finds constructive application in sinusoidal oscillator circuits, but for non-oscillator circuits it can be a serious problem.

Most operational amplifier circuits utilize negative feedback (i.e. 180° phase shift) rather than positive feedback, and so it might seem at first oscillations would not be possible in these circuits. However, the 360° of phase shift necessary for self-sustaining oscillations may occur in a negative-feedback circuit if a total of 180° additional phase shift occurs elsewhere in the opamp circuit. One operating scenario inviting additional phase shift is when the opamp’s output must drive a capacitive load. At high enough frequencies, the load’s capacitance and stray capacitances in the circuit may conspire to provide the additional 180° of phase shift necessary to make the total phase shift 360°.

The criteria necessary for self-oscillation is signal feedback with 360° of phase shift and a total signal gain of at least unity (1). If the 360° of phase shift cannot be avoided, an effective solution to the problem of unintentional oscillation is to intentionally reduce the opamp’s open-loop gain as frequency increases, so that at the frequency necessary to achieve 360° phase shift the fed-back signal has a voltage gain less than one.

Many operational amplifiers are internally compensated for this by the intentional connection of a capacitor between amplification stages. The internal schematic diagrams of three models of opamp shown in previous subsections all contain compensation capacitors. They are easy to spot, being the only capacitor in the entire diagram!

4.3.6 Gain-bandwidth product

Operational amplifier open-loop voltage gain is very high, but degrades for multiple reasons as signal frequency increases. The mathematical product of amplifier gain and signal frequency is expressed as the amplifier’s Gain Bandwidth Product, or GBWP.
4.3.7 Slew rate

The maximum rate at which the output voltage of a comparator or opamp is able to rise or fall is called the *slew rate*. If we drive a simple “voltage follower” opamp circuit with a perfect square-wave signal having instantaneous rise and fall times, what we will see on the output is a slew-limited waveform with definite rise and fall *slopes*:

Slope is always measured as a *rise over run*, so for these voltage waveforms we may represent their rising- and falling-edge slopes as a rates-of-change of voltage over time \( \frac{dV}{dt} \), the input waveform having a *infinite* rates of change, and the output waveform having *finite* rates of change. The reason for an opamp’s slew rate limitations is based on its internal current limitations and capacitances. Recall the relationship between voltage and current for any capacitance:

\[
I_C = C \frac{dV_C}{dt}
\]

If a current source drives a constant current through a capacitance, that voltage across that capacitance will either rise or fall (depending on current direction) at a constant *rate*. An opamp’s internal transistors are current-limiting devices by nature, and when this limited current energizes or de-energizes capacitance the result is a linearly-sloping waveform rather than an instantaneous jump. A major source of capacitance for any opamp is the *frequency-compensating* capacitance previously discussed, but any capacitance connected to the opamp as part of the load of course also contributes to a limited slew rate. This is why opamp datasheets always specify load capacitance when declaring maximum slew rate (e.g. the model 741 opamp is typically limited to 0.5 Volts per microsecond, or 500,000 Volts per second, while driving a load having no more than 100 picoFarads of capacitance).
4.3.8 Common-mode rejection

Operational amplifiers are differential amplifiers, and as such they are only supposed to respond to differences in voltage between their two input terminals. Ideally, any common-mode voltage is ignored (i.e. rejected) by the amplifier:

$$V_{out} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

In other words, $V_{out}$ for an ideal opamp is strictly a function of $V_{diff}$ and is entirely independent of $V_{CM}$. However, real opamps are not ideal, and as such their output voltages will be slightly affected by $V_{CM}$. We may express the relationship between $V_{out}$ and $V_{CM}$ as a gain, just as we do for differential voltage:

$$A_{CM} = \frac{\Delta V_{out}}{\Delta V_{CM}} \quad A_{diff} = \frac{\Delta V_{out}}{\Delta V_{diff}}$$

Differential voltage gain ($A_{diff}$) is typically referred to as open-loop voltage gain ($A_{OL}$) and for most operational amplifiers is an extremely large number. Common-mode voltage gain ($A_{CM}$) by contrast is much smaller (ideally being zero).

The degree to which a differential amplifier rejects a common-mode signal is typically gauged in comparison to its differential voltage gain. The ratio of $A_{diff}$ to $A_{CM}$ is called the common-mode rejection ratio, or CMRR:

$$CMRR = \frac{A_{diff}}{A_{CM}}$$

CMRR is more often found expressed as a decibel figure, so:

$$CMRR(dB) = 20 \log \left( \frac{A_{diff}}{A_{CM}} \right)$$

---

4Here, the term “common-mode” refers to a quantity shared in common by both input terminals of the amplifier. This is distinct from “differential” which by definition means something different or distinct between the input terminals.
4.4 Protecting comparator inputs from over-voltage

Electronic amplifier circuits are extremely useful devices, comprising portions of many practical integrated-circuit (IC) electronic components such as digital logic gates, comparators, operational amplifiers, signal mixers, etc. Solid-state amplifier circuits use transistors to allow one electrical signal to control another, and these transistors tend to be susceptible to damage from excessive applied signal voltage. In the case of MOSFET transistors, an excess of signal voltage may puncture the extremely thin layer of metal-oxide insulation separating the transistor’s gate terminal from its current-carrying channel. In the case of bipolar (NPN, PNP) transistors, an excess of signal voltage may break down reverse-biased PN junctions inside the transistor, often causing them to fail in “shorted” states. Excessive applied signal voltage may also cause damage to bipolar transistors if forward-biasing PN junctions to the extent that they conduct high amounts of current which may lead to thermal damage.

Sources of excessive signal voltage may be broadly categorized as electro-static discharge (ESD) or electrical over-stress (EOS). ESD happens when some object external to the circuit (including human bodies) accumulates an electro-static charge in its own capacitance, and this capacitively-stored charge is suddenly send to the circuit where it drops a voltage across circuit components high enough to cause damage. EOS is a more broad description of any over-voltage or over-current condition caused by one circuit sourcing energy to another, such as when an electrical test instrument is connected to a signal source that is too great for that instrument to handle.

A simple and effective way to limit both applied voltage and applied current to the input of any electronic amplifier is to connect a current-limiting resistor in series between the input terminal and the amplifier, as well as connect diodes between the input terminal and either DC power supply “rail” terminal:

![Amplifier diagram](image)

It matters not how the amplifier itself is internally constructed, nor what its larger purpose is – whether it be part of a digital logic gate, a comparator, an operational amplifier, or something even more complex. The purpose of these two diodes and the series resistor is to prevent any component within the amplifier from experiencing either too much voltage and/or too much current.
To understand how this protection network functions, consider the following circumstance where an excessive positive potential is applied to the input terminal by some external source. Here, “excessive” is defined as any voltage greater in magnitude than the DC power source energizing the amplifier:

![Protection network diagram](attachment:image)

The upper diode of the protection network forward-biases to permit current from the offending source to pass into the positive power bus of the amplifier. So long as the limiting resistor restricts this resulting current to a value less than what the amplifier requires for its normal operation, the voltage between the amplifier’s input terminal and the negative rail cannot exceed the DC power supply’s voltage plus the diode’s forward-voltage drop ($V_{\text{supply}} + V_F$). With a normal silicon diode this means a voltage no greater than 0.7 Volts plus the positive DC rail voltage. If Schottky diodes are used in the protection network, it means a voltage no greater than 0.4 Volts beyond the positive rail. Such mild over-voltage conditions are unlikely to cause damage to any of the amplifier’s internal transistors.

Similarly, if an excessive negative voltage is applied by an external source (“excessive” being any significant potential below that of the negative DC power supply rail):

![Protection network diagram](attachment:image)

Here the lower protection diode forward-biases and permits current from the external source to pass through the limiting resistor. While this occurs, the amplifier will never see more than that protection diode’s forward voltage drop ($V_F$) between its input terminal and the negative DC rail bus, which again is unlikely to damage any transistor inside the amplifier.

Such protection networks are often found inside of integrated circuits, sometimes without a series protection resistor in their most basic forms. Protection diodes are standard for CMOS digital logic gates to help protect their constituent MOSFET transistor gates from damage from ESD.
For some devices it is not sufficient for a diode network to clamp the input voltage to some value(s) slightly in excess of the device’s power supply rail potential(s). For example, a device operating on a 5 Volt DC supply may tolerate input voltages no lower than zero and no greater than 5 Volts. A relatively simple solution to this problem is to provide the diode network with a set of “protection rails” just shy of the actual power supply rail potentials. Consider the following example as an illustration of this technique:

Here, resistors $R_1$, $R_2$, and $R_3$ form a voltage divider to produce two electrical potentials (labeled A and B), each one diode forward-voltage drop shy of the nearest rail potentials. For example, if the DC power supply output 5 Volts and the two protection diodes each dropped 0.7 Volts when forward-biased, the voltage divider would need to be designed such that point A was +4.3 Volts and point B was +0.7 Volts with respect to the DC supply’s negative terminal. This would prompt the upper diode to turn on and clamp the amplifier’s input potential to +5 Volts if ever the input potential exceeded +5 Volts, the limit resistor dropping the rest. Likewise, the lower diode would turn on if ever the input terminal’s potential fell below the power supply’s negative rail, clamping the amplifier’s input terminal potential to exactly the same as that negative rail with $R_{limit}$ dropping the rest.

An important caveat to this strategy is that the voltage divider resistors $R_1$ and $R_3$ must be relatively small compared to the resistance of $R_{limit}$ in order to ensure that those “protection rail” potentials A and B do not vary significantly when the protection diodes begin to conduct. A general engineering design principle here is to size $R_{limit}$ at least ten times greater than either $R_1$ or $R_3$. If we size the resistors properly, this voltage-divider-based protection strategy may even be made adjustable by incorporating potentiometers into the voltage-divider network.
4.4. PROTECTING COMPARATOR INPUTS FROM OVER-VOLTAGE

When protection must be provided against extraordinarily strong sources, additional protection may be added in their form of DIACs connected between each input terminal and power supply rail:

![DIAC diagram]

DIACs are classified as *thyristors*, because once triggered into an electrically conductive state by sufficiently high voltage (and/or sufficiently high *rate of change* of voltage, $\frac{dv}{dt}$) they will remain “on” so long as sufficient “holding” current passes through them even if voltage falls far below the initial triggering value. In summary, a thyristor acts as a very effective snubbing device to tame over-voltage conditions, essentially acting as a near-short to that offending source.

Protection networks, however, are not without their disadvantages. Chief among these is their tendency to adversely affect the signal being sensed by the amplifier being protected. Ideally a protection network should not corrupt the sensed signal at all, and only come into play if and when that signal becomes strong enough to pose threat of damage to the amplifier, but this would only be true if the protection diodes (and DIACs) had no parasitic properties when non-conducting. This is unfortunately untrue, as both diodes and DIACs exhibit *parasitic capacitance* which not only has the effect of storing electrical charge that the amplifier may interpret as a voltage that should not be present, but along with the protection resistor will form a low-pass filter network preventing the amplifier from being able to fully sense rapidly-changing input signals:

![Capacitance diagram]

In other words, even with applied signals weak enough to pose absolutely no threat of harm to the amplifier, the capacitance inherent to the protection diodes will conspire with the series resistor
to “slow down” rates of rise and fall for any voltage arriving at the amplifier’s input, thus making the amplifier “think” the signal isn’t changing as rapidly as it really is.

In order to minimize the effects of parasitic capacitance within the protection diodes, we must choose diodes with as little of that capacitance as possible and also select a protection resistor with as low a value as possible that still limits maximum current to a value safe for the amplifier.
Chapter 5

Animations

Some concepts are much easier to grasp when seen in action. A simple yet effective form of animation suitable to an electronic document such as this is a “flip-book” animation where a set of pages in the document show successive frames of a simple animation. Such “flip-book” animations are designed to be viewed by paging forward (and/or back) with the document-reading software application, watching it frame-by-frame. Unlike video which may be difficult to pause at certain moments, “flip-book” animations lend themselves very well to individual frame viewing.
5.1 Comparator generating PWM waveform

One very useful application of a comparator is to generate a pulse-width modulation (PWM) waveform from a DC signal, with that magnitude of the DC signal controlling the duty cycle (i.e. on-time versus period) of the output pulses.

To do this, we connect the two inputs of a comparator to a varying (AC) signal source and the DC signal source, respectively. Ideally the AC source waveform should be linear: sawtooth or triangle waves work well for this purpose. The peak-to-peak magnitude of the AC waveform needs to span the expected range of the DC signal voltage.

The AC source’s frequency will define the frequency of the PWM output, while the DC voltage signal’s magnitude defines the PWM duty cycle.

PWM pulse signals are very useful for emulating analog control using discrete-state final-control devices (e.g. transistors operated as on-off switches for minimal power dissipation).

This animation shows the evolution of a PWM waveform for two different DC inputs, those input signal levels set by the wiper position on a potentiometer. The way this particular circuit is designed, a higher wiper position results in a lesser duty cycle. If the reverse characteristic is desired, simply swap + and − inputs on the comparator.
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. COMPARATOR GENERATING PWM WAVEFORM

\[ V_{\text{out}} = \begin{cases} +V & \text{if } V_{\text{in}} > V_{\text{ref}} \\ -V & \text{otherwise} \end{cases} \]
CHAPTER 5. ANIMATIONS

A circuit diagram showing an operational amplifier with input $V_{in}$, reference voltage $V_{ref}$, and output $V_{out}$. The diagram also includes signal lines and grid background for oscilloscope traces.
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. COMPARATOR GENERATING PWM WAVEFORM
CHAPTER 5. ANIMATIONS
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. COMPARATOR GENERATING PWM WAVEFORM

\[ V_{\text{in}} \rightarrow V_{\text{ref}} \rightarrow +V \rightarrow V_{\text{out}} \]

\[ V_{\text{out}} \]
5.1. COMPARATOR GENERATING PWM WAVEFORM

\[ V_{\text{in}} \quad \hat{\nabla} \quad V_{\text{ref}} \quad \rightarrow \quad V_{\text{out}} \]
5.1. COMPARATOR GENERATING PWM WAVEFORM

![Diagram of comparator generating PWM waveform]

The diagram shows a comparator circuit with input voltage ($V_{in}$), reference voltage ($V_{ref}$), output voltage ($V_{out}$), and the input and reference voltages illustrated on a waveform graph. The comparator generates a PWM waveform based on the comparison between $V_{in}$ and $V_{ref}$. The circuit diagram includes a voltage follower and a comparator block.
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. COMPARATOR GENERATING PWM WAVEFORM
CHAPTER 5. ANIMATIONS

[Diagram of an electronic circuit with labeled components: $V_{in}$, $V_{ref}$, $V_{out}$, and two waveforms.]
5.1. COMPARATOR GENERATING PWM WAVEFORM

\[ V_{\text{out}} = \begin{cases} +V & \text{if } V_{\text{in}} > V_{\text{ref}} \\ -V & \text{if } V_{\text{in}} < V_{\text{ref}} \end{cases} \]
5.1. COMPARATOR GENERATING PWM WAVEFORM
CHAPTER 5. ANIMATIONS
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. **COMPARATOR GENERATING PWM WAVEFORM**
5.1. COMPARATOR GENERATING PWM WAVEFORM
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5.1. COMPARATOR GENERATING PWM WAVEFORM
CHAPTER 5. ANIMATIONS
5.1. COMPARATOR GENERATING PWM WAVEFORM
5.1. COMPARATOR GENERATING PWM WAVEFORM

![Comparator Diagram]

- $V_{in}$
- $V_{ref}$
- $V_{out}$
- $+V$
- $-V$
5.1. COMPARATOR GENERATING PWM WAVEFORM

\[ V_{\text{out}} = \begin{cases} +V & \text{if } V_{\text{in}} > V_{\text{ref}} \\ -V & \text{if } V_{\text{in}} < V_{\text{ref}} \end{cases} \]
5.1. COMPARATOR GENERATING PWM WAVEFORM

![Comparator Diagram]

- $V_{in}$
- $V_{out}$
- $V_{ref}$
- $+V$
- $-V$
Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read\textsuperscript{1} the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture\textsuperscript{2}, the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

\textsuperscript{1}Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book \textit{Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms} by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

\textsuperscript{2}Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 6. QUESTIONS

General challenges following tutorial reading

• **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.

• **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

• Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

• Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

• Identify any **new concept(s)** presented in the text, and explain in your own words.

• Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

• Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

• Devise an experiment to **disprove** a plausible misconception.

• Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

• Describe any useful **problem-solving strategies** applied in the text.

• **Devise a question** of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.

- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.

- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?

- Is there more than one way to solve this problem? Which method seems best to you?

- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.

- What would you say was the most challenging part of this problem, and why was it so?

- Was any important information missing from the problem which you had to research or recall?

- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?

- Examine someone else’s solution to identify where they applied fundamental laws or principles.

- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).

- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.

- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.

- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.

- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?

- Identify where it would be easy for someone to go astray in attempting to solve this problem.

- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?

- Identify some of the challenges you faced in completing this experiment or project.
• Show how thorough documentation assisted in the completion of this experiment or project.

• Which fundamental laws or principles are key to this system’s function?

• Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.

• What will happen if (component X) fails (open/shorted/etc.)?

• What would have to occur to make this system unsafe?
6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

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3 *Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
6.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

√ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

√ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

√ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

√ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

√ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

√ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
6.1. CONCEPTUAL REASONING

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Thought experiments as a problem-solving strategy
- Amplification
- Gain
- Differential voltage signal
- Inverting input
- Noninverting input
- Integrated circuit
- Bridge network
- Pulse-Width Modulation (PWM)
- BJT principles
- Sourcing versus Sinking
Rails

Tri-state logic

Strobing

Noise

Positive feedback

Hysteresis
6.1.3 BJT as a variable resistor

A qualitative model for transistor behavior is that of a variable resistor. While grossly inaccurate in a quantitative (numbers) sense, it at least provides a means of assessing its behavior in the sense of “more conductive” or “less conductive”.

Apply this model of transistor behavior to this single-transistor amplifier circuit, and describe what happens to the collector voltage ($V_C$) and emitter voltage ($V_E$) when the input voltage ($V_{in}$) increases and decreases:

Challenges

• If $V_{in}$ were an AC voltage source rather than a DC voltage source, what would $V_C$ and $V_E$ do?
6.1.4 Differential pair behavior

Would you characterize this transistor amplifier as being *inverting* or *noninverting*, with the base terminal of transistor $Q_2$ being considered the input? Explain your answer.
Would you characterize this transistor amplifier as being *inverting* or *noninverting*, with the base terminal of transistor $Q_1$ being considered the input? Explain your answer.

**Challenges**

- What happens to the collector-emitter conductivity of transistor $Q_2$ in the second circuit as $V_{in}$ increases?
6.1.5 Determining output polarities

Determine the output voltage polarity of each comparator (with reference to ground), given the following input conditions:
Challenges

- Explain why the differential voltage gain value for each comparator is irrelevant, so long as it is arbitrarily large.
6.1.6 Light-activated load

In this circuit, a solar cell converts light into voltage for the comparator to “read” on its noninverting input. The comparator’s inverting input connects to the wiper of a potentiometer. Under what conditions does the LED energize?

Challenges

- Determine what would have to be changed in this circuit to make the LED turn on when the solar cell experiences the opposite light condition.
6.1.7 Sound-activated relay

Explain the operation of this sound-activated relay circuit:

Challenges

- How could we equip this circuit with the ability to turn the relay off once it has been turned on?
6.1.8 Comparator response to AC signal

Trace the output waveform of this comparator circuit:

Challenges

- Explain what the phrase *duty cycle* means with reference to a “square” or “pulse” waveform.
6.1.9 Solar panel tracker

Photovoltaic solar panels produce the most output power when facing directly into sunlight. To maintain proper positioning, “tracker” systems may be used to orient the panels’ direction as the sun “moves” from east to west across the sky:

One way to detect the sun’s position relative to the panel is to attach a pair of Light-Dependent Resistors (LDR’s) to the solar panel in such a way that each LDR will receive an equal amount of light only if the panel is pointed directly at the sun:
Two comparators are used to sense the differential resistance produced by these two LDR’s, and activate a tracking motor to tilt the solar panel on its axis when the differential resistance becomes too great. An “H-drive” transistor switching circuit takes the comparators’ output signals and amplifies them to drive a permanent-magnet DC motor one way or the other:

In this circuit, what guarantees that the two comparators never output a “high” (+V) voltage simultaneously, thus attempting to move the tracking motor clockwise and counter-clockwise at the same time?

**Challenges**

- Modify this circuit to use MOSFETs instead of BJTs.
6.1.10 Simple logic probe circuit

A logic probe is a very useful tool for working with digital logic circuits, providing visual indication of legitimate “high” and “low” logic states by means of LEDs. The following schematic diagram shows a logic probe circuit built using two comparators:

Identify the purpose of the two potentiometers in this circuit.

When this logic probe circuit is connected to the $V_{CC}$ and $V_{EE}$ power supply terminals of a powered TTL circuit, what voltage levels should test points TP1 and TP2 be adjusted to, in order for the probe to properly indicate “high” and “low” TTL logic states?

When this logic probe circuit is connected to the $V_{DD}$ and $V_{SS}$ power supply terminals of a powered CMOS circuit operating on a 5 Volt DC power supply, what voltage levels should test points TP1 and TP2 be adjusted to, in order for the probe to properly indicate “high” and “low” CMOS logic states?

**Challenges**

- Write a formula for calculating appropriate current-limiting resistor sizes for the two LEDs in this circuit, given the value of $+V$ and the LED forward voltage and current values.

- The logic probe circuit shown is minimal in component count. To make a more practical and reliable probe, one would probably want to have reverse-polarity protection (in case someone were to accidently connect the probe backward across the power supply) as well as decoupling for immunity against electrical noise. Add whatever necessary components you think there should be in this circuit to provide these features.
6.1.11 Differential pair as a Schmitt trigger

Comparators with positive feedback are sometimes referred to as *Schmitt triggers*. Suppose you needed a Schmitt trigger for a circuit you were building, but did not have any more integrated circuit comparators or op-amps to use. All you have available to you are discrete components. Is there any way you can think of to modify the following discrete transistor differential pair so that it behaves as a Schmitt trigger?

![Diagram of a differential pair](image)

**Challenges**

- Identify the noninverting (+) and inverting (−) input terminals on this differential pair amplifier.
- Redesign this circuit to use PNP transistors instead of NPN.
6.1.12 Relaxation oscillator circuit

This is a very common comparator-based oscillator circuit, technically of the relaxation type:

Explain how this circuit works, and what waveforms will be measured at points A and B. Be sure to make reference to \textit{RC time constants} in your explanation.

\textbf{Challenges}

- Explain how you might go about calculating the frequency of such a circuit, based on what you know about RC time constant circuits. Assume that the opamp can swing its output rail-to-rail, for simplicity.
6.1.13 Analyzing the simplified LM339 schematic

Perform a “thought experiment” on this simplified model LM339 comparator schematic, determining all transistor states and relevant voltage levels as the noninverting input is driven to a more positive potential than the inverting input:

Now, analyze the circuit for the opposite input condition: the noninverting input at a lower potential than the inverting input.

**Challenges**

- Modify this circuit to use a differential pair stage comprised of two NPN transistors instead of two PNP transistors.
6.1.14 Analyzing the full LM339 schematic

Perform a “thought experiment” on this model LM339 comparator schematic, determining all transistor states and relevant voltage levels as the noninverting input is driven to a more positive potential than the inverting input:

Now, analyze the circuit for the opposite input condition: the noninverting input at a lower potential than the inverting input.

Challenges

- Identify some features of the full model LM339 schematic that are not present in the simplified schematic seen earlier, and comment on their practical utility.
6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

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4In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
6.2. QUANTITATIVE REASONING

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro’s number ($N_A$) = $6.02214076 \times 10^{23}$ per mole (mol$^{-1}$)

Boltzmann’s constant ($k$) = $1.380649 \times 10^{-23}$ Joules per Kelvin (J/K)

Electronic charge ($e$) = $1.602176634 \times 10^{-19}$ Coulomb (C)

Faraday constant ($F$) = $96,485.33212... \times 10^{4}$ Coulombs per mole (C/mol)

Magnetic permeability of free space ($\mu_0$) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space ($\epsilon_0$) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space ($Z_0$) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant ($G$) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared (m$^3$/kg·s$^2$)

Molar gas constant ($R$) = $8.314462618...$ Joules per mole-Kelvin (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant ($h$) = $6.62607015 \times 10^{-34}$ joule-seconds (J·s)

Stefan-Boltzmann constant ($\sigma$) = $5.670374419... \times 10^{-8}$ Watts per square meter-Kelvin$^4$ (W/m$^2$·K$^4$)

Speed of light in a vacuum ($c$) = $299,792,458$ meters per second (m/s) = 186282.4 miles per second (mi/s)

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g., Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g., cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>( \text{B1} / \text{B2} )</td>
<td>( \text{km/h} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula \( \text{B1} / \text{B2} \), because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common\(^7\) arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln(), log10())

Parentheses may be used to ensure\(^8\) proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of \(ax^2 + bx + c\):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_1) = ((-B4 + \text{sqrt}((B4^2 - (4<em>B3</em>B5))) \div (2*B3))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(x_2) = ((-B4 - \text{sqrt}((B4^2 - (4<em>B3</em>B5))) \div (2*B3))</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) =</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>(b) =</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>(c) =</td>
<td>-2</td>
</tr>
</tbody>
</table>

This example is configured to compute roots\(^9\) of the polynomial \(9x^2 + 5x - 2\) because the values of \(9\), \(5\), and \(-2\) have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new \(a\), \(b\), and \(c\) coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

\(^7\)Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

\(^8\)Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

\(^9\)Reviewing some algebra here, a *root* is a value for \(x\) that yields an overall value of zero for the polynomial. For this polynomial \((9x^2 + 5x - 2)\) the two roots happen to be \(x = 0.269381\) and \(x = -0.82494\), with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>= (-B4 + C1) / C2 = sqrt((B4^2) - (4<em>B3</em>B5))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>= (-B4 - C1) / C2 = 2*B3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^\text{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
6.2.3 Thermostat threshold temperature

Calculate the amount of resistance that the thermistor must reach in order to turn the cooling fan on:

Challenges

- Suppose we wished to raise the temperature threshold at which the cooling fan turns on. Identify at least two different ways of doing this.
6.2.4 Hysteretic comparator with rail-to-rail output

Assuming the comparator in this circuit is capable of “swinging” its output fully from rail to rail, calculate the upper and lower threshold voltages given the resistor values shown:

\[ V_{UT} = \]
\[ V_{LT} = \]

**Challenges**

- Identify a practical application for giving a comparator circuit two threshold voltage values in this manner.
- Explain how to make the amount of hysteresis in this circuit adjustable.
6.2.5 Hysteretic comparator with limited output

Assuming the comparator in this circuit is only capable of “swinging” its output to within 1 volt of its power supply rail voltages, calculate the upper and lower threshold voltages given the resistor values shown:

- $V_{UT} =$
- $V_{LT} =$

Challenges

- Identify a practical application for giving a comparator circuit two threshold voltage values in this manner.
- Describe the internal circuitry difference(s) between a comparator with rail-to-rail output capability versus one lacking this capability.
6.2.6 Swept-voltage comparator simulation program

This computer program written in the C language simulates the behavior of a comparator with programmable output voltage limits, predicting $V_{out}$ given two user-input signal voltages to be compared:

```c
#include <stdio.h>

int main (void)
{
    float Vplus, Vminus, Vout;
    float Voh = 15.0, Vol = -15.0;

    printf("Enter the noninverting input voltage value: ");
    scanf("%f", &Vplus);

    printf("Enter the inverting input voltage value: ");
    scanf("%f", &Vminus);

    if (Vplus > Vminus)
        Vout = Voh;
    else
        Vout = Vol;

    printf("Output voltage = %f Volts\n", Vout);

    return 0;
}
```

Explain the purpose of each line of code in this program, and also how you could alter the simulated comparator's output voltage limits.
Next, suppose someone modifies this program to include a provision for “sweeping” the noninverting voltage signal between two different user-input limit values while outputting a comma-separated list of voltage values that could be plotted in a spreadsheet application, but abandons this editing mid-way to do something else. Examine this program and then determine how you could complete it to be functional:

```c
#include <stdio.h>

int main (void)
{
    float Vstart, Vstop, Vplus, Vminus, Vout;
    float Voh = 15.0, Vol = -15.0, Vinc = 0.1;

    printf("Enter the starting sweep voltage value: ");
    scanf("%f", &Vstart);

    printf("Enter the ending sweep voltage value: ");
    scanf("%f", &Vstop);

    printf("Enter the inverting input voltage value: ");
    scanf("%f", &Vminus);

    if (Vstart >= Vstop)
        return 1; // Exit with an error code!

    printf("Vnoninverting , Vinverting , Vout\n");

    for (Vplus = Vstart ; Vplus <= Vstop ; Vplus = Vplus + Vinc)
    {
        // UNFINISHED CODE!!!
    }

    return 0;
}
```

Test your solution by compiling and running your code, then graphing the results using a spreadsheet or other visualizing software capable of reading comma-separated variable data.

**Challenges**

- What does it mean if this program exits with a returned value of 1?
6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.
6.3.1 Faults in thermostat circuit

Predict how the operation of this thermostat circuit (where the cooling fan motor is supposed to turn on when the temperature gets too high) will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):

- Cable fails open:
- Comparator $U_1$ fails with output saturated positive:
- Resistor $R_1$ fails open:
- Cable fails shorted:
- Transistor $Q_1$ fails shorted (drain-to-source):

For each of these conditions, explain \textit{why} the resulting effects will occur.

**Challenges**

- Modify this circuit to use a BJT instead of a MOSFET.
6.3.2 Faults in another thermostat circuit

Predict how the operation of this thermostat circuit (where the cooling fan motor is supposed to turn on when the temperature gets too high) will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):

- Cable fails open:
- Comparator $U_1$ fails with output saturated positive:
- Resistor $R_1$ fails open:
- Capacitor $C_1$ fails shorted:
- Transistor $Q_1$ fails shorted (drain-to-source):

For each of these conditions, explain why the resulting effects will occur.

**Challenges**

- Modify this circuit to use a BJT instead of a MOSFET.
6.3.3 Strange comparator behavior

A student is testing a simple comparator\(^{11}\) circuit and documenting the results in a table:

<table>
<thead>
<tr>
<th>$V_{in(+)}$</th>
<th>$V_{in(-)}$</th>
<th>$V_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00 V</td>
<td>1.45 V</td>
<td>10.5 V</td>
</tr>
<tr>
<td>3.00 V</td>
<td>2.85 V</td>
<td>10.4 V</td>
</tr>
<tr>
<td>3.00 V</td>
<td>3.10 V</td>
<td>1.19 V</td>
</tr>
<tr>
<td>3.00 V</td>
<td>6.75 V</td>
<td>1.20 V</td>
</tr>
<tr>
<td>2.36 V</td>
<td>6.50 V</td>
<td>1.20 V</td>
</tr>
<tr>
<td>4.97 V</td>
<td>6.50 V</td>
<td>1.21 V</td>
</tr>
<tr>
<td>7.05 V</td>
<td>6.50 V</td>
<td>10.5 V</td>
</tr>
<tr>
<td>9.28 V</td>
<td>6.50 V</td>
<td>10.4 V</td>
</tr>
<tr>
<td>10.4 V</td>
<td>9.87 V</td>
<td>10.6 V</td>
</tr>
<tr>
<td>1.75 V</td>
<td>1.03 V</td>
<td>10.5 V</td>
</tr>
<tr>
<td>0.31 V</td>
<td>1.03 V</td>
<td>10.5 V</td>
</tr>
<tr>
<td>5.50 V</td>
<td>5.65 V</td>
<td>1.19 V</td>
</tr>
</tbody>
</table>

One of these output voltage readings is anomalous. Identify the “strange” output voltage in this table, and provide a possible explanation for it.

\(^{11}\)The “curved-end” symbol for a comparator or an operational amplifier is something more commonly seen in older schematic diagrams, and has no significance to the device’s operation.
6.3.4 Cooling fan control circuit

In this automatic cooling fan circuit, a comparator is used to turn a DC motor on and off when the sensed temperature reaches the “setpoint” established by the potentiometer:

![Circuit Diagram]

The circuit works just as it is supposed to in turning the motor on and off, but it has a strange problem: the transistor gets warm when the motor is off! Oddly enough, the transistor actually cools down when the motor turns on.

Describe what you would measure first to troubleshoot this problem. Do you have any reasonable guesses at this point as to what the problem is and how to fix it?

Challenges

- Is this a rail-to-rail output comparator?
- What could we change in the circuit to allow this range of operation and avoid latch-up?

Challenges

- What purpose does the capacitor serve in this circuit? Hint: the capacitor is not required in a “perfect world”, but it helps eliminate spurious problems in the real world!
6.3.5 Faults in sound-activated relay

Predict how the operation of this sound-activated relay circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no multiple faults):

- Zener diode $D_1$ fails open:
- Resistor $R_1$ fails open:
- Resistor $R_2$ fails open:
- Microphone voice coil fails open:
- Comparator $U_1$ fails with output saturated positive:
- Diode $D_3$ fails shorted:

Challenges

- Explain the purpose of $D_1$.
- Modify this circuit to be able to reset the relay after it gets tripped by a loud noise.
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify** what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. **Annotate** all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform **diagnostic tests** rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to **challenge** that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.
- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

• The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

• Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

• Articulate communication is fundamental to work that is complex and interdisciplinary.

• Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

• Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
APPENDIX B. INSTRUCTIONAL PHILOSOPHY

These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

1 In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2 Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3 This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge each other.

To high standards of education,

Tony R. Kuphaldt

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4As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

5Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s \TeX{} typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX{} is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, \TeX{} is *a programmer’s approach to word processing*. Since \TeX{} is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX{} makes it relatively easy to learn how other people have created their own \TeX{} documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft *Word* suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s \LaTeX{} extensions to \TeX{}

Like all true programming languages, \TeX{} is inherently extensible. So, years after the release of \TeX{} to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX{}, which is the markup language used to create all ModEL module documents. You could say that \TeX{} is to \LaTeX{} as C is to C++. This means it is permissible to use any and all \TeX{} commands within \LaTeX{} source code, and it all still works. Some of the features offered by \LaTeX{} that would be challenging to implement in \TeX{} include automatic index and table-of-content creation.

Tim Edwards’ \Xcircuit{} drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for \Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
**Gimp** graphic image manipulation program

Essentially an open-source clone of Adobe’s *Photoshop*, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although **Gimp** does offer its own scripting language (called *Script-Fu*), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

**SPICE** circuit simulation program

**SPICE** is to circuit analysis as *TeX* is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my “go to” application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang’s **ePiX** mathematical visualization programming library

This amazing project is a *C*++ library you may link to any *C/C++* code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as *Mathematica* or *Maple* to do. It should be said that **ePiX** is *not* a Computer Algebra System like *Mathematica* or *Maple*, but merely a mathematical *visualization* tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own *C/C++* code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a *C*++ programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a *C*++ library to do the same thing he accomplished something much greater.
APPENDIX C. TOOLS USED

**gnuplot** mathematical visualization software

Another open-source tool for mathematical visualization is gnuplot. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my gnuplot output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of Gimp to do rudimentary image editing, my use of gnuplot only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

**Python** programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

References


28 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.


“TLV1805 40V, Rail-to-Rail Input, Push-Pull Output, High Voltage Comparator with Shutdown”, datasheet SNOSD50, Texas Instruments Incorporated, Dallas, TX, December 2018.
Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

26 March 2024 – added another Challenge question to the “Hysteretic comparator with rail-to-rail output” Quantitative Reasoning question.

20 January 2024 – added a new Technical Reference section on protecting amplifier inputs against overvoltage from electro-static discharge (ESD) as well as electrical over-stress (EOS) from improperly connected signal sources.

12 November 2023 – included a Technical Reference section on operational amplifier imperfections.

26 October 2023 – added a new section in the Tutorial regarding the intentional omission of power supply terminals in some comparator schematics.

11 June 2023 – added a Quantitative Reasoning question challenging students to modify a computer program simulating comparator behavior so that it sweeps one voltage signal while the other is fixed.

27 March 2023 – added a Case Tutorial section on an improved-precision comparator circuit using multiple comparators along with a 3oo4 voting logic system. Also corrected a typographical error (CMNOS instead of CMOS).

12 December 2022 – added a photograph of a 14-pin DIP comparator to the Tutorial chapter.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

20 July 2022 – corrected an error in image_2841 by swapping polarities of the 2 Volt, 5 Volt, and 7 Volt sources in the lowest-left example.
23 March 2022 – added comments to the Tutorial about balanced versus unbalanced signals, as synonymous concepts with single-ended and differential signals, respectively. Also added a reference to the TLC3704 quad comparator which is pinout-identical to the LM339 but sports push-pull outputs.

30 October 2021 – replaced image_1917 with image_5627.

28 October 2021 – hid some answers that were previously shown in the non-instructor version of the “Determining output polarities” Conceptual question.

24 October 2021 – added a Conceptual Reasoning question based on a logic probe using comparators.

14 October 2021 – added a Case Tutorial section showing multiple operational amplifier experiments.

26 August 2021 – added a Case Tutorial section on making a “dot” display using comparators and XOR gates.

23 July 2021 – divided Tutorial into sections.

16 July 2021 – added Case Tutorial section showing a basic comparator demonstration circuit.

31 May 2021 – modified image_2841 to have more complex examples.

9 May 2021 – commented out or deleted empty chapters.

9 April 2021 – added a Case Tutorial section showing a comparator without hysteresis responding to a noisy signal.

2 December 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

30 April 2020 – added paragraph to the Tutorial explaining how the LM339 comparator achieves such a high voltage gain through active loading. Revised the graphic for the second Case Tutorial example, swapping around some of the component orientations to avoid misconceptions. Specifically, some students were mistakenly trying to associate the up/down position of the SPDT switch with the top/bottom locations of the + and – input labels! Also, added more text and graphics to the Tutorial to better explain a differential amplifier’s response to input signals of different polarity, and added a Conceptual Reasoning question (from the Operational Amplifiers module) for practice of the same.

3 May 2020 – minor edits to graphic image size in “Solar panel tracker” Conceptual Reasoning question.

29 April 2020 – added more Conceptual Reasoning questions, focusing on analyzing the internal schematics of the model LM339 comparator.
28 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

27 December 2019 – finished writing Tutorial, and added questions and Case Tutorial examples.

26 December 2019 – continued writing Tutorial.

22 September 2019 – document first created.
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