**Modular Electronics Learning (ModEL) Project**

**Finite State Machines**

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Chapter 1

Introduction

Digital logic that is state-based means its output depends not just on its inputs but also on some internal condition maintained by the digital circuitry. An elementary comparison between state-based logic and non-state-based logic is the comparison between a flip-flop and a gate: the flip-flop’s output(s) depend not only on the input conditions but also on whether or not the flip-flop is in its set state or its reset state. A logic gate, by comparison, simply responds immediately to its input and has no “remembered” state. Thus, state-based logic is closely associated with memory: the ability to retain some information from a prior set of input conditions.

Circuits having a finite (limited) amount of states are very useful for sequencing operations in electronic systems. Microprocessors contain finite-state machine circuits for the purpose of properly ordering operations necessary to execute instructions.

Important concepts related to state machines include latch and multivibrator circuits, memory circuits, feedback, hex dump displays of memory data, shift register circuits, clock signals, and digital programming.

Here are some good questions to ask of yourself while studying this subject:

- Why do cross-connected NOR and NAND gates (i.e. latch circuits) retain their last states?
- What is the difference between “address” and “data” as it relates to digital memory?
- What is the distinction between reading from memory versus writing to it?
- What is the purpose of the various “enable” lines on a memory IC?
- What timing requirements may be necessary for reliably reading and writing memory data?
- What do all the hex character-pairs represent in a “hex dump” display of memory data?
- How are “hex dump” displays of memory data useful in determining the future states of a finite state machine?
- How may a finite state machine be programmed to implement multiple types of counting sequences?
• Why are memory-based state machines capable of arbitrary sequences while circuits such as binary counters are not?

• What are some practical applications for a finite state machine?

A good “active reading” strategy to employ when reading the Tutorial is to examine the hex dump displays of memory contents for the finite state machine examples and follow the sequence described by that memory’s contents on your own: examine the hex dump to see where the next address will be for any given current address, repeating that process until you have convinced yourself that the circuit will indeed progress through the sequence as designed. Don’t just take the author at his word, saying that the circuit will exhibit a particular numerical sequence – rather, test this proposition for yourself!

The Programming References chapter contains a section with a simple one-page C++ program modeling the action of a simple finite state machine. When copied and pasted into a C++ compiler editor, and then compiled, this program will execute a programmed number sequence. If that sequence is edited and the program re-compiled and run, the new sequence will be displayed. This is an excellent way of experimenting with the concept of a memory-based finite state machine without building any circuits.
Chapter 2

Tutorial

Multivibrator circuits (e.g. latches, flip-flops) are the result of applying signal feedback\(^1\) to digital logic. Feedback gives the digital circuit the ability to act upon its own output, and by doing so it finds itself able to assume states not necessarily determined by external inputs. A simple example of this is the SR latch circuit built from two cross-connected NAND or NOR gates:

### Set-Reset (SR) latch

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
<th>Q</th>
<th>(\overline{Q})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Latch</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Activating the \(S\) and/or \(R\) inputs of an SR latch causes its outputs to assume definite logic levels, but leaving both \(S\) and \(R\) inputs inactive (e.g. both low for the NOR gate latch and both high for the NAND gate latch) causes this multivibrator circuit to latch and hold its last output conditions. In this “latched” mode the \(Q\) and \(\overline{Q}\) output signals depend not on the input signals but rather on the previous input signals. Another way to express this fact is to say that the multivibrator circuit has its own internal state (either “set” or “reset”) that it holds when not being externally forced. This state-based logic is the foundation of digital electronic memory.

\(^1\)Feedback is a fascinating engineering principle. It can turn a rather simple device or process into something substantially more complex. We’ve seen the effects of feedback intentionally integrated into circuit designs with some rather astounding effects, including operational amplifiers, oscillators, and Schmitt triggers.
Memory arrays are very useful circuits, able to store and retrieve data consisting of multi-bit words accessed by another multi-bit word called the *address*. Presenting an address word to a memory IC causes that circuit to access a particular group of internal memory cells, and once addressed the memory IC may either receive a data word from an external source to store as an internal set of states, or present the data word that was previously stored in it.

For example, the following diagram shows a 256x4 memory circuit where the 8-bit address is supplied by a set of toggle switches and the 4-bit data is displayed by a set of LEDs:

*Reading data 0x5 from address 0xB6*

This particular memory with its eight address lines offers 256 unique addresses, each one of those addresses holding a four-bit data word, hence the label “256x4”. The active-low “Chip Enable” (CE) input must be low in order to allow this IC to either read or write data, while the “Write/Read” (W/R) input controls whether the memory IC writes data to its four data lines or reads data from those four lines. As shown, the memory IC is enabled and it is in read mode.
Toggling the W/R control switch sets this memory IC into “write” mode where it may receive external signals into its data lines, an example of that shown in the following schematic:

Several different types of memory ICs exist, differentiated by the permanence or impermanence of their data:

- **RAM** – this memory retains data only as long as it remains powered by the DC source. When de-energized, RAM loses all its stored data.
- **Mask ROM** – this memory is programmed at the factory, retains its data indefinitely, and cannot be edited.
- **PROM** – this memory may be programmed (written) by the user, and after that it retains that data indefinitely.
- **EPROM** – this memory may be programmed (written) multiple times, but when not being programmed it retains its data indefinitely.

All digital memories regardless of type are state-based by definition because their output (read data) depends on prior (written) conditions. Like a multivibrator in its “latched” mode, memory retains its prior states.
The fact that digital memories accept both address and data signals as binary words allows us to employ feedback with interesting results. Imagine our 256x4 memory with its four data lines directly tied to four of its eight address lines while leaving the other address lines grounded:

With this feedback in place, whatever data happens to be stored within each of the first sixteen addresses (0x00 through 0x0F) serves as the next memory address for the IC to access. In other words, the memory IC tells itself where to go. If we think of memory in terms of postal addresses (either Post Office boxes or residential mailboxes), this would be analogous to opening up a box with a particular address, only to find within that box an instruction containing the address of another box. If a person follows these instructions, we may determine that person’s pattern of box-accessing by “programming” those boxes with some appropriate sequence.

For example, suppose this 256x4 memory IC was an EPROM programmed so that address 0x00 contained the data 0x01, and that address 0x01 contained the data 0x02, and that address 0x02 contained the data 0x03, and so on all the way through address 0x0F which contained the data 0x00. The resulting pattern of address-accesses would be a sixteen-state binary counting sequence. A hex dump of this particular program would appear as follows:

```
0 1 2 3 4 5 6 7 8 9 A B C D E F
00 1 2 3 4 5 6 7 8 9 A B C D E F 0
10 X X X X X X X X X X X X X X X X
* 
F0 X X X X X X X X X X X X X X X X
```

The “X” symbols represent “don’t care” states where the data contents are irrelevant because the four high-order address lines are grounded and therefore nothing but the first sixteen addresses (0x00 through 0x0F) may be read. The asterisk (*) shows us the rest of the addresses are the same (with “don’t care” content) without having to print the entire contents to the page.
A circuit built upon this concept is called a finite state machine, because it is a circuit (machine) with a finite (limited) number of states that it may occupy. From the perspective of this memory IC, each four-bit word combination (0x00 through 0x0F) is a unique state. At any given time, the address word ($A_0$ through $A_3$) is the present state and the data word ($D_0$ through $D_3$) is the next state.

One practical addition we must make to our circuit in order to ensure the memory IC reliably reads its addresses is to include a parallel shift register and a clock signal source to the feedback loop. Like all digital circuits, memory ICs exhibit propagation delays from input to output, and to ensure reliable reading of its addresses we must give the IC enough time for its data value to stabilize before feeding it back as the next address. A shift register receiving the four data line signals and sending them to the four address lines at the prompting of a clock pulse whose period is longer than the time delays of the memory IC will solve this practical problem for us. We will use an eight-bit register to synchronize all eight address lines even though we are currently only using four, because this feature will be useful to us later:

It is worth noting some alternative counting patterns available to us with this simple circuit. For example, we could make it count in reverse order:

```
0 1 2 3 4 5 6 7 8 9 A B C D E F
00 F 0 1 2 3 4 5 6 7 8 9 A B C D E
10 X X X X X X X X X X X X X X X
* F0 X X X X X X X X X X X X X X X
```
Putting one of the upper four address bits to good use by connecting it to a switchable input, we may expand our “program” stored in the memory IC to give us an up/down counter:

At first one might be tempted to question the usefulness of a finite state machine. If consider the numerous logic gates that comprise a 256x4 memory IC, it may even seem wasteful of hardware to use a memory IC to build something as simple as a binary counter that could otherwise be constructed with just a few flip-flops! This is certainly a valid conclusion, if that is all we ever wish this circuit to do. However, the advantage we gain with the complexity of a memory IC is that the counting sequence is limited not by hardware but only by the data we choose to write into that memory. So long as the sequence does not repeat any adjacent values, we may program any count sequence we wish! It is precisely this flexibility to implement any arbitrary sequence that gives finite state machines their value.

Thus, a memory-based finite state machine is the precursor of a stored-program computer. It is a device capable of executing a sequence of steps in any order of our own choosing.
A powerful tool for mathematical modeling is text-based computer programming. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, C++ and Python.
3.1 Programming in C++

One of the more popular text-based computer programming languages is called C++. This is a compiled language, which means you must create a plain-text file containing C++ code using a program called a text editor, then execute a software application called a compiler to translate your “source code” into instructions directly understandable to the computer. Here is an example of “source code” for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer’s console:

```c++
#include <iostream>
using namespace std;

int main (void)
{
    float x, y;
    x = 200;
    y = -560.5;

    cout << "This simple program performs basic arithmetic on" << endl;
    cout << "the two numbers " << x << " and " << y << " and then" << endl;
    cout << "displays the results on the computer's console." << endl;

    cout << endl;
    cout << "Sum = " << x + y << endl;
    cout << "Difference = " << x - y << endl;
    cout << "Product = " << x * y << endl;
    cout << "Quotient of " << x / y << endl;

    return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other “whitespace” is largely irrelevant in C++ code, and is included only to make the code more pleasing to view.

1 Although not included in this example, comments preceded by double-forward slash characters (//) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.
Let’s examine the C++ source code to explain what it means:

- **#include `<iostream>`** and **using namespace std;** are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }), often referred to as “curly-braces”.

- **int main (void)** labels the “Main” function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a main function at minimum, and often additional functions as well, but the main function is where execution always begins. The int declares this function will return an integer number value when complete, which helps to explain the purpose of the return 0; statement at the end of the main function: providing a numerical value of zero at the program’s completion as promised by int. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called arguments to that function. Braces surround lines of executable code belonging to a particular function.

- The float declaration reserves places in the computer’s memory for two floating-point variables, in this case the variables’ names being x and y. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. xyz would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (;) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line’s end is necessary because C++ ignores whitespace on the page and doesn’t “know” otherwise where one line ends and another begins.

- All the other instructions take the form of a cout command which prints characters to the “standard output” stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (<<) show data being sent toward the cout command. Note how verbatim text is enclosed in quotation marks, while variables such as x or mathematical expressions such as x - y are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as +, -, *, and /, respectively.

- The endl found at the end of every cout statement marks the end of a line of text printed to the computer’s console display. If not for these endl inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the cout << endl; line, which does nothing but create a blank line on the screen, for no reason other than esthetics.
After saving this source code text to a file with its own name (e.g. myprogram.cpp), you would then compile the source code into an executable file which the computer may then run. If you are using a console-based compiler such as GCC (very popular within variants of the Unix operating system\(^2\), such as Linux and Apple’s OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the GCC compiler to take your source code (myprogram.cpp) and create with it an executable file named myprogram.exe. Simply typing `.\myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio\(^3\), you may simply create a new console application “project” using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

This simple program performs basic arithmetic on the two numbers 200 and -560.5 and then displays the results on the computer’s console.

```
Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor’s screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

\(^2\)A very functional option for users of Microsoft Windows is called Cygwin, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

\(^3\)Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the main() function provided in the template, deleting the “Hello World” cout line that came with the template; (4) Type or paste any preprocessor directives (e.g. #include statements, namespace statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile (“Build”) and run your new program. Upon execution a console window will appear showing the output of your program.
learn computer programming by closely examining others’ example programs and modifying them than it is to write your own programs starting from a blank screen.
3.2 Programming in Python

Another text-based computer programming language called Python allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an interpreted language: a software application called an interpreter reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3` and pressing the Enter key:

Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> 

The >>> symbols represent the prompt within the Python interpreter “shell”, signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the >>> prompt are entries typed by the human programmer, and all lines shown without the >>> prompt are responses from the Python interpreter software:

```plaintext
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

---

4 Using version 3 of Python, which is the latest at the time of this writing.
More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which “imports” these functions from Python’s math library (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```python
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in radians rather than degrees, and how Python provides convenient functions for translating between the two. Logarithms assume a base of $e$ unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person’s first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.
Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using phasors\(^5\) as shown in the following example. Here we see Python’s interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```python
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor’s impedance (`zc`) as \(X_C \angle -90^\circ\) with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values (0 −j\(X_c\) Ω and 0 +j\(X_l\) Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω \(\angle \ -25.102^\circ\)). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in radians; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python’s interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

\(^5\)A “phasor” is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.
assignment of variables as well as a convenient text record\textsuperscript{6} of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter’s shell. For example, consider the following Python program, saved under the filename \texttt{myprogram.py}:

\begin{verbatim}
  x = 200
  y = -560.5

  print("Sum")
  print(x + y)

  print("Difference")
  print(x - y)

  print("Product")
  print(x * y)

  print("Quotient")
  print(x / y)
\end{verbatim}

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace \textit{is} significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type \texttt{python myprogram.py} and then press the Enter key at my computer console’s prompt, at which point it would display the following result:

\begin{verbatim}
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
\end{verbatim}

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

\textsuperscript{6}Like many command-line computing environments, Python’s interpreter supports “up-arrow” recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.
If you are interested in learning more about computer programming in any language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.
### 3.3 Modeling a simple finite state machine using C++

```cpp
#include <iostream>
using namespace std;

#define MAX 1000

int rom[MAX] = { 2, 3, 4, 5, 6, 7, 8, 9, 0, 0 };

int main ()
{
    int data = 1;
    int count;

    for (count = 0 ; count < 10 ; ++count)
    {
        cout << "Data = " << data << endl;

        data = rom[data]; // Looks up next state from present state

        if (data < 0 || data >= MAX)
            data = 0;
    }

    return 0;
}
```

When compiled and run, the result is as shown:

Data = 1
Data = 3
Data = 5
Data = 7
Data = 9
Data = 0
Data = 2
Data = 4
Data = 6
Data = 8
Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture, the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

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Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 4. QUESTIONS

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.

- **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

- Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

- Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

- Identify any **new concept(s)** presented in the text, and explain in your own words.

- Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

- Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

- Devise an experiment to **disprove** a plausible misconception.

- Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

- Describe any useful **problem-solving strategies** applied in the text.

- **Devise a question** of your own to challenge a reader’s comprehension of the text.
GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else’s solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.
• Show how **thorough documentation** assisted in the completion of this experiment or project.

• Which **fundamental laws or principles** are key to this system’s function?

• Identify any way(s) in which one might obtain **false or otherwise misleading measurements** from test equipment in this system.

• What will happen if *(component X)* fails (open/shorted/etc.)?

• What would have to occur to make this system **unsafe**?
4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

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3 Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
4.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

✓ Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

✓ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

✓ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

✓ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

✓ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

✓ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
4.1. CONCEPTUAL REASONING

4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Truth table
- Logic function
- NOR function
- NAND function
- Latch
- Multivibrator
- Set versus Reset
- Memory address
- Memory data
- Read versus Write
- Enable
- Hex dump
4.1.3 Generating repeated values

The following number sequence cannot be produced by this finite state machine:

1, 2, 3, 4, 5, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Explain why this is.
However, this next finite state machine *is* capable of producing that number sequence with the repeated “5” numerals:

Write a program (i.e. a “hex dump” of the memory) for this finite state machine to produce the sequence 1, 2, 3, 4, 5, 5, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 1, 2, 3, 4, 5, 5, 6, ...

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<td>00</td>
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<tr>
<td>10</td>
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<td>---</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>F0</td>
<td>---</td>
<td>---</td>
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<td>---</td>
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</tr>
</tbody>
</table>

**Challenges**

- Design a non-fsm circuit using the original 256 × 4 memory IC that can generate this sequence.
4.1.4 Sweeping LED pattern

Write a program (i.e. a “hex dump” of the memory) for this finite state machine to energize the five LEDs in the sequence shown:
4.1. CONCEPTUAL REASONING

Challenges

- Explain why the sixth data line ($D_5$) is absolutely essential to make this design of FSM able to produce this particular sequence.
4.1.5 Parallel to serial data converter

The following finite state machine is intended to take in a four-bit data word in parallel format, then output that four-bit word one bit at a time as a serial signal (followed by four more clock pulses’ worth of zero states):

Someone began writing a program (hex dump) for the PROM in this finite state machine, but did not finish the job:

| Hex | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | A  | B  | C  | D  | E  | F  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 00  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 10  | 1  | 2  | 3  | C  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 20  | 1  | 2  | B  | 4  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 30  | 1  | 2  | B  | C  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 40  | 1  | A  | 3  | 4  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 50  | 1  | A  | 3  | C  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 60  | 1  | A  | B  | 4  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 70  | 1  | A  | B  | C  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 80  | 9  | 2  | 3  | 4  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| 90  | 9  | 2  | 3  | C  | 5  | 6  | 7  | 0  | X  | X  | X  | X  | X  | X  | X  |
| A0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| C0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| D0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| E0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| F0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Complete the rest of this FSM’s program.
4.1. CONCEPTUAL REASONING

Challenges

- What will happen if the “Send” input is pulsed while a different 4-bit word is in mid-transmission?
4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

\Footnote{In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.}

\Footnote{This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.}
4.2. QUANTITATIVE REASONING

4.2.1 Miscellaneous physical constants

Note: constants shown in bold type are exact, not approximations. Values inside of parentheses show one standard deviation ($\sigma$) of uncertainty in the final digits: for example, Avogadro’s number given as 6.02214179(30) × $10^{23}$ means the center value (6.02214179 × $10^{23}$) plus or minus 0.00000030 × $10^{23}$.

Avogadro’s number ($N_A$) = 6.02214179(30) × $10^{23}$ per mole (mol$^{-1}$)

Boltzmann’s constant ($k$) = 1.3806504(24) × $10^{-23}$ Joules per Kelvin (J/K)

Electronic charge ($e$) = 1.602176487(40) × $10^{-19}$ Coulomb (C)

Faraday constant ($F$) = 9.64853399(24) × $10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space ($\mu_0$) = 1.25663706212(19) × $10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space ($\varepsilon_0$) = 8.8541878128(13) × $10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space ($Z_0$) = 376.730313668(57) Ohms (Ω)

Gravitational constant ($G$) = 6.67428(67) × $10^{-11}$ cubic meters per kilogram-seconds squared (m$^3$/kg-s$^2$)

Molar gas constant ($R$) = 8.314472(15) Joules per mole-Kelvin (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant ($h$) = 6.62606896(33) × $10^{-34}$ joule-seconds (J-s)

Stefan-Boltzmann constant ($\sigma$) = 5.670400(40) × $10^{-8}$ Watts per square meter-Kelvin$^4$ (W/m$^2$·K$^4$)

Speed of light in a vacuum ($c$) = 299792458 meters per second (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from http://physics.nist.gov/constants, National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s 2018 CODATA recommended values database.
4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g. Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>= B1 / B2</td>
<td>km/h</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

---

Spreadsheet may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln(), log10())

Parentheses may be used to ensure proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of \( ax^2 + bx + c \):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_{-1} )</td>
<td>( \frac{-B4 + \sqrt{(B4^2) - (4<em>B3</em>B5))}}{(2*B3)} )</td>
</tr>
<tr>
<td>2</td>
<td>( x_{+1} )</td>
<td>( \frac{-B4 - \sqrt{(B4^2) - (4<em>B3</em>B5))}}{(2*B3)} )</td>
</tr>
<tr>
<td>3</td>
<td>( a )</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>( b )</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>( c )</td>
<td>-2</td>
</tr>
</tbody>
</table>

This example is configured to compute roots of the polynomial \( 9x^2 + 5x - 2 \) because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new \( a \), \( b \), and \( c \) coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

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7Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

8Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

9Reviewing some algebra here, a root is a value for \( x \) that yields an overall value of zero for the polynomial. For this polynomial \( (9x^2 + 5x - 2) \) the two roots happen to be \( x = 0.269381 \) and \( x = -0.82494 \), with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>= (-B4 + C1) / C2</td>
<td>= sqrt((B4^2) - (4<em>B3</em>B5))</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>= (-B4 - C1) / C2</td>
<td>= 2*B3</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>= 9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b</td>
<td>= 5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c</td>
<td>= -2</td>
<td></td>
</tr>
</tbody>
</table>

Note how the square-root term \((y)\) is calculated in cell C1, and the denominator term \((z)\) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(\text{footnoteText}\)

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
4.2.3 Fibonacci sequence counter

The Fibonacci Sequence is a series of whole numbers beginning with 0 and 1 where each successive number is the sum of the two previous numbers. Write the first ten Fibonacci numbers and then determine if it could be implemented using the following finite state machine:

Challenges

- Suppose someone presented you with two contiguous Fibonacci numbers other than the starting values of 0 and 1. How could you determine all the preceding numbers (i.e. count backwards in the Fibonacci sequence)?
4.2.4 Prime sequence counter

Write a program (i.e. a “hex dump” of the memory) for this finite state machine to make it count prime numbers:

Challenges

- Modify this FSM circuit to count to a higher number of primes.
4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.
4.3.1 Effects of failed data lines

The following finite state machine and program is supposed to generate the count sequence shown below the diagram:

Identify the state sequence for this FSM if data line $D_2$ of the memory IC happens to fail with a "low" output signal, assuming the first state is 0.

Identify the state sequence for this FSM if data line $D_0$ of the memory IC happens to fail with a "high" output signal, assuming the first state is 1.

Identify the state sequence for this FSM if data line $D_3$ of the memory IC happens to fail with a "low" output signal, assuming the first state is 0.

**Challenges**

- Identify a method for diagnosing each fault given a multimeter or logic probe.
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify** what it is you need to solve, *identify* all relevant data, *identify* all units of measurement, *identify* any general principles or formulae linking the given information to the solution, and then *identify* any “missing pieces” to a solution. *Annotate* all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.
- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

• The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

• Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

• Articulate communication is fundamental to work that is complex and interdisciplinary.

• Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

• Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
APPENDIX B. INSTRUCTIONAL PHILOSOPHY

These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

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1 In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2 Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3 This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

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4 As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

5 Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
APPENDIX C. TOOLS USED

Donald Knuth’s \TeX \ typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, \TeX is a programmer’s approach to word processing. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft Word suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s \LaTeX \ extensions to \TeX \n
Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX, which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as C is to C++. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards’ \Xcircuit \ drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for \Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
**Gimp** graphic image manipulation program

Essentially an open-source clone of Adobe’s **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

**SPICE** circuit simulation program

**SPICE** is to circuit analysis as **\( \LaTeX \)** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for teaching students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my “go to” application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang’s **ePiX** mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is *not* a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical *visualization* tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.
Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is not part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” not be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of Gimp to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s free and that it works well.

**Python** programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a tool for myself because I use it almost daily as a calculator. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. phasor) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

References
Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

10 May 2021 – commented out or deleted empty chapters.

26 January 2021 – minor additions to the Introduction, and minor edits to the Tutorial.

31 August 2020 – added more comments to the Introduction, regarding active reading.

23 August 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions. Also, re-named “Simplified Tutorial” to “Tutorial” and similarly edited all references to it throughout the module.

2 June 2020 – added questions.

1 June 2020 – added C++ programming simulation of a simple Finite State Machine.

31 May 2020 – document first created.
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