The S Variable

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Last update = 24 October 2021

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Chapter 1

Introduction

A powerful mathematical concept useful for analyzing practically any physical system – electrical circuits included – is something called the $s$ variable. The $s$ variable is closely related to Euler’s Relation and phasor expressions of waveforms, which is why a discussion of it is included here.
Chapter 2

Full Tutorial
2.1 The meaning of the \( s \) variable

Euler’s Relation allows us to express rotating sinusoidal wave functions as imaginary exponents of \( e \). For example, \( Ae^{j\theta} \) represents a phasor\(^1\) of length \( A \) at an angle of \( \theta \) radians. \( Ae^{j\omega t} \) represents a phasor of length \( A \) rotating at a velocity of \( \omega \) radians per second at a particular instant in time \( t \). This happens to be an incredibly useful mathematical “trick” for representing sinusoidal waves in physical systems. For example, if we wished to mathematically express a sinusoidal AC voltage as a function of time with a peak voltage value of 10 Volts and a frequency of 60 hertz (377 radians per second, since \( \omega = 2\pi f \)), we could do so like this:

\[
V(t) = 10e^{j377t}
\]

Exponential functions aren’t just useful for expressing sinusoidal waves, however. They also work well for expressing rates of growth and decay, as is the case with RC and L/R time-delay circuits where exponential functions describe the charging and discharging of capacitors and inductors. Here, the exponent is a real number rather than an imaginary number: the expression \( e^{-t/\tau} \) approaching zero as time \( (t) \) increases. The Greek letter “tau” \( (\tau) \) represents the time constant of the circuit, which for capacitive circuits is the product of \( R \) and \( C \), and for inductive circuits is the quotient of \( L \) and \( R \). For example, if we wished to mathematically express the decaying voltage across a 33 \( \mu \)F capacitor initially charged to 10 Volts as it dissipates its stored energy through a 27 k\( \Omega \) resistor (the circuit having a time constant of 0.891 seconds, since \( \tau = RC \)), we could do so like this:

\[
V(t) = 10e^{-(t/0.891)}
\]

The sign of the exponential term here is very important: in this example we see it is a negative number. This tells us the function decays (approaches zero) over time, since larger positive values of \( t \) result in larger negative values of \( t/\tau \) (recall from algebra that a negative exponent is the equivalent of reciprocating the expression, so that \( e^{-x} = \frac{1}{e^x} \)). If the exponent were a real positive number, it would represent some quantity growing exponentially over time. If the exponent were zero, it would represent a constant quantity. We expect a discharging resistor-capacitor circuit to exhibit decaying voltage and current values, and so the negative exponent sign shown here makes sense.

If imaginary exponents of \( e \) represent sinusoidal waves, and real exponents of \( e \) represent growth or decay, then a complex exponent of \( e \) (having both real and imaginary parts) must represent a sinusoidal wave growing or decaying in magnitude over time. Engineers use the lower-case Greek letter “omega” \( (\omega) \) along with the imaginary operator \( j \) to represent the imaginary portion, and the lower-case Greek letter “sigma”\(^2\) \( (\sigma) \) to represent the real portion. For example, if we wished to mathematically express a sine wave AC voltage with a frequency of 60 hertz (377 radians per second) and an amplitude beginning at 10 Volts but decaying with a time constant \( (\tau) \) of 25 milliseconds \((\sigma = 1/\tau = 40 \text{ time constants per second})\), we could do so like this:

\[
V(t) = 10e^{-40t+j377t}
\]

\(^1\)A “phasor” is a complex-number expression of a sinusoidal function at some moment in time.

\(^2\)\( \sigma \) is equal to the reciprocal of the signal’s time constant \( \tau \). In other words, \( \sigma = 1/\tau \).
2.1. **THE MEANING OF THE S VARIABLE**

We may factor time from the exponential terms in this expression, since $t$ appears both in the real and imaginary parts:

$$V(t) = 10e^{(-40 + j377)t}$$

With $t$ factored out, the remaining terms $-40 + j377$ completely describe the sinusoidal wave’s characteristics. The wave’s decay rate is described by the real term ($\sigma = -40$ time constants per second), while the wave’s phase is described by the imaginary term ($j\omega = 377$ radians per second). Engineers use a single variable $s$ to represent the complex quantity $\sigma + j\omega$, such that any growing or decaying sinusoid may be expressed very succinctly as follows:

$$Ae^{st} = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t}e^{j\omega t}$$

Where,

$A =$ Initial amplitude of the sinusoid (e.g. Volts, Amperes) at time $t = 0$ (arbitrary units)

$s =$ Complex growth/decay rate and frequency (sec$^{-1}$)

$\sigma = -\frac{1}{\tau} =$ Real growth/decay rate (time constants per second, or sec$^{-1}$)

$j\omega = $ Imaginary frequency (radians per second, or sec$^{-1}$)

$t =$ Time (seconds)

Separating the expression $Ae^{\sigma t}e^{j\omega t}$ into three parts – $A$, $e^{\sigma t}$, and $e^{j\omega t}$ – we get a complete description of a rotating phasor:

$A =$ Initial amplitude of the phasor ($t = 0$)

$e^{\sigma t} =$ How much the phasor’s magnitude has grown ($\sigma > 0$) or decayed ($\sigma < 0$) at time $t$

$e^{j\omega t} =$ Unit phasor (length = 1) at time $t$
If we set \( \omega \) at some constant value and experiment with different values of \( \sigma \), we can see the effect \( \sigma \) has on the shape of the wave over time:

If we set \( \sigma \) at zero and experiment with different values of \( \omega \), we can see the effect \( \omega \) has on the shape of the wave over time:

As we will soon see, characterizing a sinusoidal response using the complex variable \( s \) allows us to mathematically describe a great many things. Not only may we describe voltage waveforms using \( s \) as shown in these simple examples, but we may also describe the response of entire physical systems including electrical filter networks, feedback control systems, servomechanisms, chemical reactions, and even nuclear chain-reactions. In fact, it is possible to map the essential characteristics of any linear system in terms of how exponentially growing, decaying, or steady sinusoidal waves affect it, and that mapping takes the form of mathematical functions of \( s \).

When engineers or technicians speak of a resonant system, they mean a circuit containing inductive and capacitive elements tending to sustain oscillations of a particular frequency (\( \omega \)). A lossless resonant system (e.g. a superconducting tank circuit, a frictionless pendulum) may be expressed by setting the real portion of \( s \) equal to zero (\( \sigma = 0 \); no growth or decay) and

\[ A e^{i(\sigma + j\omega)t} \]

where \( \omega > 0 \)

\[ A e^{i\sigma t} \]

where \( \sigma = 0 \)

\[ A e^{i\sigma t} \]

where \( \sigma < 0 \)

---

\(^3\)One value of \( \omega \) not shown in this three-panel graphic comparison is a negative frequency. This is actually not as profound as it may seem at first. All a negative value of \( \omega \) will do is ensure that the phasor will rotate in the opposite direction (clockwise, instead of counter-clockwise as phasor rotation is conventionally defined). The real portion of the sinusoid will be identical, tracing the same cosine-wave plot over time. Only the imaginary portion of the sinusoid will be different, as \( j \sin(-\theta) = -j \sin(\theta) \).
2.2. IMPEDANCE EXPRESSED USING THE S VARIABLE

letting the imaginary portion represent the resonant frequency \((j\omega = j2\pi f)\). Real-life resonant systems inevitably dissipate some energy, and so a real resonant system’s expression will have both an imaginary portion to describe resonant frequency and a negative real portion to describe the oscillation’s rate of decay over time.

Systems exhibiting a positive \(\sigma\) value are especially interesting because they represent instability: unrestrained oscillatory growth over time. An aircraft auto-pilot control system with excessive sensitivity, for example, is a system where \(\sigma\) could become positive, in which case the aircraft would begin to pitch (or yaw, or roll) in an oscillating manner until the aircraft fell out of the sky. This situation, of course, is highly undesirable.

2.2 Impedance expressed using the \(s\) variable

We know that voltage across a capacitor and current “through” a capacitor are related as follows:

\[
I = C \frac{dV}{dt}
\]

Next, we substitute an expression\(^4\) for voltage in terms of \(s\) and then use calculus to differentiate it with respect to time:

\[
I = C \frac{d}{dt}(e^{st})
\]

\[
I = sCe^{st}
\]

The ratio of \(\frac{V}{I}\) (the definition of impedance) will then be:

\[
Z_C = \frac{V}{I} = \frac{e^{st}}{sCe^{st}}
\]

\[
Z_C = \frac{1}{sC}
\]

Instead of the common scalar expression for capacitive impedance \((Z_C = \frac{1}{2\pi fC})\) which only tells us the magnitude of the impedance (in Ohms) but not the phase shift, we have a complex expression for capacitive impedance \((Z_C = \frac{1}{sC})\) describing magnitude, phase shift, and its reaction to the growth or decay of the signal.

---

\(^4\)The expression used here to represent voltage is simply \(e^{st}\). I could have used a more complete expression such as \(Ae^{st}\) (where \(A\) is the initial amplitude of the signal), but as it so happens this amplitude is irrelevant because there will be an “\(A\)” term in both the numerator and denominator of the impedance quotient. Therefore, \(A\) cancels out and is of no consequence.
Likewise, we may do the same for inductors. Recall that voltage across an inductor and current through an inductor are related as follows:

\[ V = L \frac{dI}{dt} \]

Substituting an expression for current in terms of \( s \) and using calculus to differentiate it with respect to time:

\[ V = L \frac{d}{dt} (e^{st}) \]

\[ V = sLe^{st} \]

The ratio of \( \frac{V}{I} \) (the definition of impedance) will then be:

\[ Z_L = \frac{V}{I} = \frac{sLe^{st}}{e^{st}} \]

\[ Z_L = sL \]

As with capacitors, we now have a complex expression for inductive impedance describing magnitude, phase shift, and its reaction to signal growth or decay \((Z_L = sL)\) instead of merely having a scalar expression for inductive impedance \((Z_L = 2\pi f L)\).

Resistors directly oppose current by dropping voltage, with no regard to rates of change. Therefore, there are no derivatives in the relationship between voltage across a resistor and current through a resistor:

\[ V = IR \]

If we substitute \( e^{st} \) for current into this formula, we will see that voltage must equal \( Re^{st} \). Solving for the ratio of voltage over current to define impedance:

\[ Z_R = \frac{V}{I} = \frac{Re^{st}}{e^{st}} \]

\[ Z_R = R \]

Not surprisingly, all traces of \( s \) cancel out for a pure resistor: its impedance is exactly equal to its DC resistance.

In summary:

<table>
<thead>
<tr>
<th>Inductive impedance ((Z_L))</th>
<th>Capacitive impedance ((Z_C))</th>
<th>Resistive impedance ((Z_R))</th>
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<tbody>
<tr>
<td>( sL )</td>
<td>( \frac{1}{sC} )</td>
<td>( R )</td>
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Now let’s explore these definitions of impedance using real numerical values. First, let’s consider a 22 \( \mu \)F capacitor exposed to a steady AC signal with a frequency of 500 Hz. Since the signal in this case is steady (neither growing nor decaying in magnitude), the value of \( \sigma \) will be equal to zero. \( \omega \) is equal to \( 2\pi f \), and so a frequency of 500 Hz is equal to 3141.6 radians per second. Calculating impedance is as simple as substituting these values for \( s \) and computing \( 1/sC \):

\[
Z_C = \frac{1}{sC} = \frac{1}{(\sigma + j\omega)C}
\]

\[
Z_C = \frac{1}{(0 + j3141.6 \text{ sec}^{-1})(22 \times 10^{-6} \text{ F})}
\]

\[
Z_C = \frac{1}{j0.0691}
\]

\[
Z_C = -\frac{j}{0.0691}
\]

\[
Z_C = 0 - j14.469 \Omega \quad (\text{rectangular notation})
\]

\[
Z_C = 14.469 \Omega \angle -90^\circ \quad (\text{polar notation})
\]

Thus, the impedance of this capacitor will be 14.469 Ohms at a phase angle of \(-90^\circ\). The purely imaginary nature of this impedance (its orthogonal phase shift between voltage and current) tells us there is no net power dissipated by the capacitor. Rather, the capacitor spends its time alternately absorbing and releasing energy to and from the circuit.
Next, we will consider the case of a 150 mH inductor exposed to an exponentially rising DC signal with a time constant (τ) of 5 seconds. 5 seconds per time constant (τ) is equal to 0.2 time constants per second (σ). Since the signal in this case is DC and not AC, the value of ω will be equal to zero. Calculating impedance, once again, is as simple as substituting these values for s and computing sL:

\[ Z_L = sL = (\sigma + j\omega)L \]

\[ Z_L = (0.2 + j0 \text{ sec}^{-1})(150 \times 10^{-3} \text{ H}) \]

\[ Z_L = 0.03 + j0 \Omega \quad \text{(rectangular notation)} \]

\[ Z_L = 0.03 \Omega \angle 0^\circ \quad \text{(polar notation)} \]

Thus, the impedance of this inductor will be 0.03 Ohms at a phase angle of 0°. The purely real nature of this impedance (i.e. no phase shift between voltage and current) tells us energy will be continually absorbed by the inductor, and for this reason it will be seen by the rest of the circuit as though it were a resistor dissipating energy for however long the signal continues to exponentially grow.

A phase shift of 0 degrees for a reactive component such as an inductor may come as a surprise to students accustomed to thinking of inductive impedances always having 90 degree phase shifts! However, the application of the complex variable s to impedance mathematically demonstrates we can indeed have conditions of no phase shift given just the right circumstances. This makes conceptual sense as well if we consider how inductors store energy: if the current through an inductor increases exponentially over time, never reversing direction, it means the inductor’s magnetic field will always be growing and therefore absorbing more energy from the rest of the circuit.
We see something even more interesting happen when we subject a reactive component to a *decaying* DC signal. Take for example a 33,000 µF capacitor exposed to a decaying DC signal with a time constant of 65 milliseconds. 65 milliseconds per time constant (τ) is equal to 15.38 time constants per second (σ). Once again ω will be zero because this is a non-oscillating signal. Calculating capacitive impedance:

\[
Z_C = \frac{1}{sC} = \frac{1}{(\sigma + j\omega)C}
\]

\[
Z_C = \frac{1}{(-15.38 + j0 \text{ sec}^{-1})(33000 \times 10^{-6} \text{ F})}
\]

\[
Z_C = \frac{1}{-0.508}
\]

\[
Z_C = -1.970 + j0 \Omega \text{ (rectangular notation)}
\]

\[
Z_C = 1.970 \Omega \angle 180^\circ \text{ (polar notation)}
\]

A *negative* real impedance figure represents a phase shift of 180° between voltage and current. Once again, this may surprise students of electronics who are accustomed to thinking of capacitive impedances *always* having phase shifts of −90 degrees. What a 180 degree phase shift means is the direction of current with respect to voltage polarity has the capacitor functioning as an energy *source* rather than as a *load*. If we consider what happens to a capacitor when it discharges, the 180 degree phase shift makes sense: current flowing in this direction depletes the capacitor’s plates of stored charge, which means the electric field within the capacitor weakens over time as it releases that energy to the rest of the circuit.
2.3 Transfer function analysis

An extremely important topic in engineering is that of transfer functions. Simply defined, a transfer function is the ratio of output to input for any physical system, usually with both the output and input being mathematical functions of $s$. In other words, we express both the output of a system and the corresponding input of that system in terms of exponentially growing/decaying sinusoidal waves and then solve for the ratio of those two expressions.

Unfortunately, the teaching of transfer functions and their relation to real-world phenomena is often obfuscated by a heavy emphasis on mathematics. The intent of this section is to introduce this concept in a way that is very “gentle” and continually referenced to real-world applications. If I can do anything to help lift the veil of mystery surrounding concepts such as transfer functions, the $s$ variable, and pole-zero plots, then technicians as well as engineers will be able to appreciate the power of this analytical technique and be able to exchange more ideas in the same “language”.

A simple example of a transfer function is the gain of an electronic amplifier. As all students of electronics learn, “gain” is the ratio of output signal to input signal for a circuit. Beginning students learn to represent circuit gains as scalar values (e.g. “The amplifier has a voltage gain of 24”), first as plain ratios and later as decibel figures (e.g. “The amplifier has a voltage gain of 27.6 dB”). One limitation of this approach is that it oversimplifies the situation when the gain of the circuit in question varies with the frequency and/or the growth/decay rate of the signal, which is quite often the case. If we take the engineering approach of expressing output and input signals as functions of $s$, we obtain a more complete picture of that circuit’s behavior over a wide range of conditions.

Another simple example of a transfer function is what we have just seen in this book: the impedance of a reactive electrical component such as a capacitor or an inductor. Here, the ratio in question is between voltage and current. If we consider current through the component to be the “input” signal and voltage across the component to be the “output” signal – both expressed in terms of $s$ – then impedance $Z(s) = \frac{V(s)}{I(s)}$ is the transfer function for that component. This raises an important point about transfer functions: what we define as the “input” and the “output” of the system is quite arbitrary, so long as there is an actual relationship between the two signals.

If we write generalized output/input transfer functions of $s$ for an AC circuit, we may mathematically analyze that transfer function to gain insight into the behavior and characteristics of that circuit. Features of transfer functions of interest to us include:

- **Zeros**: any value(s) of $s$ resulting in a zero value for the transfer function (i.e. zero gain)
- **Poles**: any value(s) of $s$ resulting in an infinite value for the transfer function (i.e. maximum gain)

An AC circuit’s zeros tell us where the circuit is unresponsive to input stimuli. An AC circuit’s poles tell us where the circuit is able to generate an output signal with no input stimulus (i.e. its natural or un-driven mode(s) of response).

In order to clearly understand the concept of transfer functions, practical examples are very helpful. Here we will explore some very simple AC circuits in order to grasp what transfer functions are and how they benefit system analysis.
2.3. TRANSFER FUNCTION ANALYSIS

2.3.1 Example: LR low-pass filter circuit

First, let’s begin with a simple low-pass filter circuit comprised of an inductor and a resistor connected in series:

The impedance of each component as a function of $s$ is shown in the diagram: the inductor’s impedance is $sL$ while the resistor’s impedance is simply $R$. It should be clear to any student of electronics that these two components will function as a voltage divider, with the output voltage being some fraction of the input voltage. Knowing this, we may write a transfer function for this circuit based on the voltage divider formula, which tells us the ratio of output voltage to input voltage is the same as the ratio of output impedance to total impedance:

$$\text{Transfer function} = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{R}{R + sL} = \frac{R}{R + (\sigma + j\omega)L}$$

This transfer function allows us to calculate the “gain” of the system for any given value of $s$, which brings us to the next step of our analysis. At this point we will ask ourselves three questions:

1. How does this system respond when $s = 0$?

2. What value(s) of $s$ make the transfer function approach a value of zero?

3. What value(s) of $s$ make the transfer function approach a value of infinity?

The first of these questions refers to a condition where we apply a steady DC signal to the input of the system. If $s = 0$ then both $\sigma$ and $\omega$ must each be equal to zero. A value of zero for $\sigma$ means the signal is neither growing nor decaying over time, but remains at some unchanging value. A value of zero for $\omega$ means the signal is not oscillating. These two conditions can only refer to a steady DC signal applied to the circuit. Substituting zero for $s$ we get:

$$\frac{R}{R + 0L} = \frac{R}{R} = 1$$

---

What we are really doing here is applying a problem-solving technique called limiting cases. This is where we simplify the analysis of some system by considering scenarios where the mathematical quantities are easy to compute.
Therefore the transfer function of this circuit is unity (1) under DC conditions. This is precisely what we would expect given an inductor connected in series with a resistor, with output voltage taken across the resistor. If there is no change in the applied signal, then the inductor’s magnetic field will be unchanging as well, which means it will drop zero voltage (assuming a pure inductor with no wire resistance) leaving the entire input voltage dropped across the resistor.

The second question refers to a condition where the output signal of this circuit is zero. Any values of $s$ resulting in zero output from the system are called the zeros of the transfer function. Examining the transfer function for this particular low-pass LR filter circuit, we see that this can only be true if $s$ becomes infinitely large, because $s$ is located in the denominator of the fraction:

$$\frac{R}{R \pm \infty L} = 0$$

This is consistent with the behavior of a low-pass filter: as frequency ($\omega$) increases, the filter’s output signal diminishes. The transfer function doesn’t just tell us how this circuit will respond to change in frequency, however – it also tells us how the circuit will respond to growing or decaying signals too. Here, we see that infinitely large $\sigma$ values also result in zero output: the inductor, which tends to oppose any current exhibiting a high rate of change, doesn’t allow much voltage to develop across the resistor if the input signal is growing or decaying very rapidly.

The third question refers to a condition where either the transfer function’s numerator approaches infinity or its denominator approaches zero. Any values of $s$ having this result are called the poles of the transfer function. Since the numerator in this particular case is a constant ($R$), only a denominator value of zero could cause the transfer function to reach infinity:

$$\frac{R}{R + sL} = \infty \text{ only if } R + sL = 0$$

If the necessary condition for a “pole” is that $R + sL = 0$, then we may solve for $s$ as follows:

$$R + sL = 0$$

$$sL = -R$$

$$s = -\frac{R}{L}$$

Thus, this transfer function for this simple low-pass filter circuit has one pole located at $s = -R/L$. Since both $R$ and $L$ are real numbers (not imaginary) with positive values, then the value of $s$ for the pole must be a real number with a negative value. In other words, the solution for $s$ at this pole is all $\sigma$ and no $\omega$: this refers to an exponentially decaying DC signal.
It is important at this point to consider what this “pole” condition means in real life. The notion that a circuit is able to produce an output signal with zero input signal may sound absurd, but it makes sense if the circuit in question has the ability to store and release energy. In this particular circuit, the inductor is the energy-storing component, and it is able to produce a voltage drop across the resistor with zero input voltage in its “discharging” mode.

An illustration helps make this clear. If the “pole” condition is such that $V_{in}(s) = 0$, we may show this by short-circuiting the input of our filter circuit to ensure a zero-input condition:

Assuming the inductor has been “charged” with energy previous to the short-circuiting of the input, an output voltage will surely develop across the resistor as the inductor discharges. In other words, the inductor behaves as an electrical source while the resistor behaves as an electrical load: connected in series they must of course share the same current, but their respective voltages are equal in magnitude and opposing in polarity in accordance with Kirchhoff’s Voltage Law. Furthermore, the value of $s$ in this “pole” condition tells us exactly how rapidly the output signal will decay: it will do so at a rate $\sigma = -\frac{R}{L}$. Recall that the growth/decay term of the $s$ variable the reciprocal of the system’s time constant ($\sigma = 1/\tau$). Therefore, a $\sigma$ value of $R/L$ is equivalent to a time constant of $L/R$, which as all beginning students of electronics learn is how we calculate the time constant for a simple inductor-resistor circuit.
Transfer functions are easier to understand when graphically plotted as three-dimensional surfaces: the real and imaginary portions of the $s$ variable occupying the horizontal axes, and the magnitude of the transfer function fraction displayed as height. Here is a pole-zero plot of this low-pass filter circuit’s transfer function, with a resistor value of $R = 5 \, \Omega$ and an inductor value of $L = 10 \, \text{H}$:

\[
\frac{R}{R + sL} = \frac{5}{5 + s10}
\]

This surface plot makes the meaning of the term “pole” quite obvious: the shape of the function looks just like a rubber mat stretched up at one point by a physical pole. Here, the “pole” rises to an infinite\(^6\) height at a value of $s$ where $\sigma = -0.5$ time constants per second and $\omega = 0$ radians per second. The surface is seen to decrease in height at all edges of the plot, as $\sigma$ and $\omega$ increase in value.

The “zero” of this transfer function is not as obvious as the pole, since the function’s value does not equal zero unless and until $s$ becomes infinite, which of course cannot be plotted on any finite domain. Suffice to say that the zero of this transfer function lies in all horizontal directions at an infinite distance away from the plot’s origin (center), explaining why the surface slopes down to zero everywhere with increasing distance from the pole.

\[^6\text{Of course, the mathematical plotting software cannot show a pole of truly infinite height, and so the pole has been truncated. This is why it appears to have a “flat” top.}\]
One of the valuable insights provided by a three-dimensional pole-zero plot is the system’s response to an input signal of constant magnitude and varying frequency. This is commonly referred to as the frequency response of the system, its graphical representation called a Bode plot. We may trace the Bode plot for this system by revealing a cross-sectional slice of the three-dimensional surface along the plane where $\sigma = 0$ (i.e. showing how the system responds to sinusoidal waves of varying frequency that don’t grow or decay over time):

$$\frac{R}{R + sL} = \frac{5}{5 + s10}$$

Only one-half of the pole-zero surface has been plotted here, in order to better reveal the cross-section along the $j\omega$ axis. The bold, red curve traces the edge of the transfer function surface as it begins at zero frequency (DC) to increasingly positive values of $j\omega$. The red trace is therefore the Bode plot for this low-pass filter, starting at a maximum value of 1 ($V_{out} = V_{in}$ for a DC input signal) and approaching zero as frequency increases.
As insightful as three-dimensional pole-zero plots are, they are laborious to plot by hand, and even with the aid of a computer may require significant time to set up. For this reason, pole-zero plots have traditionally been drawn in a two-dimensional rather than three-dimensional format, from a “bird’s eye” view looking down at the \( s \) plane. Since this view hides any features of height, poles and zeros are instead located on the \( s \) plane by \( \times \) and \( \circ \) symbols, respectively. An example of a traditional pole-zero plot for our low-pass filter appears here:

\[
\begin{align*}
\text{Pole at } s &= (-0.5 + j0) \\
\end{align*}
\]

Admittedly, this type of pole-zero plot is much less interesting to look at than a three-dimensional surface plotted by computer, but nevertheless contains useful information about the system. The single pole lying on the real (\( \sigma \)) axis tells us the system will not self-oscillate (i.e. \( \omega = 0 \) at the pole), and that it is inherently stable: when subjected to a pulse, its natural tendency is to decay to a stable value over time (i.e. \( \sigma < 0 \)).

It should be noted that transfer functions and pole-zero plots apply to much more than just filter circuits. In fact, any physical system having the same “low-pass” characteristic as this filter circuit is describable by the same transfer function and the same pole-zero plots. Electric circuits just happen to be convenient applications because their individual component characteristics are so easy to represent as functions of \( s \). However, if we are able to characterize the components of a different physical system in the same terms\(^8\), the same mathematical tools apply.

---

\(^7\)My first pole-zero plot using the \texttt{ePiX} \texttt{C++} mathematical visualization library took several hours to get it just right. Subsequent plots went a lot faster, of course, but they still require substantial amounts of time to adjust for a useful and aesthetically pleasing appearance.

\(^8\)A powerful mathematical technique known as a Laplace Transform does this very thing: translate any differential equation describing a physical system into functions of \( s \), which may then be analyzed in terms of transfer functions and pole-zero plots.
2.3.2 Example: RC high-pass filter circuit

For our next exploratory example we will consider another simple filter circuit, this time comprised of a capacitor and a resistor, with the output signal taken across the resistor. As before, we may derive a transfer function by expressing $V_{out}/V_{in}$ as a ratio of the resistor’s impedance to the total series resistor-capacitor impedance (treating this as a voltage divider circuit):

$$Z(s) = \frac{R}{sC}$$

$$T ransfer \ function = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{R}{\left(\sigma + j\omega\right)C}$$

After writing this initial transfer function based on component impedances, we will algebraically manipulate it to eliminate compound fractions. This will aid our analysis of the circuit’s DC response, zeros, and poles:

$$\frac{R}{R + \frac{1}{sC}}$$

$$\frac{R}{\frac{sRC}{sC} + \frac{1}{sC}}$$

$$\frac{R}{\frac{1 + sRC}{sC}}$$

$$sRC\frac{1 + sRC}{1 + \frac{sRC}{sC}}$$

This transfer function allows us to calculate the “gain” of the system for any given value of $s$, which brings us to the next step of our analysis. Once again we will ask ourselves three questions about the transfer function:

1. How does this system respond when $s = 0$?

2. What value(s) of $s$ make the transfer function approach a value of zero?

3. What value(s) of $s$ make the transfer function approach a value of infinity?
In answer to the first question, we see that the transfer function is equal to zero when \( s = 0 \):

\[
\frac{0RC}{1 + 0RC} = \frac{0}{1} = 0
\]

Of course, a value of 0 for \( s \) means exposure to a steady DC signal: one that neither grows nor decays over time, nor oscillates. Therefore, this resistor-capacitor circuit will output zero voltage when exposed to a purely DC input signal. This makes conceptual sense when we examine the circuit itself: a DC input signal voltage means the capacitor will not experience any change in voltage over time, which means it will not pass any current along to the resistor. With no current through the resistor, there will be no output voltage. Thus, the capacitor “blocks” the DC input signal, preventing it from reaching the output. This behavior is exactly what we would expect from such a circuit, which any student of electronics should immediately recognize as being a simple high-pass filter: DC is a condition of zero frequency, which should be completely blocked by any filter circuit with a high-pass characteristic.

The answer to our first question is also the answer to the second question: “what value of \( s \) makes the transfer function equal to zero?” Here we see that it is only at a value of \( s = 0 \) that the entire transfer function’s value will be zero. Any other values for \( s \) – even infinite – yield non-zero results. In contrast to the last circuit (the resistor-inductor low-pass filter) this circuit exhibits a singular “zero” point in its transfer function: one specific location on the pole-zero plot where the function’s value diminishes to nothing.

When we consider the third question (“What value(s) of \( s \) make the transfer function approach a value of infinity?”) we proceed the same as before: by finding value(s) of \( s \) which will make the denominator of the transfer function fraction equal to zero. If we set the denominator portion equal to zero and solve for \( s \), we will obtain the pole for the circuit:

\[
1 + sRC = 0
\]

\[
sRC = -1
\]

\[
s = -\frac{1}{RC}
\]

We know that both \( R \) and \( C \) are real numbers, not imaginary. This tells us that \( s \) will likewise be a real number at the pole. That is to say, \( s \) will be comprised of all \( \sigma \) and no \( \omega \). The fact that the value of \( \sigma \) is negative tells us the pole represents a condition of exponential decay, just the same as in the case of the resistor-inductor low-pass filter. As before, this means the circuit will produce an output voltage signal with no\(^9\) input voltage signal when the rate of signal decay is \( \sigma = -1/RC \).

Recall that the rate of decay in the \( s \) variable (\( \sigma \)) is nothing more than the reciprocal of the system’s time constant (\( \tau \)). Thus, a rate of decay equal to \( 1/RC \) equates to a time constant \( \tau = RC \).

\(^9\)As before, this counter-intuitive condition is possible only because the capacitor in this circuit has the ability to store energy. If the capacitor is charged by some previous input signal event and then allowed to discharge through the resistor, it becomes possible for this circuit to develop an output voltage even with short-circuited input terminals.
which as all electronics students know is how we calculate the time constant for any simple resistor-capacitor circuit.

Using a computer to plot a three-dimensional representation of this transfer function, we clearly see both the pole and the zero as singularities. Here I have assumed a 10 Ω resistor and a 0.2 F capacitor to place the pole at the same location as with the low-pass filter circuit $s = -0.5 + j0$, for an equitable comparison:

$$\frac{sRC}{1+sRC} = \frac{s(10)(0.2)}{1+s(10)(0.2)}$$

Here we see both the pole at $s = -0.5 + j0$ and the zero at $s = 0 + j0$ quite clearly: the pole is a singular point of infinite height while the zero is a singular point of zero height. The three-dimensional surface of the transfer function looks like a rubber sheet that has been stretched to an infinite height at the pole and stretched to ground level at the zero.
As in the last example, we may re-plot the transfer function in a way that shows a cross-sectional view at \( \sigma = 0 \) in order to reveal the frequency response of this high-pass filter circuit:

\[
\frac{sRC}{1 + sRC} = \frac{s(10)(0.2)}{1 + s(10)(0.2)}
\]

Once again the bold, red curve traces the edge of the transfer function surface as it begins at zero frequency (DC) to increasingly positive values of \( j\omega \). The red trace is therefore the Bode plot for this high-pass filter, starting at a minimum value of 0 (\( V_{out} = 0 \) for a DC input signal) and approaching unity (1) as frequency increases. Naturally, this is the type of response we would expect to see exhibited by a high-pass filter circuit.
A more traditional two-dimensional pole-zero plot for this circuit locates the pole with a “×” symbol and the zero with a “○” symbol:
2.3.3 Example: LC “tank” circuit

Next, we will explore the transfer function for a tank circuit, comprised of a capacitor and an inductor. We will assume the use of pure reactances here with no electrical resistance or other energy losses of any kind, just to analyze an ideal case. The output voltage in this particular circuit will be taken across the inductor:

\[
\begin{align*}
&\text{C} & Z(s) = \frac{1}{sC} \\
&\text{L} & Z(s) = sL
\end{align*}
\]

Writing the transfer function for this tank circuit is (once again) a matter of expressing the ratio between the output component’s impedance versus the total circuit impedance:

\[
\text{Transfer function} = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{sL}{sL + \frac{1}{sC}}
\]

Algebraically manipulating this function to eliminate compound fractions:

\[
\begin{align*}
\frac{sL}{sL + \frac{1}{sC}} & = \frac{sL}{sL + \frac{1}{sC}} \\
& = \frac{sL}{s^2LC + 1} \\
& = \frac{sL}{s^2LC + 1}
\end{align*}
\]

Note how this transfer function contains \(s^2\) terms rather than \(s\) terms. This makes it a second-order function, which will yield very different results on the pole-zero plot than what we saw with either the resistor-inductor or resistor-capacitor filter circuits.
Asking ourselves the same three questions again:

1. How does this system respond when \( s = 0 \)?
2. What value(s) of \( s \) make the transfer function approach a value of zero?
3. What value(s) of \( s \) make the transfer function approach a value of infinity?

In answer to the first question, we see that the transfer function is equal to zero when \( s = 0 \):

\[
\frac{s^2LC}{s^2LC + 1} = \frac{0}{0 + 1} = \frac{0}{1} = 0
\]

As with the RC low-pass filter, its response at DC also happens to be a “zero” for the transfer function. With a DC input signal, the output signal of this circuit will be zero Volts.

In order to find poles for this transfer function, we must solve for values of \( s \) that will make the denominator term of the transfer function equal to zero:

\[
s^2LC + 1 = 0
\]

\[
s^2LC = -1
\]

\[
s^2 = -\frac{1}{LC}
\]

\[
s = \pm j\sqrt{\frac{1}{LC}}
\]

Given the fact that both \( L \) and \( C \) are real, positive numbers, and therefore solving for \( s \) requires we take the square root of a negative real number, we see that the value of \( s \) must be imaginary. We also see here that there are two poles in this transfer function: one at \( s = 0 + j\sqrt{\frac{1}{LC}} \) and another at \( s = 0 - j\sqrt{\frac{1}{LC}} \).
Using a computer to plot a three-dimensional representation of this transfer function, we clearly see a single zero at $s = 0$ and two poles symmetrically positioned along the $j\omega$ axis. Here I have assumed a 0.2 F capacitor and a 5 H inductor for component values:

$$\frac{s^2LC}{s^2LC+1}$$

The two poles located on the $j\omega$ axis (one at $s = 0 + j1$ and the other at $s = 0 - j1$) tell us the circuit is able to generate an oscillating output signal ($\omega = 1$ radian per second frequency) at constant magnitude ($\sigma = 0$) with no input signal. This is only possible because we have assumed a perfect capacitor and a perfect inductor with no energy losses whatsoever. If we charge up either or both of these components and then immediately short-circuit the input of the circuit to ensure $V_{in} = 0$, it will oscillate at its resonant frequency forever.

The two poles located on the $j\omega$ axis (one at $s = 0 + j1$ and the other at $s = 0 - j1$) tell us the circuit is able to generate an oscillating output signal ($\omega = 1$ radian per second frequency) at constant magnitude ($\sigma = 0$) with no input signal. This is only possible because we have assumed a perfect capacitor and a perfect inductor with no energy losses whatsoever. If we charge up either or both of these components and then immediately short-circuit the input of the circuit to ensure $V_{in} = 0$, it will oscillate at its resonant frequency forever.

---

10. The two solutions for $\omega$ (one at $+1$ radian per second and the other at $-1$ radian per second) merely indicate the circuit is able to oscillate “forward” as well as “backward”. In other words, it is able to oscillate sinusoidally where the positive peak occurs at time $t = 0$ ($+1$ rad/sec) as well as oscillate sinusoidally where the negative peak occurs at time $t = 0$ ($-1$ rad/sec). We will find that solutions for $s$ in general are symmetrical about the real axis, meaning if there is any solution for $s$ requiring an imaginary number value, there will be two of them: one with a positive imaginary value and the other with a negative imaginary value.
Earlier we noted that the poles in this circuit were \( s = 0 + j\sqrt{\frac{1}{LC}} \) and \( s = 0 - j\sqrt{\frac{1}{LC}} \). In other words, its resonant frequency is \( \omega = \sqrt{\frac{1}{LC}} \). Recalling that the definition for \( \omega \) is radians of rotation per second, and that there are \( 2\pi \) radians in one complete revolution (cycle), we can derive the familiar resonant frequency formula for a simple LC circuit:

\[
\omega = \sqrt{\frac{1}{LC}}
\]

... substituting \( 2\pi f \) for \( \omega \) ...

\[
2\pi f = \sqrt{\frac{1}{LC}}
\]

\[
f = \frac{1}{2\pi\sqrt{LC}}
\]

Taking a cross-section of this surface plot at \( \sigma = 0 \) to obtain a frequency response (Bode plot) of the LC tank circuit, we see the output of this circuit begin at zero when the frequency (\( \omega \)) is zero, then the output peaks at the resonant frequency (\( \omega = 1 \) rad/sec), then the output approaches unity (1) as frequency increases past resonance:
A more traditional two-dimensional pole-zero plot for this circuit locates shows the zero and the two poles using “○” and “×” symbols:
2.3. TRANSFER FUNCTION ANALYSIS

2.3.4 Example: RLC band-pass filter circuit

For our next example circuit, we will add a resistor in series with the inductor and capacitor to explore its effects on the transfer function. Taking our output voltage across the resistor, we should expect to see band-pass filtering behavior from this circuit, with maximum voltage developing across the resistor at one frequency where the inductor’s and capacitor’s impedances cancel:

\[
\begin{align*}
C & : Z(s) = \frac{1}{sC} \\
L & : Z(s) = sL \\
R & : Z(s) = R
\end{align*}
\]

As usual, the transfer function for this circuit is the ratio between the output component’s impedance \((R)\) and the total series impedance, functioning as a voltage divider:

\[
\text{Transfer function} = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{R}{R + sL + \frac{1}{sC}}
\]

Algebraically manipulating this function to eliminate compound fractions:

\[
\begin{align*}
R & \\
R + sL + \frac{1}{sC} & \\
\frac{R}{sRC + \frac{s^2LC}{sC} + \frac{1}{sC}} & \\
\frac{R}{sRC + s^2LC + 1} & \\
\frac{sRC}{sRC + s^2LC + 1} & \\
\end{align*}
\]

As with the pure tank circuit analyzed previously, we can see that this LC circuit exhibits a second-order transfer function because it contains an \(s^2\) term. Running through our three questions again:

1. How does this system respond when \(s = 0\)?
2. What value(s) of \(s\) make the transfer function approach a value of zero?
3. What value(s) of \(s\) make the transfer function approach a value of infinity?
The answers to the first two questions are one and the same: the numerator of the transfer function will be zero when \( s = 0 \), this being the single zero of the function. Recalling that a condition of \( s = 0 \) represents a DC input signal (no growth or decay over time, and no oscillation), this makes perfect sense: the presence of the DC-blocking series capacitor in this circuit ensures the output voltage under steady-state conditions must be zero.

In answering the third question to identify any poles for this circuit, we encounter a more complicated mathematical problem than seen with previous example circuits. The denominator of the transfer function’s fraction is a second-degree polynomial in the variable \( s \). As you may recall from your study of algebra, any solution resulting in a polynomial having an over-all value of zero is called a root of that polynomial expression. Since this particular expression is found in the denominator of the transfer function where we know zero values mark poles of the system, and solutions for \( s \) resulting are roots of the polynomial, then roots of the expression \( sRC + s^2LC + 1 \) must mark the locations of the poles on the \( s \) plane.

A very useful algebraic tool for finding roots of a second-degree polynomial expression is the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Where,

- \( ax^2 + bx + c \) is a polynomial expression
- \( x \) is the independent variable of that polynomial expression
- \( a \) is the coefficient of the second-degree \( (x^2) \) term
- \( b \) is the coefficient of the first-degree \( (x) \) term
- \( c \) is the coefficient of the zero-degree (constant) term

Reviewing the denominator of our transfer function again, we see that \( s \) is the independent variable, and so \( LC \) must be the “\( a \)” coefficient, \( RC \) must be the “\( b \)” coefficient, and \( 1 \) must be the “\( c \)” coefficient. Substituting these variables into the quadratic formula will give us a formula for computing the poles of this resistor-inductor-capacitor circuit:

\[
s = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}
\]

Perhaps the most interesting part of this formula is what lies beneath the radicand (square-root) symbol: \( (RC)^2 - 4LC \). This portion of the quadratic formula is called the discriminant, and its value determines both the number of roots as well as their real or imaginary \(^1\) character. If the discriminant is equal to zero, there will be a single real root for our polynomial and therefore only one pole for our circuit. If the discriminant is greater than zero (i.e. a positive value), then there will be two real roots and therefore two poles lying on the \( \sigma \) axis (i.e. no imaginary \( j\omega \) parts). If the discriminant is less than zero (i.e. a negative value), then there will be two complex roots for our polynomial and therefore two complex poles having both real and imaginary parts.

Let us consider for a moment what the sign of the discriminant means in practical terms. A pole that is purely real means a value for \( s \) that is all \( \sigma \) and no \( \omega \): representing a condition of growth

---

\(^1\)The only way to obtain a purely imaginary root for this polynomial is for the “\( b \)” coefficient to be equal to zero. For our example circuit, it means either \( R \) or \( C \) would have to be zero, which is impossible if both of those components are present and functioning. Thus, our RLC filter circuit will have either real poles or complex poles.
or decay but with no oscillation. This is similar to what we saw with the LR or RC low/high pass filter circuits, where the circuit in a state of discharge could generate an output signal even with its input terminals shorted to ensure no input signal.

If we have a positive value for the discriminant which yields two real poles, it means two different possible values for $\sigma$ (rate of growth/decay) that could occur with no signal input to the circuit. This behavior is only possible with two energy-storing components in the circuit: a kind of double time-constant where different portions of the circuit discharge at different rates. The lack of any imaginary part within $s$ means the circuit still will not self-oscillate.

If we have a negative value for the discriminant which yields two complex poles, it means two different values for $s$ both having real and imaginary parts. Since the real part ($\sigma$) represents growth/decay while the imaginary part ($\omega$) represents oscillation, complex poles tell us the circuit will be able to self-oscillate but not at a constant magnitude as with an ideal (lossless) tank circuit. In fact, intuition should tell us these complex poles must have negative real values representing decaying oscillations, because it would violate the Law of Energy Conservation for our circuit to self-oscillate with increasing magnitude.

Looking at the discriminant $(RC)^2 - 4LC$ we see that it is possible to push the circuit into any one of these three modes of operation merely by adjusting the value of $R$ and leaving both $L$ and $C$ unchanged. If we wish to calculate the critical value of $R$ necessary to produce a single real pole for any given values of $L$ and $C$, we may set the discriminant equal to zero and algebraically solve for $R$ as follows:

\[
(RC)^2 - 4LC = 0
\]

\[
(RC)^2 = 4LC
\]

\[
RC = \sqrt{4LC}
\]

\[
RC = 2\sqrt{LC}
\]

\[
R = \frac{2\sqrt{LC}}{C}
\]

\[
R = \frac{2\sqrt{LC}}{\sqrt{C}\sqrt{C}}
\]

\[
R = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{L}{C}}
\]

This critical value of $R$ resulting in one real pole is the minimum amount of resistance necessary to prevent self-oscillation. If the circuit is operating at this point, it is said to be critically damped. Larger values of $R$ will result in multiple real poles, where the circuit is said to be over-damped. Smaller values of $R$ will permit some self-oscillation to occur, and the circuit is said to be under-damped.
Sometimes electrical engineers intentionally install resistors into circuits containing both inductance and capacitance for the express purpose of damping oscillations. In such cases, the resistor is called an anti-resonance resistor, because its purpose is to combat resonant oscillations that would otherwise occur as the inductive and capacitive elements of the circuit exchange energy back and forth with each other. If the engineer’s intent is to install just enough resistance into the circuit to prevent oscillations without creating unnecessary time delays, then the best value of the resistor will be that which causes critical damping.

Recall that the subject of transfer functions, poles, and zeros applies to any system that is both linear (i.e. its output and input magnitudes will be proportional to each other for all conditions) and time-invariant (i.e. the essential characteristics of the system do not change with the passage of time), not just AC circuits. Mechanical systems, feedback control loops, and many other physical systems may be characterized in the same way using the same mathematical tools. This particular subject of damping is extremely important in applications where oscillations are detrimental. Consider the design of an automobile’s suspension system, where the complementary energy-storing phenomena of spring tension and vehicle mass give rise to oscillations following impact with a disturbance in the road surface. It is the job of the shock absorber to act as the “resistor” in this system and dissipate energy in order to minimize oscillations following a bump in the road. An under-sized shock absorber won’t do a good enough job dissipating the energy of the disturbance, and so the vehicle’s suspension will exhibit complex poles (i.e. there will be some lingering oscillations following a bump). An over-sized shock absorber will be too “stiff” and allow too much of the bump’s energy to pass through to the vehicle frame and passengers. A perfectly-sized shock absorber, however, will “critically damp” the system to completely prevent oscillation while presenting the smoothest ride possible.

In order to fully illustrate the characteristics of this circuit’s transfer function, we will do so for three different resistor values: one where $R$ yields critical damping (one real pole), one where $R$ makes the circuit over-damped (two real poles), and one where $R$ makes the circuit under-damped (two complex poles). We will use three-dimensional plotting to show the transfer function response in each case. To be consistent with our former tank circuit example, we will assume the same capacitor value of 0.2 Farads and the same inductor value of 5 Henrys. The resistor value will be modified in each case to create a different damping condition.
First, the critically-damped example, with a resistor value of 10 Ohms:

\[
\begin{align*}
    s &= \frac{R}{sRC + s^2LC + 1} \\
    L &= 5 ; C = 0.2 \\
    R &= 10
\end{align*}
\]

Critically damped

As expected, a single zero appears at \( s = 0 \), and a single \( s^2 \) pole at \( s = -1 + j0 \). Thus, this circuit has a decay rate of -1 time constants per second (\( \tau = 1 \) second) as seen when we use the quadratic formula to solve for \( s \):

\[
s = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}
\]

\[
s = \frac{-(10)(0.2) \pm \sqrt{[(10)(0.2)]^2 - (4)(5)(0.2)}}{(2)(5)(0.2)}
\]

\[
s = \frac{-2 \pm \sqrt{0}}{2} = (-1 + j0) \sec^{-1}
\]

Interestingly, only \( L \) and \( R \) determine the decay rate (\( \sigma \), the real part of \( s \)) in the critically damped condition. This is clear to see if we set the discriminant to zero in the quadratic formula and look for variables to cancel:

\[
s = \frac{-RC \pm \sqrt{0}}{2LC} = \frac{-R}{2L}
\]

\(^{12}\) Or, one might argue there are two superimposed poles, one at \( s = -1 + j0 \) and another at \( s = -1 - j0 \).
Next, we will plot the same transfer function with a larger resistor value (15 Ohms) to ensure over-damping:

\[
\frac{sRC}{sRC + s^2LC + 1}
\]

\[L = 5; C = 0.2\]
\[R = 15\]

*Over-damped*

We clearly see two poles\(^\text{13}\) centered along the \(\sigma\) axis in this plot, representing the two real roots of the transfer function’s denominator. Again, we will use the quadratic formula to solve for these two values of \(s\):

\[
s = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}
\]

\[
s = \frac{-(15)(0.2) \pm \sqrt{[(15)(0.2)]^2 - (4)(5)(0.2)}}{(2)(5)(0.2)}
\]

\[
s = \frac{-3 \pm \sqrt{3^2 - 4}}{2}
\]

\[
s = \frac{-3 + \sqrt{5}}{2} = (-0.382 + j0) \text{ sec}^{-1}
\]

\[
s = \frac{-3 - \sqrt{5}}{2} = (-2.618 + j0) \text{ sec}^{-1}
\]

These real poles represent two different decay rates (time constants) for the over-damped circuit: a fast decay rate of \(\sigma = -2.618 \text{ sec}^{-1}\) and a slow decay rate of \(\sigma = -0.382 \text{ sec}^{-1}\), the slower of these two decay rates dominating the circuit’s transient response over long periods of time.

\(^\text{13}\)The center of the pole farthest from the plot’s origin actually lies outside the plotted area, which is why that pole appears to be vertically sliced. This plot’s domain was limited to the same values (±2) as previous plots for the sake of visual continuity, the compromise here being an incomplete mapping of one pole.
Next, we will plot the same transfer function with a smaller resistor value (5 Ohms) to ensure under-damping:

\[
\frac{sRC}{sRC + s^2LC + 1}
\]

\[L = 5\]
\[C = 0.2\]
\[R = 5\]

Under-damped

We clearly see two poles once again, but neither of them are located on an axis. These represent two complex values for \(s\) describing the circuit’s behavior with zero input. The imaginary \((j\omega)\) part of \(s\) tells us the circuit has the ability to self-oscillate. The negative, real \((\sigma)\) part of \(s\) tells us these oscillations decrease in magnitude over time. Using the quadratic formula to solve for these two poles:

\[
s = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}
\]

\[s = \frac{-(5)(0.2) \pm \sqrt{(5)(0.2)^2 - (4)(5)(0.2)}}{(2)(5)(0.2)}
\]

\[s = \frac{-1 \pm \sqrt{1}^2 - 4}{2}
\]

\[s = \frac{-1 + \sqrt{-3}}{2} = (-0.5 + j0.866) \text{ sec}^{-1}
\]

\[s = \frac{-1 - \sqrt{-3}}{2} = (-0.5 - j0.866) \text{ sec}^{-1}
\]

The calculated \(\omega\) value of 0.866 radians per second is slower than the 1 radian per second resonant frequency calculated for the pure tank circuit having the same \(L\) and \(C\) values, revealing that the damping resistor skews the “center” frequency of this RLC band-pass filter. The calculated \(\sigma\) value of \(-0.5\) time constants per second (equivalent to a time constant of \(\tau = 2\) seconds) describes the
rate at which the sinusoidal oscillations decay in magnitude. Here as well we see the under-damped decay rate \( \sigma = -0.5 \text{ sec}^{-1} \) is slower than the critically damped decay rate \( \sigma = -1 \text{ sec}^{-1} \).

If we compare two-dimensional pole-zero plots for each of the three resistor values in this RLC circuit, we may contrast the over-damped, critically-damped, and under-damped responses:

- **Over-damped** \((R = 15 \Omega)\)
  - Pole at \(s = (-0.382 + j0)\)
  - Pole at \(s = (-2.618 + j0)\)
  - Zero at \(s = (0 + j0)\)

- **Critically damped** \((R = 10 \Omega)\)
  - Pole at \(s = (-0.5 + j0.866)\)
  - Pole at \(s = (-0.5 - j0.866)\)
  - Zero at \(s = (0 + j0)\)

- **Under-damped** \((R = 5 \Omega)\)
  - Pole at \(s = (-0.5 + j0.866)\)
  - Pole at \(s = (-0.5 - j0.866)\)
  - Zero at \(s = (0 + j0)\)
2.3. TRANSFER FUNCTION ANALYSIS

If we take cross-sectional plots of the transfer function at \( \sigma = 0 \) to show the frequency response of this RLC band-pass filter, we see the response become “sharper” (more selective) as the resistor’s value decreases and the poles move closer to the \( j\omega \) axis. Electronic technicians relate this to the quality factor or \( Q \) of the band-pass filter circuit, the circuit exhibiting a higher “quality” of band-pass selection as the ratio of reactance to resistance increases:

\[
\begin{align*}
\text{R} &= 15 \\
\text{Over-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 10 \\
\text{Critically damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 8 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 7 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 5 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 4 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 3 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 2 \\
\text{Under-damped}
\end{align*}
\]

\[
\begin{align*}
\text{R} &= 1 \\
\text{Under-damped}
\end{align*}
\]

Although each and every pole in a pole-zero plot is has the same (infinite) height, poles grow narrower when moved farther away from each other, and wider when closely spaced. As the resistance in this circuit decreases and the poles move farther away from each other and closer to the \( j\omega \) axis, their widths narrow and the frequency response curve’s peak becomes narrower and steeper.
2.4 Summary

Here is a summary of some of the major concepts important to transfer function analysis:

- The $s$ variable is an expression of growing/decaying sinusoidal waves, comprised of a real part and an imaginary part ($s = \sigma + j\omega$). The real part of $s$ ($\sigma$) is the growth/decay rate, telling us how rapidly the signal grows or decays over time, with positive values of $\sigma$ representing growth and negative values of $\sigma$ representing decay. This growth/decay rate is the reciprocal of time constant ($\sigma = 1/\tau$), and is measured in reciprocal units of time (time constants per second, or sec$^{-1}$). The imaginary part of $s$ ($j\omega$) represents the frequency of the sinusoidal quantity, measured in radians per second (sec$^{-1}$).

- An important assumption we make when analyzing any system’s transfer function(s) is that the system is linear (i.e. its output and input magnitudes will be proportional to each other for all conditions) and time-invariant (i.e. the essential characteristics of the system do not change with the passage of time). If we wish to analyze a non-linear system using these tools, we must limit ourselves to ranges of operation where the system’s response is approximately linear, and then accept small errors between the results of our analysis and the system’s real-life response.

- For any linear time-invariant system (an “LTI” system), $s$ is descriptive throughout the system. In other words, for a certain value of $s$ describing the input to this system, that same value of $s$ will also describe the output of that system.

- A transfer function is an expression of a systems’ gain, measured as a ratio of output over input. In engineering, transfer functions are typically mathematical functions of $s$ (i.e. $s$ is the independent variable in the formula). When expressed in this way, the transfer function for a system tells us how much gain the system will have for any given value of $s$.

- Transfer functions are useful for analyzing the behavior of electric circuits, but they are not limited to this application. Any linear system, whether it be electrical, mechanical, chemical, or otherwise, may be characterized by transfer functions and analyzed using the same mathematical techniques. Thus, transfer functions and the $s$ variable are general tools, not limited to electric circuit analysis.

- A zero is any value of $s$ that results in the transfer function having a value of zero (i.e. zero gain, or no output for any magnitude of input). This tells us where the system will be least responsive. On a three-dimensional pole-zero plot, each zero appears as a low point where the surface touches the $s$ plane. On a traditional two-dimensional pole-zero plot, each zero is marked with a circle symbol (○). We may solve for the zero(s) of a system by solving for value(s) of $s$ that will make the numerator of the transfer function equal to zero, since the numerator of the transfer function represents the output term of the system.

- A pole is any value of $s$ that results in the transfer function having an infinite value (i.e. maximum gain, yielding an output without any input). This tells us what the system is capable
of doing when it is not being “driven” by any input stimulus. Poles are typically associated with energy-storing elements in a passive system, because the only way an unpowered system could possibly generate an output with zero input is if there are energy-storing elements within that system discharging themselves to the output. On a three-dimensional pole-zero plot, each pole appears as a vertical spike on the surface reaching to infinity. On a traditional two-dimensional pole-zero plot, each pole is marked with a (×) symbol. We may solve for the pole(s) of a system by solving for value(s) of s that will make the denominator of the transfer function equal to zero, since the denominator of the transfer function represents the input term of the system.

- **Second-order** systems are capable of self-oscillation. This is revealed by poles having imaginary values. These oscillations may be completely undamped (i.e. s is entirely imaginary, with \(σ = 0\)), in which case the system is able to oscillate forever on its own. If energy-dissipating elements are present in a second-order system, the oscillations will be damped (i.e. decay in magnitude over time).

- An **under-damped** system exhibits complex poles, with s having both imaginary \((jω)\) frequency values and real \(σ\) decay values. This means the system can self-oscillate, but only with decreasing magnitude over time.

- A **critically damped** system is one having just enough dissipative behavior to completely prevent self-oscillation, exhibiting a single pole having only a real \(σ\) value and no imaginary \((jω)\) value.

- An **over-damped** system is one having excessive dissipation, exhibiting multiple real poles. Each of these real poles represents a different decay rate \(σ\) or time constant \((τ = 1/σ)\) in the system. When these decay rates differ in value substantially from one another, the slowest one will dominate the behavior of the system over long periods of time.
Chapter 3

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read\(^1\) the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture\(^2\), the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

\(^1\)Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

\(^2\)Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 3. QUESTIONS

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an *intelligent child*: as simple as you can without compromising too much accuracy.

- **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

- Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

- Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

- Identify any **new concept(s)** presented in the text, and explain in your own words.

- Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

- Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

- Devise an experiment to **disprove** a plausible misconception.

- Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

- Describe any useful **problem-solving strategies** applied in the text.

- **Devise a question** of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.

- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.

- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?

- Is there more than one way to solve this problem? Which method seems best to you?

- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.

- What would you say was the most challenging part of this problem, and why was it so?

- Was any important information missing from the problem which you had to research or recall?

- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?

- Examine someone else’s solution to identify where they applied fundamental laws or principles.

- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).

- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.

- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.

- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.

- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?

- Identify where it would be easy for someone to go astray in attempting to solve this problem.

- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?

- Identify some of the challenges you faced in completing this experiment or project.
CHAPTER 3. QUESTIONS

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system’s function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?
3.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking\(^3\). In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your \textit{reasoning} than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

\(^3\text{Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.}\)
3.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

✓ Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

✓ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

✓ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

✓ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

✓ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

✓ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
3.1.2 Foundational concepts
Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- **Energy**
- **Conservation of Energy**
- **Limiting cases as a problem-solving strategy**
- **Other Concept**

3.1.3 First conceptual question
This is the text of the question!

- **Challenges**
  - ???.
  - ???.
  - ???.

3.1.4 Second conceptual question
This is the text of the question!

- **Challenges**
  - ???.
  - ???.
  - ???.
3.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!  

---

4In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
3.2. QUANTITATIVE REASONING

3.2.1 Miscellaneous physical constants

Note: constants shown in bold type are exact, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, Avogadro’s number given as \(6.02214179(30) \times 10^{23}\) means the center value \((6.02214179 \times 10^{23})\) plus or minus \(0.00000030 \times 10^{23}\).

Avogadro’s number \(\left( N_A \right) = 6.02214179(30) \times 10^{23} \) per mole (mol\(^{-1}\))

Boltzmann’s constant \(\left( k \right) = 1.3806504(24) \times 10^{-23} \) Joules per Kelvin (J/K)

Electronic charge \(\left( e \right) = 1.602176487(40) \times 10^{-19} \) Coulomb (C)

Faraday constant \(\left( F \right) = 9.64853399(24) \times 10^{4} \) Coulombs per mole (C/mol)

Magnetic permeability of free space \(\left( \mu_0 \right) = 1.25663706212(19) \times 10^{-6} \) Henrys per meter (H/m)

Electric permittivity of free space \(\left( \epsilon_0 \right) = 8.8541878128(13) \times 10^{-12} \) Farads per meter (F/m)

Characteristic impedance of free space \(\left( Z_0 \right) = 376.730313668(57) \) Ohms (Ω)

Gravitational constant \(\left( G \right) = 6.67428(67) \times 10^{-11} \) cubic meters per kilogram-seconds squared (m\(^3\)/kg-s\(^2\))

Molar gas constant \(\left( R \right) = 8.314472(15) \) Joules per mole-Kelvin (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant \(\left( h \right) = 6.62606896(33) \times 10^{-34} \) joule-seconds (J-s)

Stefan-Boltzmann constant \(\left( \sigma \right) = 5.670400(40) \times 10^{-8} \) Watts per square meter-Kelvin\(^4\) (W/m\(^2\)-K\(^4\))

Speed of light in a vacuum \(\left( c \right) = 299792458 \text{ meters per second} \) (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from http://physics.nist.gov/constant, National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s 2018 CODATA recommended values database.
3.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g., Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g., cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>= B1 / B2</td>
<td>km/h</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula =B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common arithmetic operations available for your use in a spreadsheet include the following:
• Addition (+)
• Subtraction (-)
• Multiplication (*)
• Division (/)
• Powers (^)
• Square roots (sqrt())
• Logarithms (ln(), log10())

Parentheses may be used to ensure proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of \( ax^2 + bx + c \):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

This example is configured to compute roots of the polynomial \( 9x^2 + 5x - 2 \) because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new \( a \), \( b \), and \( c \) coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

7Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

8Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

9Reviewing some algebra here, a root is a value for \( x \) that yields an overall value of zero for the polynomial. For this polynomial \( (9x^2 + 5x - 2) \) the two roots happen to be \( x = 0.269381 \) and \( x = -0.82494 \), with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>((-B4 + C1) / C2)</td>
<td>(\sqrt{(B4^2) - (4<em>B3</em>B5)})</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>((-B4 - C1) / C2)</td>
<td>(2*B3)</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

Note how the square-root term \((y)\) is calculated in cell \(C1\), and the denominator term \((z)\) in cell \(C2\). This makes the two final formulae (in cells \(B1\) and \(B2\)) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^\text{10}\) — all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
3.2. QUANTITATIVE REASONING

3.2.3 First quantitative problem

This is a description of the problem!

<table>
<thead>
<tr>
<th>Challenges</th>
</tr>
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3.2.4 Second quantitative problem

This is a description of the problem!

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<th>Challenges</th>
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3.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

3.3.1 First diagnostic scenario

This is a description of the scenario!

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3.3.2 Second diagnostic scenario

This is a description of the scenario!

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Chapter 4

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

4.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.
4.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. “Live” work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to never come into electrical contact with an energized conductor, no matter what the circuit’s voltage level! Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

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1Professor Charles Dalziel published a research paper in 1961 called “The Deleterious Effects of Electric Shock” detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel’s subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn’t require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

2By “electrical contact” I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

3Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it generally wise (and always a good habit to build) to “power down” any circuit before making contact between it and your body.
contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be firmly strain-relieved (e.g. “cord grips” used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.

- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use “touch-safe” terminal connections with recessed metal parts to minimize risk of accidental contact.

- Always provide overcurrent protection in any circuit you build. Always. This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.

- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. Always. A fuse does no good if the wire or printed circuit board trace will “blow” before it does!

- Always bond metal enclosures to Earth ground for any line-powered circuit. Always. Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one’s body bridging between the Earth and the enclosure.

- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.

- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a “hot” line conductor.

- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.

- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.

- When in doubt, ask an expert. If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.
### 4.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- **Avoid resistor values less than 1 kΩ or greater than 100 kΩ**, unless such values are definitely necessary. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter’s non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter’s finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- **Ensure all electrical connections are low-resistance and physically rugged.** For this reason, one should avoid *compression splices* (e.g. “butt” connectors), solderless breadboards, and wires that are simply twisted together.

- **Build your circuit with testing in mind.** For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- **Design permanent projects with maintenance in mind.** The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work.** Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. *Talk to yourself* when solving problems. These simple steps clarify thought and simplify identification of errors.

---

4 An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

5 Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.
4.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems. They are also quite useful as a learning tool, and so I highly recommend their use in lieu of solderless breadboards. Terminal blocks provide highly reliable connections capable of withstanding significant voltage and current magnitudes, and they force the builder to think very carefully about component layout which is an important mental practice. Terminal blocks that mount on standard 35 mm DIN rail are made in a wide range of types and sizes, some with built-in disconnecting switches, some with built-in components such as rectifying diodes and fuseholders, all of which facilitate practical circuit construction.

I recommend every student of electricity build their own terminal block array for use in constructing experimental circuits, consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a number, starting from 0. An example is shown in the following photograph and illustration:

Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others provide buttons for this task which may be pressed using the tip of any suitable tool.

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6Solderless breadboards are preferable for complicated electronic circuits with multiple integrated “chip” components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for “chip” circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates surface-mount components, which solderless breadboards do not.

7DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

8Sometimes referred to as equipotential, same-potential, or potential distribution terminal blocks.

9The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.
The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:

![Schematic diagram](image1)

![Pictorial diagram](image2)

Numbering on the terminal blocks provides a very natural translation to SPICE\textsuperscript{10} netlists, where component connections are identified by terminal number:

* Series-parallel resistor circuit

\begin{verbatim}
 v1 1 0 dc 6
 r1 2 5 7100
 r2 5 8 2200
 r3 2 8 3300
 r4 8 11 4700
 rjmp1 1 2 0.01
 rjmp2 0 11 0.01
 .op
 .end
\end{verbatim}

Note the use of “jumper” resistances $\textit{rjmp1}$ and $\textit{rjmp2}$ to describe the wire connections between terminals 1 and 2 and between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm realistic for short pieces of wire.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a “wiring sequence” may be thought of as a \textit{non-graphical description} of an electric circuit, and is exceptionally easy to follow.

\textsuperscript{10}SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of \textit{netlists} which are text files listing each component type, connection node numbers, and component values.
4.1. **RECOMMENDED PRACTICES**

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and “ice-cube” style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel\(^\text{11}\). This “terminal block board” hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT “ice-cube” relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:

![Terminal Block Board](image)

Four plastic-bottomed “feet” support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord’s ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer’s screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer’s 12 Volt AC output. The perforated holes happen to be on \(\frac{1}{4}\) inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a “terminal block board” is an inexpensive\(^\text{12}\) yet highly flexible means to construct physically robust circuits using industrial wiring practices.

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\(^{11}\) An electrical subpanel is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional breadboard which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

\(^{12}\) At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.
4.1.4 Conducting experiments

An experiment is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the scientific method, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an hypothesis, defined by the American Heritage Dictionary of the English Language as:

An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning, (b) A premise from which a conclusion is drawn, (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an educated guess about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, incorrect hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of learning than about the status of being correct. In order for an hypothesis to be valid, it must be testable, which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the “baseline” variable set is called the control, while the experiment set up with different value(s) is called the test or experimental.

For some hypotheses a viable alternative to a physical experiment is a computer-simulated experiment or even a thought experiment. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available.

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13 Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scientists may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is funded than anything else – but scientific method itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of being wrong. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one’s hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – real science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

14 This is the principle of falsifiability: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can never be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

15 A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting “data” from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.
Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

• The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit any experimental results, and therefore the experiment cannot teach you anything about the hypothesis.

• Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!

• Never discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even “bad” data holds useful information, and that someone else may be able to uncover its value even if you do not.

• Prioritize quantitative data over qualitative data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).

• Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.

• Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve exact values for any variable. Quantify the amount of uncertainty (i.e. the “tolerance” of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!

• Always remember that scientific confirmation is provisional – no number of “successful” experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, truth is elusive but error is within reach.

• Remember that scientific method is about learning, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.
The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm’s Law.

Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage DC power supply. Use an ammeter in series to measure resistor current and a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/or pose a burn hazard, while excessive voltage poses an electric shock hazard. 30 Volts is a safe maximum voltage for laboratory practices, and according to Joule’s Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts \((P = V^2 / R)\), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:

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<th>Current (mA)</th>
<th>Voltage (V)</th>
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<td>5.400</td>
<td>5.206</td>
<td>14.00</td>
<td>13.49</td>
</tr>
</tbody>
</table>

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected exactly one milliAmpere per Volt the actual current was usually less than that. The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts) to a high of 1037.81 (at 14 Volts), but this represents a variance of only \(-0.0365\%\) to \(+0.0541\%\) from the average, indicating a very consistent proportionality -- results consistent with Ohm’s Law.

ERROR SOURCES: one major source of error is the resistor’s value itself. I did not measure it, but simply assumed color bands of brown-black-red meant exactly 1000 Ohms. Based on the data I think the true resistance is closer to 1037 Ohms. Another possible explanation is multimeter calibration error. However, neither explains the small positive and negative variances from the average. This might be due to electrical noise, a good test being to repeat the same experiment to see if the variances are the same or different. Noise should generate slightly different results every time.
The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm’s Law using a computer (SPICE) simulation.

Planning Time/Date = 12:32 on 14 February 2019

**HYPOTHESIS:** for any given resistor, the current through that resistor should be exactly proportional to the voltage impressed across it.

**PROCEDURE:** write a SPICE netlist with a single DC voltage source and single 1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis from 0 Volts to 25 Volts in 5 Volt increments.

```
* SPICE circuit
 v1 1 0 dc
 r1 1 0 1000
 .dc v1 0 25 5
 .print dc v(1) i(v1)
 .end
```

**RISKS AND MITIGATION:** none.

**DATA COLLECTED:**

```
DC transfer characteristic Thu Feb 14 13:05:08 2019

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```

Analysis Time/Date = 13:06 on 14 February 2019

**ANALYSIS:** perfect agreement between data and hypothesis -- current is precisely 1/1000 of the applied voltage for all values. Anything other than perfect agreement would have probably meant my netlist was incorrect. The negative current values surprised me, but it seems this is just how SPICE interprets normal current through a DC voltage source.

**ERROR SOURCES:** none.
As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm’s Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE’s convention of denoting regular current through a DC voltage source as being negative in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things do not go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let’s try to set up an “up” counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician’s and every engineer’s education.

### 4.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one’s abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project do?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead*.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project’s goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.
4.2 Experiment: (first experiment)

The purpose of this experiment is to . . .

You will need the following components and materials for this experiment . . .

- 
- 
- 

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Formulate an hypothesis (i.e. an educated guess) as to . . .

Design your experiment in such a way as to clearly test your hypothesis. What evidence would agree with your hypothesis? What evidence would disprove your hypothesis? What type and quantity of data must you collect in order to have sufficient evidence?

One of the essential requirements of any scientific test is that it be controlled to guard against false conclusions. This means running multiple instances of the experiment in order to gather comparative data.

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

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For example, if we were trying to determine the effect of light exposure on plant growth, we would need to grow multiple plants under different light conditions while maintaining all other conditions (e.g. temperature, humidity, soil type, etc.) equal. In experiments where the hypothesis is very narrowly defined, we would need to compare the results of the hypothesized condition (“test”) against a lack of the hypothesized condition (“control”). For example, if our hypothesis was that bean sprouts grow best when watered with the athletic beverage Gatorade, we would need to grow several bean sprouts under different conditions: some irrigated with plain water and others irrigated with Gatorade. The plain-water sprouts would constitute our “control” group while the Gatorade-irrigated sprouts would constitute our “test” group. The hypothesis of enhanced growth under Gatorade could only be confirmed, and provisionally at that, if a majority of the test sprouts grew better than a majority of the control sprouts.
4.3 Project: (first project)

This is a description of the project!

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Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

• **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

• **Identify** what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. **Annotation** all diagrams with this data.

• **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

• **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

• **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

• **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

• **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.
- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

• The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

• Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

• Articulate communication is fundamental to work that is complex and interdisciplinary.

• Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

• Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

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1In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

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4 As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.  
5 Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s \TeX\ typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX\ is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, \TeX\ *is a programmer’s approach to word processing*. Since \TeX\ is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX\ makes it relatively easy to learn how other people have created their own \TeX\ documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft *Word* suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s \LaTeX\ extensions to \TeX\

Like all true programming languages, \TeX\ is inherently extensible. So, years after the release of \TeX\ to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX, which is the markup language used to create all ModEL module documents. You could say that \TeX\ is to \LaTeX\ as C is to C++. This means it is permissible to use any and all \TeX\ commands within \LaTeX\ source code, and it all still works. Some of the features offered by \LaTeX\ that would be challenging to implement in \TeX\ include automatic index and table-of-content creation.

Tim Edwards’ \Xcircuit\ drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for \Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
**Gimp** graphic image manipulation program

Essentially an open-source clone of Adobe’s **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

**SPICE** circuit simulation program

**SPICE** is to circuit analysis as **T\TeX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is **free**, it is **fast**, it is **reliable**, and it is a fantastic tool for **teaching** students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my “go to” application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang’s **ePiX** mathematical visualization programming library

This amazing project is a C++ library you may link to any C/C++ code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is not a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical visualization tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own C/C++ code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a C++ programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a C++ library to do the same thing he accomplished something much greater.
**gnuplot** mathematical visualization software

Another open-source tool for mathematical visualization is **gnuplot**. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my **gnuplot** output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of **Gimp** to do rudimentary image editing, my use of **gnuplot** only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

**Python** programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

References


Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

24 October 2021 – minor edit in the RC high-pass filter example section of the Full Tutorial, expanding $s$ to be $\sigma + j\omega$ in the introductory transfer function equation.

21 January 2020 – added a new section to the beginning of the Full Tutorial.

July 2018 – document first created.
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