**Superposition Theorem**

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Chapter 1

Introduction

A network theorem is a useful tool for simplifying the analysis of some system, provable on the basis of more fundamental principles and/or theorems. Network theorems are proven tools useful for quickly and easily analyzing complex electrical networks without being limited to fundamental rules such as Ohm’s Law and Kirchhoff’s Laws.

The Superposition Theorem is one of those network theorems. It is, perhaps, the easiest network theorem to understand: the amount of voltage or current in any given portion of a multi-source electrical network is the sum total (i.e. superposition) of the individual effects of each source (considered one at a time) at that portion. To summarize, the whole is equal to the sum of its parts.

Important concepts related to the Superposition Theorem include voltage sources, current sources, internal resistance, the effects of opens versus shorts, light versus heavy load conditions, Ohm’s Law, Joule’s Law, linear versus non-linear functions, equivalent electrical networks, and substitution of source resistances.

Here are some good questions to ask of yourself while studying this subject:

• In what ways do voltage sources differ from current sources?
• How much internal resistance does an ideal voltage source possess?
• How much internal resistance does an ideal current source possess?
• What defines the “heaviness” of a load, for voltage sources and current sources alike?
• How would you describe the application of the Superposition Theorem to any given circuit, in your own words?
• What parameters in a circuit is the Superposition Theorem useful in solving for?
• What parameters in a circuit is the Superposition Theorem not useful in solving for?
• What does it mean for a mathematical function to be linear?
• Why did the Tutorial apply the Superposition theorem to circuits so elementary that they did not require any network theorems to analyze? What educational value is there in doing that?

• What must be done with all unused sources when applying the Superposition Theorem to a network?

• What is the definition of a series network?

• What is the definition of a parallel network?

• What is meant by the phrase “exchanging tedium for complexity” in regard to problem-solving strategies?
Chapter 2

Tutorial
2.1 Review of ideal and real sources

A perfect voltage source outputs constant voltage regardless of how much or little current passes through it: no matter the rate of electrical charge carriers flowing through a perfect voltage source, each charge carrier gains the exact same amount of energy from the source. Real voltage sources cannot maintain absolutely constant voltage over wide variations in current, and so we often model real voltage sources as ideal voltage sources connected in series with internal resistance:

The voltage source maintains constant voltage regardless of the load

The voltage source's output decreases as load current increases

The closer to ideal a voltage source is, the less its internal resistance. An ideal voltage source is considered to have zero internal resistance.
Similarly, a perfect current source outputs constant current regardless of how much or little voltage appears across its terminals: no matter how much energy is given to each electrical charge carrier as it passes through the source, the rate that charge carriers move through will remain constant. Real current sources cannot maintain absolutely constant current over wide variations in voltage, and so we often model real current sources as ideal current sources connected in parallel with internal resistance:

![Diagram of ideal and real current sources](image)

The current source maintains constant current regardless of the load.

The closer to ideal a current source is, the more its internal resistance. An ideal current source is considered to have infinite internal resistance.

Real sources, both voltage and current, approach ideal behavior as their loads becomes lighter (i.e. dissipate less power). For a voltage source, a “light” load is one drawing little or no current, such as an open-circuit. For a current source, a “light” load is one dropping little to no voltage, such as a short-circuit. If you examine the schematic models of real voltage and current sources previously shown, and imagine their respective minimum-load conditions (i.e. open load for the voltage source and shorted load for the current source) you will see that in both cases the internal resistance becomes irrelevant. For the voltage source, a load drawing no current means that the source’s $R_{\text{internal}}$ drops no voltage, leaving full source voltage at the output terminals. For the current source, a load dropping no voltage means that the source’s $R_{\text{internal}}$ passes no current, leaving full source current at the output terminals.

While the internal resistances of real voltage and current sources have many practical implications, the important point to recognize for this tutorial is that voltage sources ideally have no internal resistance while current sources ideally have infinite internal resistance. In particular, some electrical network theorems require this knowledge to apply, as one must consider the internal resistance of each source as a factor in the theorem.
2.2 The Superposition Theorem

One network theorem particularly useful for circuits containing more than one electrical source is the Superposition Theorem. It may be stated as such:

Any given voltage or current in a multi-source network is equal to the sum (superposition) of that same voltage or current resulting from each source considered one at a time.

Suppose you are tasked with determining some voltage or current quantity in a complex multi-source network. According to the Superposition Theorem, all you have to do is treat that network as though it only has one source component (necessarily simplifying the network) for as many times as there are sources, and add the voltage/current values computed while each of those sources was in effect. In other words, we regard a multi-source electrical network as the superposition of multiple single-source networks analyzed one source at a time.

There is one important caveat for the Superposition Theorem, namely that it only applies to linearly proportional quantities. Networks comprised exclusively of components obeying Ohm’s Law are a good fit for Superposition because Ohm’s Law implies a direct proportionality between voltage and current. Many types of semiconductor components, by contrast, are nonlinear and therefore unsuitable for the Superposition Theorem. Likewise, certain quantities such as power ($P$) do not lend themselves to the Superposition Theorem because it scales nonlinearly as a function of voltage or current ($P = I^2R$ and $P = \frac{V^2}{R}$).

Another important consideration when applying the Superposition Theorem is that all the other sources in the circuit must be replaced by their equivalent internal resistance values when analyzing the circuit one source at a time. This is why the tutorial began with a discussion of ideal versus real voltage and current sources, and their corresponding internal resistance values. One must know that voltage sources have low internal resistance (ideally zero) and that current sources have high internal resistance (ideally infinite) in order to properly perform these substitutions.

We will explore the application of the Superposition Theorem to various resistor-source networks in the following sections.

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1 For the purpose of this discussion, a “nonlinear” component is one where voltage and current are not directly proportional to each other at any portion of the component’s range. Another way to consider this characteristic would be to think of such components as having unstable resistance values. Not only are semiconductor PN junctions quite nonlinear, but so are some non-semiconductor devices such as incandescent lamps (where filament resistance changes with temperature, which in turn is affected by voltage and current). Some texts also add the condition of bilateral symmetry to the use of the Superposition Theorem, meaning that components must behave the same when energized in both polarities. However, the concept of linearity applied across a range of values both positive and negative covers this criterion. If a component does not exhibit bilateral symmetry, then it cannot be linear across its entire range of permitted values.
2.3 Example: Series-aiding voltage sources

The following circuit is so simple it doesn’t require the use of the Superposition Theorem, but we will begin with this anyway because it will make the concept easy to understand:

\[ I = \frac{V_T}{R} = \frac{6 \text{ V} + 14 \text{ V}}{2000 \text{ } \Omega} = 10 \text{ mA} \]

Let us now prove we can get the same results using the Superposition Theorem. What we must do is analyze how this circuit would function with just one source active at a time, the other source “disabled” by replacing it with its equivalent internal resistance. Since these are voltage sources, and we have not been instructed that they are anything but ideal voltage sources, we will substitute a short-circuit (i.e. a length of wire) in place of each being disabled:

In each analysis, we use Ohm’s Law to calculate current \( I = \frac{V}{R} \), 3 mA resulting from the 6 V source and 7 mA resulting from the 14 V source. Superimposing these two currents – 3 mA and 7 mA both flowing in the same direction (down) – we get a sum of 10 mA (down), just as we expected. Resistor voltage drops also superimpose correctly, with 6 V (+ on top and − on bottom) and 14 V (+ on top and − on bottom) summing to make 20 V (+ on top and − on bottom).

Note how power dissipations do not superimpose correctly. Using \( P = IV \) we get 18 mW with just the 6 V source and 98 mW with just the 14 V source, but much more than the sum (a true value of 200 mW versus a superimposed sum of only 116 mW) with both sources active. Superposition does not work here because power is nonlinear with respect to applied voltage in a resistive network.
2.4 Example: Series-opposing voltage sources

Our next circuit is also simple enough that the Superposition Theorem is unnecessary, but again it should prove instructive:

With the two voltage sources opposing, the net voltage experienced by the resistor will be 8 V, resulting in a current of 4 mA (flowing up). Now that we know the correct result for resistor voltage and resistor current in this dual-source circuit, let us apply Superposition Theorem to verify we get the same results:

- Superimposing currents: 3 mA down plus 7 mA up = 4 mA up
- Superimposing voltages: 6 V (+ top) plus 14 V (− top) = 8 V (− top)

Power dissipations once again do not superimpose correctly. The single-source dissipation values of 18 mW and 98 mW (negative power?) most certainly do not add up to (−)80 mW, as we know the true power dissipation with both sources active will be 32 mW ($P = IV$ with 8 V and 4 mA at the resistor)!
2.5 Example: Two paralleled batteries

Having applied the Superposition Theorem to two simplistic circuits, let us now apply it to a circuit worthy of the theorem. We will choose for our example a circuit where one resistive load is powered by two parallel-connected batteries. The batteries’ voltages are unequal, as are their internal resistances:

![Circuit Diagram]

If we re-draw the circuit to model each real battery as a combination of an ideal voltage source with a series-connected resistance, we will see why we need the Superposition Theorem to calculate voltages and currents:

![Modelled Circuit Diagram]

A simplifying Theorem is necessary to analyze this circuit because no two resistances are directly in series or directly in parallel with each other, and therefore we cannot compute a total resistance. While it should be clear that both batteries contribute to the powering of the resistor, it is not intuitive exactly how much of a contribution each battery makes.

Our analysis of this circuit will proceed similarly to the previous examples: we will consider the effects of each voltage source one at a time on circuit voltages and currents, then algebraically sum those voltages and currents to determine voltage and current in the circuit with both sources active. Since the batteries are real and not ideal voltage sources, when we disable each one we will maintain its internal resistance in place while shorting the ideal-source portion of the model.

Despite the fact that our basic procedure for applying Superposition Theorem to this circuit will remain the same as the prior examples, more calculations will be necessary simply due to the fact that this circuit contains more resistors than either of the previous examples. May this stand as a general caveat: just because a network theorem simplifies the complexity of a problem, doesn’t mean the solution will not be tedious. In fact, a great many problem-solving techniques function by

\[2\text{Recall that the definition of a series network is one where all components constitute a single path for current (i.e. all components are connected in-line with each other). Recall as well that a parallel network is one where all components span the same two sets of electrically-common points (i.e. all components are connected across one another). The resistors in this dual-battery circuit fail to meet either criterion, and so are neither in series nor in parallel with each other.}\]
substituting tedium for complexity, converting a problem too difficult for you to solve at all into a series of smaller problems you can solve.

First, we will analyze this circuit with just the 15 V battery active:

![Circuit Diagram](image)

With the 18 V source disabled, the three resistors form a series-parallel network with the 25 Ω and 2 Ω in parallel, and the 1 Ω in series with that parallel subnetwork, all powered by the 15 V source. We could calculate all voltage drops and all currents, but for the sake of simplicity we will only calculate currents. Once we superimpose all the calculated current values, we will be able to determine voltage drops from those:

![Circuit Diagram](image)

Summarizing the results:

- Current through 15 V battery = 5.260 A (right)
- Current through 18 V battery = 4.870 A (right)
- Current through 25 Ω load = 0.390 A (down)
Next, we will analyze this circuit with just the 18 V battery active:

With the 15 V source disabled, the three resistors form a different series-parallel network with the 25 Ω and 1 Ω in parallel, and the 2 Ω in series with that parallel subnetwork, all powered by the 18 V source. Calculating currents as before:

Superimposing these results with the previous:

- Current through 15 V battery = 5.260 A (right) − 5.844 A (left) = 0.584 A (left)
- Current through 18 V battery = 4.870 A (right) − 6.078 A (left) = 1.208 A (left)
- Current through 25 Ω load = 0.390 A (down) + 0.234 A (down) = 0.623 A (down)

Annotating our original schematic diagram with these currents, we see that the 15 V battery actually functions as a load while the 18 V battery is the only operating source in the circuit:

Terminal voltage for both batteries, equal to load voltage assuming connecting wires of negligible resistance, will be 15.584 Volts as predicted by Ohm’s Law (\(V = IR = (0.623 \text{ A})(25 \Omega) = 15.584 \text{ V}\)).

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3Recall the functional definition of an electrical load, where electric charge carriers enter at higher energy (+) than they exit (−). A source draws charge carriers in at lower energy (−) and ejects them out at higher energy (+). In this dual-battery circuit, the 18 V “source” battery is depleting its internal energy reserve as it discharges, while the 15 V “load” battery is being charged by the 18 V battery.
2.6 Example: Voltage source and current source

The last example circuit we will examine is the following, containing both a voltage source and a current source:

![Circuit Diagram]

As with the “Two paralleled batteries” example, this circuit really demands a network theorem, as it is impossible to regard it as a simple series-parallel network. With both sources in place, it is impossible to condense all resistances into a single equivalent resistance as we would do in a normal series-parallel resistor network.

For our first step, we will analyze the circuit with only the voltage source active, replacing the current source with its equivalent internal resistance (i.e. open). Note the careful marking of voltage polarities and current arrow directions as well as magnitudes, as these will become important when we superimpose the calculated quantities:

![First Analysis Diagram]

Next, we will analyze the circuit with only the current source active, replacing the voltage source with its equivalent internal resistance (i.e. short):

![Second Analysis Diagram]
Superimposing the calculated currents:

- 6 V source: $1.176 \text{ A (left)} + 0.353 \text{ A (left)} = \textbf{1.529 A (left)}$
- 2 A source: $0 \text{ A} + 2 \text{ A (up)} = \textbf{2 A (up)}$
- 10 Ω resistor: $0.353 \text{ A (up)} - 0.824 \text{ A (down)} = \textbf{0.471 A (down)}$

Superimposing the calculated voltages:

- 2 A source: $3.529 \text{ V (+ down)} - 12.235 \text{ V (+ up)} = \textbf{8.706 V (+ up)}$
- 4 Ω resistor: $1.412 \text{ V (+ right)} + 4.706 \text{ V (+ right)} = \textbf{6.118 V (+ right)}$
- 3 Ω resistor: $1.059 \text{ V (+ left)} + 3.529 \text{ (+ left)} = \textbf{4.588 V (+ left)}$
- 10 Ω resistor: $3.529 \text{ V (+ down)} - 8.235 \text{ V (+ up)} = \textbf{4.706 V (+ up)}$
- 2 Ω resistor: $0 \text{ V} + 4 \text{ V (+ right)} = \textbf{4 V (+ right)}$

Tedious though the solution may be, at least each step of it is possible with just the tools of series-parallel circuit analysis. As was mentioned previously, this is the hallmark of many problem-solving techniques: exchanging tedium for complexity. It should be evident just why the Superposition Theorem made this formerly intractable problem amenable to ordinary series-parallel analysis as we examine the schematic diagram at each step: by disabling all but one source at a time, the circuit contains fewer components and therefore becomes simpler.
Chapter 3

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.
3.1 Superposition of AC signals

This module’s Tutorial has focused on Superposition as a concept within DC circuits, whereby DC voltages and/or currents resulting from multiple sources may be superimposed to obtain the aggregate result of those sources working simultaneously. However, the Superposition Theorem is not limited to direct-current (DC) circuits, but also applies to alternating-current (AC) circuits as well, so long as the components in question behave linearly.

Consider a simple AC circuit powered by two series-connected sinusoidal voltage sources of differing frequency. The sum of these two AC sources will not be sinusoidal, and this complicates analysis of the circuit:

Computing voltages and currents in any resistor-capacitor-inductor network is easy enough using phasor mathematics to represent simple sine-shaped AC waveforms. However, as we see in the graphical plot, the sum $V_1 + V_2$ is most definitely not a simple sine wave and so phasor techniques fail.

However, since we know resistors and capacitors and inductors are all linear components, we know that the Superposition Theorem applies. This means we may calculate any voltage or current in this network by calculating that voltage or current twice (once for each of the two AC sources considered by itself with the other source disabled) and then summing the results of those two calculations. The resulting AC voltage or current, like $V_1 + V_2$, will not be a simple sine wave, but at least we are able to compute it using mathematical techniques that work only with simple sine waves.
This basic concept of superimposed sinusoidal AC voltages/currents has many theoretical and practical applications, including the following:

- **The Fourier series** – This is the theoretical equivalence between any repeating (“periodic”) function and the sum (i.e. superposition) of multiple sinusoidal functions of integer-multiple frequencies to the original waveform’s over-all period. For example, a square-wave function is mathematically equivalent to \( \sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t) + \cdots + \frac{1}{n} \sin(n\omega t) \). The remarkable thing about the Fourier series is that, at least in theory, we may simulate any periodic waveform whatsoever as a particular series of sine and cosine functions at different frequencies and amplitudes. Some of these series may be infinite, as for square waves, but others are not.

- **Harmonics** – This is really just an expression of the Fourier series. Any non-sinusoidal voltage or current waveform in a circuit acts like the sum (i.e. superposition) of multiple sinusoidal waveforms at different amplitudes and frequencies. A remarkable phenomenon is when some change made to a single-source AC circuit distorts the wave-shape of an AC voltage or current so that it no longer resembles a perfect sinusoid, that circuit begins to behave as though it is being powered by multiple superimposed AC sources at different frequencies and amplitudes rather than by a single source! A computational technique called a Fourier Transform (FT) is able to compute those sinusoidal signals, thereby “separating” a non-sinusoidal signal into a collection of definite sinusoidal signals.

- **Symmetrical components** – This is the theoretical equivalence between an imbalanced set of phasors in a three-phase AC circuit and the sum (i.e. superposition) of three sets of balanced phasors with specific properties: one set rotating the normal direction, another set rotating backwards, and a third set where all three phasors share the same angle. Similar to harmonics, these “other” phasor sets “appear” any time a circuit alteration causes the normally-balanced phasors to become imbalanced (e.g. during fault conditions in a three-phase AC power network).

- **Biasing** – This is where a DC “offset” or “bias” signal is added to (i.e. superimposed with) an AC signal, usually for the purpose of “tricking” some semiconductor device to remain in its conductive state throughout the AC signal’s waveform.

- **Audio mixing** – An electronic device used in many professional audio performance and recording settings is a mixing console, whereby AC signals from a variety of individual sound sources are summed together to produce a “mixed” signal representing the whole audio experience. The audio signals originate from such devices as microphones and musical instrument “pickup” sensors, with the relative “weighting” of each signal manually adjusted by manually-adjusted potentiometers. A person with the title of sound engineer makes adjustments to these potentiometers to achieve the over-all desired effect, which is an aesthetically proper balance of sound sources. The audio power signal sent to the loudspeakers (or to the recording device) represents a superposition of every audio channel’s signal fed to the mixing console, the result (ideally) being the same as though every audience member were simultaneously listening to each sound produced by the performers.
Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read the tutorials and attempt to answer questions on their own prior to the instructor’s interaction with them. In place of lecture, the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is independence, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of experimentation and computer simulation (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of metacognition within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

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1. Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

2. Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 4. QUESTIONS

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.

- **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

- Where did the text make the most sense to you? What was it about the text’s presentation that made it clear?

- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.

- Identify any new concept(s) presented in the text, and explain in your own words.

- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.

- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

- Devise an experiment to disprove a plausible misconception.

- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

- Describe any useful problem-solving strategies applied in the text.

- Devise a question of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else’s solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.
• Show how thorough documentation assisted in the completion of this experiment or project.
• Which fundamental laws or principles are key to this system’s function?
• Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
• What will happen if (component X) fails (open/shorted/etc.)?
• What would have to occur to make this system unsafe?
4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

3Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
4.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

- **√** Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

- **√** Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

- **√** Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

- **√** Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

- **√** Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

- **√** Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
4.1. CONCEPTUAL REASONING

4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Network theorem
- Series
- Parallel
- Electrical source
- Electrical load
- Voltage source
- Current source
- Strategy for analyzing a series-parallel resistor circuit
- Superposition
4.1.3 1-5 VDC versus 4-20 mA signaling

Electrical signals are frequently used in industrial control applications to communicate information from one device to another. An example of this is motor speed control, where a computer outputs a speed command signal to a motor “drive” circuit, which then provides metered power to an electric motor:

Two common standards for analog control signals are 1-5 Volts DC and 4-20 mA DC. In either case, the motor will spin faster when this signal from the computer grows in magnitude (1 volt = motor stopped, 5 Volts = motor runs at full speed; or 4 mA = motor stopped, 20 mA = motor runs at full speed).
At first, it would seem as though the choice between 1-5 Volts and 4-20 mA as control signal standards is arbitrary. However, one of these standards exhibits much greater immunity to induced noise along the two-wire cable than the other. Shown here are two equivalent schematics for these signal standards, complete with an AC voltage source in series to represent the “noise” voltage picked up along the cable’s length:

Use the Superposition Theorem to qualitatively determine which signal standard drops the greatest amount of noise voltage across the motor drive input’s resistance, thereby most affecting the motor speed control.

**Challenges**

- What undesirable effects do you think noise superimposed on the DC signal cable would have on motor speed control?

- Why do you suppose the 1-5 volt signal system requires a much greater input impedance (1 MΩ) than the 4-20 mA signal system? What might happen to the voltage signal received at the motor drive’s input terminals if the input resistance were much less?
4.1.4 Mixed AC/DC network

Sketch the approximate waveform of this circuit’s output signal ($V_{out}$) on the screen of the oscilloscope:
Sketch the approximate waveform of this circuit’s output signal ($V_{out}$) on the screen of the oscilloscope:

Challenges

- Explain why it is acceptable to use a polarized (polarity-sensitive) capacitor in this circuit when it is clearly connected to a source of AC. Why is it not damaged by the AC voltage when used like this?
4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

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4 In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5 This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
4.2. QUANTITATIVE REASONING

4.2.1 Miscellaneous physical constants

Note: constants shown in bold type are exact, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, Avogadro’s number given as 6.02214179(30) × 10^{23} means the center value (6.02214179 × 10^{23}) plus or minus 0.00000030 × 10^{23}.

Avogadro’s number \( (N_A) = 6.02214179(30) \times 10^{23} \) per mole (mol\(^{-1}\))

Boltzmann’s constant \( (k) = 1.3806504(24) \times 10^{-23} \) Joules per Kelvin (J/K)

Electronic charge \( (e) = 1.602176487(40) \times 10^{-19} \) Coulomb (C)

Faraday constant \( (F) = 9.64853399(24) \times 10^{4} \) Coulombs per mole (C/mol)

Magnetic permeability of free space \( (\mu_0) = 1.25663706212(19) \times 10^{-6} \) Henrys per meter (H/m)

Electric permittivity of free space \( (\varepsilon_0) = 8.8541878128(13) \times 10^{-12} \) Farads per meter (F/m)

Characteristic impedance of free space \( (Z_0) = 376.730313668(57) \) Ohms (Ω)

Gravitational constant \( (G) = 6.67428(67) \times 10^{-11} \) cubic meters per kilogram-seconds squared (m\(^3\)/kg-s\(^2\))

Molar gas constant \( (R) = 8.314472(15) \) Joules per mole-Kelvin (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant \( (h) = 6.62606896(33) \times 10^{-34} \) joule-seconds (J-s)

Stefan-Boltzmann constant \( (\sigma) = 5.670400(40) \times 10^{-8} \) Watts per square meter-Kelvin\(^4\) (W/m\(^2\)-K\(^4\))

Speed of light in a vacuum \( (c) = 299792458 \text{ meters per second} \) (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from http://physics.nist.gov/constants, National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s 2018 CODATA recommended values database.
4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g., Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g., cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Average speed</td>
<td>= B1 / B2</td>
<td>km/h</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln(), log10())

Parentheses may be used to ensure proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of \(ax^2 + bx + c\):

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_{-1} = \frac{-B4 + \text{sqrt}((B4^2) - (4<em>B3</em>B5)))}{(2*B3)})</td>
</tr>
<tr>
<td>2</td>
<td>(x_{+2} = \frac{-B4 - \text{sqrt}((B4^2) - (4<em>B3</em>B5)))}{(2*B3)})</td>
</tr>
<tr>
<td>3</td>
<td>(a = )</td>
</tr>
<tr>
<td>4</td>
<td>(b = )</td>
</tr>
<tr>
<td>5</td>
<td>(c = )</td>
</tr>
</tbody>
</table>

This example is configured to compute roots of the polynomial \(9x^2 + 5x - 2\) because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new \(a\), \(b\), and \(c\) coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

---

Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

Reviewing some algebra here, a root is a value for \(x\) that yields an overall value of zero for the polynomial. For this polynomial \((9x^2 + 5x - 2)\) the two roots happen to be \(x = 0.269381\) and \(x = -0.82494\), with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_1 )</td>
<td>( = \frac{(-B4 + C1)}{C2} )</td>
</tr>
<tr>
<td>2</td>
<td>( x_2 )</td>
<td>( = \frac{(-B4 - C1)}{C2} )</td>
</tr>
<tr>
<td>3</td>
<td>a  =</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>b  =</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>c  =</td>
<td>-2</td>
</tr>
</tbody>
</table>

Note how the square-root term \((y)\) is calculated in cell \(C1\), and the denominator term \((z)\) in cell \(C2\). This makes the two final formulae (in cells \(B1\) and \(B2\)) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^\text{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

\(^\text{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
4.2.3 Two-source, three-resistor network

Explain in your own words how to apply the Superposition Theorem to calculate the amount of current through the load resistor in this circuit, and calculate its value:

Challenges

- Describe how the voltage source and current source are regarded differently from one another in your application of the Superposition Theorem.

- Is either source in this circuit actually functioning as a load? How can we tell?
4.2.4 Wind turbine and battery network

A wind turbine generator and a storage battery work together to supply DC power to an electric lamp. Calculate the amount of current through each of these three components, given the values shown in the schematic diagram. Assume internal resistances of the generator and battery to be negligible:

\[ I_{batt} = \quad I_{bulb} = \quad I_{gen} = \]

**Challenges**

- Is either the generator or the battery in this circuit functioning as a load? How can we tell?
- Would we ever desire the battery to function as a load?
- Would we ever desire the generator to function as a load?
4.2.5 Two generators powering a heating element

Use the Superposition Theorem to calculate the amount of current going through the 55 Ω heater element. Ignore all wire and connection resistances, only considering the resistance of each fuse in addition to the heater element resistance:

Challenges

- Explain how you could use the Superposition Theorem to calculate current going through the short length of wire connecting the two generator fuses together.
- Is either generator in this circuit functioning as a load? How can we tell?
4.2.6 Two generators powering a motor

Suppose a DC generator is powering an electric motor, the latter we will model as a 100 Ω resistor:

\[
\begin{align*}
\text{Generator} & \quad \text{Motor} \\
R_{\text{wire}} & \quad r_{\text{gen}} & \quad V_{\text{gen}} & \quad 475 \text{ V} \\
0.1 \Omega & \quad 0.3 \Omega & \quad & \\
\end{align*}
\]

Calculate the amount of current this generator will supply to the motor and the voltage measured across the motor’s terminals, taking into account all the resistances shown (generator internal resistance \(r_{\text{gen}}\), wiring resistances \(R_{\text{wire}}\), and the motor’s equivalent resistance).

Now suppose we connect an identical generator in parallel with the first, using connecting wire so short that we may safely discount its additional resistance:

\[
\begin{align*}
\text{Generator} & \quad \text{Generator} \quad \text{Motor} \\
R_{\text{wire}} & \quad r_{\text{gen}} & \quad V_{\text{gen}} & \quad 475 \text{ V} \\
0.1 \Omega & \quad 0.3 \Omega & \quad & \\
\end{align*}
\]

Use the Superposition Theorem to re-calculate the motor current and motor terminal voltage, commenting on how these figures compare with the first calculation (using only one generator).

**Challenges**

- How much current does each generator supply to the circuit when there are two generators connected in parallel?
- Is either generator in this circuit functioning as a load? How can we tell?
4.2.7 One generator and two batteries

Calculate the current through each battery using the Superposition Theorem. Feel free to ignore all wire and connection and battery internal resistances – only consider the resistance of each fuse:

---

**Challenges**

- Identify any safety hazards that could arise as a result of excessive resistance in the fuse holders (e.g., corrosion build-up on the metal tabs where the fuse clips in to the fuse-holder).
- Are the two batteries behaving as loads, sources, or one of each?
4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.
4.3. DIAGNOSTIC REASONING

4.3.1 Actually disabling a source

When applying the Superposition Theorem, we consider how a multi-source circuit will behave while only being powered by one source at a time. Identify a realistic fault that could result in this circuit actually having only one active source (while the other source is disabled in the same manner as we assume for Superposition analyses):

Challenges

• What would happen if the heater element failed open?
Chapter 5

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

5.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.
5.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. “Live” work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to never come into electrical contact with an energized conductor, no matter what the circuit’s voltage level! Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

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1 Professor Charles Dalziel published a research paper in 1961 called “The Deleterious Effects of Electric Shock” detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel’s subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn’t require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

2 By “electrical contact” I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

3 Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it generally wise (and always a good habit to build) to “power down” any circuit before making contact between it and your body.
5.1. **RECOMMENDED PRACTICES**

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. “cord grips” used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.

- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use “touch-safe” terminal connections with recessed metal parts to minimize risk of accidental contact.

- Always provide overcurrent protection in any circuit you build. *Always*. This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.

- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always*. A fuse does no good if the wire or printed circuit board trace will “blow” before it does!

- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always*. Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one’s body bridging between the Earth and the enclosure.

- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.

- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a “hot” line conductor.

- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.

- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.

- When in doubt, *ask an expert*. If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.
5.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their
own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely
  necessary\(^4\). Resistances below 1 kΩ may draw excessive current if directly connected to
  a voltage source of significant magnitude, and may also complicate the task of accurately
  measuring current since any ammeter's non-zero resistance inserted in series with a low-value
  circuit resistor will significantly alter the total resistance and thereby skew the measurement.
  Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's
  finite resistance connected in parallel with a high-value circuit resistor will significantly alter
  the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values
  should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one
  should avoid compression splices (e.g. "butt" connectors), solderless breadboards\(^5\), and wires
  that are simply twisted together.

- Build your circuit with testing in mind. For example, provide convenient connection points
  for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with maintenance in mind. The more convenient you make
  maintenance tasks, the more likely they will get done.

- **Always document and save your work.** Circuits lacking schematic diagrams are more
difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler
when a schematic diagram precedes construction. Experimental results are easier to interpret
when comprehensively recorded. Consider modern videorecording technology for this purpose
where appropriate.

- Record your steps when troubleshooting. **Talk to yourself** when solving problems. These
  simple steps clarify thought and simplify identification of errors.

\(^4\)An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage
drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose
an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an
electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing
damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to
the person (i.e. shock).

\(^5\)Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many
components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after
frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit
boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary
connections may be made.
5.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems. They are also quite useful as a learning tool, and so I highly recommend their use in lieu of solderless breadboards. Terminal blocks provide highly reliable connections capable of withstanding significant voltage and current magnitudes, and they force the builder to think very carefully about component layout which is an important mental practice. Terminal blocks that mount on standard 35 mm DIN rail are made in a wide range of types and sizes, some with built-in disconnecting switches, some with built-in components such as rectifying diodes and fuseholders, all of which facilitate practical circuit construction.

I recommend every student of electricity build their own terminal block array for use in constructing experimental circuits, consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a number, starting from 0. An example is shown in the following photograph and illustration:

Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others provide buttons for this task which may be pressed using the tip of any suitable tool.

---

*Solderless breadboards are preferable for complicated electronic circuits with multiple integrated “chip” components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for “chip” circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates surface-mount components, which solderless breadboards do not.

*DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

*Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

*The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.
The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:

Numbering on the terminal blocks provides a very natural translation to SPICE\textsuperscript{10} netlists, where component connections are identified by terminal number:

\* Series-parallel resistor circuit

```
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rjmp1 1 2 0.01
rjmp2 0 11 0.01
.op
.end
```

Note the use of “jumper” resistances \texttt{rjmp1} and \texttt{rjmp2} to describe the wire connections between terminals 1 and 2 and between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm realistic for short pieces of wire.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a “wiring sequence” may be thought of as a non-graphical description of an electric circuit, and is exceptionally easy to follow.

\textsuperscript{10}SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of netlists which are text files listing each component type, connection node numbers, and component values.
5.1. RECOMMENDED PRACTICES

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and “ice-cube” style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel. This “terminal block board” hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT “ice-cube” relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:

Four plastic-bottomed “feet” support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord’s ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer’s screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer’s 12 Volt AC output. The perforated holes happen to be on \( \frac{1}{4} \) inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a “terminal block board” is an inexpensive yet highly flexible means to construct physically robust circuits using industrial wiring practices.

---

11 An electrical subpanel is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional breadboard which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

12 At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.
5.1.4 Conducting experiments

An experiment is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the scientific method, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an hypothesis, defined by the American Heritage Dictionary of the English Language as:

An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an educated guess about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, incorrect hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of learning than about the status of being correct. In order for an hypothesis to be valid, it must be testable, which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the “baseline” variable set is called the control, while the experiment set up with different value(s) is called the test or experimental.

For some hypotheses a viable alternative to a physical experiment is a computer-simulated experiment or even a thought experiment. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available.

Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scientists may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is funded than anything else – but scientific method itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of being wrong. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one’s hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – real science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

This is the principle of falsifiability: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can never be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting “data” from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.
5.1. **RECOMMENDED PRACTICES**

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.

- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!

- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even “bad” data holds useful information, and that someone else may be able to uncover its value even if you do not.

- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).

- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.

- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the “tolerance” of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!

- Always remember that scientific confirmation is provisional – no number of “successful” experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*

- Remember that scientific method is about *learning,* first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.
The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage DC power supply. Use an ammeter in series to measure resistor current and a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/or pose a burn hazard, while excessive voltage poses an electric shock hazard. 30 Volts is a safe maximum voltage for laboratory practices, and according to Joule’s Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts ($P = V^2 / R$), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Current</th>
</tr>
</thead>
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<tr>
<td>0.000 V</td>
<td>0.000 mA</td>
</tr>
<tr>
<td>2.700 V</td>
<td>2.603 mA</td>
</tr>
<tr>
<td>5.400 V</td>
<td>5.206 mA</td>
</tr>
<tr>
<td>8.100 V</td>
<td>7.812 mA</td>
</tr>
<tr>
<td>10.00 V</td>
<td>9.643 mA</td>
</tr>
<tr>
<td>14.00 V</td>
<td>13.49 mA</td>
</tr>
</tbody>
</table>

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected exactly one milliAmpere per Volt the actual current was usually less than that. The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts) to a high of 1037.81 (at 14 Volts), but this represents a variance of only -0.0365% to +0.0541% from the average, indicating a very consistent proportionality -- results consistent with Ohm’s Law.

ERROR SOURCES: one major source of error is the resistor’s value itself. I did not measure it, but simply assumed color bands of brown-black-red meant exactly 1000 Ohms. Based on the data I think the true resistance is closer to 1037 Ohms. Another possible explanation is multimeter calibration error. However, neither explains the small positive and negative variances from the average. This might be due to electrical noise, a good test being to repeat the same experiment to see if the variances are the same or different. Noise should generate slightly different results every time.
5.1. RECOMMENDED PRACTICES

The following is an example of a well-planned and executed virtual experiment, in this case demonstrating Ohm’s Law using a computer (SPICE) simulation.

Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single 1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis from 0 Volts to 25 Volts in 5 Volt increments.

   * SPICE circuit
     v1 1 0 dc
     r1 1 0 1000
     .dc v1 0 25 5
     .print dc v(1) i(v1)
     .end

RISKS AND MITIGATION: none.

DATA COLLECTED:

   DC transfer characteristic Thu Feb 14 13:05:08 2019

<table>
<thead>
<tr>
<th>Index</th>
<th>v-sweep</th>
<th>v(1)</th>
<th>v1#branch</th>
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<tr>
<td>5</td>
<td>2.500000e+01</td>
<td>2.5000000e+01</td>
<td>-2.500000e-02</td>
</tr>
</tbody>
</table>

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely 1/1000 of the applied voltage for all values. Anything other than perfect agreement would have probably meant my netlist was incorrect. The negative current values surprised me, but it seems this is just how SPICE interprets normal current through a DC voltage source.

ERROR SOURCES: none.
As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm’s Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE’s convention of denoting regular current through a DC voltage source as being negative in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things do not go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. Which terminals of this switch connect to the NO versus NC contacts?)
- System testing (e.g. How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?)
- Learning programming languages (e.g. Let’s try to set up an “up” counter function in this PLC!)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician’s and every engineer’s education.

5.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one’s abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project do?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, plan ahead.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of scope creep: the tendency to modify the project’s goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.
5.2 Experiment: Confirming the Superposition Theorem

Conduct an experiment to confirm the validity of the Superposition Theorem for a linear network.

**EXPERIMENT CHECKLIST:**

- **Prior to experimentation:**
  - √ Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - √ Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
  - √ Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.

- **During experimentation:**
  - √ Safe practices followed at all times (e.g. no contact with energized circuit).
  - √ Correct equipment usage according to manufacturer’s recommendations.
  - √ All data collected, ideally quantitative with full precision (i.e. no rounding).

- **After each experimental run:**
  - √ If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - √ Identify any uncontrolled sources of error in the experiment.

- **After all experimental re-runs:**
  - √ Save all data for future reference.
  - √ Write an analysis of experimental results and lessons learned.

---

**Challenges**

- Science is an *iterative* process, and for this reason is never complete. Following the results of your experiment, what would you propose for your *next* hypothesis and *next* experimental procedure? Hint: if your experiment produced any unexpected results, exploring those unexpected results is often a very good basis for the next experiment!

- ???.
- ???.

---
5.3 Experiment: Confirming a limit of the Superposition Theorem

Conduct an experiment to confirm the fact that the Superposition Theorem does not apply to a nonlinear network.

**EXPERIMENT CHECKLIST:**

- **Prior to experimentation:**
  - Write a hypothesis (i.e., a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g., schematic diagrams, programming code).
  - Identify any risks (e.g., shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.

- **During experimentation:**
  - Safe practices followed at all times (e.g., no contact with energized circuit).
  - Correct equipment usage according to manufacturer’s recommendations.
  - All data collected, ideally quantitative with full precision (i.e., no rounding).

- **After each experimental run:**
  - If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - Identify any uncontrolled sources of error in the experiment.

- **After all experimental re-runs:**
  - Save all data for future reference.
  - Write an analysis of experimental results and lessons learned.

**Challenges**

- Science is an iterative process, and for this reason is never complete. Following the results of your experiment, what would you propose for your next hypothesis and next experimental procedure? Hint: if your experiment produced any unexpected results, exploring those unexpected results is often a very good basis for the next experiment!

- ???

- ???
5.4 Project: (first project)

This is a description of the project!

**PROJECT CHECKLIST:**

- **Prior to construction:**
  - ✓ Prototype diagram(s) and description of project scope.
  - ✓ Risk assessment/mitigation plan.
  - ✓ Timeline and action plan.

- **During construction:**
  - ✓ Safe work habits (e.g. no contact made with energized circuit at any time).
  - ✓ Correct equipment usage according to manufacturer’s recommendations.
  - ✓ Timeline and action plan amended as necessary.
  - ✓ Maintain the originally-planned project scope (i.e. avoid adding features!).

- **After completion:**
  - ✓ All functions tested against original plan.
  - ✓ Full, accurate, and appropriate documentation of all project details.
  - ✓ Complete bill of materials.
  - ✓ Written summary of lessons learned.

<table>
<thead>
<tr>
<th>Challenges</th>
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<tr>
<td>• ???</td>
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<td>• ???</td>
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<td>• ???</td>
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</tbody>
</table>
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify what it is you need to solve,** identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. **Annotate** all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform **diagnostic tests** rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to **challenge** that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.

- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.

- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).

- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?

- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.

- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does **not** mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

• The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

• Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

• Articulate communication is fundamental to work that is complex and interdisciplinary.

• Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

• Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

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1In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
APPENDIX B. INSTRUCTIONAL PHILOSOPHY

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

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4 As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

5 Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s TeX typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TeX *is a programmer’s approach to word processing*. Since TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TeX makes it relatively easy to learn how other people have created their own TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft Word suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s LaTeX extensions to TeX

Like all true programming languages, TeX is inherently extensible. So, years after the release of TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LaTeX, which is the markup language used to create all ModEL module documents. You could say that TeX is to LaTeX as C is to C++. This means it is permissible to use any and all TeX commands within LaTeX source code, and it all still works. Some of the features offered by LaTeX that would be challenging to implement in TeX include automatic index and table-of-content creation.

Tim Edwards’ Xcircuit drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
Gimp graphic image manipulation program

Essentially an open-source clone of Adobe’s PhotoShop, I use Gimp to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although Gimp does offer its own scripting language (called Script-Fu), I have never had occasion to use it. Thus, my utilization of Gimp to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as TeX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer SPICE for the following reasons: it is free, it is fast, it is reliable, and it is a fantastic tool for teaching students of electricity and electronics how to write simple code. I happen to use rather old versions of SPICE, version 2g6 being my “go to” application when I only require text-based output. NGSPICE (version 26), which is based on Berkeley SPICE version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all SPICE example netlists I strive to use coding conventions compatible with all SPICE versions.

Andrew D. Hwang’s ePiX mathematical visualization programming library

This amazing project is a C++ library you may link to any C/C++ code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as Mathematica or Maple to do. It should be said that ePiX is not a Computer Algebra System like Mathematica or Maple, but merely a mathematical visualization tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own C/C++ code!), but it can graph the results, and it does so beautifully. What I really admire about ePiX is that it is a C++ programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a C++ library to do the same thing he accomplished something much greater.
**APPENDIX C. TOOLS USED**

**gnuplot** mathematical visualization software

Another open-source tool for mathematical visualization is **gnuplot**. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my **gnuplot** output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of **Gimp** to do rudimentary image editing, my use of **gnuplot** only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

**Python** programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

References
Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

7 March 2022 – added some questions to the Introduction chapter.

11 October 2021 – added some instructor notes.

9 July 2021 – replaced some TeX-style italicizing markup with LaTeX-style.

18 March 2021 – minor additions to the Introduction, and minor edits to the Tutorial. Also added some labels to image 2033.

8 October 2020 – minor additions to the Introduction, and minor edits to the Tutorial.

29 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

17 April 2020 – added more sections to the Tutorial.

12 September 2019 – altered capitalization of “Theorem” in two places.

11 September 2019 – added Technical Reference on superposition as a concept applicable to AC signals of differing frequency. Also added another Example circuit to the Tutorial, and corrected some typographical errors.

5 September 2019 – completed Tutorial and began adding questions.

3 September 2019 – continued writing Tutorial.

30 August 2019 – document first created.
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