

N 69 19943

Code 1
87

30p.

MTP-AERO-63-39
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SURVEY AND CLASSIFICATION OF EARTH-MOON TRAJECTORIES
BASED ON NEWLY DISCOVERED PROPERTIES

By

R. F. Hoelker and N. J. Braud



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ABSTRACT

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The field of earth-moon trajectories is numerically surveyed and systematically described. The description is based on a classification according to equal time period trajectories. Classes of trajectories of constant transit time are related to position and velocity data of their perigee and periselenium terminals. The position loci of the perigee terminals are ordered by a phase angle relationship with respect to a central point. It is shown that trajectories departing from stations of constant phase envelop the moon densely and collect, after passing their periselenium, at a common point which is called the vertex. Other characteristics are exhibited.

The survey is accomplished by utilizing a smallest characteristic element, which, due to its topologically invariant behavior, constitutes a powerful tool for generating classes of dimensions larger than that of its own.

The survey covers by application of Miele's Image Theorem also the trajectories from moon to earth.

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Presented at the AIAA Summer Meeting
June 17-20, 1963 in Los Angeles, California

FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION

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SUMMARY

The field of earth-moon trajectories is numerically surveyed and systematically described. The description is based on a classification according to equal time period trajectories. Classes of trajectories of constant transit time are related to position and velocity data of their perigee and periselenium terminals. The position loci of the perigee terminals are ordered by a phase angle relationship with respect to a central point. It is shown that trajectories departing from stations of constant phase envelop the moon densely and collect, after passing their periselenium, at a common point which is called the vertex. Other characteristics are exhibited.

The survey is accomplished by utilizing a smallest characteristic element, which, due to its topologically invariant behavior, constitutes a powerful tool for generating classes of dimensions larger than that of its own.

The survey covers by application of Miele's Image Theorem also the trajectories from moon to earth.

SECTION I. INTRODUCTION

The objectives of the study reported here were to generate a full survey of the field of trajectories between earth and moon and to present this survey in a systematic way.

For the description of the field, it was aimed at creating notions that are geometrical, since it is believed that these notions are grasped easier and retained longer.

The means of arriving at the survey were numerical and experimental, with generation of concepts and classifications during the process of data collection. The trajectory calculations were done by Cowell's method.

The range of parameters investigated is sufficiently large to encompass all cases of present day interest of application.

SECTION II. DESCRIPTION OF APPROACH

Solving the general problem of flights from earth to moon (which we call "earth-moon transits" from now on) by numerical means requires surveying the transits for representative ranges of all parameters involved. Studying the variety of parameters involved lets the task seem almost hopeless. The parameters are:

1. Gravitational, as caused by the number of celestial bodies, the non-uniformity of the bodies, the varying distance of the bodies,
2. Graphical, as caused by the movement or rotation of the earth (geography) or of the moon (selenography),
3. State-descriptive, as three for position description and three for velocity description, referring to coordinate systems at earth and moon,
4. Time-descriptive, as referring to time of departure from earth or time of arrival at moon or to the time period of transit duration.

It will be shown in the following that one can gain full control of the volume of earth-moon transits. This is

done by the differentiation between influential parameters and "weak" ones and by a stepwise building-up of patterns of sub-classes of transits. The fact that the sub-classes are easily imaginable helps the reader in creating a pictorial or geometric concept of the full volume.

We will reduce the problem temporarily to a much simpler one by freezing the parameters of weak influence to a fixed value which may also be zero, solve for the remaining core of the problem, and subsequently stepwise thaw the frozen parameters to explore their influence. The thawing process, however, is carried through in this paper with only the more interesting parameters.

The reduction of the problem proceeds in the following way.

1. Mass of the celestial bodies other than earth and moon is temporarily neglected.

2. Earth and moon are considered temporarily as homogeneous masses with central gravitational fields.

3. Motion of the earth-moon system is temporarily considered as that of the restricted three body problem with the following constraints:

Earth-moon distance:	3.85080×10^5 km.
Earth-barycenter distance:	4.6747195×10^3 km.
Earth mass:	5.904×10^{24} kg.
Moon mass:	7.255×10^{22} kg.
Earth radius:	6370 km.
Moon radius:	1738.3 km.
Revolution period of earth-moon system:	27.524318 days

4. The geographical and selenographic coordinate system is replaced by a system that is referenced to the plane of the earth-moon motion and to the earth-moon line. It is called the MEP-System ("Moon-Earth-Plane"). The MEP-System is flight-mechanically the only significant system. Expressing our study-results in a geographical net system requires then simple coordinate transformations.

The MEP-System: On earth and moon, MEP-equators are defined as the intersections of the MEP with the bodies. MEP-poles are perpendicular to the MEP through the centers of the body with the north direction in agreement with the positive direction of the rotational velocity vector of the earth-moon line. On both bodies, the MEP-latitude circles are defined in consistency with the definitions of MEP-equators and MEP-poles, positive latitude extending from the equators to the north poles. MEP-longitudes are measured by great half-circles from pole to pole, the zero longitude at the moon being the nearest to the earth, at the earth being farthest from the moon. Longitudes are measured eastward through 360 degrees, east being defined as forming a right screw system with north. The MEP-System rotates with the earth-moon line. This coordinate system is illustrated on Figure 1.

5. The behavior of transits is described by reference to their state variables at the points of nearest approach to earth and to moon. These points are called perigee and periselenium (for brevity, "perisel"). Alternately the terms "departure" and "arrival", or collectively "terminals" are used, though transits will also be studied beyond their terminals. Since the flight path is horizontal at a terminal, one of the six state-variables is frozen. Note that the terminal points may lie inside the volume of earth or moon, or both.

6. Consideration of absolute time for departure from earth or for arrival at moon is rendered unnecessary by the fact that the terminals are position-wise referenced to the MEP-System. The notion of "transit-time" remains as an important parameter and shall be the time-period spent in transit between the terminals, that is, between perigee and perisel.

7. We freeze temporarily the radii (or altitudes) of the terminals:

Perigee radius = 6555.0 km.
 or Perigee altitude = 185.0 km.
 Perisel radius = 1923.3 km.
 or Perisel altitude = 185.0 km.

8. The transit time will be temporarily frozen to the period of 72 hours.

The general problem is now reduced to the form: survey the transits from earth to moon which have their perigee at 185 km above the earth's surface and perisel at 185 km above the moon's surface and require a transit time of 72 hours between these terminals.

SECTION III. THE SMALLEST CHARACTERISTIC ELEMENT

The class of all 72 hour transits between the two terminal altitudes is still too large to be treated summarily and to serve as easily conceivable memory-piece. Rather, we construct a class of the next lower order which serves then as the generating element for the larger class. The class of lower order may be looked upon as the "smallest characteristic element."

To introduce this element, we proceed from the two-dimensional case. We search numerically for those transits that lie completely in the moon-earth plane ("embedded transits") and satisfy the listed conditions of terminal altitudes and time. The parameters available for this isolation problem are longitude and velocity magnitude at the terminals. Selecting values at one terminal determines the values at the other terminal. After we (arbitrarily) agree to calculate our transits in the direction from earth to moon, we utilize at the earth the parameters of longitude and velocity to isolate for the two values of arrival altitude and transit time.

We may leave the earth "co-rotationally" and "counter-rotationally" (these terms are used here in preference to the conventional astronomical terms "direct" and "retrograde"). To construct our characteristic element, we select the co-rotational direction. Two solutions are found: one being co-rotational at the moon, the other counter-rotational. These are plotted on Figure 2. Note that the longitude values of the two departure points are 47.11° and 48.33° , leaving a distance between them of 1.22° .

The crucial step for constructing the smallest characteristic element is following the desire to construct a smallest family of transits that completely envelops the moon in three-dimensional space. For this purpose we allow the velocity vectors at perigee to point out of the moon-earth plane, while the locations of the perigee points are still kept in this plane. The isolation is done by choosing stepwise a longitude and varying velocity magnitude and azimuth (or inclination) to satisfy perisel altitude and transit time.

On Figure 3 this family of transits is shown.. The velocity vectors have a largest relative azimuth from the earth-moon plane of $\pm 5.4^\circ$. These largest-azimuth-connected transits pass the moon (or "by-fly" the moon) in polar directions.

SECTION IV. THE VERTEX-CONCEPT

An all-important notion may be introduced here which not only helps in obtaining easier mental control of the larger class of transits which we are going to discuss shortly, but is also of eminent practical value.

For illustration of the following, the reader is referred to Figure 4. Studying the family of transits that was constructed in the last chapter with respect to its behavior at the moon, we recognize that the perisel locations form a nearly perfect circle, and that the transits, if continued beyond their perisel points, cross through a small volume which, for all practical purposes, may be called a point, and that this point lies on the line from the center of the moon through the center of the perisel circle. We refer to the small volume of transit crossings as the "vertex" of this family of transits. Where there is no cause for misunderstanding, we will use the term "vertex" also for the projection of the vertex point on the surface of the moon. The phenomenon of bundling of a family through a small volume is referred to as "verticing" or "vertexing."

Note that for the family of transits shown, the vertex is at latitude zero and at longitude 130° .

To summarize the development up to this point, we may say:

By varying the departure location one-dimensionally and allowing velocity magnitude and azimuth to be varied, we constructed a one-parameter family (or class) of 72-hour transits that envelop the moon densely, have a virtually circular locus-curve of perisels and are vertexing on a radial line through the center of the perisel circle.

A family constructed in this way is the smallest characteristic element of the larger class of all 72-hour transits.

In the course of our numerical survey, we have found that such a "generating element" exists for all classes of transits of constant transit times studied. The range investigated is from 60-hour transits to 96-hour transits.

We proceed now to produce the complete 72-hour survey utilizing the generating element.

SECTION V. PROCEDURE OF GENERATING THE FULL SURVEY OF THE 72-HOUR TRANSITS

As yet, the departure locations were frozen to a one-dimensional region, i.e., MEP-latitude zero. Expanding it into the second dimension can be done in various ways and is to a degree arbitrary. We have studied, beside the way we propose later, the method of varying systematically the latitude of the departure locations. We found that some convenient features which we would like to expound are not easily demonstrated by latitude variation; hence, this was abandoned in favor of the phase-angle variation.

This method of phase-angle variation may be explained in the following way.

In Section III, we started the development of the smallest characteristic element by referring to two "embedded" transits departing the earth co-rotationally. Actually, there is another pair of embedded transits that depart from the earth counter-rotationally, one of which arrives at the moon co-rotationally, the other counter-rotationally. Associated to these two transits, there is also a segment on the MEP-equator from which an enveloping family of transits can be started by varying azimuth and velocity. The observation of the existence of two opposite segments for departure locations gives rise to defining a center-point C between these segments, connecting the point with the center of the earth, and considering this radial line as the nodal line for great circles along which the search for possible departure points can be made. This procedure is schematically shown on Figure 5.

Since our discussions from now on are mainly referring to this pencil of great circles, it is expedient to introduce a few new terms and a coordinate reference system directly related to the pencil of circles.

We look at this pencil as one of half-circles rather than of full circles. Each half-circle starts at the center-point C and ends at the point diametrically opposite to C.

The radial line from C through the center of the earth is the common line of the half-planes associated to the half-circles.

Any point on the spherical surface corresponding to 185 km altitude can now be defined by two coordinates, first the particular half-circle on which it lies, and second, the distance it has from the center-point C. The half-circle identification is done by giving the inclination of the half-circle plane to the equator plane, more specifically to the half-plane of the equator that is east of the point C. We start by going northward and like to count the inclination through 360° . Since this is uncommon to inclination measurements, we replace the term "inclination" with the term "phase-angle" or simply "phase."

The second angle which we need for defining a point on the spherical surface is the angle measured at the center of the earth between the radius to the center C and the radius to the point of interest. We call this angle "the distance angle" or simply "distance."

Figure 5 shows the defined angles pictorially.

We owe still an exact definition of the location of the center-point C, but take allowance to delay this for a moment and proceed with the task of creating the full survey of the 72-hour transits with the terminal altitude constraints as discussed before.

In principal, for every point on the spherical surface of 185 km above the earth, the problem is to be answered: Does a transit exist that has its perigee at this point and satisfies the constraint of perisel altitude and transit time? If there were more than one transit from a departure point, all of them would have to be recognized.

The freedom of state variables is two-dimensional with respect to position (altitude is frozen) and two-dimensional with respect to velocity (path angle is frozen to horizontal).

The method of proceeding in the survey is suggested by the earlier derivation of a smallest characteristic element. If we look back at this now, we recognize that the family derived there is the answer to our problem for the case that the phase angle is frozen to zero degrees. What is left to make the survey complete is the opening of the phase

angle up to 360° . Thus, we choose sufficiently small step increases of the phase angle, and for each chosen phase angle, the points along the half-circle are checked for "departure allowance." The check for departure allowance is in principle a running through all combinations of the two remaining degrees of freedom, azimuth and magnitude of velocity, to find all transits starting from this point that satisfy transit time and perisel-altitude.

Utilizing the concept and reference system introduced before, the results of this search procedure can now be presented in easily comprehensible form.

For each frozen phase angle (which is identical to saying: along each fixed half-circle of our pencil) we find that all transits originating at this half-circle constitute a family which has the same characteristics that were exhibited by the smallest characteristic element originally derived for the zero phase angle departures.

Specifically, (a) the locations from which transits are possible constitute a small segment on the half-circle, extending over a few degrees only, (b) the volume of velocity-directions of the transits are centered with the direction of the half-circle plane out of which the transits originate. The largest angle which a velocity vector makes with its associated circle-plane is less than 6 degrees.

For the arrival geometry at the moon, we find: (a) Each family of transits starting from a half-circle of fixed phase angle envelops the moon densely in a one-parameter fashion (no double points prior to reaching the vertex). (b) The perisel points of a fixed-phase transit-class form a locus curve that is virtually a circle. (c) All transits of a fixed-phase transit-class collect in a vertex. The radius from the moon's center to the vertex goes through the center of the associated perisel-circle. (d) The location and the magnitude of the perisel circles and the location of the associated vertices are functions of the departure phase angle.

Before proceeding with a quantitative survey of the results, we would like to fill the gap, left open before, concerning the exact determination of the center-point of the pencil of half-circles. It is understood that this center-point is in the MEP-equator plane. For its longitude determination, we would like to bring this in relation to a

distinguished transit. There is one transit that departs from its perigee in exact northerly direction and arrives at its perisel from exact northerly direction. We select then the longitude of the center-point C such that it is overflown by this transit when the transit is continued backwards beyond its perigee.

SECTION VI. QUANTITATIVE DESCRIPTION OF THE CLASS OF 72-HOUR TRANSITS

To create a feeling for the magnitude of the area on earth from which departures of the 72-hour class (with the listed constraints) are feasible, we refer to Figure 6 which shows in geometrical fashion the loci of possible departure stations and the near earth geometry of the three sub-classes that depart from locations with phase angles of 0, 90, and 180 degrees. The area of departure stations has the shape of a belt that covers across its full extent about 15 degrees of longitude and latitude. Its strip-thickness varies from about 1.5 degrees at its eastern part to less than one degree at its western part. The center of the belt is located near 41 degree MEP-longitude.

The same figure (#6) shows the arrival geometry near moon of the three sub-classes of transits. (This picture had to be distorted somewhat from the true geometry in order to bring out more clearly the salient features.) Shown is the geometry of the three sub-classes after they pass their perisel-circle up to the points where they converge to the vertices. These features are the key-features of sub-classes, well suited for acquiring mental control of their behavior in the large. The vertices are marked by the phase angle ϕ to which the sub-class was associated at earth. We notice that MEP-northern departures result in MEP-southern vertex locations.

The next figure (#7) shows once more the essential features at earth and moon, but in a slightly more abstract form which offers also a quantitative reading. The belt of departure stations is shown on the MEP-grid of longitudes and latitudes. The arrival characteristics on the moon are shown by the locations of some perisel-circles on the moon's MEP-grid and by the locations of the vertex-points. In addition to the single perisel-circles, the whole area of points that offer a perisel possibility is shaded. Perisel-circles and vertex-points are again labeled by the phase angle ϕ of earth departures belonging to this family.

The area of all perisel locations also has the shape of a belt. Its total extension covers longitude and latitude differences of about 120 degrees. Its strip-thickness varies from a few degrees at the east side to about 20 degrees at the west side of it.

A remark may be made here with respect to a symmetry law in the restricted three-body problem, which probably is self-evident to the reader. To every transit there exists a second transit that is generated by reflection on the earth-moon plane. This law is observed here in the symmetry with respect to the earth-moon plane of the departure belt, the area of perisels, and the locus-curve of vertices.

A complete survey of the characteristics of the 72-hour transits would now contain information about all state-variables as function of other state-variables. Though this material was generated in the course of the survey, we consider the publication of it to be outside of the scope of this paper. It was felt that the principal aim of this paper should be the exposition of the perigee belt, the perisel area, the vertex concept, and the striking functional dependence between vertex location and departure phase angle. Information beyond this will be published in a separate comprehensive report.

A preview to a particular feature, however, might be included here briefly. This concerns a type of symmetry that concerns the topologically identical behavior of the transits near earth to those near moon. (We might call this for lack of a better term "the body-symmetry.") This refers to the possibilities to construct vertices for departing transits at earth and a coordination of departure vertices to arrival stations of equal phase-angles at the moon. Thus, the whole structure elaborated on in the previous parts of this paper can well be reversed with respect to earth and moon. An elaboration on this feature is also deferred to a later report.

SECTION VII. SURVEY OF TRANSITS RELATED TO TRANSIT TIMES BETWEEN 60 AND 96 HOURS

A practical way and maybe the only possible way to expand a numerical investigation in dimension is the selection of representative values of the parameter to be varied and apply stepwise to all selected values the same type of investigation which was applied before to the initially frozen value of the parameter.

This is the way in which we now proceed to find a solution to the transit problem over an extended range of transit times. Of interest to the application field at the present time is the range between the 60 hour transits and the 96 hour transits. This range will be sampled at four points, namely at 60, 72, 84, and 96 hours. The coverage which was described before for the class of 72-hour transits is in a similar fashion produced for the three other selected transit times.

This investigation did not encounter features that were structurally (or topologically) different from those found and exhibited here for the 72-hour class. Learning of this may be disappointing for the scientist who is out for the exciting discoveries of pathological behavior of functions, but is pleasing to the engineer whose task it is to design for a variety of applications.

Because of the structural affinity of the sampled classes to the 72-hour class, there is no point here in expounding detailed steps of the investigations. Rather, it suffices to compare for the four classes the key features. These obviously are the locations of the terminals, as the belt of perigee locations and the area of perisels with the coordinated vertex positions. Figure 8 shows the earth terminals for the four classes. Notice is given to the longitudinal shift of them and their shrinking in over-all diameter as transit times increase. Figures 9 and 10 depict the areas of perisels and their vertices. The coordination of the vertex-points to the phase-angle of the corresponding departure stations is following the pattern as shown on Figure 7 for the 72-hour class.

Detailed quantitative surveys will be published in a separate publication.

SECTION VIII. CONCLUDING REMARKS

The reader will recognize that thawing other parameters and investigating their effects on the transit behaviors will now be a routine procedure. The authors, therefore, will refrain here from discussing these effects.

A few features of practical interest may be pointed out instead.

1. The significance of the vertex phenomenon lies in the fact that the vertex position determines by its latitude datum the smallest inclination that any transit of the coordinated family may have on the moon with the moon's equator. This holds for the MEP-coordinate system as well as the true selenographic coordinate system. Thus, the smallest possible inclination of an orbit around the moon, generated by braking at perisel without plane change maneuvers, is controlled by the departure phase-angle at perigee. (The limitation in inclinations on the moon to finite values for certain classes of transits is a well known fact. Among other references, reports by the Jet Propulsion Laboratory pointed this fact out quite clearly.)

2. A survey of free return transits of the two symmetrical kinds can be derived from the present survey by evaluating the transits for having perisel either at the earth-moon axis (first kind) or having the perisel circle tangential to the zero longitude or 180° -longitude circle at the moon (second kind). This study requires the variation of the terminal altitudes. (See Reference 1.)

3. It is of importance to note that the survey of earth-moon transits contains implicitly also the survey of the reverse transits (from moon to earth). This is following from the image theorems as developed by Angelo Miele. (See Reference 2.) The classification by means of transit times is preserved in the earth-bound direction. Thus, no new survey need be made. All that is required is changing certain sign-definitions.

4. Key features, in general, were shown here in geometric pictures. This was done purposely to enhance a "feel" for their characteristic behavior. Plotting these features on diagrams in parametric form facilitates a cataloguing for reference in any practical problem.

5. The plotted data can be "curve-fitted" and expressed as algebraic functions of desired parameters. The resulting structure of formulas constitutes then the equivalent of an analytic solution.

6. The researcher who is specializing in exploiting the large automatic computers for the solution of the earth-moon flight problem sees his ultimate goal in expressing also the time histories of transits in formulas that are constructed by curve-fitting methods.

MEP-RELATED SYSTEM COORDINATES

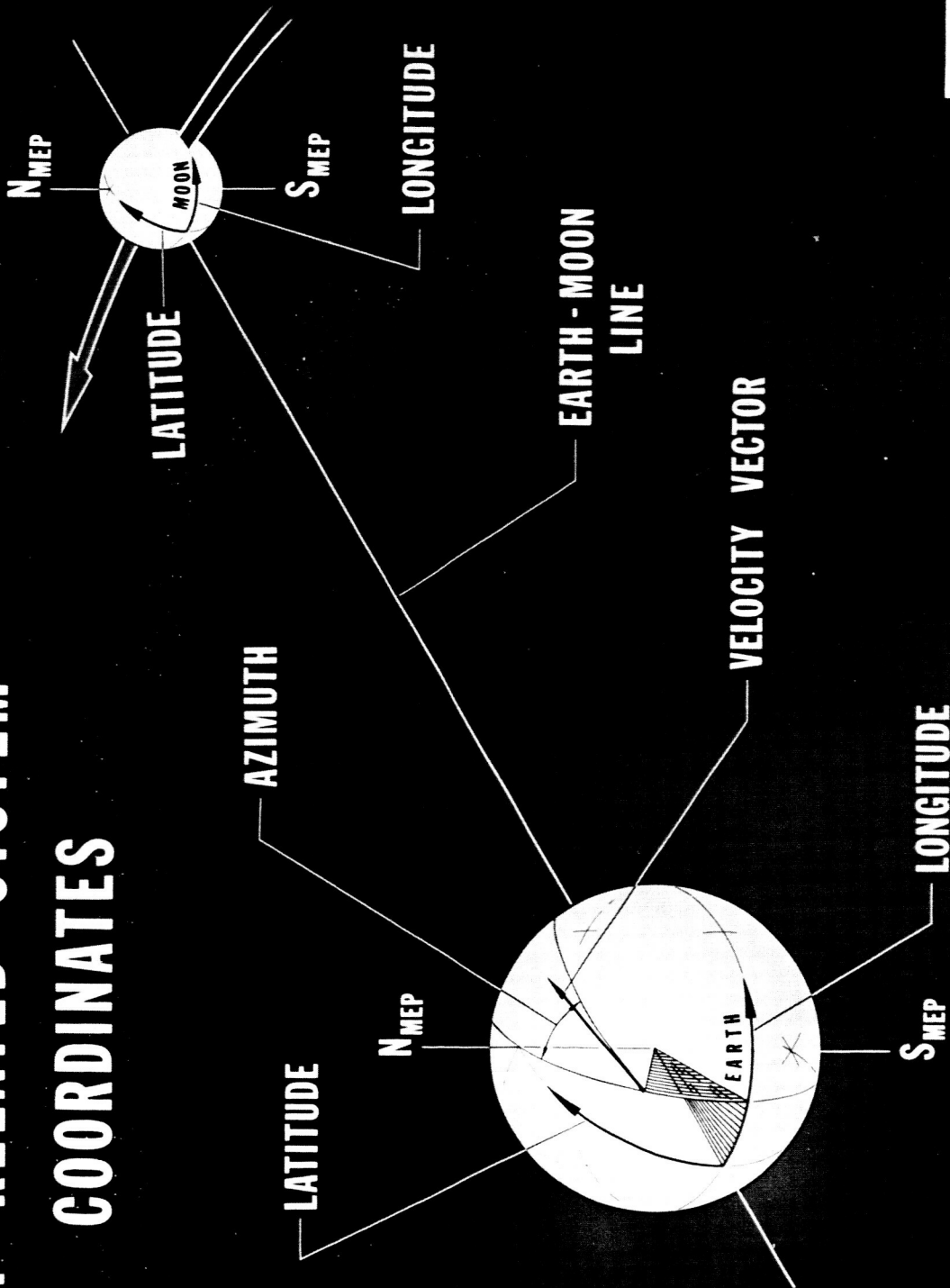


FIG. 1.

ACKNOWLEDGEMENT

The authors wish to express their gratitude and appreciation to Mr. Gary P. Herring, Mathematician in the Aeroballistics Division, for his indefatigable efforts in preparing, processing, and evaluating the data required for this survey.

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2. Miele, Angelo, "Theorem of Image Trajectories in the Earth-Moon Space," Boeing Scientific Research Laboratories, Flight Sciences Laboratory Report-No. 21, dated January 1960.

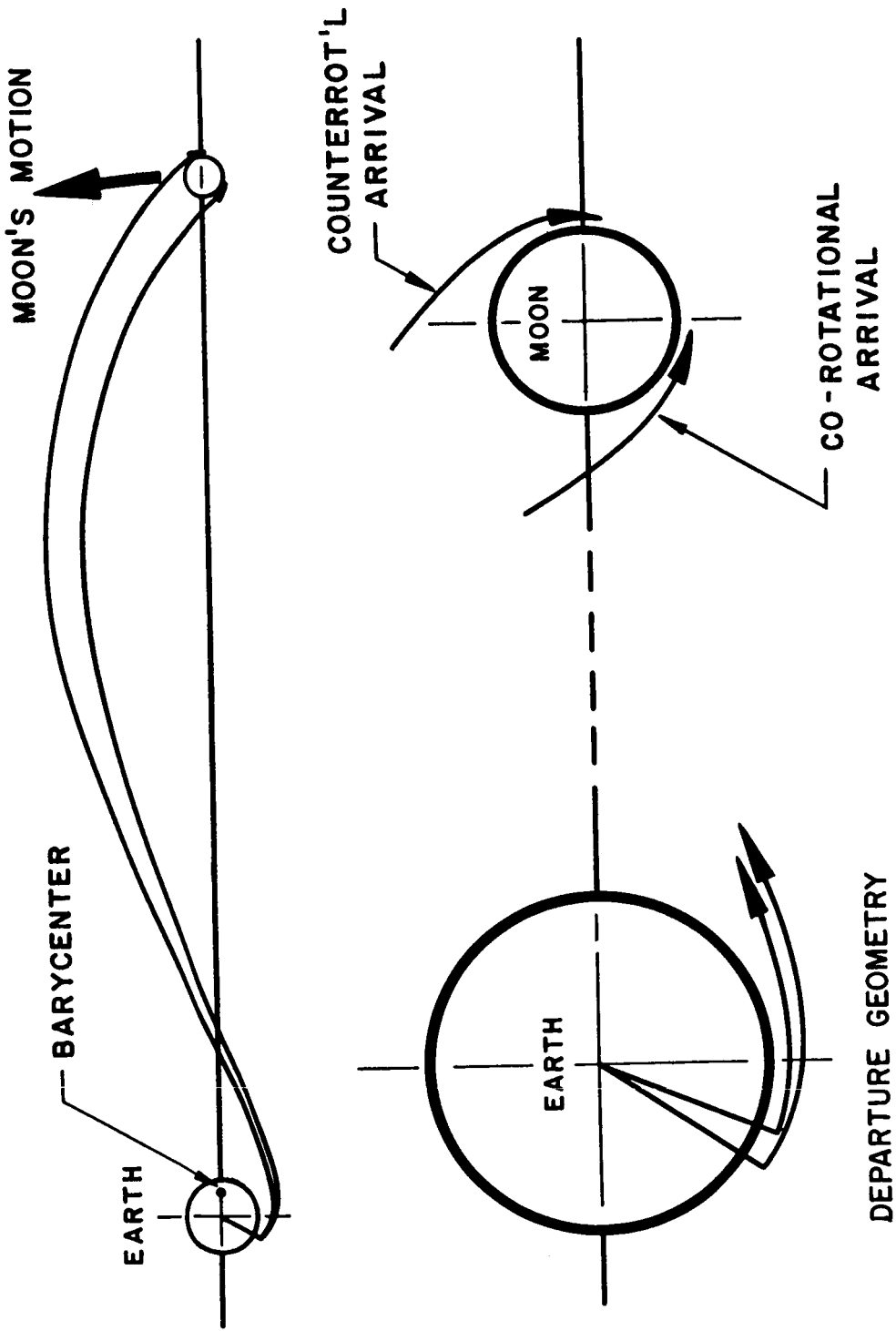


FIG. 2. THE TWO "EMBEDDED" TRANSITS
DEPARTING CO-ROTATIONALLY

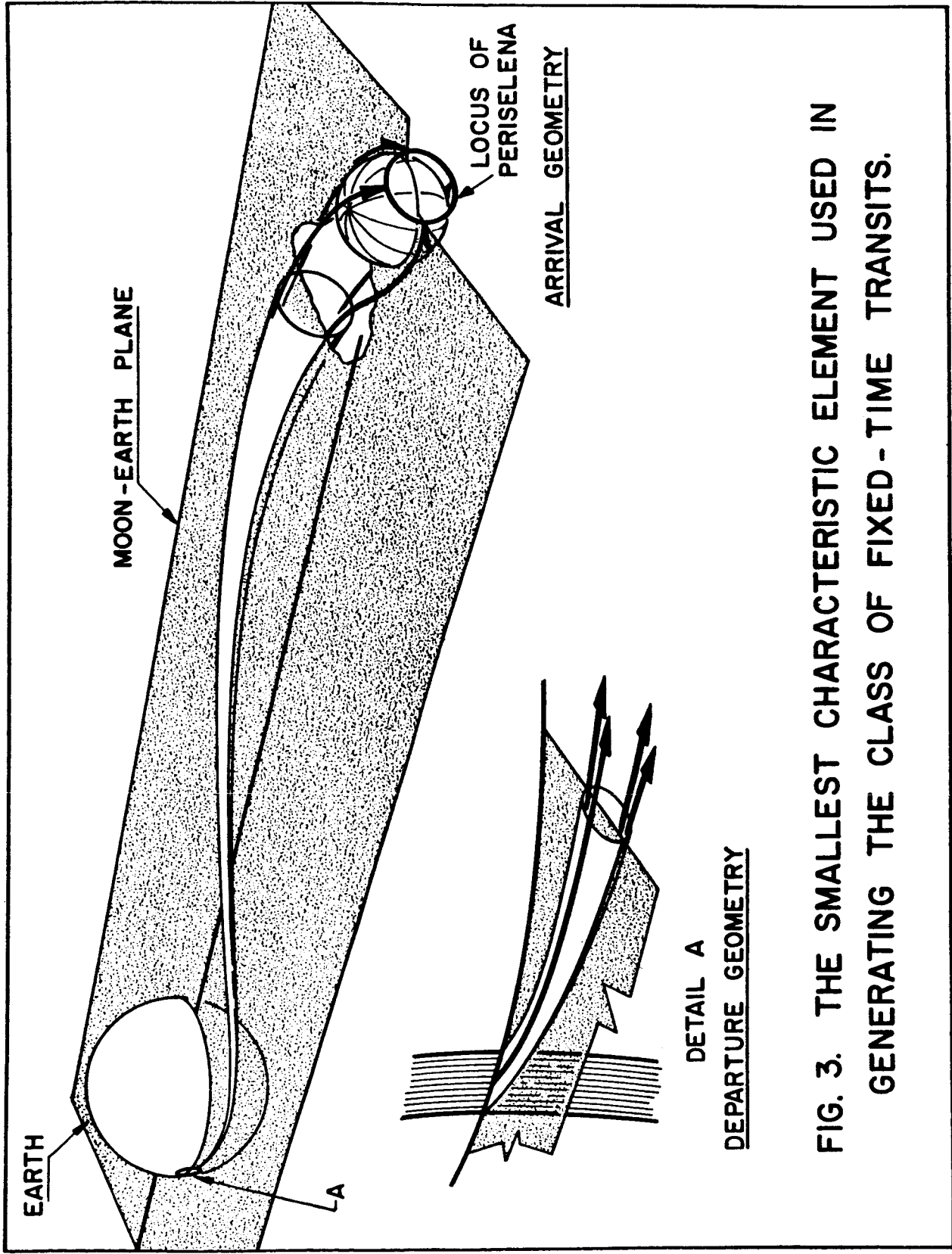
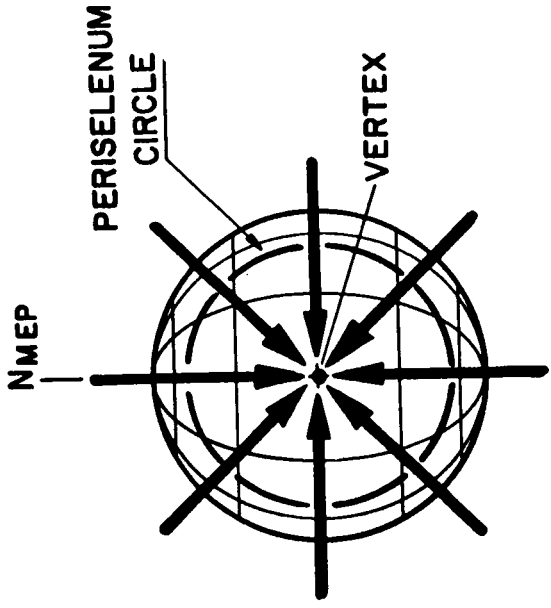
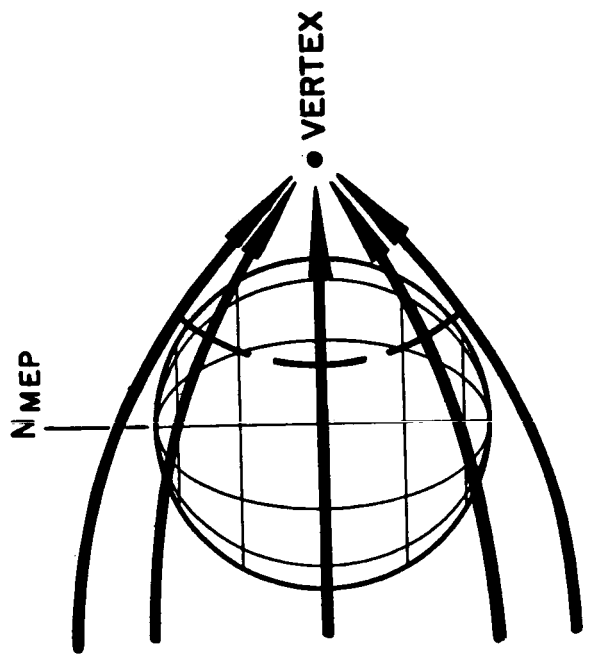


FIG. 3. THE SMALLEST CHARACTERISTIC ELEMENT USED IN GENERATING THE CLASS OF FIXED-TIME TRANSITS.



VERTEX - CENTERED VIEW
OF TRANSITS



SIDE VIEW OF
TRANSITS NEAR MOON

FIG. 4. THE VERTEX - CONCEPT

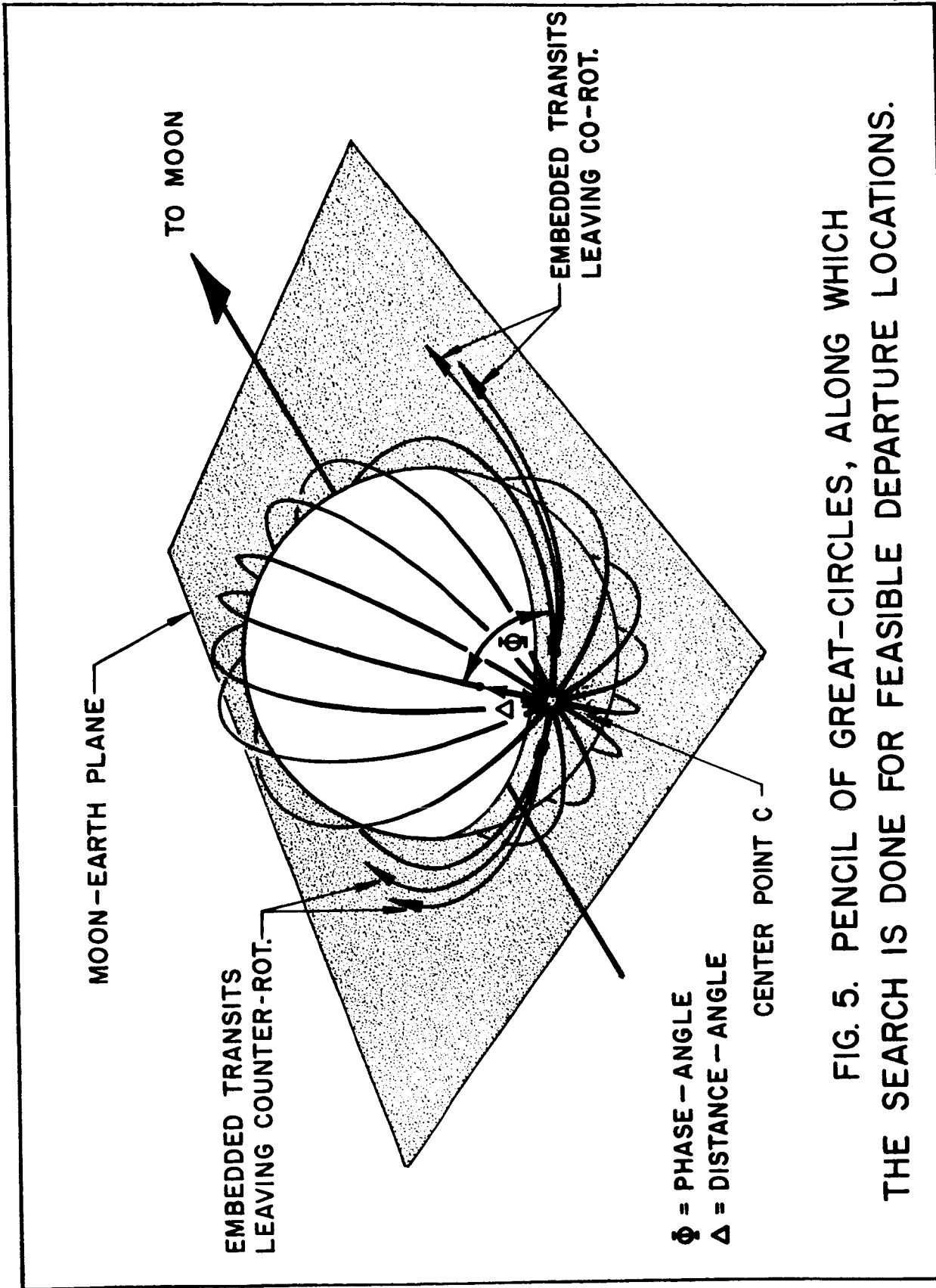


FIG. 5. PENCIL OF GREAT-CIRCLES, ALONG WHICH THE SEARCH IS DONE FOR FEASIBLE DEPARTURE LOCATIONS.

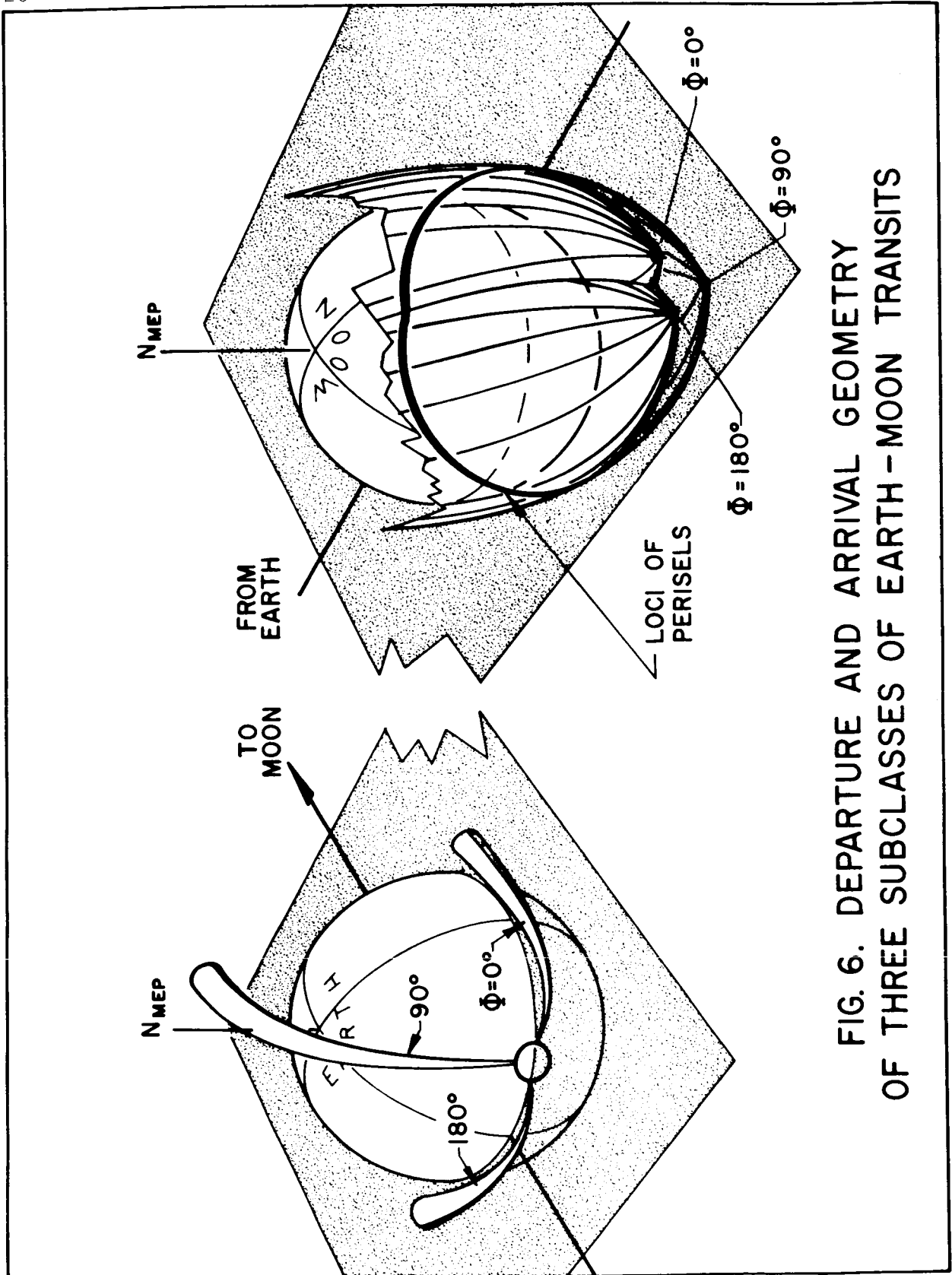


FIG. 6. DEPARTURE AND ARRIVAL GEOMETRY OF THREE SUBCLASSES OF EARTH - MOON TRANSITS

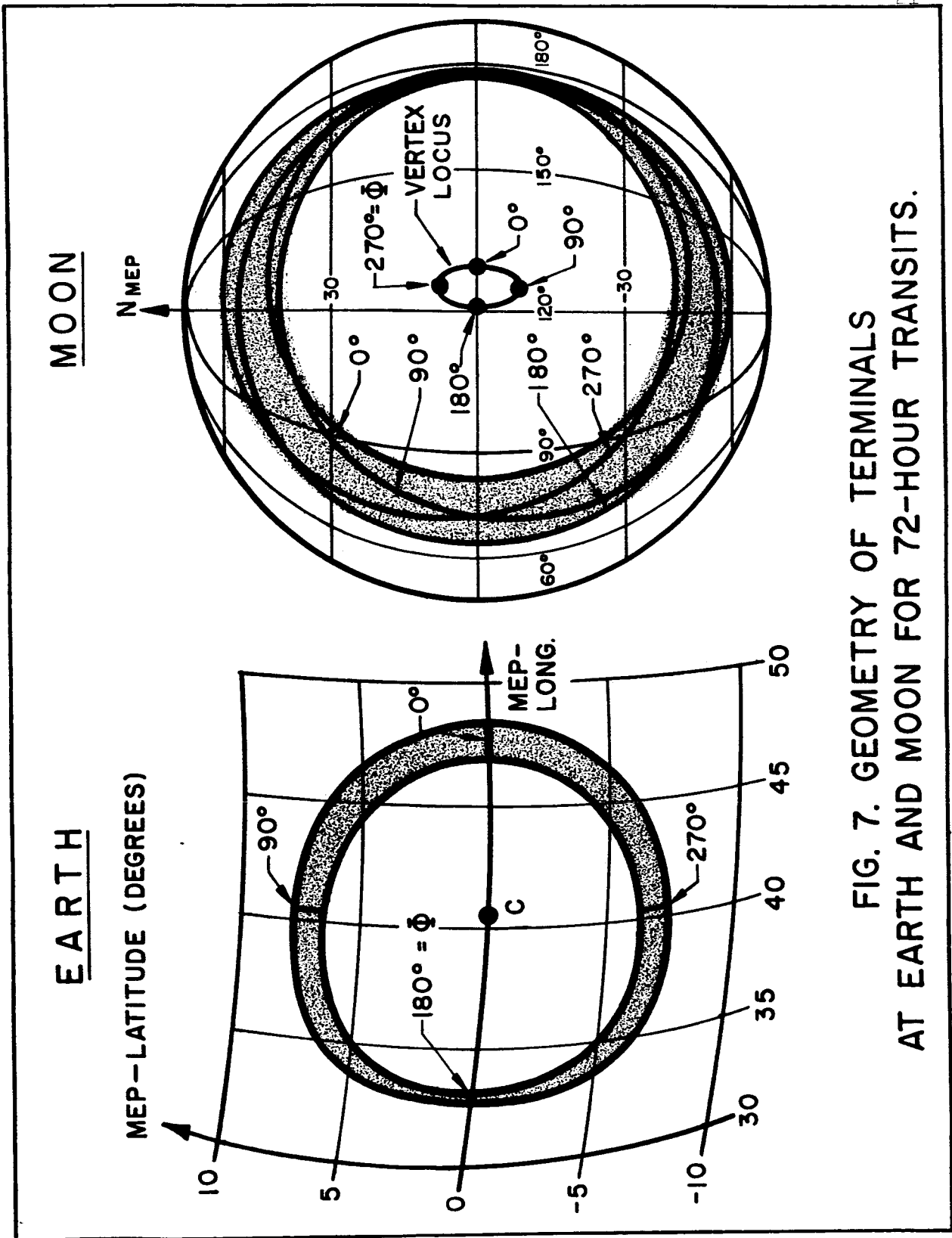


FIG. 7. GEOMETRY OF TERMINALS AT EARTH AND MOON FOR 72-HOUR TRANSITS.

TRANSITS DEFINED BY: DEPARTURE AT PERIGEE AT ALTITUDE OF 185 KM AND ARRIVAL AT PERISELENUM AT ALTITUDE OF 185 KM.

EARTH MEP-LATITUDE (DEG)

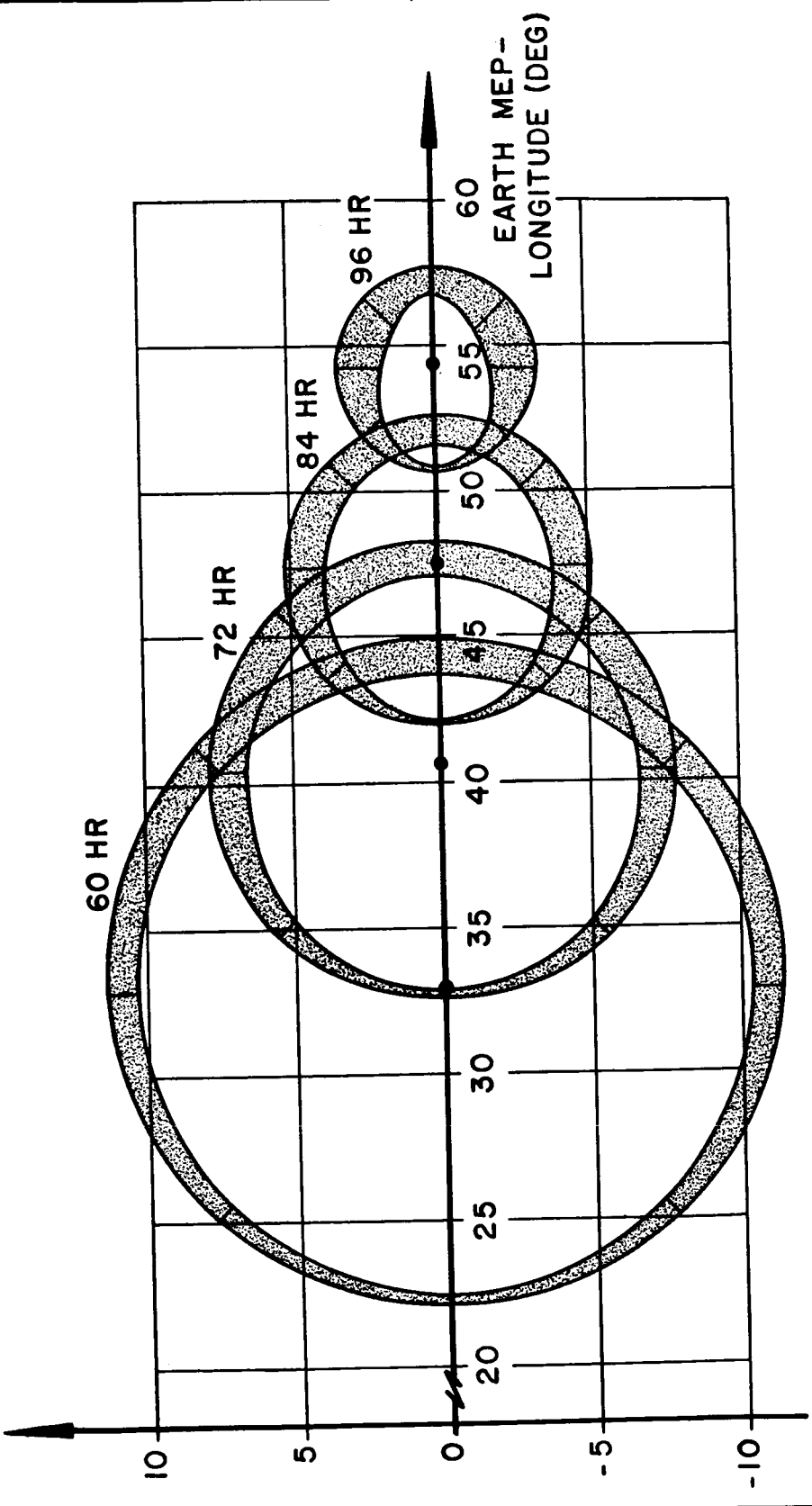
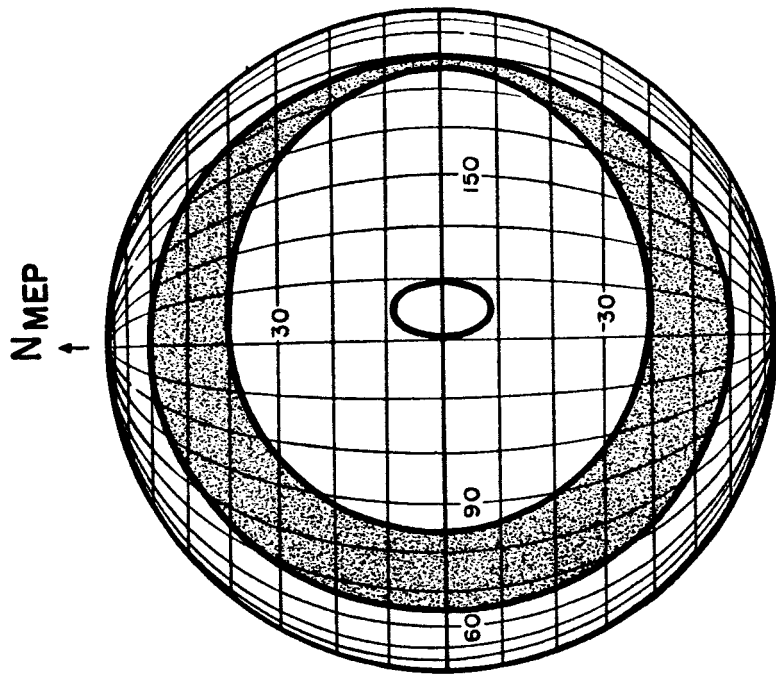
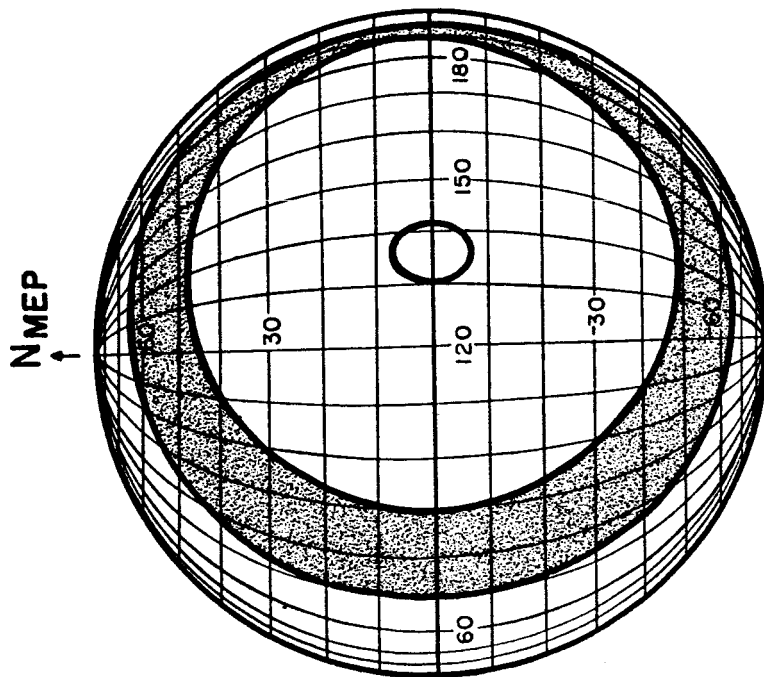


FIG. 8. SURVEY OF DEPARTURE LOCI AT EARTH FOR THE TRANSIT CLASSES OF 60, 72, 84, AND 96 HOURS



72 HR



60 HR

FIG. 9. SURVEY OF ARRIVAL AREAS AT MOON FOR TRANSIT CLASSES OF 60 AND 72 HOURS.

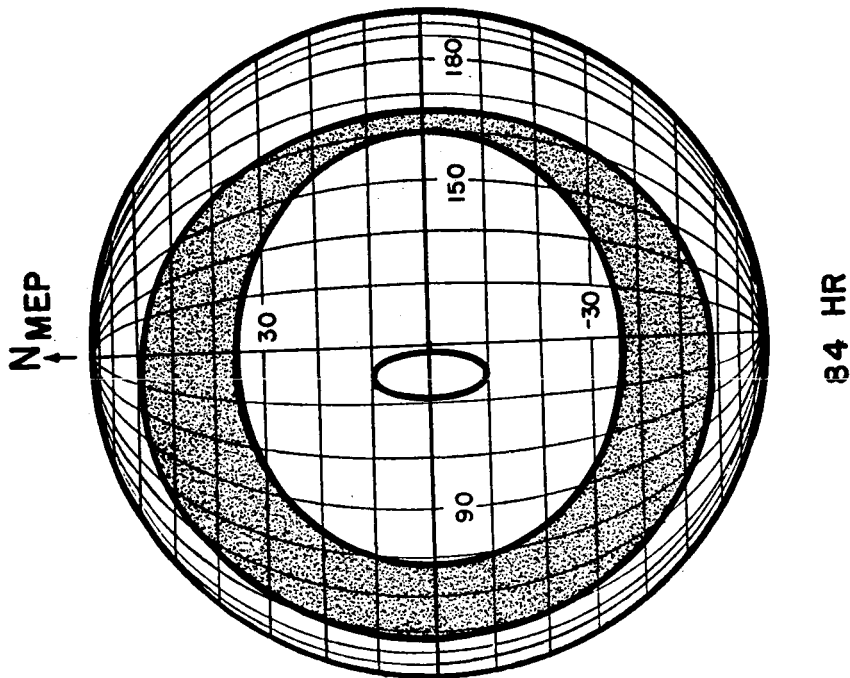
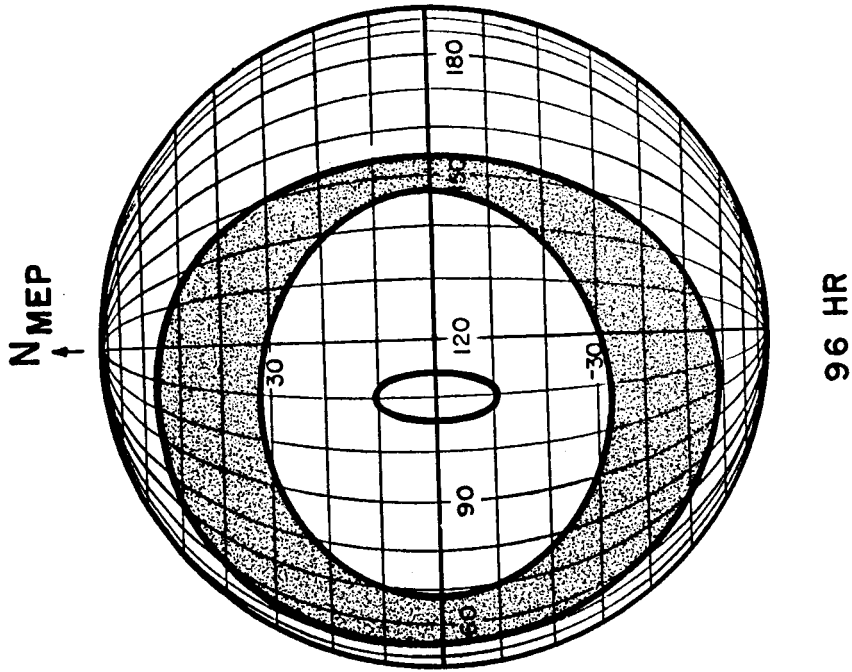


FIG. 10. SURVEY OF ARRIVAL AREAS AT MOON FOR TRANSIT CLASSES OF 84 AND 96 HOURS.

APPROVAL

SURVEY AND CLASSIFICATION OF EARTH-MOON TRAJECTORIES
BASED ON NEWLY DISCOVERED PROPERTIES

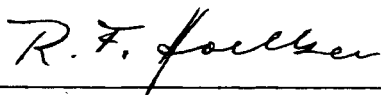
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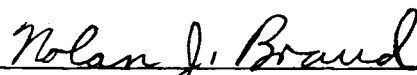
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