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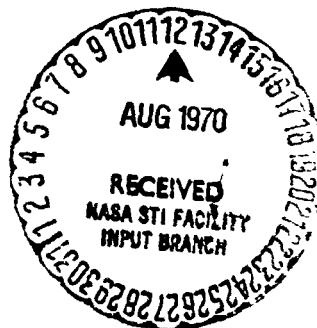
USE OF THE LANDING POINT DESIGNATOR TO  
LAND THE LUNAR MODULE TO A GIVEN  
TARGET

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## SUMMARY

This document presents some theoretical results pertaining to the problem of making efficient use of the landing point designator (LPD) to land the lunar module (LM) to a given target. In particular, the question of how to bias the initial aim point so as to make the most effective use of the LPD is considered. The procedure for determining the initial aim point bias is presented along with example applications. Results are given in terms of the probability of reaching the target within the LPD  $\Delta V$  budget.

## INTRODUCTION

The accuracy to which the onboard Apollo G&N system can negotiate a fully automatic LM landing is presented in reference 1. In the reference, the downrange and crossrange  $1\sigma$  values for dispersions are given and are approximately 3100 ft and 2400 ft respectively. With these dispersions it can be shown that the probability of making an automatic landing to a target area the size of the Dome of the Rock is only about 2.5%. From this it becomes obvious that the landing of the LM to a specific target point, or small area, will most likely entail the use of the LPD.

The purpose of this study was to determine the procedure for making the most effective use of the LPD to land the LM to a given target. Emphasis was placed on determining how to bias the initial aim point so as to maximize the probability of getting to the target with an acceptable redesignation.

The study includes several example problems with the results being given in terms of the probability of reaching the target within the LPD  $\Delta V$  budget.

## ANALYSIS

### Statement of the Problem

Simply stated, the problem to be considered is that of determining the initial aim point bias which maximizes the probability of landing the LM to a specific target within the LPD  $\Delta V$  budget.

In order for the results to be meaningful, certain ground rules associated with the use of the LPD must be observed. These ground rules are detailed in the next section.

### LPD Operational Constraints

Pilot Field of View. The process of making a landing point redesignation is briefly described in the following way. During the final portion of the LM descent, the point for which the LM is targeted is determined by the command pilot through the alignment of the LPD reticle in his window. If the pilot wishes to designate a different landing site, he uses the LPD reticle to determine the required change in the line of sight, and converts this to a hand controller input. The onboard computer interprets this input as a command to retarget. Since the LM configuration is such that the command pilot field of view is essentially restricted to an area of the lunar surface from the LM trajectory plane to the pilot's left, it should be clear that redesignations to the right would be highly undesirable.

Target Visibility. The dynamics of the LM descent trajectory are such that a short redesignation causes the vehicle to pitch back. This could lead to loss of target visibility which also would be highly undesirable.

Fuel Budget. In general, any redesignation requiring more fuel than is available should be ruled out. The current  $\Delta V$  budget for the LPD is 90 ft/sec.

LPD Ground Rules. In determining an initial aim point bias, the two ground rules observed in the study are:

1. Only long and/or left redesignations are to be considered.
2. Redesignations must be within the  $\Delta V$  budget.

### Solution

Before going to the solution of more general problems, it is felt that examination of a special case is particularly helpful in getting some feeling for how the problem is solved and what type of results are to be expected.

Special Case. The geometry for this case is presented in figure 1. It is assumed that there are no out-of-plane dispersions and that the range dispersion about the initial aim point,  $O$ , is normally distributed with zero mean. The distribution function,  $p(x)$ , is shown in the figure.

Let the LM approach from left to right and let the true target be denoted by the point  $A$ . Let the distance corresponding to a total  $\Delta V$  budget redesignation be denoted by  $p$ .

It should be clear that if the LM were going to land at any point in the interval  $(A-\rho, A)$ , the vehicle could be retargeted (with a long redesignation) to land at A within the  $\Delta V$  budget. Conversely, if the LM were going to land outside this interval, the redesignation would either violate the visibility constraint, exceed the  $\Delta V$  budget, or both.

Thus, the problem of maximizing the probability of making a long redesignation within the  $\Delta V$  budget is equivalent to placing the initial aim point, O, relative to A in such a way as to maximize the probability that the LM will land in the interval  $(A-\rho, A)$  without a redesignation.

The solution to this equivalent problem can be determined by inspection in the following way. The probability that the LM will be targeted for a point in the interval is given by

$$\text{Pr} = \int_{A-\rho}^A p(x)dx \quad (1)$$

which is simply the area under the normal distribution curve, over the interval  $(A-\rho, A)$ . From the geometry of the normal distribution curve it should be clear that this area is maximized by placing the aim point O at the center of the interval.

Thus, the solution for this special case is to place the initial aim point short of the true target by a distance equal to one-half the total LPD range capability.

To illustrate the benefit derived from targeting in this manner, consider the following example. Let  $\rho = 2\sigma$  (which is reasonable for the LM LPD capability) and suppose that the initial aim point is the true target. Then the probability of getting to the target with a long redesignation within the  $\Delta V$  budget is 48%. However, placing the initial aim point properly results in a corresponding probability of 68%. This represents an increase in the probability of mission success of about 40%. While this is paid for by increasing the mean LPD  $\Delta V$  expenditure from zero to one-half of that available, it seems justifiable in that the  $\Delta V$  would be used in the task for which it is intended.

The next section presents the more general problem of getting to a target point when out-of-plane dispersions are included.

General Target Point Case. The geometry for this case is given in figure 2. The downrange and crossrange axes are denoted by X and Y respectively. The true target point is denoted by A, and the initial aim point is denoted by O. The quarter circle region labeled R is assumed to be the set of all points from which the target can be reached by a long and/or left redesignation within the  $\Delta V$  budget.

Once again the problem is to locate O relative to A so as to maximize the probability that the LM would land in R without a redesignation.

The straightforward method for solving the problem is to write the probability as

$$P_r = \frac{1}{2\pi\sigma_x\sigma_y} \iint_R \exp \left\{ -\frac{1}{2} \left( \frac{x-\mu_x}{\sigma_x} \right)^2 + \left( \frac{y-\mu_y}{\sigma_y} \right)^2 \right\} dy dx \quad (2)$$

where  $\mu_x$  and  $\mu_y$  are the coordinates of the initial aim point, and then maximize the probability with respect to  $\mu_x$  and  $\mu_y$ . Unfortunately, this problem is intractable in terms of obtaining an analytical solution, and a digital computer has been used to obtain results.

A simple but useful approximation to the location of the initial aim point has been found to be given by

$$\mu_x^* = \frac{\sigma_x P}{\sigma_x + \sqrt{\sigma_x^2 + \sigma_y^2}} \quad (3)$$

$$\mu_y^* = \frac{\sigma_y P}{\sigma_y + \sqrt{\sigma_x^2 + \sigma_y^2}} \quad (4)$$

where  $\mu_x^*$  and  $\mu_y^*$  are the distances short and to the right, respectively, at which the initial aim point is placed. A minimal amount of computation has indicated that (3) and (4) approximate the location of the optimal aim point very accurately, and the probability obtained using these expressions is within about 1% of the maximum. Thus for all practical purposes, these equations would appear to be adequate.

Special Target Area Case. This case corresponds to the problem of using the LPD to land the LM at any point in a proposed target area. It is assumed that the target is mapped in a mosaic fashion with rectangular maps of a fixed size and overlap fraction,  $\alpha$ . The geometric of the problem is shown in figure 3. The length and width of the maps are denoted by  $l$  and  $w$  respectively. The overlap is noted by  $\alpha l$  and  $\alpha w$ . The shaded area of figure 3 denotes the additional area to which the LM may be targeted and still obtain a point in the mapped area using the LPD.

Once again the problem is simply that of locating the initial aim point relative to the region (inclusive of shaded area represented in figure 3) so as to maximize the integral of the probability density function over the region. It is to be noted that for a large total mapped area compared to the additional area for redesignation, that the total region may be considered a rectangle of length,

$$L = \frac{3n+1}{4} l + p \quad (5)$$

(n = number of overlays), and width,

$$W = \frac{3n+1}{4} w + p \quad (6)$$

such that the initial aim point becomes the geometric center of the rectangle.

#### RESULTS AND DISCUSSION

General Target Point. Results for the general target point case are presented in figures 4, 5, 6, and 7. Figure 4 is a plot of probability versus aim point location for the case of  $\sigma_x = \sigma_y = 2400$  ft, and  $p = 7250$  ft. For the case of equal  $\sigma$ 's, the aim point should be on the axis of symmetry of the footprint shown in figure 2. Thus figure 4 shows how the probability varies as the aim point is moved away from A along the axis of symmetry. As shown in the figure, putting the aim point at the true target gives a probability of mission success of about 25%. However, using the optimal aim point results in a probability of about 65%.

It should be noted that the aim point determined by equations (3) and (4) is sufficiently close to the optimal aim point and results in a probability within 1% of the maximum. Since the equations are exact for either  $\sigma_x$  or  $\sigma_y$  equal to zero, one would expect the equal  $\sigma$  case to result in the largest error. However, the figure indicates that even in the "worst case" the error is essentially negligible.

Figure 5 is a non-dimensional plot of aim point location versus the ratio of the  $\sigma$ 's. The way to use the plot is to make a ratio with  $\sigma_x$  and  $\sigma_y$  which is less than or equal to unity. Then if  $\xi = \sigma_x/\sigma_y \leq 1$ , the upper and lower parts of the curve are  $\mu_x^*/p$  and  $\mu_y^*/p$  respectively. If  $\xi = \sigma_y/\sigma_x \leq 1$ , the upper and lower parts of the curve are  $\mu_x^*/p$  and  $\mu_y^*/p$  respectively.

Figure 6 is a plot of probability of mission success versus  $\sigma_x$  for  $p = 7250$  ft and  $\sigma_y = 2400$  ft. The motivation for this plot is explained in the following way. Suppose the target point was the center of the crater Copernicus, and suppose the onboard navigation system could update range to go by recognizing passage over the crater rim in the landing radar data. Then it is possible that the standard deviation for range,  $\sigma_x$ , could be reduced below the presently estimated 3100 ft. To do this would require a knowledge of the lunar terrain to some level of accuracy along with some additional sophistication of the navigation program in the computer. Thus there is the question: "What is the trade-off?" The figure shows that a point of diminishing return is reached at about  $\sigma_x = 800$  ft, where the probability is 85%. Thus for a redesignation radius of 7250 ft and  $\sigma_y = 2400$  ft, knowledge of the lunar terrain, landing radar accuracy, and computer program complexity would not have to go beyond the levels required to reduce  $\sigma_x$  to about 800 ft.

Figure 7 is a plot of probability of mission success versus redesignation footprint radius,  $p$ , for the case of  $\sigma_x = 3100$  ft and  $\sigma_y = 2400$  ft. For a 90 ft/sec  $\Delta V$  budget and redesignation at an altitude of 7,000 ft, the corresponding value of  $p$  is 7250 ft and the maximum probability of success is about 55%. However, less  $\Delta V$ , or redesignation at a lower altitude would result in smaller values of both  $p$  and probability.

Special Target Area. The probabilities of landing in a mapped area are given in figures 8 and 9. In generating these figures it was assumed that 8" x 10.5" maps would be put together as shown in figure 3 with an overlap fraction,  $\alpha$ , of 1/4.

Figure 8 gives the probability of landing in the mapped area versus the square root of the number of maps for cases with the initial aim point being the geometric center of the area and no redesignation capability. The cases are for different map scales,  $D$ . The plot only shows the obvious fact that the LM is more likely to land in a given large area than in a given small area.

Figure 9a is a plot similar to figure 8 with the exception being that of including LPD capability. By comparing figure 9a with figure 8, it can be seen that efficient use of the LPD makes a significant reduction in the number of maps required to realize a 99% probability of landing in the mapped area. In particular, the number of 1/2500 maps is reduced from 121 to 36.

Figure 9b is a plot of probability versus the square root of the number of maps for the case where the initial aim point was set on the basis of having a redesignation at an altitude of 7,000 ft, but in reality the redesignation occurs at an altitude of 5,000 ft. A comparison of figure 9b with 9a shows that the loss of probability due to a late redesignation is essentially negligible for high probability cases.

Figure 9c is similar to 9a **except** that the data is for a planned redesignation at 5,000 ft. Examination of figures 9a, 9b, and 9c shows that probability of mission success is not critically dependent upon altitude of redesignation provided the probability is very high to begin with.

#### CONCLUDING REMARKS

A theoretical study was made to determine the procedure for making effective use of the LPD to land at a point or a small area. The procedure was determined and applied to numerical examples. All of the numerical results were based on current estimates of LM LPD and Apollo G&N capabilities and adhered to the constraint that only long and/or left redesignations within the fuel budget were allowable.

It was found that effective use of the LPD in landing to a target point or area results in a significant increase in the probability of mission success in cases of interest.

In maximizing the probability of getting to a specific target point, there is an expected  $\Delta V$  cost of approximately one-half of the total  $\Delta V$  budget for redesignation. However, this would seem to be justifiable in that the fuel would be used in the task for which it is intended.

In any practical situation of landing to a mapped area, there would be no expected  $\Delta V$  cost associated with maximizing the probability of landing in the area. This is due to the fact that in practice the mapped area would be large enough to contain the initial aim point, and on the average a redesignation would not be required to get into the area.

It was found that the altitude at which redesignation occurs is not a highly critical parameter provided it occurs while the LPD still has a significant redesignation capability.



#### REFERENCE

1. Compilation of material presented at a Lunar Landing Site Selection Briefing at the Manned Spacecraft Center, Houston, Texas, March 8, 1967. This is a NASA document for internal use only.

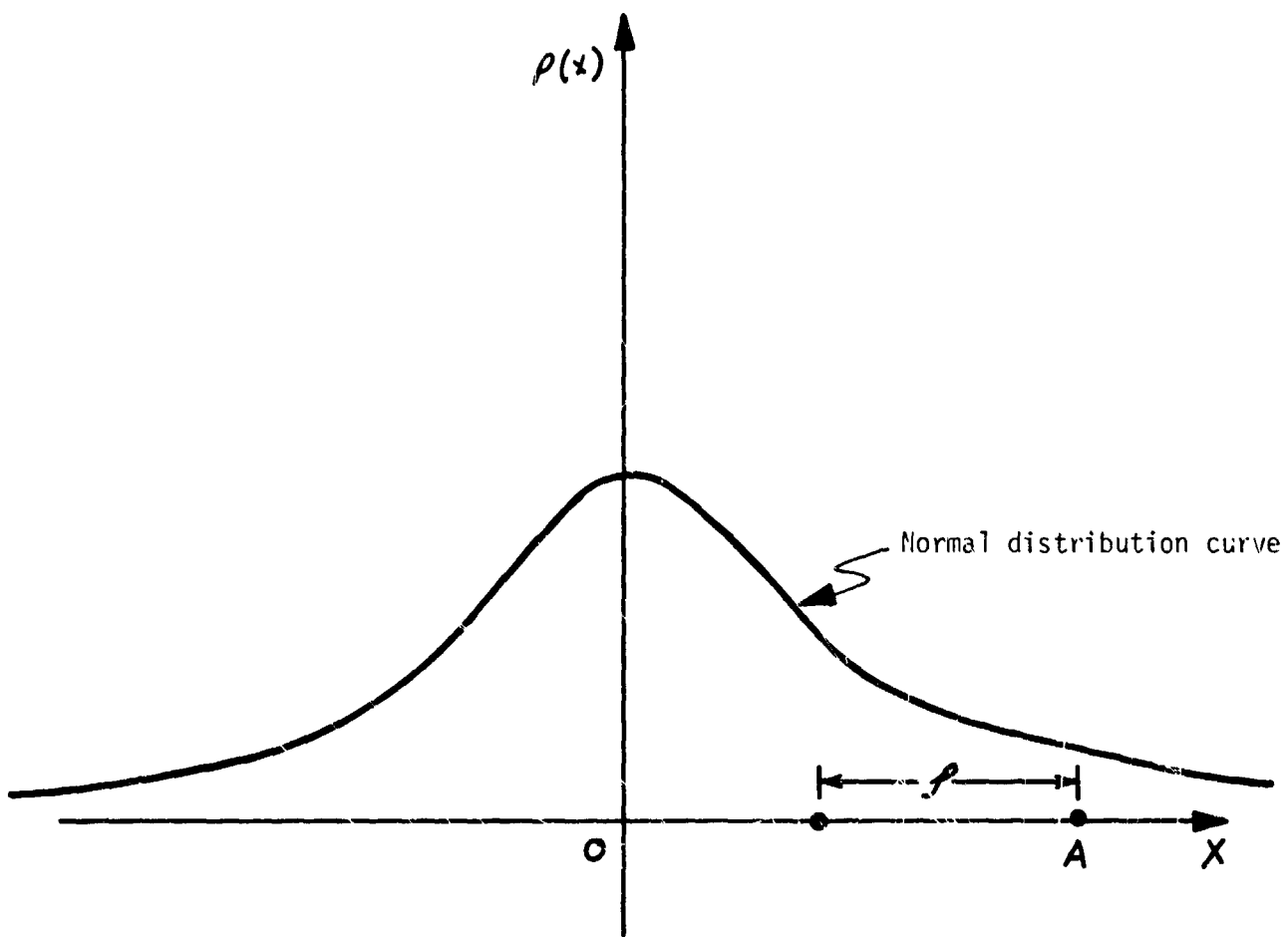


Figure 1. Geometry of Planar Targeting Problem

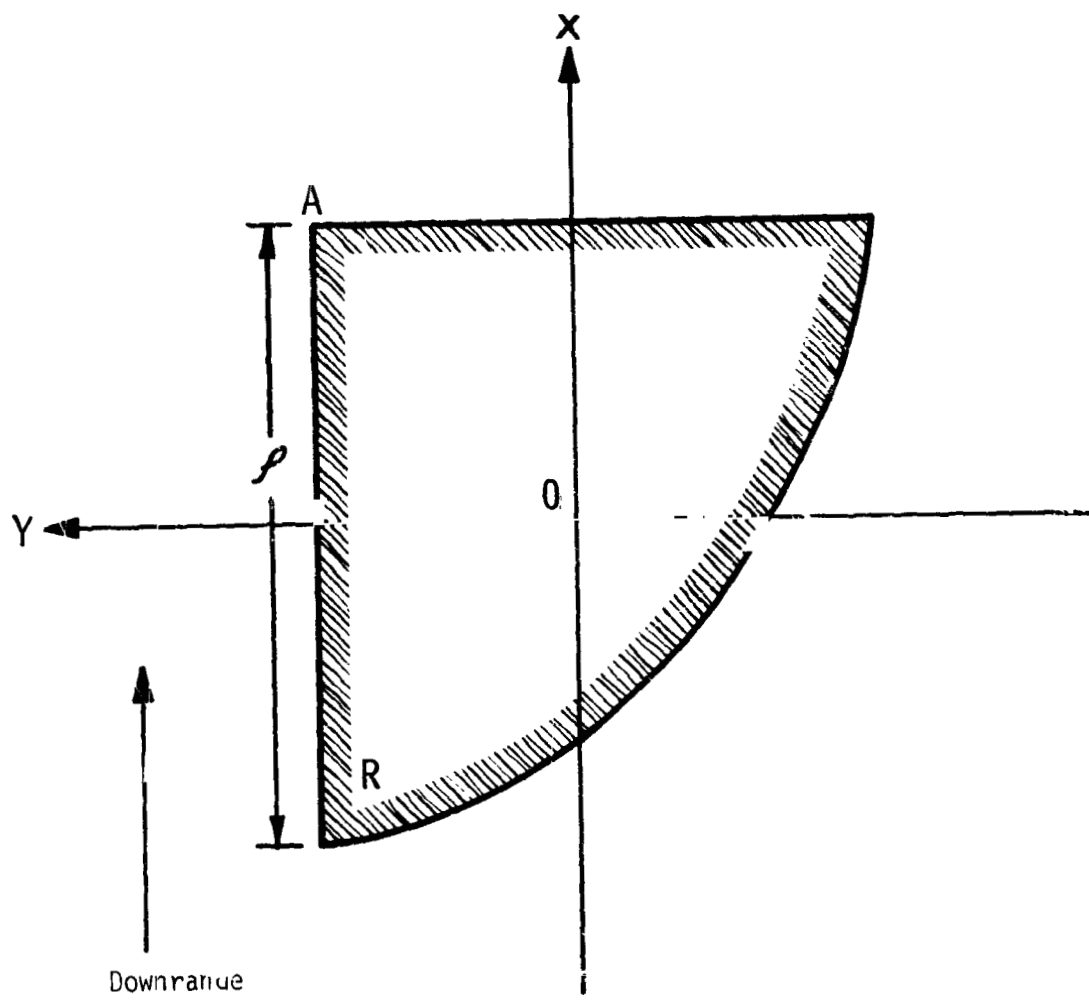


Figure 2. Geometry of General Target Point Problem

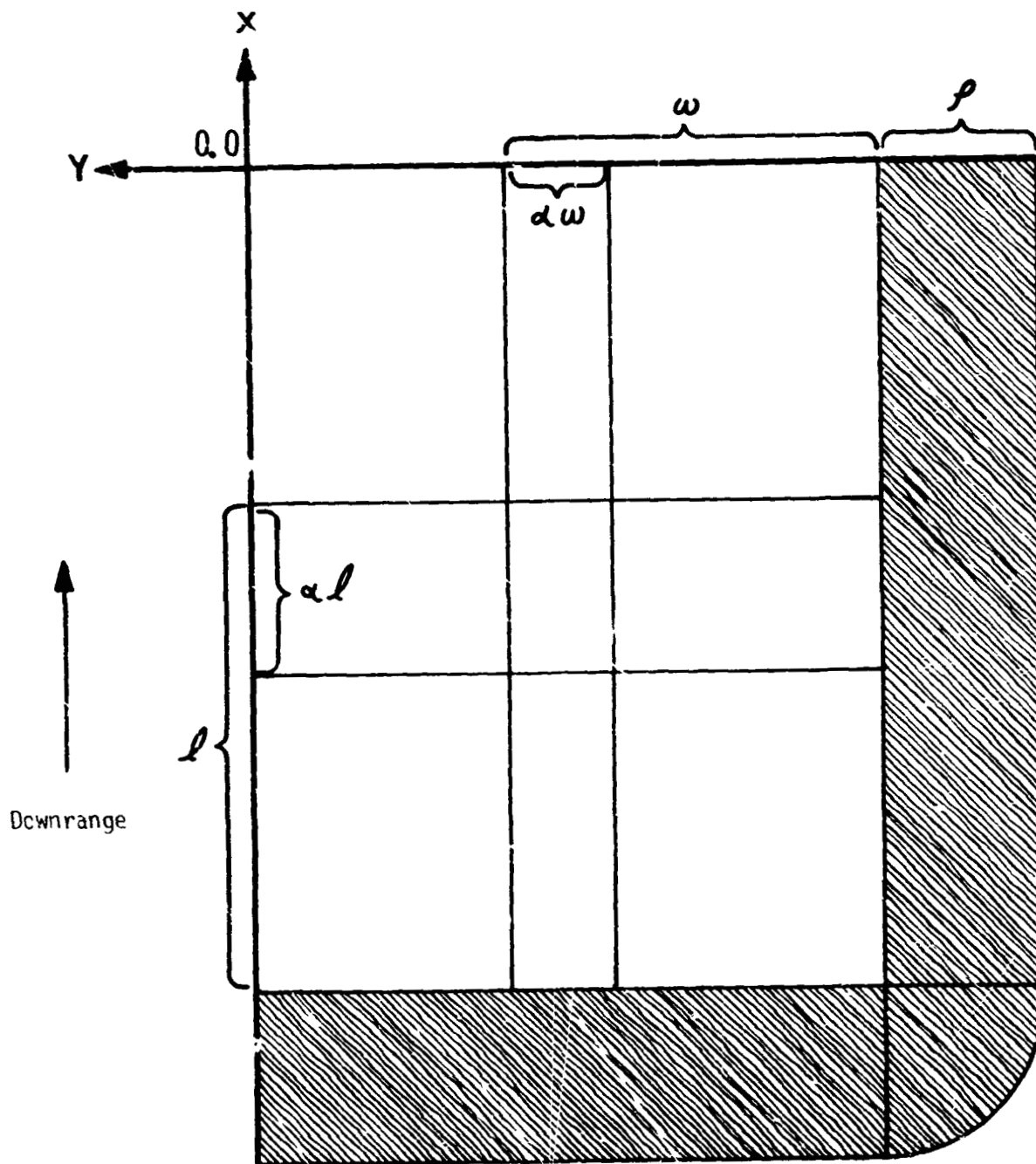


Figure 3. Geometry of Special Target Area Problem

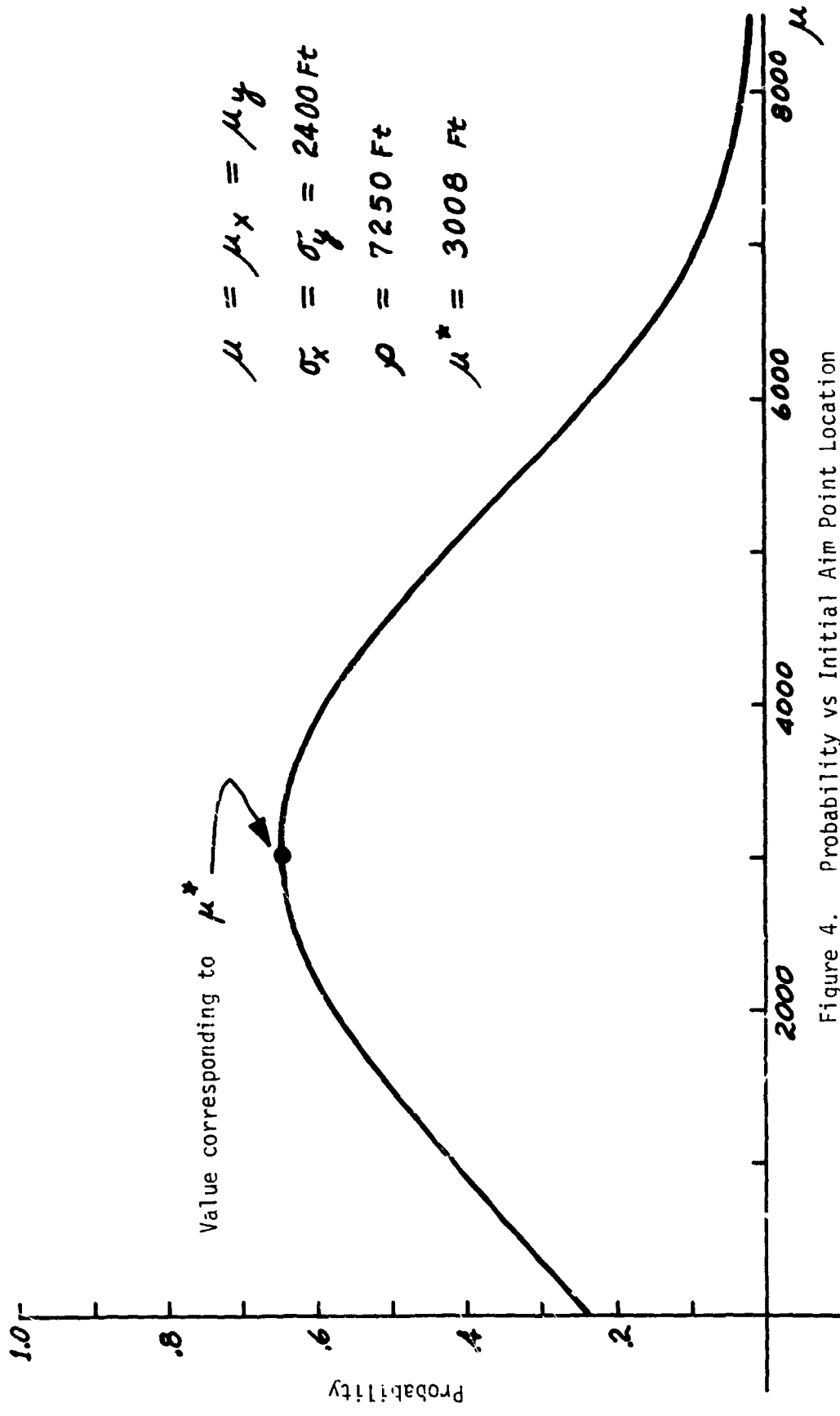


Figure 4. Probability vs Initial Aim Point Location

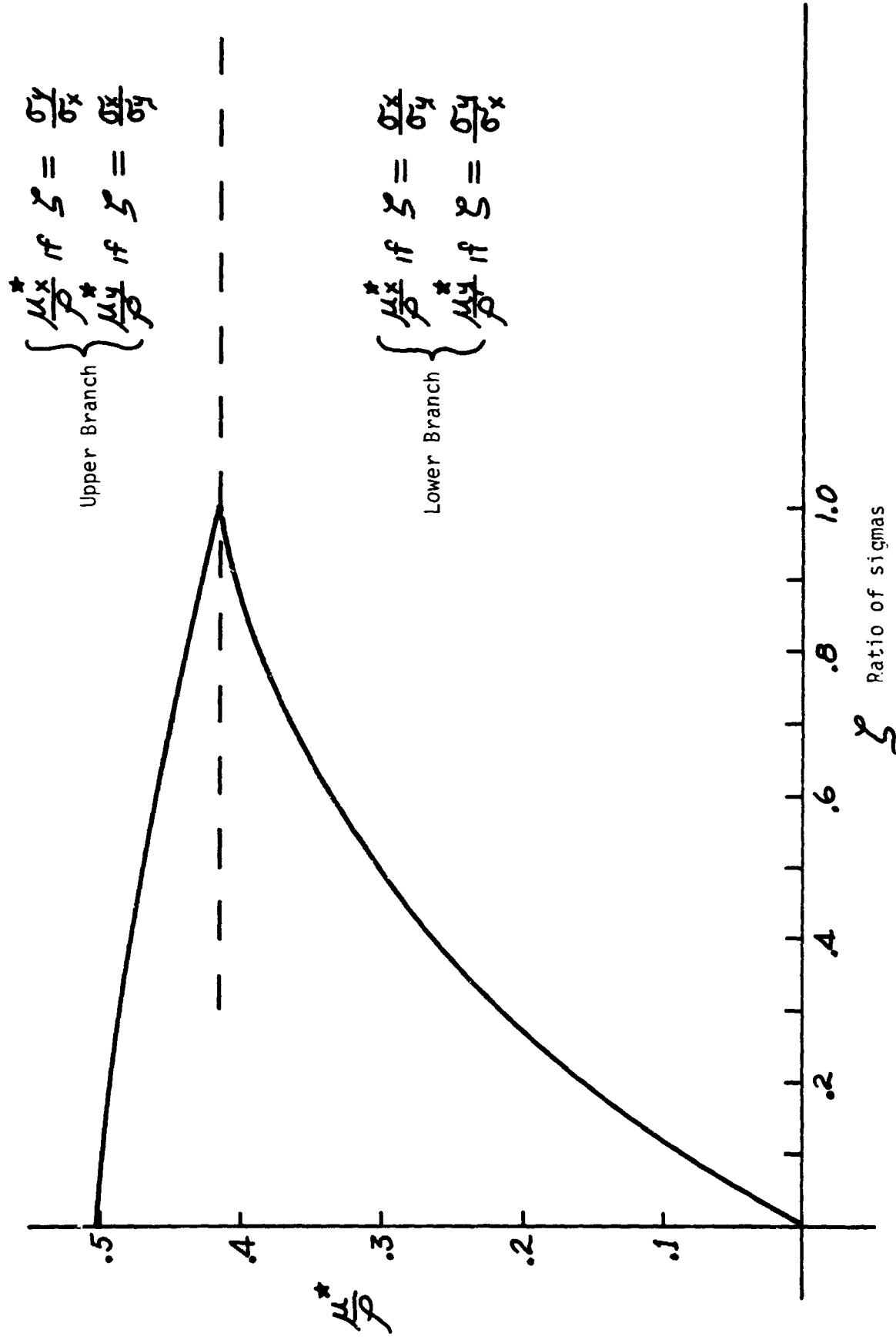


Figure 5. Non-Dimensional Plot for Determining Initial Aim Point Coordinates

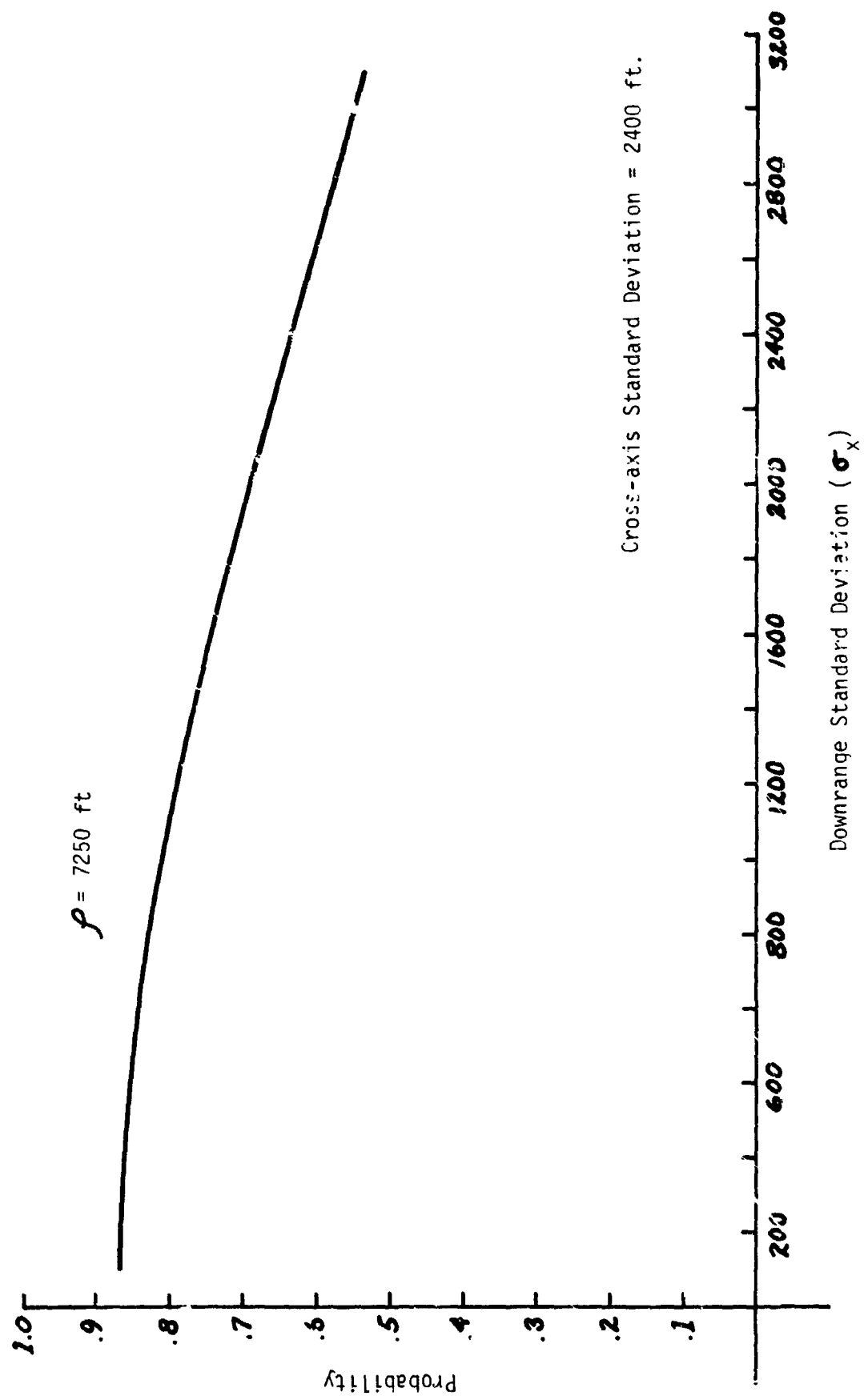


Figure 6. Probability vs Downrange Deviation

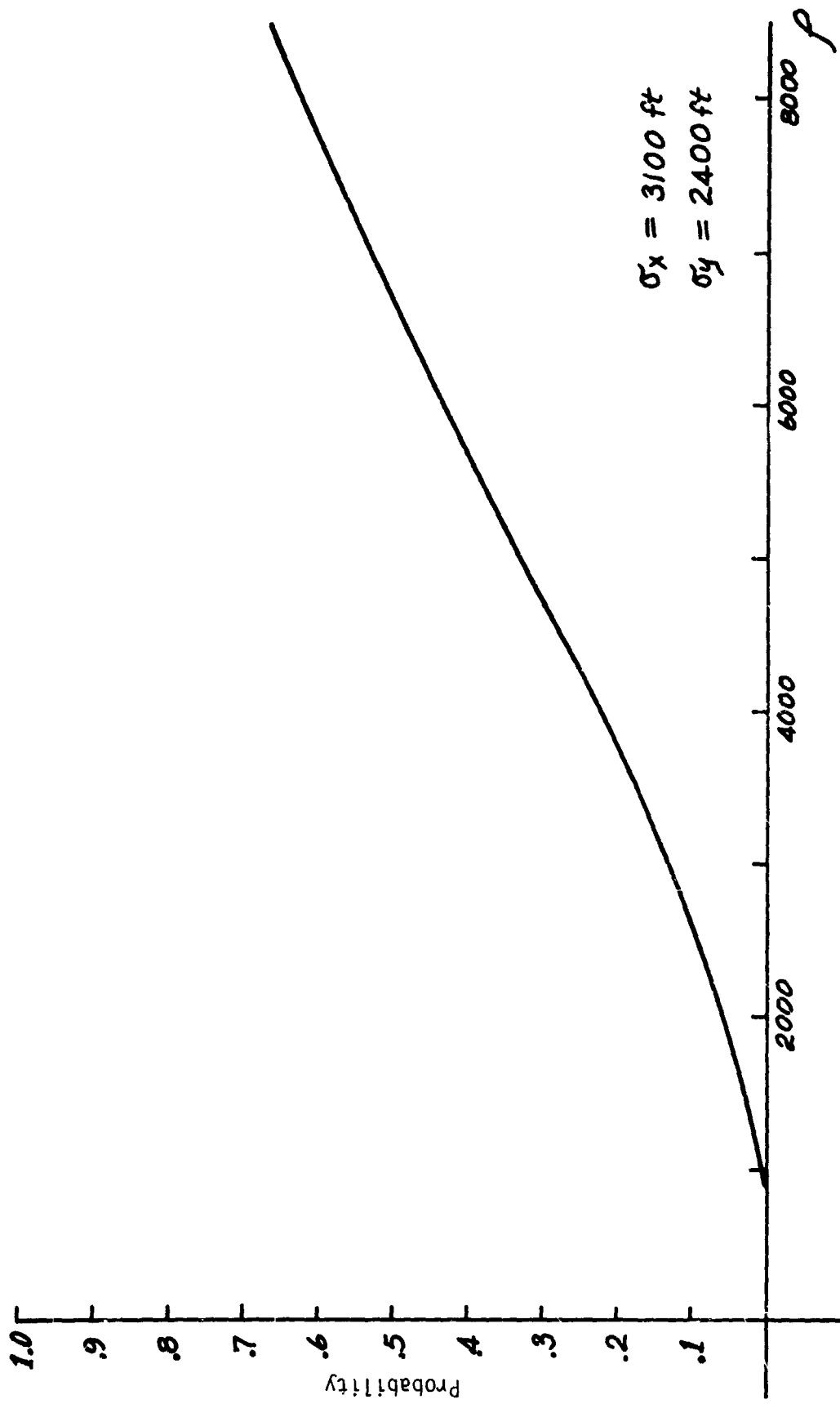


Figure 7. Probability vs Redesignation Footprint Radius



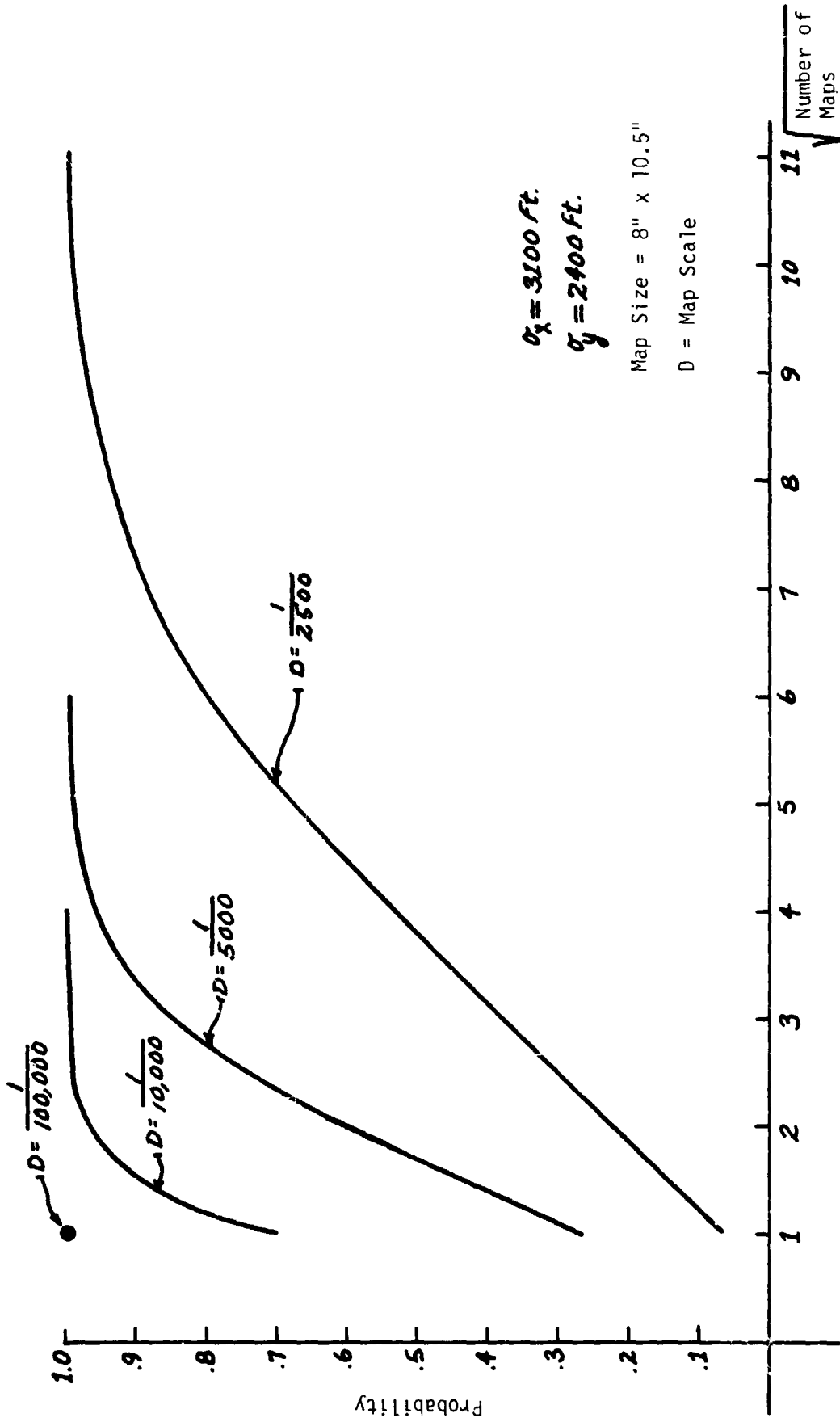


Figure 3. Probability of Landing in Mapped Area without Pedesignation

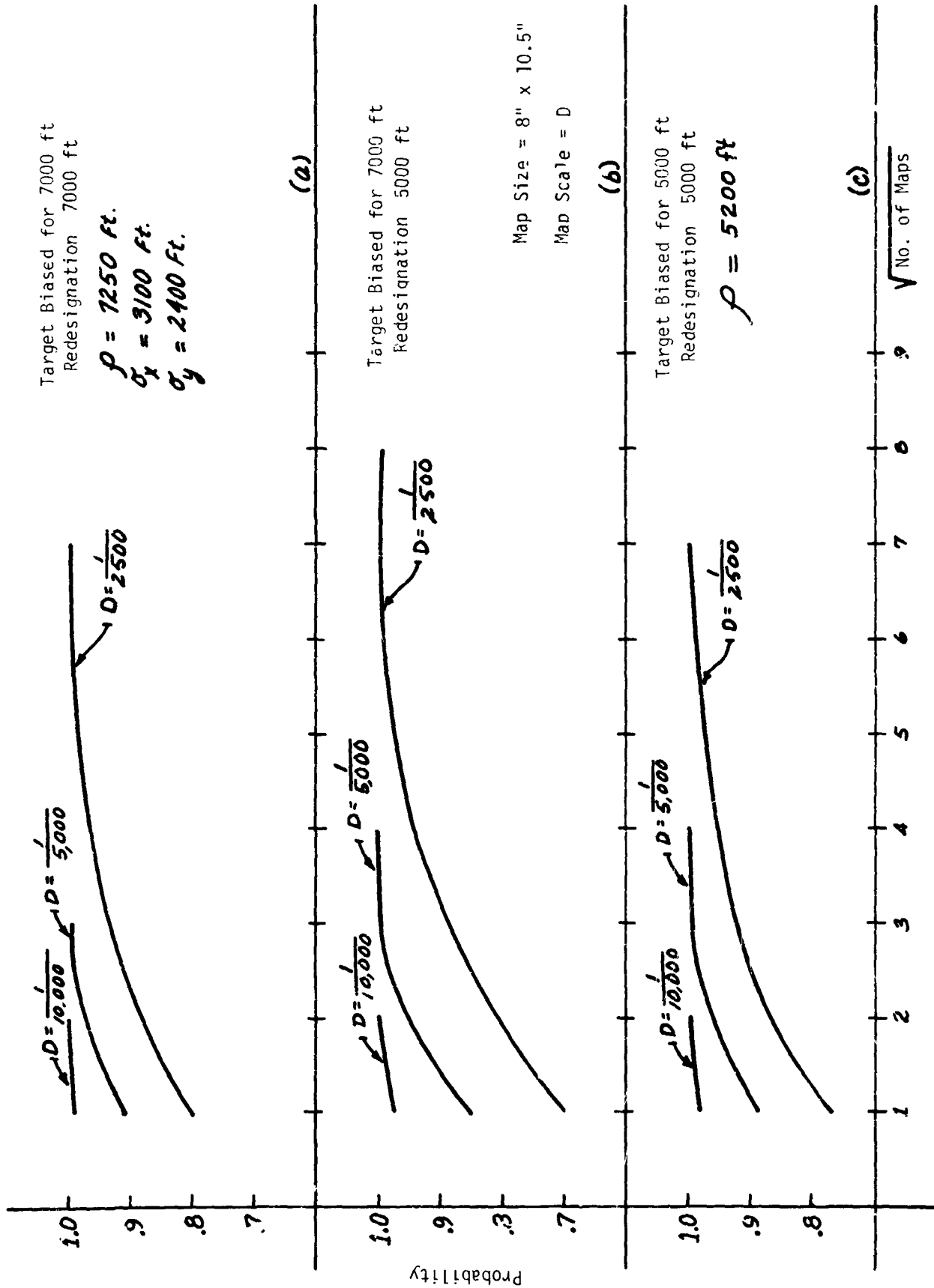


Figure 9. Special Target Area Case