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# APOLLO

## GUIDANCE AND NAVIGATION

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FACILITY FORM 602	<u>265 15369</u>	(THRU)
	<u>38</u>	(CODE)
	<u>08</u>	(CATEGORY)
	(ACCESSION NUMBER)	
	(PAGES)	
	(NASA CR OR TMX OR AD NUMBER)	



# MIT

# INSTRUMENTATION LABORATORY

(E-1539) THERMAL PROPERTIES OF SOME  
STRUCTURAL MEMBERS FOR SPACE-BORNE COMPUTER  
ASSEMBLIES (Massachusetts Inst. of Tech.)  
39 p

N78-70231

Unclas  
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ΟΡΓΑΝΙΣΜΟΣ ΕΡΕΥΝΑΣ ΚΑΙ ΠΡΟΒΛΕΨΗΣ

ΕΡΕΥΝΗΤΙΚΟ ΕΡΓΑΣΤΗΡΙΟ

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# APOLLO

## GUIDANCE AND NAVIGATION

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E-1539  
Thermal Properties of  
Some Structural Members for  
Space-Borne Computer Assemblies  
by  
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March 1964

AVAILABLE TO NASA HEADQUARTERS ONLY

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## ACKNOWLEDGMENT

This report was prepared under DSR Project 55-191, sponsored by the Manned Spacecraft Center of the National Aeronautics and Space Administration through Contract NAS 9-153.

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E-1539

THERMAL PROPERTIES OF  
SOME STRUCTURAL MEMBERS  
FOR SPACE-BORNE COMPUTER ASSEMBLIES

ABSTRACT

Current practice in the design of missile and space-borne computer assemblies uses a chassis-and-module type of construction, wherein a considerable amount of structural metal is used. The amount of metal required in chassis, or tray parts is a function of a number of considerations, one of which is the heat conducting properties of these parts. This report is intended to acquaint computer assembly designers with the thermal conduction properties of some major chassis configurations which have been used, or are applicable to use in computer assemblies.

Idealized models are developed to display the thermal properties of three basic arrangements for cooling computer trays. The first involves one-dimensional heat conduction, and heat removal via straight ribs, or strips contiguous with the tray. The others involve two-dimensional conduction, and heat removal via lugs in one case, and rings, or closed loop rib patterns in the other case. The models are compared, and the possibility of reducing thermal resistance through a dispersal, or subdivision of the cooling effect, is shown. The thermal design of a simple fin, or thermal spreader plate, is also discussed, since this heat transfer element is often either an integral part of a tray design, or else the cooling element to which a tray thermal load is coupled.

The report concludes with an idealized analytical model to show the general effects of tray design on the thermal weight requirements in an assembly involving a typical configuration, and several different heat conduction processes.

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## THERMAL PROPERTIES OF SOME STRUCTURAL MEMBERS FOR SPACE-BORNE COMPUTER ASSEMBLIES

### I. Introduction

Two earlier reports have considered some of the general problems involved in the thermal analysis and thermal design of heat generating circuit structures wherein the most important mode of heat removal is thermal conduction\*. The present study is more specific, considering some particular types of thermal conduction problems which occur in the major metal structural members of typical computer assemblies. As presently designed, computer structures for missile or spacecraft environments contain a considerable amount of metal framing and/or paneling. These metal members provide the long-range structural integrity of assemblies based on the "chassis-and-module" or "tray-and-module" system of circuit building. At the same time, they may perform other important functions, such as serving as a connector-pin frame, enclosing interconnection wiring, or serving as a heat conduction path for cooling the circuit. The amount and kinds of metal used for these purposes are usually derived primarily from considerations of rigidity or of manufacturing practice. As a result, the amount of metal used is usually greatly in excess of that which would theoretically be required if it could be configured primarily to perform the heat transfer function. As a matter of practical fact, however, most of this metal must be configured for other reasons, and the heat transfer function is taken up as a late-phase modification to a nearly fixed design. In this circumstance, one may conceivably be faced with a design which is thermally unsound, or which can be cured of its ills only (if at all) by unwanted increases in metal weight, or by the sacrifice in elegance of some of its non-thermal functions.

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\* See Refs 1 and 2 in Section V. References

This report is intended to acquaint the circuit structure designer with some of the thermal design characteristics of currently-used major structural parts, and thereby encourage the early incorporation of thermal requirements into design thinking. The particular parts to be considered are those called "trays" which function, in effect, as chassis for a number of smaller circuit structures, or "modules", and which also often serve as heat collectors for those modules. The trays are not only the largest single-piece metal members of a typical aerospace computer, but account for an appreciable fraction of the total metal bulk as well. It therefore follows that an appreciation for their thermal design, and the factors which influence it, is central to the general design trade-offs between weight, manufacturability, and structural characteristics of the computer as a whole.

In order to avoid the tedium of presenting the subject of tray thermal design in terms of specific examples, the approach will be generalized, and given in the form of somewhat idealized physical models. The reader, it is believed, will have less difficulty in perceiving the application of these model concepts to specific cases than would be encountered in attempting to generalize from specific examples. The use of this general model technique also makes easier the demonstration of some broad principles of thermal design. Since our primary objective is the presentation of such principles, the mathematics used to derive them will be given only in summary form.

## II. Connector Plates or Tray Models

An idealized model of a typical connector plate, or tray, with its modules in place, is represented in Fig. 1. Heat generated by the operation of circuit components in the modules is conducted through module structure and transferred into the tray. The heat then conducts through the tray, to those portions of it provided with an externally-driven cooling effect. In a space vacuum

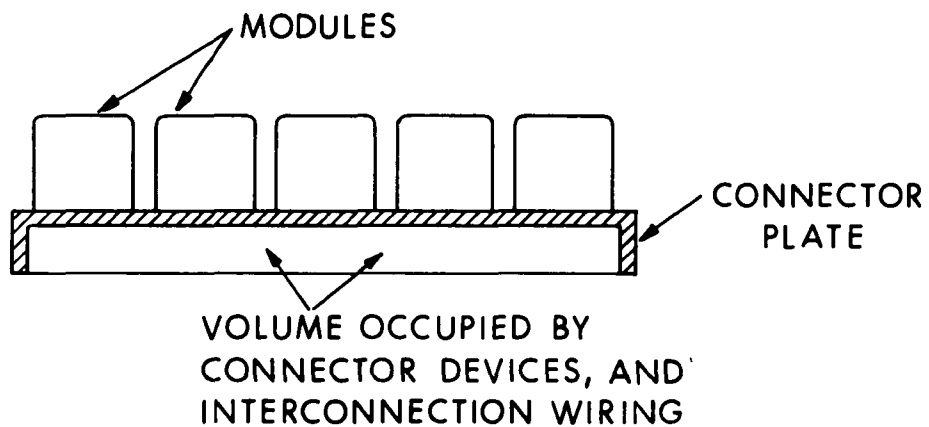
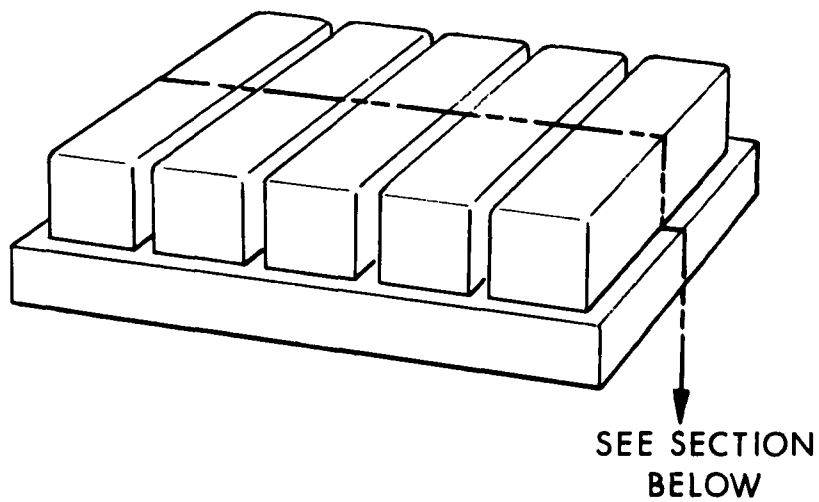
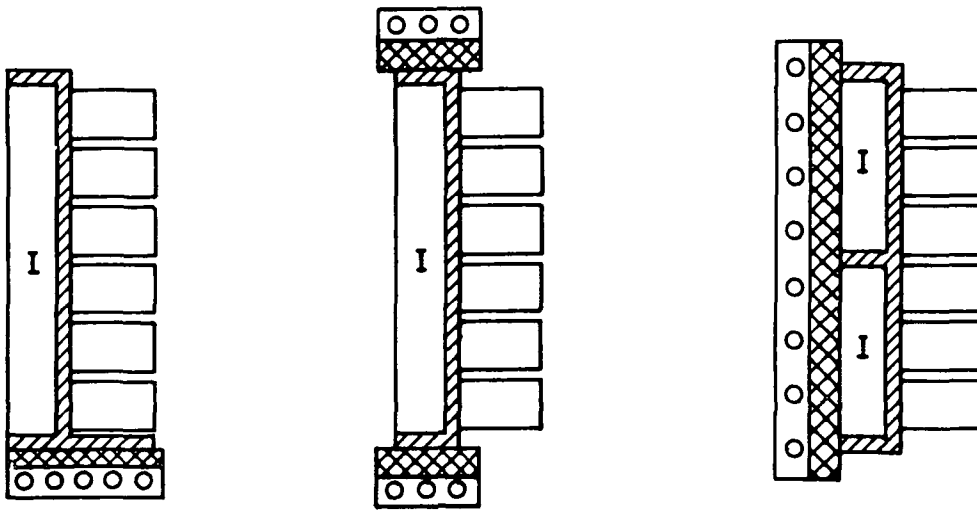
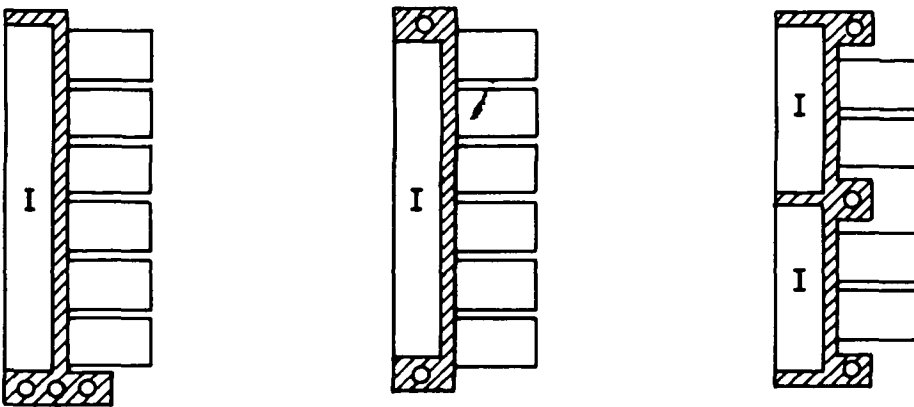


Fig. 1 Schematic representation of a tray and module assembly



(a) USING LIQUID-CIRCULATING PLATES AND THERMAL INTERFACE MATERIALS



(b) USING LIQUID-CIRCULATING INTEGRAL PASSAGES

Fig 2 Schematic sections of a few physical arrangements for tray cooling

environment the normal operating temperatures for electronic equipment demand that nearly all of the heat generated by an assembly be disposed of in this manner. For ambients other than vacuum, where the temperature is less than that of the equipment, only a fraction of the generated heat may be handled in the tray conduction process. The externally-driven cooling effect could assume a variety of forms. Two of the most commonly considered are: 1) a liquid-circulating "cold plate" attached to some portion of the tray surface, such as the bottom edges or ribs, and using some thermal interface material to provide satisfactory thermal contact, and 2) liquid-circulating cooling passages internal to the tray metal, but concentrated, as in the rib structure. Some of many possible physical arrangements of these two cooling effects are illustrated in schematic section in Fig. 2. These schematic drawings are sufficient to illustrate that the tray is a multi-purpose structure. One of its major faces is a placement area for modules, while the other is a placement area for the interconnection wiring associated with those modules. The placement of cooling passages, or coolable surface area anywhere on these major faces serves to encroach upon the usefulness of the tray as a circuit element. Further, the volume used for interconnection wiring is usually filled, at best, with an electrical insulator of low thermal conductivity, so that a cooling effect applied as in Fig. 2 (a), (right end) is relatively ineffectual, except where it contacts the tray's metal ribbing.

Thus, it is apparent that the areas of possible application of cooling effect to a tray are quite limited, making it necessary that the tray be designed to carry thermal loads from their application zones to restricted zones of heat removal. The thermal conduction mechanics involved in such a process are reasonably simple to analyze for some typical geometries of interest, and are presented below.

### Case A. The Strip-Cooled Plate

The first geometry considered is that of a strip-cooled plate with a uniformly distributed thermal load, as illustrated in Fig. 3. The modules might be plugged into such a tray (details not indicated) on the side opposite the ribs, or on the side with the ribs, and between them. In either case, the interconnection wiring would be located on the face opposite the modules. The cooling effect is assumed to be applied to the ribs, either to their bottom edges, or by means of liquid coolant channels drilled through them (not indicated in the figure). In either case, heat which is generated by the modules, and assumed to be uniformly distributed over the tray area, must be conducted by the tray deck to the areas shown single-shaded in the figure. For simplicity and generality, we analyze an area of the tray deck labelled "A" from the upper figure, such that  $2x_0$  is the centerline spacing between ribs. It is assumed that the complete tray may be composed of such elemental units, as in fact it is if  $L_y$  is an integral multiple of  $2x_0$ .

For heat flow toward the rib area in the tray deck, the temperature drop required to sustain heat flow over a length  $dx$  is given by Fourier's conduction law as,

$$d\theta = - p(x) \frac{dx}{kA}, \quad (1)$$

where the local heat flow rate is, (in the negative direction),

$$p(x) = - \int_x^{x_0} 2 x_0 p dx, \quad (2)$$

for a heat load per unit tray surface area of  $p$ . The tray thermal conduction section is  $2x_0 t$  for a tray thickness  $t$ , and the thermal conductivity of the tray metal is denoted by  $k$ . Thus, the total temperature difference between a point located at  $x_0$  and one at

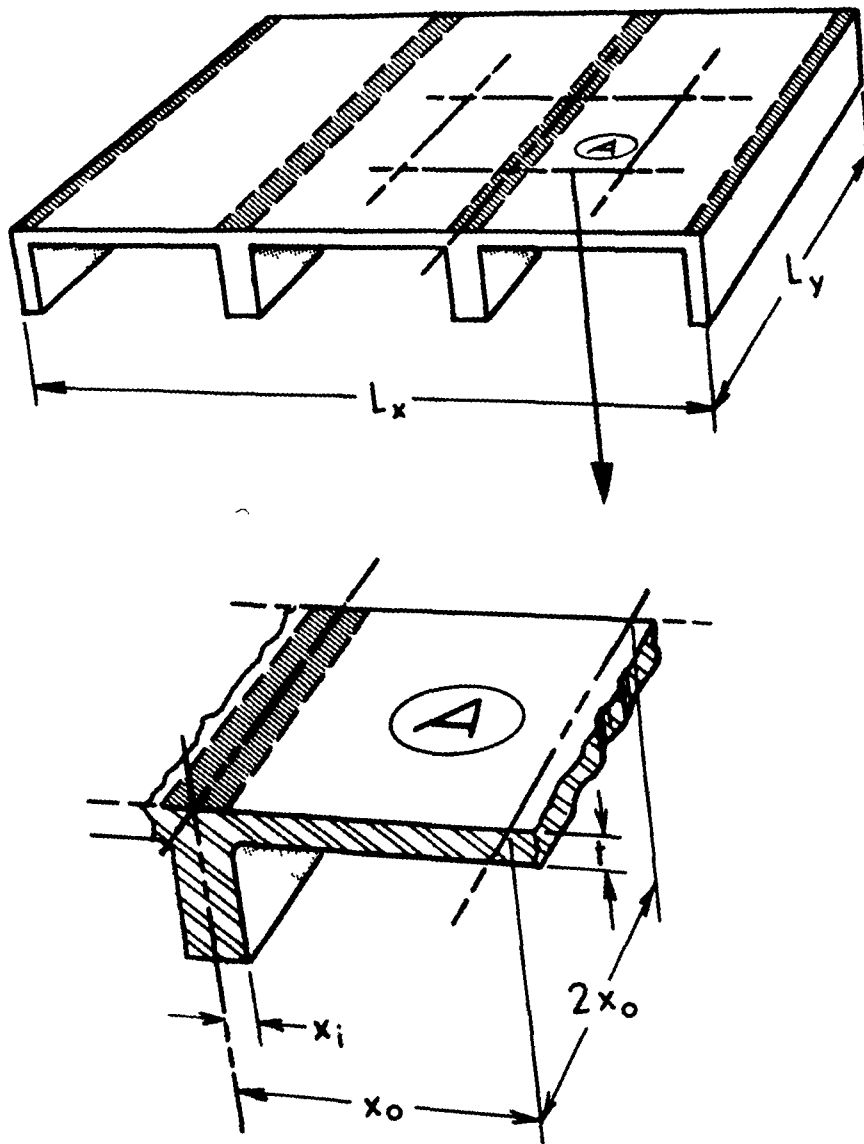


Fig 3 A channel-type tray where in the cooling effect is applied to strips located at the ribs.

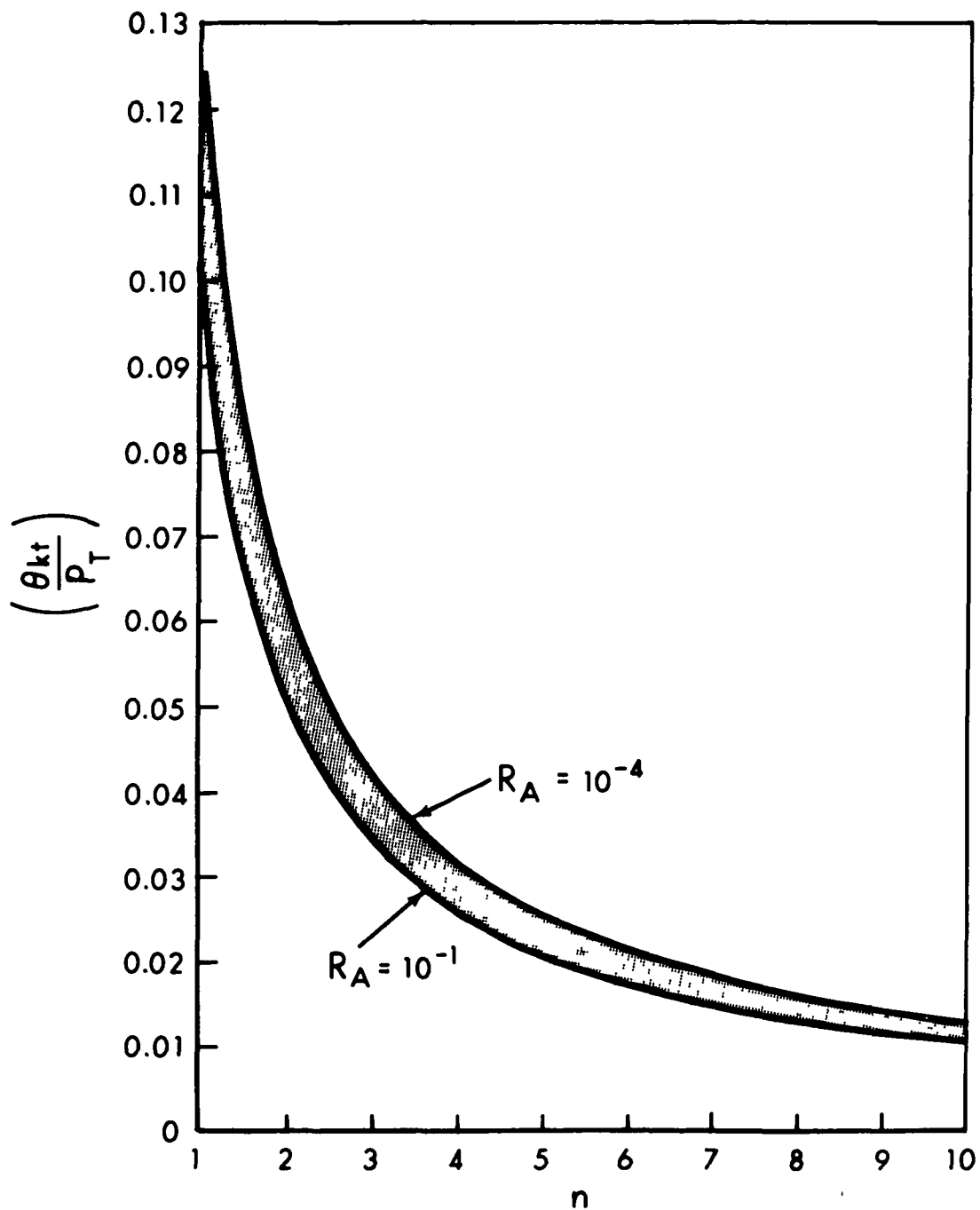


Fig. 4 The dimensionless thermal resistance of a strip-cooled tray as a function of its number of rib-centered fields ( $n$ ) and the rib-to-gross area ratio  $R_{A_i}$



the edge of the rib area is,

$$\theta = \left(\frac{p}{2kt}\right) (x_0 - x_1)^2. \quad (3)$$

Then, since the total power applied to the elemental area is  $2p x_0^2$ , and that applied to an equal area on the other side of the same rib is the same, we define two such paired areas as a "field" and note that the total power in the field, measuring  $2x_0$  on a side, is

$$P_F = 4 p x_0^2. \quad (4)$$

Thus, by substituting in Eq. (3) we obtain,

$$\theta = \frac{P_F}{8kt} \left(\frac{x_0 - x_1}{x_0}\right)^2. \quad (5)$$

If the entire tray is composed of  $n$  such fields, then  $P_F = P_T/n$ , and,

$$\left(\frac{\theta kt}{P_T}\right) = \left(\frac{1}{8n}\right) (1 - R_A)^2, \quad (6)$$

where the ratio of rib-blocked area to total area is  $R_A = (x_1/x_0)$ . Or, Eq. (6) may be expressed in the form

$$\left(\frac{\theta kt}{P_T}\right) = \left(\frac{1}{n}\right) \phi (R_A), \quad (7)$$

showing that the dimensionless form of the thermal spreading resistance ( $\theta kt/P_T$ ) for uniform thermal loading varies inversely with the number of fields, and as a function of the rib-to-gross area ratio. Equation (7) is plotted for some values of interest in Fig. 4, where it may be seen that the most influential variable is the number of fields  $n$ , with the area ratio having very little influence over the range of  $R_A$  of practical interest.

Thus, we may conclude that for a strip-cooled, or ribbed tray, the number of ribs to which cooling can be applied strongly influences its thermal resistance, while the area taken by those ribs can be very small, without loss of effectiveness. If the designer is unwilling to install a multiple-strip cooling effect, he may evaluate the alternative means of achieving the same thermal resistance, which is by thickening the tray deck. From Eq (7), since  $\phi (R_A)$  is essentially constant for the  $R_A$  of interest, we may write,

$$\left(\frac{\theta}{P_T}\right) \approx \left(\frac{K}{tn}\right), \quad (8)$$

where  $K$  is a constant, depending on fixed  $R_A$  and thermal conductivity. Hence, if the tray deck thickness required for  $n = 5$  is 0.125 in, the value required to achieve the same thermal resistance for  $n = 1$  is  $t = 0.625$  in. In evaluating a proposed tray design, the approximate value of  $n$  for a tray with equally spaced ribs is computed from

$$n = \left(\frac{A_{xy}}{4x_o^2}\right) = \left(\frac{L_x L_y}{4x_o^2}\right) \quad (9)$$

where  $A_{xy}$  is the gross area of the tray face, and  $2x_o$  is the rib spacing. Thus, for a tray exactly  $2x_o$  long in the dimension  $L_y$  (see Fig. 3)  $n$  is exactly the number of ribs.\* For trays of other proportions, it is given by Eq (9), and is the number of ribs that a tray of the same area, but with  $L_y = 2x_o$ , would have. The important quantity is, of course,  $2x_o$ , but we choose to express the result in terms of the number of fields  $n$  in order to facilitate comparison of the strip-cooled tray with the other physical cooling arrangements, to be considered next.

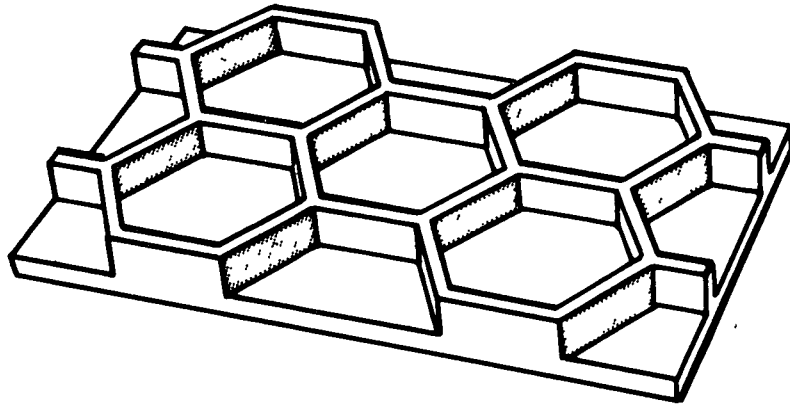
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\*Note that side ribs are considered as half-ribs

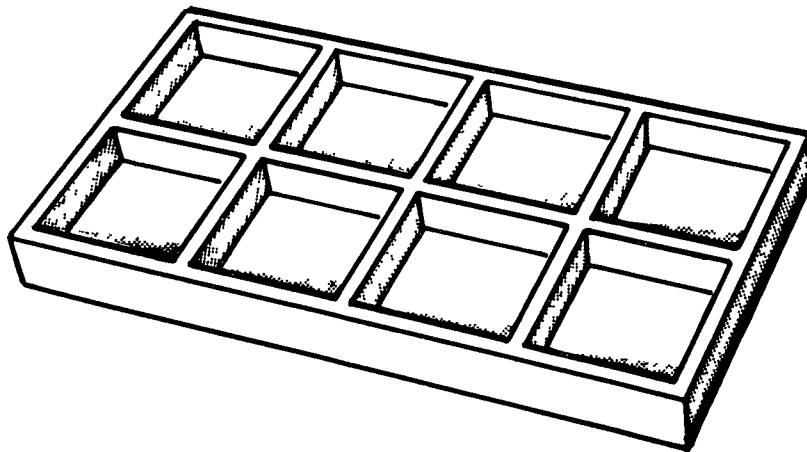
## Case B. The Ring-Cooled Plate

The case of the strip-cooled tray, just considered, involved only one-dimensional heat conduction. We next consider the first of the two-dimensional cases, which is that of the ring-cooled tray, or plate. In the sense to be used here, the term "ring" denotes a closed rib pattern completely encircling a portion of tray deck, or a field. Two of the infinitely many possible ring-type ribbing patterns are shown as examples in Fig. 5 with the rib patterns uppermost. As with the channel type tray of Fig. 3, one might visualize these trays as having modules placed on the deck surface, either on the ribbed, or unribbed face, with interconnection devices occupying the space projecting from the deck face opposite the modules.

With heat loaded uniformly to the tray deck, and cooling effect applied to the rib pattern, such as to the free edges, the heat will flow in the deck in a two-dimensional pattern toward the ribs. The cooling effect is thus applied at the level of the tray deck in a system of closed polygons, or rings. For the sake of mathematical simplicity, we shall confine our analysis to ribbing patterns such as the hexagonal and the square, which can be represented, without large error, by circles of equivalent deck area. Figure 6 shows three possible geometric rib field patterns, together with the size of the corresponding circle of equal area. Clearly, the conduction of heat from the field into a hexagonal boundary would correspond quite closely to that to the circular boundary, while that to the square or triangular boundaries would be successively more approximate in its behavior to that for the corresponding circle. Thus, we consider each field, irrespective of its true shape, to be modeled by a concentric circle of equal area, for the purpose of calculating its approximate thermal resistance.



a. HEXAGONAL OR HONEYCOMB



b. SQUARE OR WAFFLE-IRON

Fig. 5 Examples of possible rib configurations in ring-cooled trays.

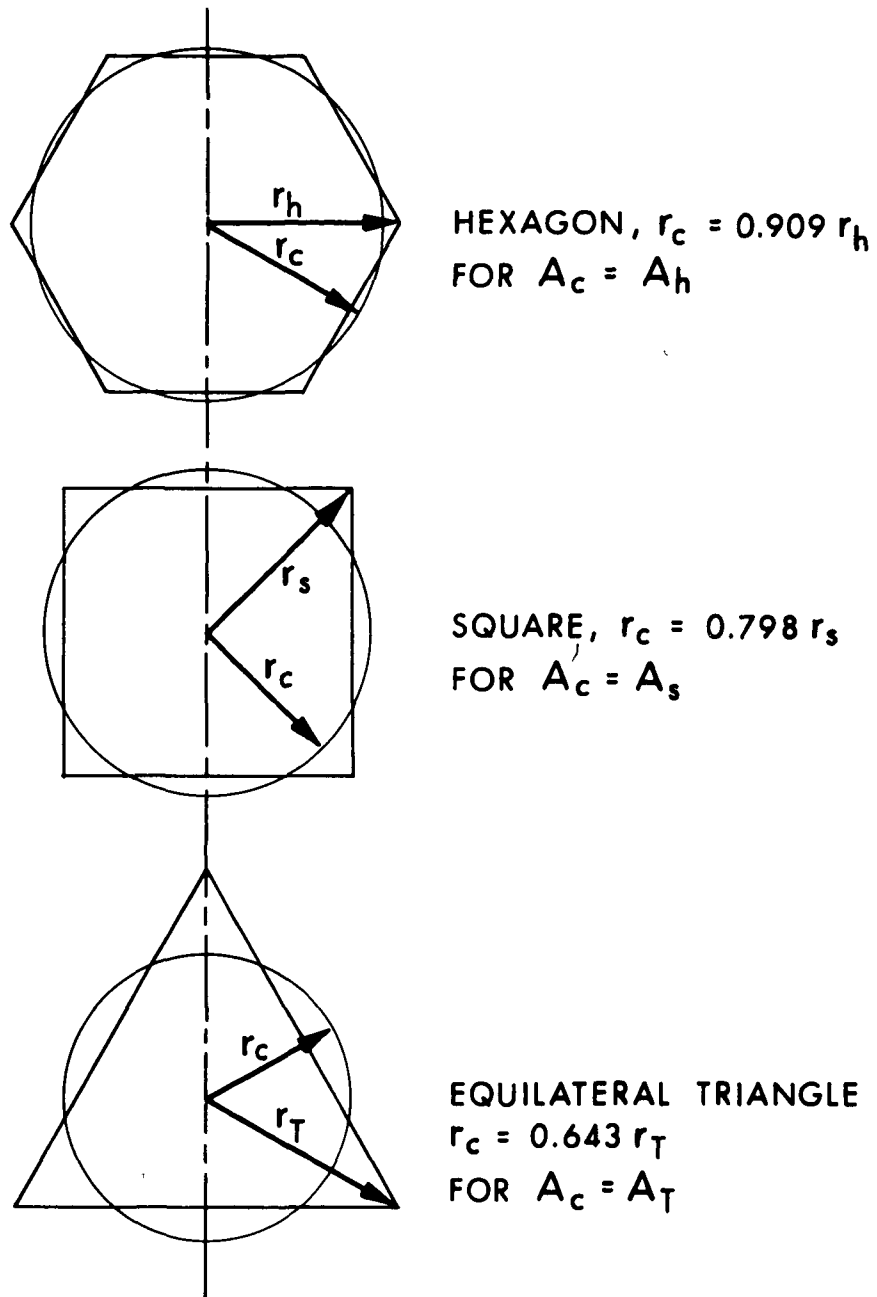


Fig. 6 The correspondence between some possible field patterns and their circular fields of equal area.

The derivation of a thermal resistance is analogous to that used previously, thus,

$$d\theta = -p(r) \frac{dr}{kA}, \quad (10)$$

where

$$p(r) = \int_0^r p \cdot 2\pi r dr = p\pi r^2,$$

and where  $A = 2\pi r t$ . Thus, the total temperature difference between the center of a circular field and the inside edge of its cooling ring is given by,

$$\theta = -\left(\frac{p}{4kt}\right) r_1^2 \quad (11)$$

But, the total field area includes the rib thickness assignable to each, so that, if we assign  $r_o > r_1$ , to include the share of rib area for the field, the field power load is,

$$P_F = p\pi r_o^2. \quad (12)$$

Thus,

$$\theta = -\frac{P_F}{4\pi kt} \left(\frac{r_1}{r_o}\right)^2 \quad (13)$$

The ratio of rib-blocked to total deck area for this geometry is,

$$R_A = \frac{\pi(r_o^2 - r_1^2)}{\pi r_o^2}. \quad (14)$$

Therefore, since  $P_T = n P_F$ , substituting and rearranging of Eq (13) gives,

$$\left(\frac{\theta kt}{P_T}\right) = -\left(\frac{1}{n}\right) \left(\frac{1 - R_A}{4\pi}\right), \quad (15)$$

which is of a form similar to Eq (7). The negative sign has arisen simply because the heat flow is now in the same direction as the direction of increase in the length coordinate,  $r$ , whereas for the strip-cooled tray, the opposite was the case. The signs may be ignored if we simply remember that  $\theta$  is a temperature difference between the rib-cooled area and the point in the field most remote from that area, the latter always being the point of higher temperature.

The dimensionless thermal resistance given by Eq (15) is plotted in Fig. 7 as a function of the number of fields  $n$  for a total power loading, uniformly distributed, of  $P_T$ , and as a function of  $R_A$ . The result is quite similar to that shown earlier in Fig. 4, except that the two-dimensional heat flow of the ring-cooled tray results in a lower value of resistance.

#### Case C. The Spot-Cooled Plate

The final specific geometry to be considered for tray deck cooling arrangements is that of the spot-cooled tray with a uniformly-distributed thermal load. This situation arises in practice when a tray deck is fitted with an occasional or regular array of bosses or posts which communicate thermally and structurally between the tray deck and a cooler surface. We model this situation analytically by a circular field, concentric with the post or boss, as illustrated in Fig. 8. The circular field has the same area as fields of a more realistic geometry which would be concentric with their cooling posts. Two such tray designs are indicated in Fig. 9. It may be noted that these trays are, in a sense, the "negatives" of those shown in Fig. 5, in that the direction of heat flow is centripetal in Fig 9, but centrifugal in Fig. 5. Proceeding as before,

$$d\theta = - p (r) \frac{dr}{kA} , \quad (16)$$

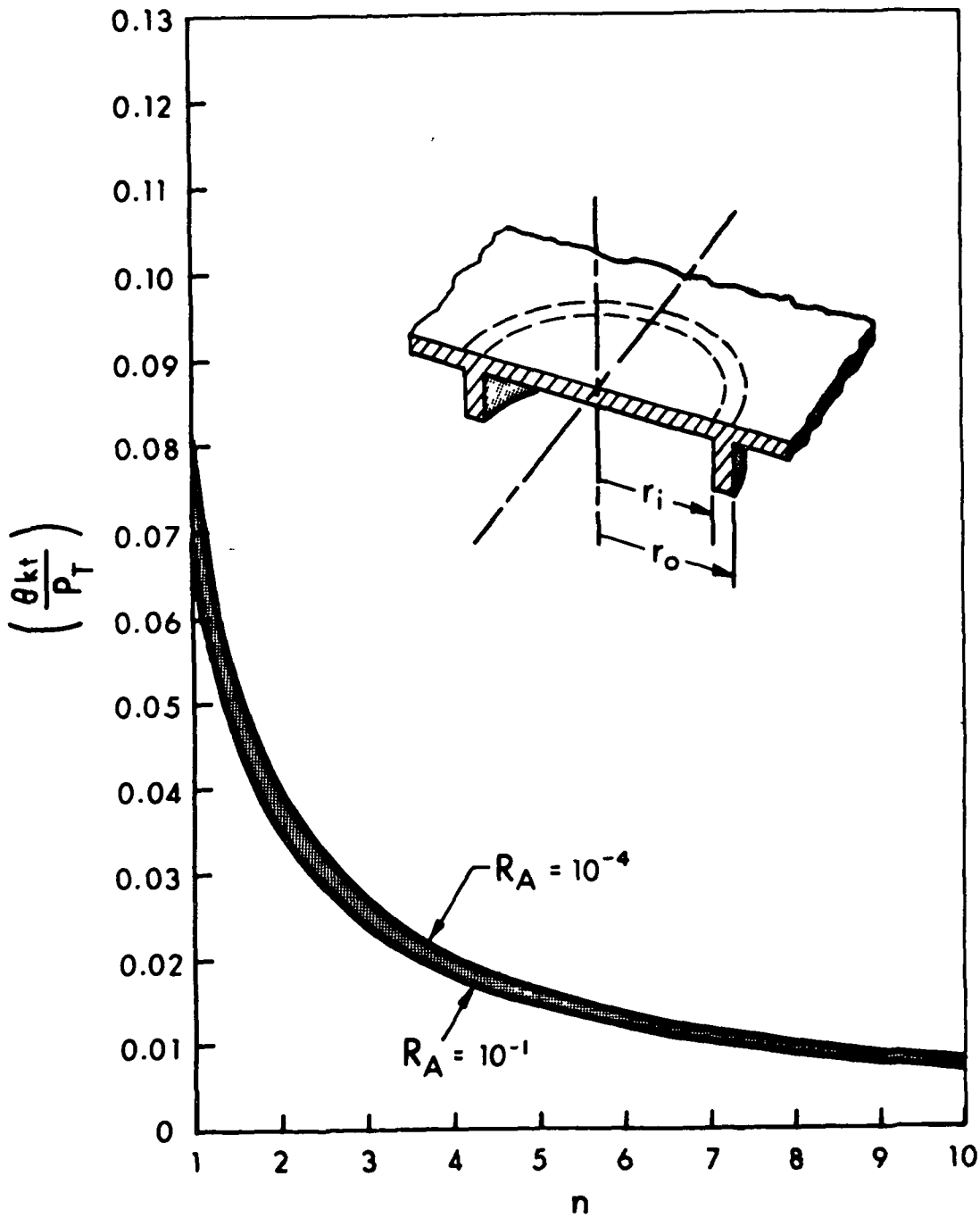


Fig. 7 The dimensionless thermal resistance of a ring-cooled tray, as a function of its number of fields (modeled as equivalent area circles)  $n$ , and the rib-to-gross area ratio  $R_A$ .



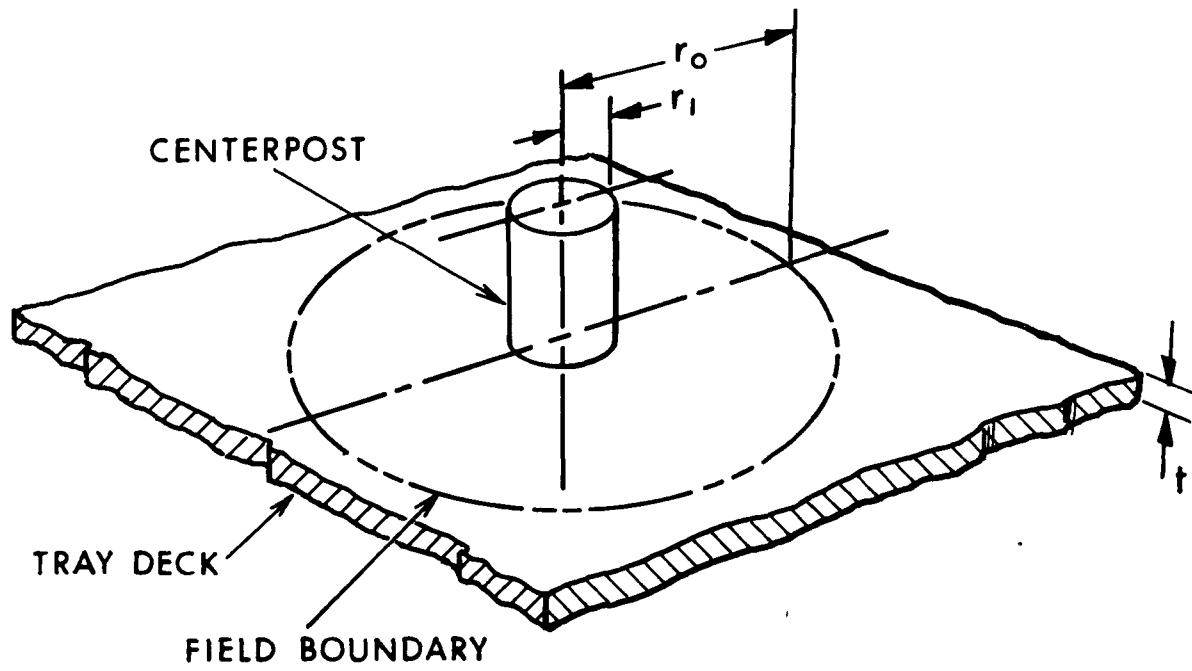


Fig 8 The circular model of one field in a spot-cooled tray in which the field, encircled by the section-dashed line, is cooled by conduction to the centerpost

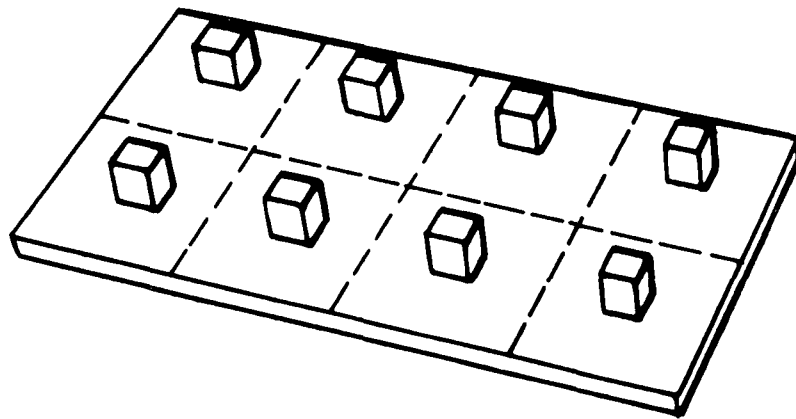
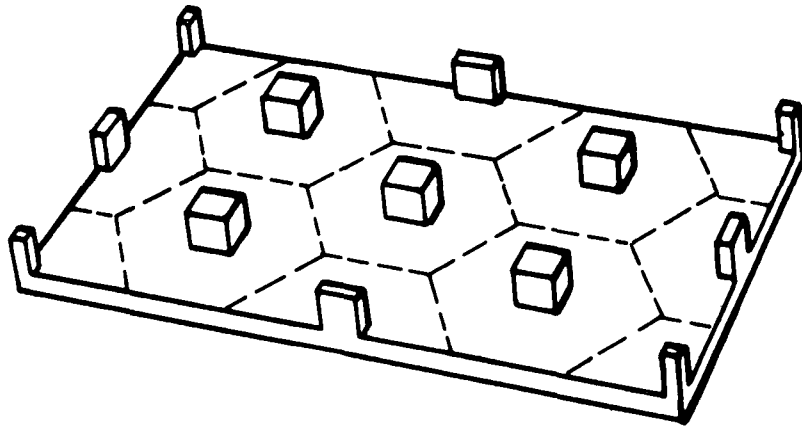


Fig. 9 Examples of possible post-placement arrangements in spot-cooled trays, with the corresponding field boundaries indicated by the dashed lines.

where

$$p(r) = - \int_r^{r_o} p 2\pi r dr,$$

(since the direction of heat flow is opposite to positive  $dr$ ) and where  $A = 2\pi r t$ . Whence, the maximum temperature difference in the field is given by

$$\theta = \left( \frac{pr_1^2}{2kt} \right) \left[ \left( \frac{r_o^2}{r_1^2} \right) \ln \left( \frac{r_o}{r_1} \right) - \frac{1}{2} \left( \frac{r_o^2}{r_1^2} - 1 \right) \right] \quad (17)$$

But, since  $P_F = p\pi r_o^2$ , and  $R_A = (r_1^2/r_o^2)$ , and  $P_T = n P_F$ , we may substitute and rearrange to,

$$\left( \frac{\theta kt}{P_T} \right) = \left( \frac{1}{n} \right) \left( \frac{1}{4\pi} \right) \left[ \ln \left( \frac{1}{R_A} \right) + R_A - 1 \right], \quad (18)$$

which is of a form similar to Eq (7). A plot of (18) for some values of  $n$  and  $R_A$  of practical interest is given in Fig. 10, where it is apparent that the value of  $R_A$  has a much stronger influence on the thermal resistance of a spot-cooled tray than was the case for the strip-cooled or the ring-cooled structure. In addition, the thermal resistance of the spot-cooled arrangement is somewhat larger than those of the other arrangements, since the band of values from Fig. 4 (strip-cooled) straddles the line for  $R_A = 10^{-1}$  in Fig. 10, while the data for the ring-cooled tray are even smaller in magnitude.

Thus, a comparison of three basic arrangements for applying partial area cooling effect to a tray indicates that the resulting thermal resistance in a tray deck subject to uniform thermal load increases in the order of the ring-cooled, strip-cooled, and spot-cooled arrangements, where comparable areas of cooling effect are applied. In addition, each arrangement

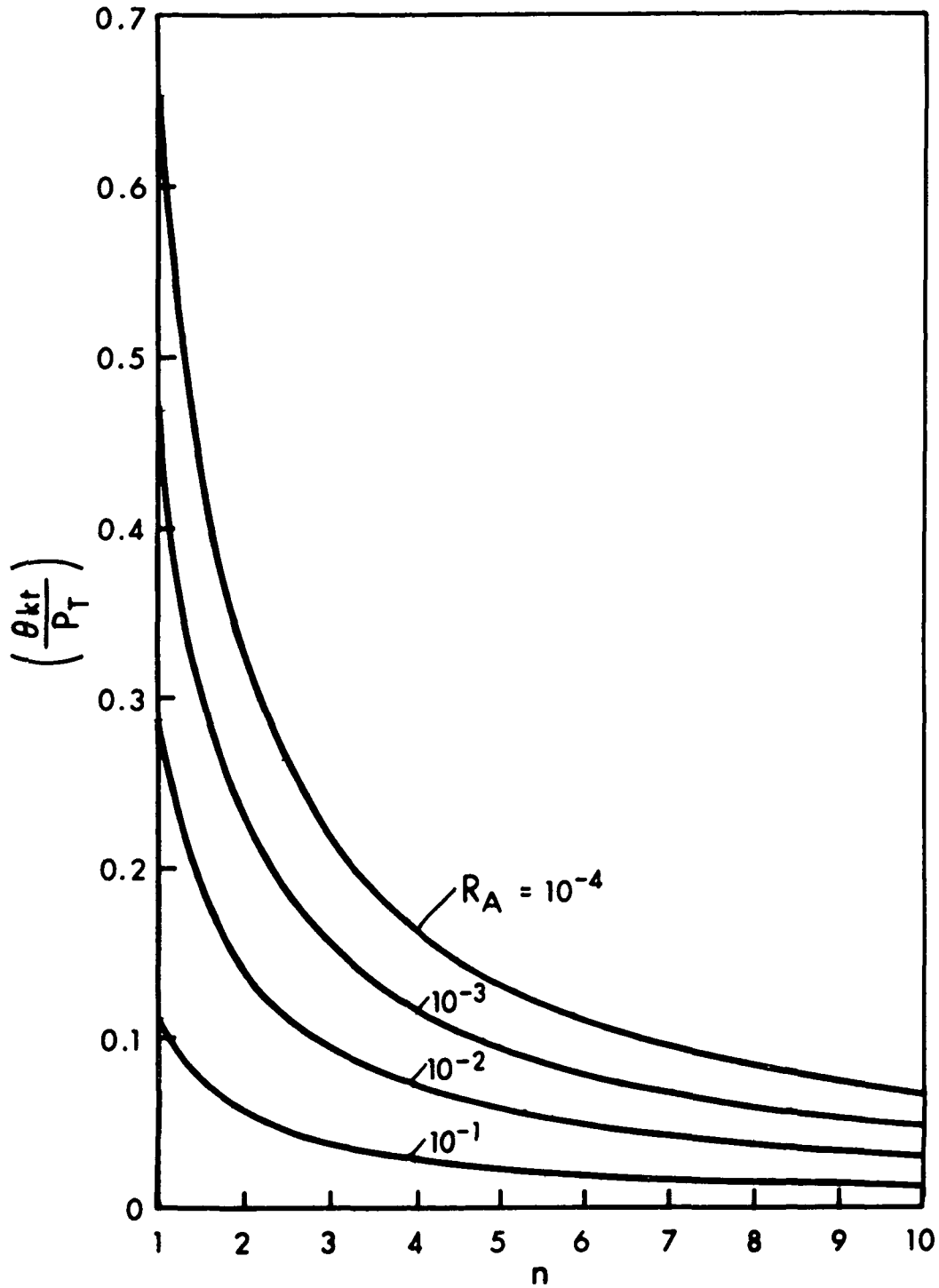


Fig. 10 The dimensionless thermal resistance of a spot-cooled tray, as a function of the number of its spot-centered fields,  $n$ , and the spot-to-gross area ratio  $R_A$ .

results in a thermal resistance which is an inverse function of the number of times the gross tray area is uniformly subdivided into smaller, but geometrically similar units, or fields. Unfortunately, the designs which minimize the thermal resistance (as with ring-cooling and a large  $n$  value) are precisely those which most limit the usefulness of a tray from the viewpoint of other functions. Consider, for example, the difficulty of an automated wire-wrapping operation on a tray of ring-cooled design as compared to one of spot-cooled design, assuming that the interconnection is committed to the ribbed side of the tray. The rapid approximate evaluation of the thermal properties of some design alternatives may be done with the aid of Figures 4, 7, and 10.

It is appropriate to remark on the establishment of the value of  $R_A$ . For a gross tray area of  $A_T$ , we define

$$R_A = \left( \frac{A_C}{A_T} \right) \quad (19)$$

where  $A_C$  is the area devoted to ribs, posts, or ring structures. This area is the thermal cross-section available to conduct the heat collected by the tray to the cooling medium, whether it be externally applied, as to the post ends, or be internal, as through coolant passages in straight ribs. If the distance from the centerline of the tray deck to the cooling effect is  $L_Z$ , then the temperature drop required to sustain this conduction process is given by,

$$\theta_Z = \left( \frac{P_T L_Z}{k A_C} \right) = \left( \frac{P_T L_Z}{k R_A A_T} \right) \quad (20)$$

So that,

$$R_A = \left( \frac{P_T L_Z}{k \theta_Z A_T} \right) \quad (21)$$

Whence, a designer may determine what value of  $R_A$  is required from a thermal viewpoint, if he can formulate an estimate of how much temperature difference is tolerable for the rib conduction process. We would ordinarily expect that structural and fabrication requirements would require a larger value of  $R_A$  than that found from Eq (21).

### III. A Spreader Plate, or Fin Model

It does not always happen that the conduction processes following heat removal from the tray deck are those of simple conduction through a rib or post. This fact is obvious from Fig. 2, where some more complicated situations are indicated. Whereas the tray deck itself could be treated as a strip-cooled problem, the cooling effect is not always applied to the lower edges, or by internal passages to a rib. Situations such as the first two in Fig. 2 (a) are examples of the use of "spreaders", or "fins", which are required whenever the conductance between cooled surfaces and the external heat removal medium is low, as can occur when using some elastic or quasi-elastic types of thermal interface materials. The low conductance requires a large contact area to transfer the heat at an acceptable temperature differential. Since the large area cannot usually be provided by thickening ribs to give broad edges, it may become necessary to cool from the side of a rib (as in Fig 2 (a) center), or to add ribbing, or fins (as in 2 (a) left) whose function is solely that of heat transfer, and whose location does not interfere with other functions of the structure. Of the many possible combinations of geometry and cooling effect, we select one for analysis which will display some of the general thermal design characteristics of fins, or spreaders.

A simple thermal fin has a function opposite to that of the tray deck. Whereas the tray deck accumulates heat loading from the sources, or modules, and conveys it to some local disposal area, the fin receives it from a local area and conveys it, while

unloading, to an external cooling effect. Thus, the relationships developed above might be used to design fins, with the temperature differentials reversed, except that the loading conditions are not appropriate. Whereas the uniform thermal loading of a tray deck is a reasonable (if not always realized) approximation to consider, the uniform unloading of a fin into an external cooling effect is much less likely. There are many unloading conditions which could be realized, but for simplicity we shall consider only one, which is that of heat removal from the fin through an interface of constant conductance, into an isothermal sink. For added simplicity in the analysis, we consider only a fin of constant thickness, with one-dimensional heat flow.

A section of the fin of length  $L$  and width  $w$  is illustrated in Fig. 11. Heat flows from the rib into the fin, and thence along the fin length  $L$ , while simultaneously draining to the sink via the thermal interface material. An actual configuration could either involve a thermal interface material and an isothermal surface, as are shown, or could involve a flowing liquid coolant which contacts the under face of the fin. In the latter instance the thermal interface is simulated by the fluid film at the fin boundary layer, and the isothermal surface is simulated by the average free stream condition of the fluid. The one-dimensional thermal mechanics are described by the well-known relationship,

$$\left(\frac{d^2\theta}{dx^2}\right) - \left(\frac{C}{kt}\right)\theta = 0, \quad (22)$$

wherein  $\theta$  denotes the temperature difference between the fin and the isothermal surface,  $C$  denotes the interfacial thermal conductance, and  $k$  denotes the thermal conductivity of the fin material. Solving with appropriate boundary conditions, and applying the fact that

$$P = \int_0^L Cw\theta dx \quad (23)$$

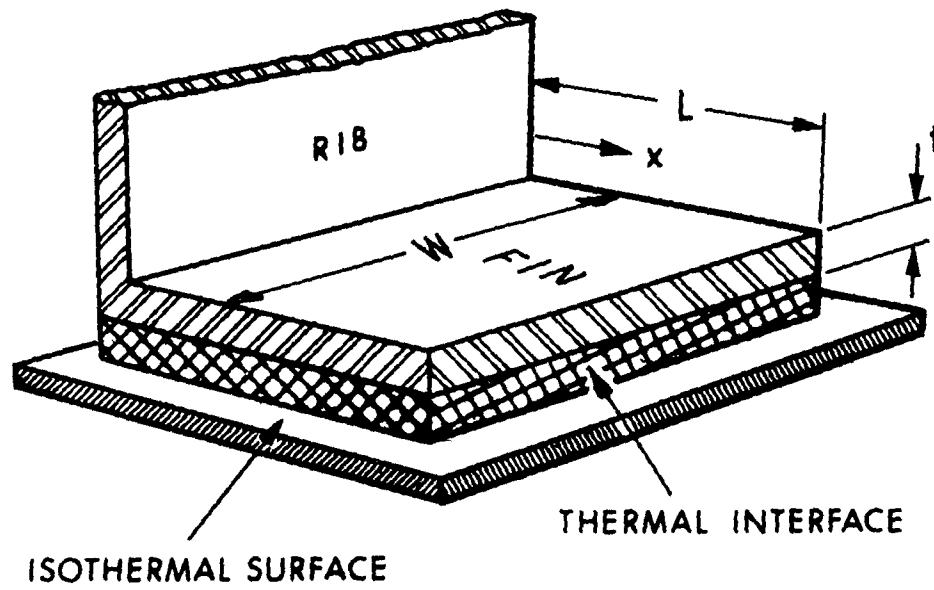


Fig. 11 Model of the single straight fin, for unloading heat through its lower surface into an isothermal sink, via the thermal interface



is the heat dissipated by a width  $w$  of the fin, one may obtain

$$\left(\frac{P}{Cw\theta_{\max}}\right) = \left(\frac{1}{a}\right) \tanh (aL) \quad (24)$$

where  $\theta_{\max}$  is the temperature difference at the root section of the fin, and where

$$a = (C/kt)^{1/2}.$$

Therefore, since the function  $\tanh (aL)$  has a maximum possible value of one, we may write

$$P \leq \sqrt{Ckt} w \theta \quad (25)$$

This relationship may be used in the preliminary design of fin arrangements to determine the lower limiting thermal resistance achievable as,

$$\left(\frac{\theta}{P}\right) \geq \frac{1}{w \sqrt{Ckt}} \quad (26)$$

Thus, for a known value of  $C$  and  $k$ , and having selected a fin thickness  $t$  and width  $w$ , no fin length  $L$ , however great, can result in a thermal resistance less than given by Eq (26). If the value obtained is too large to be acceptable, this relationship indicates the efficacy of changing the various design features to obtain a lower value. When a design has proceeded to a more detailed stage, one may revert to the use of Eq (24) for more exact evaluation of a fin thermal resistance. In this connection, it is useful to introduce the concept of fin effectiveness. We note that if a fin were of a material having infinite thermal conductivity, it would be able to conduct heat without a drop in temperature along its length, so that the heat disposed of to the interface would be,

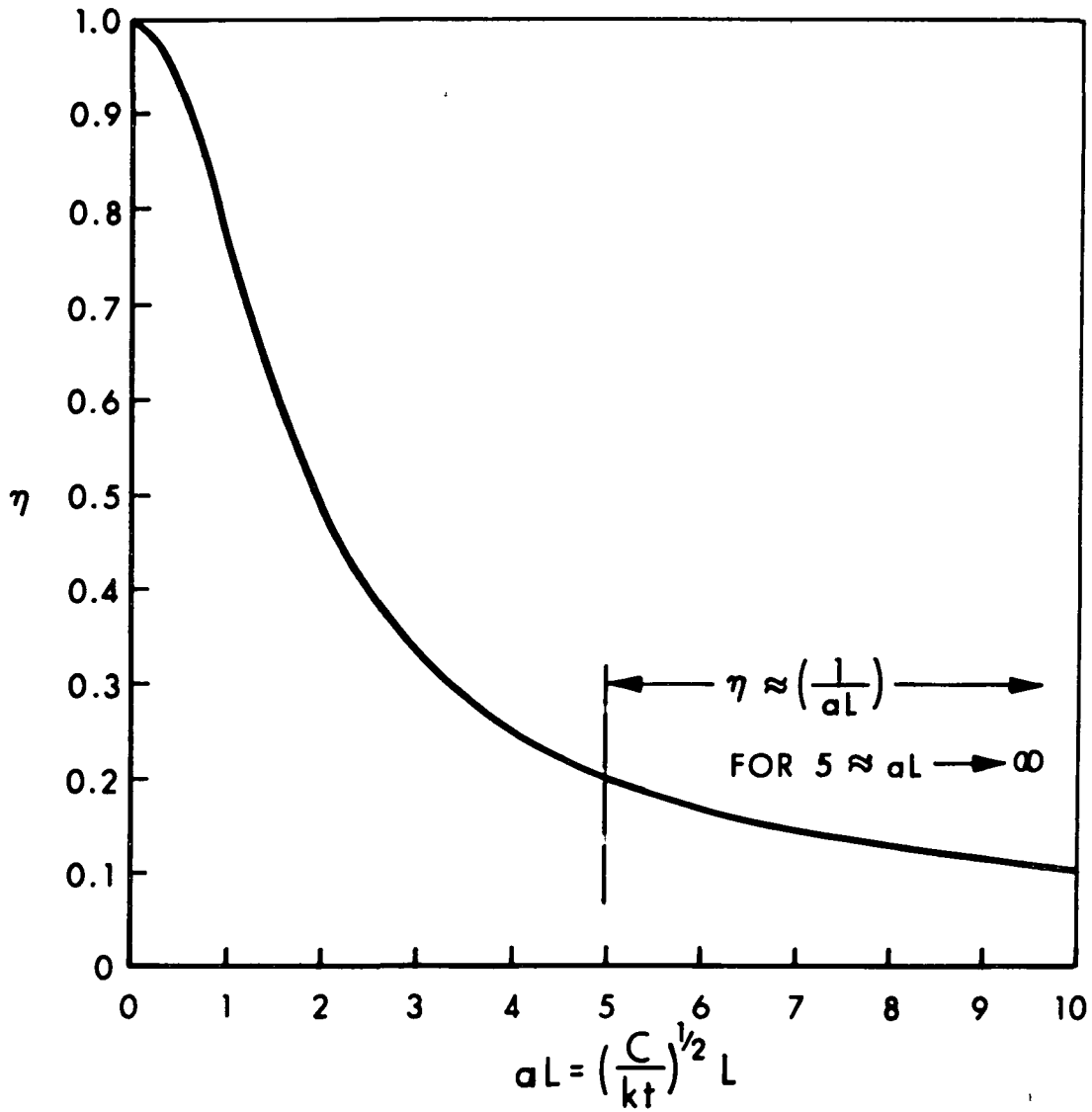


Fig. 12 Plot of the effectiveness of a straight fin as a function of its design parameters.

$$P_{\infty} = Cw \theta_{\max} L. \quad (27)$$

Using Eq (24), we find ,

$$\eta = \frac{P}{P_{\infty}} = \left(\frac{1}{aL}\right) \tanh (aL) \quad (28)$$

where  $\eta$  is the fin effectiveness, and where the performance of a real fin is given by

$$P = \eta P_{\infty} = \eta Cw \theta_{\max} L. \quad (29)$$

Figure 12 is a plot of Eq (28) which may be used together with Eq (29) to evaluate the effect of alterations to the design values of  $k$ ,  $t$ , and  $L$  on the fin performance.

#### IV. Conclusion

The models presented to describe the thermal mechanics of some major computer structural members have been somewhat idealized and simplified, in order to enhance their generality, and to lessen the tedium of the mathematics involved. They are, nevertheless, specific to certain classes of structural geometry, and consider only the spreading resistances associated with chassis parts. The non-specialist may have difficulty in developing an appreciation, in proper perspective, of the influence of these part designs on the general thermal design problem in computers. We therefore conclude with a model development which is aimed at displaying some more general information about the thermal design of structural arrangements wherein several heat conduction processes are involved.

Consider a sectional portion of a tray and cold plate assembly, as indicated in Fig. 13. Heat generation is assumed to take place above the surface EAC, being loaded to that surface,

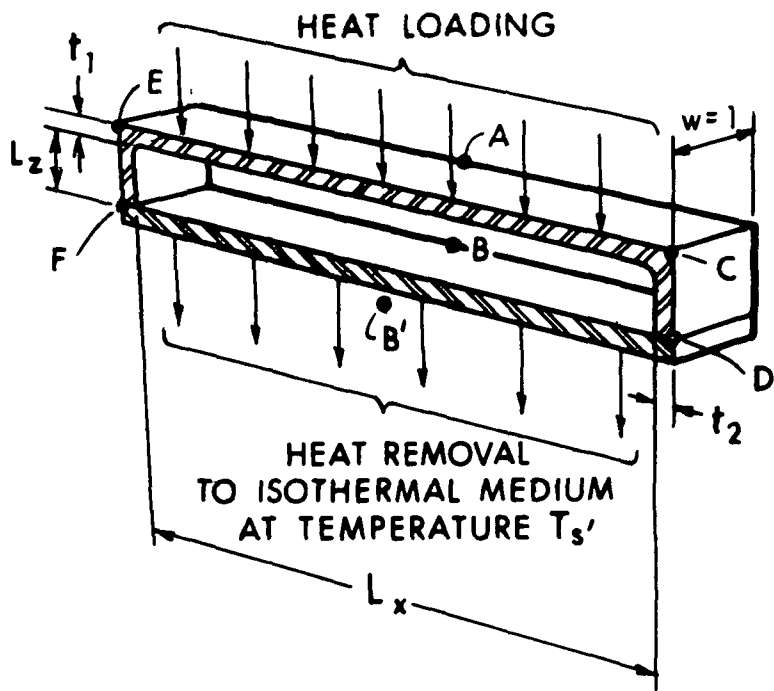


Fig. 13 Model configuration for discussing the thermal weight requirements of a design.

and heat removal is assumed from the surface FBD. The heat transfer processes involved are 1) accumulation and conduction in the tray deck EAC, 2) conduction down the members EF and CD, 3) conduction through a separable interface above points F and D, and 4) conduction in and disposal from the plate FBD (here assumed to be non-isothermal, for generality). Although this pattern of heat flow is complex, it is assumed that its basic character cannot be changed, without disruption of other functions of the structure. Assuming that the structure must operate to thermal satisfaction on a fixed total temperature difference  $T_A - T_B = \theta_T$ , we shall investigate the over-all effect on the thermal weight requirement of the structure of rearranging the members EF and CD, without changing the enclosed, useful volume CDEF, except by subdividing it with vertical planes. We shall then indicate briefly the effect on the temperature difference if the weight were not changed.

Let the total heat loading to a strip of the tray deck of unit width ( $w = 1$ ) be  $P_s$ . Then, for a symmetrical loading, half the heat would flow each way from the center. Let this heat loading be distributed in some manner as a function of the  $x$  coordinate, then the total temperature difference in the tray deck between points A and C would be, at most,

$$\theta_{1T} = \frac{\left(\frac{P_s}{2}\right) \left(\frac{Lx}{2}\right)}{kA}, \quad (30)$$

where  $A = wt = 1 \cdot t$ . This maximum occurs only for heat load concentration near A, and for any other possible distribution, we may write,

$$\theta_{1T} = f_{px} \left(\frac{P_s Lx}{4 k A}\right), \quad (31)$$

where  $f_{px} < 1$ , and has a unique value for any unique distribution of the load over the length ( $L_x/2$ ).

If  $L_x$  be subdivided nearly uniformly, and the load is nearly uniformly distributed, then within each subdivision the load is,

$$P \leq P_s \approx \left(\frac{P_s}{n}\right), \quad (32)$$

where  $n$  is the number of subdivisions, and the length ( $x$ -direction) of these is

$$x_d < L_x \approx \left(\frac{L_x}{n}\right). \quad (33)$$

So that,

$$\theta_{1T} = f_{px} \left(\frac{P_s L_x}{4kt_1}\right) \left(\frac{1}{n^2}\right). \quad (34)$$

But, the total quantity of material required in the strip of tray deck is  $Q = L_x t_1 \cdot 1$ , so that,

$$Q_1 = f_{px} \left(\frac{P_s x_m^2}{4k \theta_{1T}}\right), \quad (35)$$

where  $x_m = (L_x/n)$ . Thus, the quantity of material for a particular thermal design can be represented by

$$Q_1 = K_1 x_m^2, \quad (36)$$

where  $x_m$  is the modular spacing of cooling members, or subdividers, which cool the deck. For a uniform spacing,  $x_m$  would be that spacing. For a non-uniform spacing, it would be necessary to base the design on the largest value.

For heat conduction down the cooling legs, it is obvious that the two shown in Fig. 13 give rise to a temperature drop of,

$$\theta_{2T} = \left( \frac{P_s L_z}{2kt_2} \right). \quad (37)$$

Since  $L_z$  is constant, in order to carry this same total heat load at fixed  $\theta_{2T}$  in a larger number of legs in parallel, the same total amount of material would have to be used, even though distributed over a larger number of members of thickness  $t_{2n} < t_2$ . Therefore,

$$Q_2 = K_2 \text{ (constant)} \quad (38)$$

Further, it is clear that the process of conduction through an interface joint, as at D and F, if redistributed among a number of smaller joints of the same total area, does not affect the material used in any part.

Finally, we assume that the lower plate is a fin, unloading heat through a constant conductance into an isothermal medium. By subdividing  $L_x$ , we shorten the fin length, and hence its area, while reducing the loading by the same amount. The fin loading per unit area is therefore reduced, but by Fig. 12, the fin efficiency is raised. Therefore, we could decrease the fin thickness, to restore the original value of  $\theta_{3T}$ , and therefore save material. Hence,

$$Q_3 = \phi_3(x_m) \quad (39)$$

where  $\phi_3$  is a function which is monotonic increasing in  $x_m$ .

The total structural material in the system is therefore described by,

$$Q_T = Q_1 + Q_2 + Q_3 = K_1 x_m^2 + K_2 + \phi_3(x_m). \quad (40)$$

Hence, based on a thermal design requirement, the structure of this system can invariably be reduced in weight if the degree of subdivision is increased. Obviously, in a system where most of the structural weight is tray deck, or heat accumulator plate, the system weight variation could approach variation as the square of the modular dimension used in subdivision of the cooling effect to that accumulator.

Since the redistribution of the cooling legs to a smaller modular spacing has made possible the reduction of the structural weight at the same value of  $\theta_T$ , it follows that an alternative would be the use of the original weight of structure, with a decrease in  $\theta_T$ . By analogy to Eq (40) the temperature difference can be expressed as,

$$\theta_T = \theta_{1T} + \theta_{2T} + \theta_{2T'} + \theta_{3T}, \quad (41)$$

where  $\theta_{2T'}$  represents the temperature difference required to drive heat through the separable joints. Therefore,

$$\theta_T = K_1 x_m^2 + (K_2 + K_{2'}) + \psi_3(x_m) \quad (42)$$

where  $\psi_3$  is a function which is monotonic increasing in  $x_m$ . Obviously, the use of a weight in the tray structure which results in a smaller value of  $\theta_T$  could be used as a basis for weight reductions in structures other than the tray, if thermal design weight additions had been applied to such other structures.

It is important to recognize that the example just considered is one which involves several typical types of heat conduction problems, and a rather specific one-dimensional configuration, but that it is not specific to the non-thermal functions of that configuration. In simplest terms, the structural function might be described as that of enclosing the useful volume CDEF, and the



transport of heat around or through it. The volume itself might be used otherwise for various purposes, such as housing interconnection wiring or housing electronic modules. It is also important to recognize that the particular combination of shapes and heat transfer processes discussed in connection with Fig. 13 is not unique in being susceptible to a weight saving by subdivision. The essential requirement for saving structural weight or reducing the temperature difference by means of structural rearrangement is simply that the rearrangement must operate on the system so as to reduce the thermal resistance of it. There is a potential opportunity for doing this in any configuration whose heat conduction paths are not along the shortest straight lines between the heat sources and the heat sink.

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