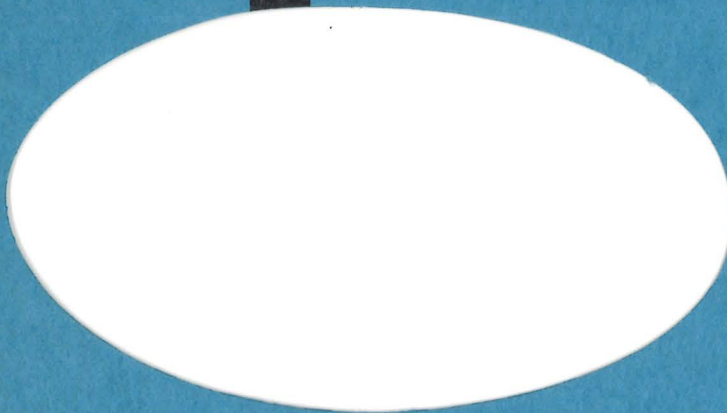


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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

# APOLLO

GUIDANCE AND NAVIGATION



**MIT** INSTRUMENTATION  
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## GUIDANCE, NAVIGATION AND CONTROL

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R-557

GUIDANCE SYSTEM OPERATIONS PLAN  
 FOR MANNED LM EARTH ORBITAL MISSIONS  
 USING PROGRAM SUNDANCE (REV. 306)

SECTION 5 GUIDANCE EQUATIONS  
 (Rev. 2)

DECEMBER 1968

**MIT** INSTRUMENTATION  
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R-557

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(Rev. 2)

December 1968

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PCR - (PCN\*)

TITLE

Incorporated in Revision 1

410.1	*	RR Monitor Routine Control Flags
412	*	Remove Landing Site IMU Alignment Option from P-52
414	*	Section 5 Update Based on MIT and MSC Reviews
415.1	*	Prevent Display Conflict During RR Mode Switching

Incorporated in Revision 2

103		Deletion of CSM Rendezvous Targeting by the LM
500	*	RR Search Termination
573.2	*	Definition of $A_X$ and $A_Y$ in Section 5 of GSOP
587.1		APS and DPS Tail-off Constants
612		Raise Thresholds for Delta V Monitor
657	*	Sundance GSOP Section 5 Update
658		RR Shaft / Trunnion Bias Modification by Crew

Date: 30 December, 1968

REVISION INDEX COVER SHEET  
GUIDANCE SYSTEM OPERATIONS PLAN

GSOP #R-557 Title: For Manned CM Earth Orbital and Lunar  
Missions Using Program Sundance Revision 306

Section # 5 Title: Guidance Equations (Revision 2)

This publication is a revision of the previous issue of this document to describe Program Sundance, Revision 306. All PCR's and PCN's approved by NASA for Sundance have been incorporated. In addition many editorial changes have been included. PCR and PCN changes are indicated by denoting the applicable number at the bottom of the page and indicating the location of the change by a black line █ on the side of the page. Editorial corrections (not covered by PCR or PCN) are denoted by black dots ∴.

Below is a list of the PCR's and PCN's incorporated in this GSOP. In addition, for convenience, the PCR's and PCN's included in previous issues are listed.

PCR - (PCN*)	TITLE
	<u>Incorporated in Revision 1</u>
7	IMU Alignment Program Change
30	DPS Trim Time in Erasable Memory
80.1	* State Vector Synchronization
95.1	RR Trunnion Angle Display Sign Change
97.1	RR Monitor Routine Program Alarm Elimination
100	* Eliminate Rate Test From Thrust Vector Filter
124.1	* RR Search Routine Modification
144.1	* Reduction of Ullage Duration and $\Delta V$ Monitor Threshold
146.1	* Emergency Termination of Integration Function
164.2	Mode II Attitude Error Displays
173.2	* RMS Position and Velocity Error Display
181.1	* Reduction of Sun Occultation Cone
182	* Section 5 Powered Flight Update
183.1	* Section 5 Control Data Reference Update
188	Reduce LGC Executive and Control Processor Load
191.1	* Correct GSOP Definition of RR Range Low Scale Discrete
409.1	* Transfer of RR from Auto Track to LGC Mode

SECTION 5  
GUIDANCE EQUATIONS

5.1        INTRODUCTION

5.1.1     GENERAL COMMENTS

The purpose of this section is to present the Guidance and Navigation Computation Routines associated with a manned LM Apollo Earth-Orbital Mission. These Routines are utilized by the Programs outlined in Section 4 where astronaut and other subsystem interface and operational requirements are described. The guidance and navigation equations and procedures presented in Section 5 are primarily concerned with mission type programs representing the current LM PGNCS Computer (LGC) capability. A restricted number of LGC service type program operations which involve computation requirements are also included.

The LM PGNCS Computer (LGC) guidance and navigation equations for earth orbital missions are presented in the following five categories:

- Section 5.2    Coasting Flight Navigation Routines
- Section 5.3    Powered Flight Navigation and  
                  Guidance Routines
- Section 5.4    Targeting Routines
- Section 5.5    Basic Subroutines
- Section 5.6    General Service Routines

Guidance equation parameters required for program initialization and operation are listed in Section 5.7. These selected parameters are stored in the LGC erasable memory. General constants used in the equations of this volume are presented in Section 5.8.



A complete table of contents for Section 5 is presented in the following Section 5.1.2.1. A cross-reference between the LGC programs and routines of Section 4 that are described in Section 5 is listed in Section 5.1.2.2. It should be noted that the LM earth orbital mission program SUNDANCE is a subset of the LM lunar landing mission program LUMINARY. In the following Section 5 table of contents and text, missing section numbers correspond to LUMINARY programs that were deleted to form the SUNDANCE program.

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The Guidance System Operations Plan is published as seven separate volumes (sections) as listed below:

Section 1	Pre-Launch
Section 2	Data Links
Section 3	Digital Auto-Pilots
Section 4	Operational Modes
Section 5	Guidance Equations
Section 6	Control Data
Section 7	Error Analyses

This volume constitutes a control document to govern the structure of the manned LM Earth Orbital Mission, using SUNDANCE, including PGNCs interfaces with the flight crew and Mission Control Center.

Revisions to this plan which reflect changes in the above control items require NASA approval.

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#### 5.1.4 COORDINATE SYSTEMS

There are six major coordinate systems used in the navigation and guidance programs. These six coordinate systems are defined individually in the following descriptions, and referenced to control specifications of Section 5.8.2 where applicable. Any other coordinate system used in any particular LGC program is defined in the individual section describing that program.

##### 5.1.4.1 Basic Reference Coordinate System

The Basic Reference Coordinate System is an orthogonal inertial coordinate system whose origin is located at either the moon or the earth center of mass. The orientation of this coordinate system is defined by the line of intersection of the mean earth equatorial plane and the mean orbit of the earth (the ecliptic) at the beginning of the Besselian year which starts January 0.525, 1969. The X-axis ( $u_{XI}$ ) is along this intersection with the positive sense in the direction of the ascending node of the ecliptic on the equator (the equinox), the Z-axis ( $u_{ZI}$ ) is along the mean earth north pole, and the Y-axis ( $u_{YI}$ ) completes the right-handed triad. The Basic Reference Coordinate System is presented in Ref. 1 of Sec. 5.8.2 as Standard Coordinate System 4, Geocentric Inertial in Fig. A-4.

During earth-orbital missions in which the LM is active near the earth, the Basic Reference Coordinate System will be earth-centered. All navigation stars and lunar-solar ephemerides are referenced to this coordinate system. All vehicle state vectors are referenced to this system during coasting or free fall phases of the mission.

##### 5.1.4.2 IMU Stable Member or Platform Coordinate System

The orthogonal inertial coordinate system defined by the PGNCs inertial measurement unit (IMU) is dependent upon the current IMU alignment. There are many possible alignments during a mission, but the primary IMU alignment orientations described in Section 5.6.3.4 are summarized below and designated by the subscript SM:

1. Preferred Alignment

$$\underline{u}_{XSM} = \underline{u}_{TD}$$

$$\underline{u}_{YSM} = \text{UNIT} (\underline{u}_{XSM} \times \underline{r}) \quad (5.1.1)$$

$$\underline{u}_{ZSM} = \underline{u}_{XSM} \times \underline{u}_{YSM}$$

where:

$$\left. \begin{array}{l} \underline{u}_{XSM} \\ \underline{u}_{YSM} \\ \underline{u}_{ZSM} \end{array} \right\} \begin{array}{l} \text{IMU stable member coordinate unit} \\ \text{vectors referenced to the Basic Re-} \\ \text{ference Coordinate System} \end{array}$$

$\underline{u}_{TD}$  = unit vector in desired thrust direction  
at ignition

$\underline{r}$  = position vector at ignition

2. Nominal Alignment (Local Vertical)

$$\underline{u}_{XSM} = \text{UNIT} (\underline{r}) \text{ at } t_{\text{align}}$$

$$\underline{u}_{YSM} = \text{UNIT} (\underline{v} \times \underline{r}) \quad (5.1.2)$$

$$\underline{u}_{ZSM} = \underline{u}_{XSM} \times \underline{u}_{YSM}$$

where  $\underline{r}$  and  $\underline{v}$  represent the vehicle state vector at the alignment time,  $t_{\text{align}}$ .

The origin of the IMU Stable Member Coordinate System is the center of the IMU stable member.

#### 5.1.4.3 Vehicle or Body Coordinate System

The Vehicle or Body Coordinate System is the orthogonal coordinate system used for the LM structural body. The origin of this coordinate system is 200 inches below the LM ascent stage base. The X-axis ( $\underline{u}_{XB}$ ) lies along the longitudinal axis (centerline of the transfer tunnel) of the LM, positive in the nominal DPS thrust direction. The Z-axis ( $\underline{u}_{ZB}$ ) is parallel to the centerline of the exit hatch and directed forward from the design eye. The Y-axis ( $\underline{u}_{YB}$ ) completes the right-handed triad. This coordinate system is defined in Ref. 1 of Section 5.8.2 as the Standard Coordinate System 8d, LEM Structural Body Axis in Fig. A-8d.

#### 5.1.4.4 Earth-Fixed Coordinate System

The Earth-Fixed Coordinate System is an orthogonal rotating coordinate system whose origin is at the center of mass of the earth. This coordinate system is shown in Ref. 1 of Sec. 5.8.2 as the Standard Coordinate System 1, Geographic Polar in Fig. A-1. The Z-axis of this coordinate system is defined to be along the earth's true rotational or polar axis. The X-axis is defined to be along the intersection of the prime (Greenwich) meridian and the equatorial plane of the earth, and the Y-axis is in the equatorial plane and completes the right-handed triad.

#### 5.1.4.5 Moon-Fixed Coordinate System

The Moon-Fixed Coordinate System is an orthogonal rotating coordinate system whose origin is at the center of mass of the moon. This coordinate system is shown in Ref. 1 of Sec. 5.8.2 as the Standard Coordinate System 2, Selenographic Polar in Fig. A-2. The Z-axis is defined to be along the true polar or rotation axis of the moon, the X-axis is through the mean center of the apparent disk or along the intersection of the meridian of  $0^{\circ}$  longitude and the equatorial plane of the moon, and the Y-axis is in the equatorial plane and completes the right-handed triad.

#### 5.1.4.6 Navigation Base Coordinate System

The Navigation Base Coordinate System is an orthogonal coordinate system whose origin and axis orientation are defined by the three mounting points between the PGNCS navigation base and the LM vehicle structure. These mounting point locations are defined in Section G-G of the GAEC-MIT Interface Control Document LID280-10004 (Ref. 13 of Sec 5.8.2). The  $Y_{NB}$  axis is defined by the centers of the two upper mounting points along line F of Ref. 13, with the positive direction in the same general direction as the LM+Y vehicle axis. The  $X_{NB}$  axis is defined by a line through the center of the lower mounting point (point K of Ref. 13) and perpendicular to the  $Y_{NB}$  axis. The positive  $X_{NB}$  direction is in the same general sense as the LM+X vehicle axis. The  $+Z_{NB}$  defined as  $X_{NB} \times Y_{NB}$  to complete the right-handed triad. The Navigation Base Coordinate System is approximately parallel with the LM Vehicle Coordinate System.

5.1.5 GENERAL DEFINITIONS AND CONVENTIONS

In this section the definitions of and the conventions for using a selected number of parameters are given. Although virtually all of the information in this section appears elsewhere in this document, this section provides a summary of facts which are distributed among various other sections.

5.1.5.1 Error Transition Matrix Maintenance

5.1.5.1.1 Definitions

The error transition matrix (W matrix) is defined in Section 5.2.2.4 and is used in processing navigation measurement data. Control of the W matrix is maintained by means of the flag RENDWFLG (see Section 5.2.4.2.2). If RENDWFLG is equal to one, then the W matrix is valid for processing rendezvous navigation data; while this flag being equal to zero indicates that the W matrix is invalid.

5.1.5.1.2 W Matrix Control Flag

The W matrix control flag is maintained according to the following rules:

1. RENDWFLG is initially zero.
2. A CSM state vector update from the ground causes the flag to be zeroed.
3. A LM in-flight state vector update from the ground causes the flag to be zeroed. An update of the landing site vector when the LM is on the lunar surface does not cause the flag to be zeroed.

4. There exists a special DSKY verb by which the astronaut can zero the flag.
5. Indication to the LGC that the LM is on the lunar surface causes RENDWFLG to be zeroed.
6. Initialization of the W matrix for rendezvous navigation causes RENDWFLG to be set to one. There exists in erasable memory one set of initialization parameters for the W matrix consisting of four elements: one each for position, velocity, RR shaft bias, and RR trunnion bias. At the time each set of rendezvous navigation data is processed, RENDWFLG is tested. If the flag is found to be zero, then the W matrix is initialized consistent with the erasable parameters, and the flag is set to one. See Section 5.2.4.2.2 for more complete details of this initialization procedure.
7. There exists a special DSKY verb which causes the RMS position and velocity errors to be computed from the W matrix and displayed. At this display the astronaut can enter new W matrix position and velocity initialization parameters, at which RENDWFLG is zeroed. Item 6 above then applies. See Section 5.6.19 for more complete details of this procedure.

### 5.1.5.1.3 W Matrix Extrapolation

Extrapolation of the W matrix is described in Section 5.2.2.4. Required in this extrapolation is the specification of the appropriate vehicle's state vector with which the W matrix is extrapolated. This extrapolation occurs during programs P-00 and P-20; and at the conclusion of programs P-40, P-41, P-42, and P-47. The conventions under which the extrapolation occurs during each of these programs are as follows:

P-00 : If RENDWFLG is equal to one, the W matrix is extrapolated with the LM state vector if the LM is in flight, and with the CSM state vector if the LM is on the lunar surface. The W matrix is not extrapolated if the flag is equal to zero. (See Section 5.6.11).

P-20 : The W matrix is extrapolated with the state vector that is being updated if RENDWFLG is equal to one, and not extrapolated if RENDWFLG is equal to zero. (See Section 5.2.4.2.2).

P-40 }  
P-41 } : The result of the maneuver will be a  
P-42 } final state vector at the end-of-maneuver  
P-47 } time  $t_F$ . The LM state vector that existed  
before the maneuver program will still  
exist; and, cotemporal with it, there will  
also be the CSM state vector and the W  
matrix. The following steps are performed  
after the Average-G Routine (Sec. 5.3.2)  
is terminated:

1. If the W matrix control flag is equal to one, the old LM state vector and the W matrix are extrapolated to  $t_F$ .
2. The CSM state vector is extrapolated to time  $t_F$ .
3. The LM state vector is initialized to the end-of-maneuver state vector.

If a computation overflow occurs during any of the above W matrix extrapolations, a program alarm will result, the W matrix control flag will be zeroed, and the extrapolation of the state vector will continue without the W matrix.



5.1.5.2 Altitude Parameter Convention

In the following programs and routines the display parameter of the vehicle altitude or trajectory pericenter or apocenter altitude is measured with respect to the earth launch pad radius magnitude,  $r_{LP}$ , in earth orbit cases, or the lunar landing site radius magnitude,  $r_{LS}$ , in lunar orbit cases. The earth launch pad radius parameter,  $r_{LP}$ , is stored in fixed memory and the lunar landing site radius,  $r_{LS}$ , is the magnitude of the landing site vector,  $r_{LS}$ , available in erasable memory.

P-21	Ground Track Determination	Sec. 5.6.5
P-30	External $\Delta V$ Maneuver Guidance	Sec. 5.3.3.3.1
P-34 & P74	TPI Pre-Thrust Program	Sec. 5.4.4.4
R-30	Orbit Parameter Display Routine	Sec. 5.6.17

## 5.2 COASTING FLIGHT NAVIGATION

### 5.2.1 GENERAL COMMENTS

The LGC Coasting Flight Navigation Routines which are presented in Sections 5.2.2 through 5.2.4 are used during non-thrusting phases of the Apollo mission. The basic objective of the navigation routines is to maintain estimates of the position and velocity vectors of both the CSM and the LM. Let  $\underline{r}$  and  $\underline{v}$  be the estimates of a vehicle's position and velocity vectors, respectively. Then, the six-dimensional state vector,  $\underline{x}$ , of the spacecraft is defined by

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \end{pmatrix}$$

Coasting Flight Navigation is accomplished by extrapolating the state vector,  $\underline{x}$ , by means of the Coasting Integration Routine (Section 5.2.2), and updating or modifying this estimated state using Rendezvous Radar (RR) tracking data by the recursive method of navigation (Sections 5.2.3 - 5.2.4).

The Coasting Integration Routine (Section 5.2.2) is used by other navigation and targeting routines to extrapolate the following:

- 1) Present estimated LM state vector
- 2) Present estimated CSM state vector
- 3) An arbitrary specified state vector, such as the predicted result of a maneuver

State vector extrapolation is accomplished by means of Encke's method of differential accelerations. The motion of a spacecraft is dominated by the conic orbit which would result if the spacecraft were in a central force field. In Encke's method the differential equations for the deviations from conic motion are integrated numerically. This technique is in contrast to a numerical integration of the differential equations for the total motion, and it provides a more accurate orbit extrapolation. The numerical integration is accomplished by means of Nystrom's method which gives fourth-order accuracy while requiring only three computations of the derivatives per time step. The usual fourth-order Runge-Kutta integration methods require four derivative computations per time step.

Regardless of the accuracy of the state vector extrapolation, errors in the initial conditions will propagate and soon grow to intolerable size. Thus, it is necessary periodically to obtain additional data in the form of either new state vector estimates or modifications to the current state vector estimates. These state vector modifications are computed from navigation data obtained by means of navigation measurements.

The LM PGNCS uses RR tracking data to compute state vector changes. Navigation measurement data are used to update state vector estimates during the rendezvous navigation procedure which will be used normally during all LM-CSM rendezvous phases.

In order to achieve desired rendezvous objectives, it is necessary to expand the rendezvous navigation procedure to nine dimensions, and to include in the estimation the constant RR angle biases. The estimated state vector that is used in rendezvous navigation is given by

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \\ \underline{\text{bias}} \end{pmatrix}$$

where  $\underline{r}$  and  $\underline{v}$  are the estimated position and velocity vectors of either the LM or the CSM, and  $\underline{\text{bias}}$  is a vector whose components are the estimates of the RR angle biases. Normally the LM state vector is updated, but the astronaut can select the CSM update mode. The selection of the vehicle update mode is based primarily upon which vehicle's state vector is most accurately known initially, and which vehicle is controlling the rendezvous maneuvers.

In order to estimate the RR angle biases, it is necessary to restrict the LM attitude during RR tracking. This attitude restriction involves controlling the LM +Z-axis to be within  $30^\circ$  of the tracking line-of-sight and is described in Section 5.2.4.1.

Navigation data is incorporated into the state vector estimates by means of the Measurement Incorporation Routine (Section 5.2.3) which has both six- and nine-dimensional modes. The six-dimensional mode is not used in this restricted Earth Orbital Mission Program. The Measurement Incorporation Routine is a subroutine of the Rendezvous Navigation Routine (Section 5.2.4.2). A simplified functional diagram of the Rendezvous Navigation Program which uses these routines is given in Fig. 2.1-1.

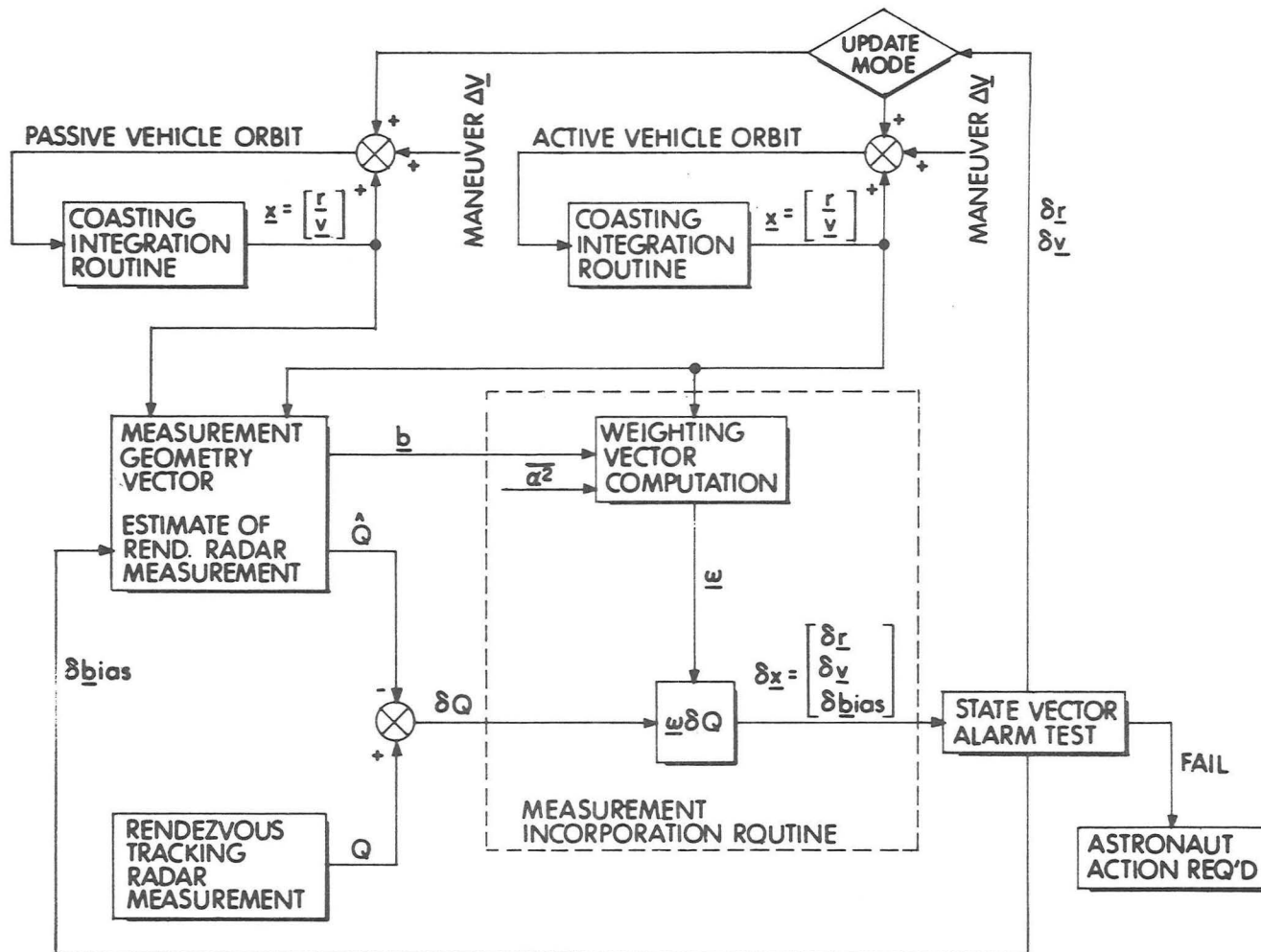


Figure 2. 1-1 Simplified LGC Rendezvous Navigation Functional Diagram

In rendezvous navigation, estimated LM and CSM position and velocity vectors are obtained at required times by means of the Coasting Integration Routine (Section 5.2.2). The Measurement Incorporation Routine (Section 5.2.3) is used to incorporate the measurement data into the state vector estimates.

The Navigation procedure, which is illustrated in simplified form in Fig. 2.1-1, involves computing an estimated tracking measurement,  $\hat{Q}$ , based on the current state vector estimates. This estimated measurement is then compared with the actual tracking measurement  $Q$  (RR tracking data in the LGC) to form a measured deviation  $\delta Q$ . A statistical weighting vector,  $\underline{\omega}$ , is computed from statistical knowledge of state vector uncertainties and tracking performance,  $\overline{\alpha^2}$ , plus a geometry vector,  $\underline{b}$ , determined by the type of measurement being made. The weighting vector,  $\underline{\omega}$ , is defined such that a statistical optimum linear estimate of the deviation,  $\delta \underline{x}$ , from the estimated state vector is obtained when the weighting vector is multiplied by the measured deviation  $\delta Q$ . The vectors  $\underline{\omega}$ ,  $\underline{b}$  and  $\delta \underline{x}$  are of six or nine dimensions depending upon the dimension of the state vector being estimated.

In an attempt to prevent unacceptably large incorrect state vector changes, certain validity tests have been included in the LGC Rendezvous Navigation Routine. In this routine measurement data is processed periodically (approximately once per minute), and it is desirable that the CSM be tracked during the entire rendezvous phase up to the manual terminal maneuver. If the magnitudes of the changes in the estimated position and velocity vectors,  $\delta r$  and  $\delta v$ , respectively, are both less than preset tracking alarm levels, then the selected vehicle's state

vector is automatically updated by the computed deviation,  $\delta \underline{x}$ , and no special display is presented, except that the tracking measurement counter is incremented by one. If either  $\delta r$  or  $\delta v$  exceeds its alarm level, then the state vector is not updated, and the astronaut is alerted to this condition by a special display of  $\delta r$  and  $\delta v$ .

In this case the astronaut should place the RR under manual control and make the necessary radar operating and side lobe checks to verify main lobe lock-on and tracking conditions. After the tracking has been verified, and navigation data has again been acquired, the astronaut has the option of commanding a state vector update if the tracking alarm is again exceeded, or of repeating further RR checks before incorporating the measurement data. If the astronaut cannot verify the tracking, then he can terminate the program and try to achieve tracking conditions at a later time.

The tracking alarm criterion is incorporated in the navigation routine to alert the astronaut to the fact that the state vector update is larger than normally expected, and to prevent the estimated state vector from automatically being updated in such cases. The update occurs only by specific command of the astronaut. The tracking alarm level beyond which updating is suspended is primarily chosen to avoid false acquisition and tracking conditions. There is a low probability that the alarm level will be exceeded in the LGC if the estimated state vectors are essentially correct, since the RR Designation and Data Read Routines have partial internal checks for side-lobe acquisition before tracking data are incorporated in the navigation routine.

The most probable condition for the state vector update alarm level being exceeded in the LGC is, therefore, initial acquisition and tracking in the case where a poor estimate of either the CSM or LM state vector exists. In this case the astronaut would have to command the initial state vector update, after which the tracking alarm level would seldom be exceeded during the remainder of the navigation phase. It should be noted that this statement is true only if the estimated state vector of the active vehicle performing a powered rendezvous maneuver is updated by the Average-G Routine in the case of the LM being the active vehicle, or by a DSKY entry (R-32) of the maneuver  $\Delta V$  if the CSM is the active vehicle (Section 5.6.16).

The displayed values of  $\delta r$  and  $\delta v$  which have not passed the tracking alarm test will depend upon the statistical parameters stored in the LGC and upon the following types of errors:

- Type 1: Errors in the current state vector estimates
- Type 2: Errors in alignment of the IMU
- Type 3: Reasonable RR tracking performance errors
- Type 4: A PGNCS or RR failure resulting in false acquisition

The existence of Type 1 errors is precisely the reason that the RR tracking is being done. It is the function of the navigation to decrease Type 1 errors in the presence of noise in the form of errors of Types 2 and 3. Since the RR tracking should not be performed unless the IMU is well aligned and the PGNCS and RR are functioning properly, it follows that the purpose of the state vector change validity check is to discover a Type 4 error. As previously mentioned, there is a low probability of this type of error occurring.



Based upon the last time that the state vector was updated and when the IMU last was realigned, very crude reasonable values for  $\delta r$  and  $\delta v$  can be generated by the astronaut. The LGC will provide no information to assist the astronaut in his estimates of reasonable values for  $\delta r$  and  $\delta v$ .

The parameters required to initialize the Rendezvous Navigation Routine (Section 5.2.4.2) are the initial estimated LM and CSM state vectors, initial state vector estimation error covariance matrices in the form of prestored diagonal error transition matrices (as defined in Section 5.2.2.4), and a priori measurement error variances. The basic input to the navigation routines is RR tracking data which is automatically acquired by the Data Read Routine. The primary results of the navigation routine are the estimated LM and CSM state vectors. The various guidance targeting modes outlined in Section 5.4 are based on the state vector estimates which result from these navigation routines.

## 5. 2. 2 COASTING INTEGRATION ROUTINE\*

### 5. 2. 2. 1 General Comments

During all coasting phase navigation procedures, an extrapolation of position and velocity by numerical integration of the equations of motion is required. The basic equation may be written in the form

$$\frac{d^2}{dt^2} \underline{r}(t) + \frac{\mu_P^{**}}{r^3} \underline{r}(t) = \underline{a}_d(t) \quad (2. 2. 1)$$

where  $\mu_P$  is the gravitational constant of the primary body, and  $\underline{a}_d(t)$  is the vector acceleration which prevents the motion of the vehicle (CSM or LM) from being precisely a conic with focus at the center of the primary body. The Coasting Integration Routine is a precision integration routine in which all significant perturbation effects are included. The form of the disturbing acceleration  $\underline{a}_d(t)$  depends on the phase of the mission.

An approximate extrapolation of a vehicle state vector in which the disturbing acceleration,  $\underline{a}_d(t)$  of Eq. (2. 2. 1), is set to zero may be accomplished by means of the Kepler Subroutine (Section 5. 5. 5).

The LGC Coasting Integration Routine is restricted to earth or lunar orbit and is not to be used in cislunar-midcourse space. The routine does not contain the capability of computing the gravitational perturbations of the sun or the other body (moon or earth); and, therefore, cannot provide accurate midcourse integration.

### 5. 2. 2. 2 Encke's Method

If  $\underline{a}_d$  is small compared with the central force field, direct integration of Eq. (2. 2. 1) is inefficient. Therefore, the extrapolation will be accomplished using the technique of differential accelerations attributed to Encke.

\* This section does not reflect the entire LGC Coasting Integration Routine which is identical to that contained in the CMC. These two routines have been kept identical for the purpose of efficient flight program production. However, certain control constants are set in the LGC so that the midcourse perturbation calculations are locked out. Only those equations which can be executed are documented here.

\*\*In the remainder of Section 5. 2 the subscript P will denote primary body (earth or moon). When the body is known, then the subscripts E and M will be used for earth and moon, respectively. The vehicle will be indicated by the subscripts C for CSM and L for LM.

At time  $t_0$  the position and velocity vectors,  $\underline{r}_0$  and  $\underline{v}_0$ , define an osculating conic orbit. The position and velocity vectors in the conic orbit,  $\underline{r}_{\text{con}}(t)$  and  $\underline{v}_{\text{con}}(t)$ , respectively, will deviate by a small amount from the actual position and velocity vectors.

The conic position and velocity at time  $t$  are computed as shown in Section 5.5.5. Required in this calculation is the variable  $x$  which is the root of Kepler's equation. In order to minimize the number of iterations required in solving Kepler's equation, an estimate of the correct solution for  $x$  is obtained as follows:

Let

$$\tau = t - t_0 \quad (2.2.2)$$

During the previous computation cycle the values

$$\begin{aligned} \underline{r}' &= \underline{r}_{\text{con}} \left( \tau - \frac{\Delta t}{2} \right) \\ \underline{v}' &= \underline{v}_{\text{con}} \left( \tau - \frac{\Delta t}{2} \right) \\ x' &= x \left( \tau - \frac{\Delta t}{2} \right) \end{aligned} \quad (2.2.3)$$

were computed. A trial value of  $x(\tau)$  is obtained from

$$x_t = x' + s \left[ 1 - \gamma s (1 - 2 \gamma s) - \frac{1}{6} \left( \frac{1}{r'} - \alpha \right) s^2 \right] \quad (2.2.4)$$

where

$$\begin{aligned} s &= \frac{\sqrt{\mu_P}}{r'} \left( \frac{\Delta t}{2} \right) \\ \gamma &= \frac{\underline{r}' \cdot \underline{v}'}{2r' \sqrt{\mu_P}} \\ \alpha &= \frac{2}{r_0} - \frac{v_0^2}{\mu_P} \end{aligned} \quad (2.2.5)$$

Equations (2.2.4) and (2.2.5) are also used to establish a preliminary estimate of  $\underline{x}$  for the Kepler Subroutine when it is used to determine an approximate state vector extrapolation in one step. For this procedure

$$\begin{aligned}\underline{r}' &= \underline{r}_0 \\ \underline{v}' &= \underline{v}_0 \\ \underline{x}' &= 0 \\ \frac{\Delta t}{2} &= \tau\end{aligned}$$

After specification of  $\underline{r}_0$ ,  $\underline{v}_0$ ,  $\underline{x}_t$  and  $\tau$ , the Kepler subroutine (Section 5.5.5) is used to compute  $\underline{r}_{\text{con}}(\tau)$ ,  $\underline{v}_{\text{con}}(\tau)$ , and  $\underline{x}(\tau)$ .

The true position and velocity vectors will deviate from the conic position and velocity since  $\underline{a}_d$  is not zero. Let

$$\underline{r}(t) = \underline{\delta}(t) + \underline{r}_{\text{con}}(t) \tag{2.2.6}$$

$$\underline{v}(t) = \underline{\nu}(t) + \underline{v}_{\text{con}}(t)$$

where  $\underline{\delta}(t)$  and  $\underline{\nu}(t)$  are the position and velocity deviations from the conic. The deviation vector  $\underline{\delta}(t)$  satisfies the differential equation

$$\frac{d^2}{dt^2} \underline{\delta}(t) = -\frac{\mu_P}{r_{\text{con}}^3(t)} \left[ f(q) \underline{r}(t) + \underline{\delta}(t) \right] + \underline{a}_d(t) \tag{2.2.7}$$

subject to the initial conditions

$$\underline{\delta}(t_0) = \underline{0}, \quad \underline{\nu}(t_0) = \underline{0} \tag{2.2.8}$$

where

$$q = \frac{(\underline{\delta} - 2\underline{r}) \cdot \underline{\delta}}{r^2} \tag{2.2.9}$$

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1 + q)^{3/2}} \quad (2.2.10)$$

The first term on the right-hand side of Eq. (2.2.7) must remain small, i. e., of the same order as  $\underline{a}_d(t)$ , if the method is to be efficient. As the deviation vector  $\underline{\delta}(t)$  grows in magnitude, this term will eventually increase in size. Therefore, in order to maintain the efficiency of the method, a new osculating conic orbit should be defined by the total position and velocity vectors  $\underline{r}(t)$  and  $\underline{v}(t)$ . The process of selecting a new conic orbit from which to calculate deviations is called rectification. When rectification occurs, the initial conditions for the differential equation for  $\underline{\delta}(t)$ , as well as the variables  $\tau$  and  $x$ , are again zero.

### 5.2.2.3 Disturbing Acceleration

The form of the disturbing acceleration  $\underline{a}_d(t)$  that is used in Eq. (2.2.1) depends on the phase of the mission. In earth or lunar orbit, only the gravitational perturbations arising from the non-spherical shape of the primary body need be considered. Let  $\underline{a}_{dP}$  be the acceleration due to the non-spherical gravitational perturbations of the primary body. Then, for the earth

$$\underline{a}_{dE} = \frac{\mu_E}{r^2} \sum_{i=2}^4 J_{iE} \left( \frac{r_E}{r} \right)^i \left[ P'_{i+1} (\cos \phi) \underline{u}_r - P'_i (\cos \phi) \underline{u}_z \right] \quad (2.2.11)$$

where

$$\begin{aligned}
P_2'(\cos \phi) &= 3 \cos \phi \\
P_3'(\cos \phi) &= \frac{1}{2} (15 \cos^2 \phi - 3) \\
P_4'(\cos \phi) &= \frac{1}{3} (7 \cos \phi P_3' - 4 P_2') \\
P_5'(\cos \phi) &= \frac{1}{4} (9 \cos \phi P_4' - 5 P_3')
\end{aligned}
\tag{2.2.12}$$

are the derivatives of Legendre polynomials,

$$\cos \phi = \underline{u}_r \cdot \underline{u}_z \tag{2.2.13}$$

$$\underline{u}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and  $J_2, J_3, J_4$  are the coefficients of the second, third, and fourth harmonics of the earth's potential function. The vectors  $\underline{u}_r$  and  $\underline{u}_z$  are unit vectors in the direction of  $\underline{r}$  and the polar axis of the earth, respectively, and  $r_E$  is the equatorial radius of the earth.

In the case of the moon

$$\begin{aligned}
\underline{a}_{dM} &= \frac{\mu_M}{r^2} \left\{ \sum_{i=2}^4 J_{iM} \left( \frac{r_M}{r} \right)^i \left[ P_{i+1}'(\cos \phi) \underline{u}_r - P_i'(\cos \phi) \underline{u}_z \right] \right. \\
&\quad \left. + 3 J_{22M} \left( \frac{r_M}{r} \right)^2 \left[ 4 \frac{x_M y_M}{x_M^2 + y_M^2} (\underline{u}_r \times \underline{u}_z) + \frac{x_M^2 - y_M^2}{x_M^2 + y_M^2} ((5 \cos^2 \phi - 3) \underline{u}_r - 2 \cos \phi \underline{u}_z) \right] \right\}
\end{aligned}
\tag{2.2.14}$$

where  $r_M$  is the mean lunar radius,  $J_{22M}$  is the coefficient of the term in the moon's gravitational potential function which describes the assymetry of the moon about its polar axis, and  $x_M$  and  $y_M$  are the X and Y components of  $\underline{r}$  expressed in moon-fixed coordinates. The other terms in Eq. (2.2.14) have definitions analogous to those in Eq. (2.2.11). The variables  $x_M$ ,  $y_M$ , and  $\underline{u}_z$  are computed by means of the Planetary Inertial Orientation Subroutine (Section 5.5.2).

#### 5.2.2.4 Error Transition Matrix

The position and velocity vectors as maintained in the computer are only estimates of the true values. As part of the navigation technique it is necessary also to maintain statistical data in the computer to aid in the processing of navigation measurements.

If  $\underline{\epsilon}(t)$  and  $\underline{\eta}(t)$  are the errors in the estimates of the position and velocity vectors, respectively, then the six-dimensional correlation matrix  $E(t)$  is defined by

$$E_6(t) = \begin{pmatrix} \overline{\underline{\epsilon}(t) \underline{\epsilon}(t)^T} & \overline{\underline{\epsilon}(t) \underline{\eta}(t)^T} \\ \overline{\underline{\eta}(t) \underline{\epsilon}(t)^T} & \overline{\underline{\eta}(t) \underline{\eta}(t)^T} \end{pmatrix} \quad (2.2.15)$$

In certain applications it becomes necessary to expand the state vector and the correlation matrix to more than six dimensions so as to include estimation of landmark locations in the CMC during orbit navigation, and rendezvous radar tracking biases in the LGC during the rendezvous navigation procedure. For this purpose a nine-dimensional correlation matrix is defined as follows :

$$E(t) = \begin{pmatrix} & & \overline{\underline{\epsilon}(t) \underline{\beta}^T} \\ & E_6(t) & \overline{\underline{\eta}(t) \underline{\beta}^T} \\ \overline{\underline{\beta} \underline{\epsilon}(t)^T} & \overline{\underline{\beta} \underline{\eta}(t)^T} & \overline{\underline{\beta} \underline{\beta}^T} \end{pmatrix} \quad (2.2.16)$$

where the components of the three-dimensional vector  $\underline{\beta}$  are the errors in the estimates of three variables which are estimated in addition to the components of the spacecraft state vector.

In order to take full advantage of the operations provided by the interpreter in the computer, the correlation matrix will be restricted to either six or nine dimensions. If, in some navigation procedure, only one or two additional items are to be estimated, then a sufficient number of dummy variables will be added to the desired seven- or eight-dimensional state vector to make it nine-dimensional.



Rather than use the correlation matrix in the navigation procedure, it is more convenient to utilize a matrix  $W(t)$ , called the error transition matrix, and defined by

$$E(t) = W(t) W(t)^T \quad (2.2.17)$$

Extrapolation of the nine-dimensional matrix  $W(t)$  is made by direct numerical integration of the differential equation

$$\frac{d}{dt} W(t) = \begin{pmatrix} O & I & O \\ G(t) & O & O \\ O & O & O \end{pmatrix} W(t) \quad (2.2.18)$$

where  $G(t)$  is the three-dimensional gravity gradient matrix, and  $I$  and  $O$  are the three-dimensional identity and zero matrices, respectively. If the  $W$  matrix is partitioned as

$$W = \begin{pmatrix} w_0 & w_1 & \dots & w_8 \\ w_9 & w_{10} & \dots & w_{17} \\ w_{18} & w_{19} & \dots & w_{26} \end{pmatrix} \quad (2.2.19)$$

then,

$$\left. \begin{aligned}
 \frac{d}{dt} \underline{w}_i(t) &= \underline{w}_{i+9}(t) \\
 \frac{d}{dt} \underline{w}_{i+9}(t) &= G(t) \underline{w}_i(t) \\
 \frac{d}{dt} \underline{w}_{i+18}(t) &= \underline{0}
 \end{aligned} \right\} \quad i = 0, 1, \dots, 8 \quad (2.2.20)$$

The extrapolation may be accomplished by successively integrating the vector differential equations

$$\frac{d^2}{dt^2} \underline{w}_i(t) = G(t) \underline{w}_i(t) \quad i = 0, 1, \dots, 8 \quad (2.2.21)$$

The gravity gradient matrix  $G(t)$  for earth or lunar orbit is given by

$$G(t) = \frac{\mu_P}{r^5(t)} \left[ 3 \underline{r}(t) \underline{r}(t)^T - r^2(t) I \right] \quad (2.2.22)$$

Thus, if D is the dimension of the matrix W(t) for the given navigation procedure, the differential equations for the  $\underline{w}_i(t)$  vectors are

$$\frac{d^2}{dt^2} \underline{w}_i(t) = \frac{\mu_P}{r^3(t)} \left\{ 3 \left[ \underline{u}_r(t) \cdot \underline{w}_i(t) \right] \underline{u}_r(t) - \underline{w}_i(t) \right\} \quad (2.2.23)$$

$$(i = 0, 1, \dots, D-1)$$

where  $\underline{u}_r(t)$  is a unit vector in the direction of  $\underline{r}(t)$ .

It is possible for a computation overflow to occur during the W matrix integration if any element of the matrix exceeds its maximum value. This event is extremely unlikely because of the large scale factors chosen. The overflow occurs if

- 1) any element of the position part (upper third) of the W matrix becomes equal to or greater than  $2^{19}m$ ,

or

- 2) any element of the velocity part (middle third) of the W matrix becomes equal to or greater than one m/csec.

In addition, each element of the bias part (lower third) of the matrix must remain less than  $2^{-5}$  radian, but this part does not change during integration.

If overflow should occur, an alarm results, and either new state vector estimates must be obtained from RTCC, or a sufficient number of navigation measurements must be made before the state vectors are used in any targeting or maneuver programs.

### 5. 2. 2. 5 Numerical Integration Method

The extrapolation of navigational data requires the solution of a number of second-order vector differential equations, specifically Eqs. (2. 2. 7) and (2. 2. 23). These are all special cases of the form

$$\frac{d^2}{dt^2} \underline{y} = \underline{f}(\underline{y}, t) \quad (2. 2. 24)$$

Nystrom's method is particularly well suited to this form and gives an integration method of fourth-order accuracy. The second-order system is written

$$\frac{d}{dt} \underline{y} = \underline{z} \quad (2. 2. 25)$$

$$\frac{d}{dt} \underline{z} = \underline{f}(\underline{y}, t)$$

and the formulas are summarized below.

$$\begin{aligned} \underline{y}_{n+1} &= \underline{y}_n + \underline{\phi}(\underline{y}_n) \Delta t \\ \underline{z}_{n+1} &= \underline{z}_n + \underline{\psi}(\underline{y}_n) \Delta t \\ \underline{\phi}(\underline{y}_n) &= \underline{z}_n + \frac{1}{6} (\underline{k}_1 + 2\underline{k}_2) \Delta t \\ \underline{\psi}(\underline{y}_n) &= \frac{1}{6} (\underline{k}_1 + 4\underline{k}_2 + \underline{k}_3) \\ \underline{k}_1 &= \underline{f}(\underline{y}_n, t_n) \\ \underline{k}_2 &= \underline{f}\left(\underline{y}_n + \frac{1}{2} \underline{z}_n \Delta t + \frac{1}{8} \underline{k}_1 (\Delta t)^2, t_n + \frac{1}{2} \Delta t\right) \\ \underline{k}_3 &= \underline{f}\left(\underline{y}_n + \underline{z}_n \Delta t + \frac{1}{2} \underline{k}_2 (\Delta t)^2, t_n + \Delta t\right) \end{aligned} \quad (2. 2. 26)$$

For efficient use of computer storage as well as computing time the computations are performed in the following order:

- 1) Equation (2.2.7) is solved using the Nystrom formulas Eqs.(2.2.26). It is necessary to preserve the values of the vector  $\underline{r}$  at times  $t_n$ ,  $t_n + \Delta t/2$ ,  $t_n + \Delta t$  for use in the solution of Eqs.(2.2.23).
- 2) Equations (2.2.23) are solved one-at-a-time using Eqs. (2.2.26) together with the values of  $\underline{r}$  which resulted from the first step.

The variable  $\Delta t$  is the integration time step and should not be confused with  $\tau$ , the time since rectification. The maximum value for  $\Delta t$  which can be used for precision integration,  $\Delta t_{\max}$ , is computed from

$$\Delta t_{\max} = \text{minimum} \left( 0.3 \frac{r_{\text{con}}^{3/2}}{\sqrt{\mu_P}}, 4000 \text{ sec.} \right) \quad (2.2.27)$$

#### 5.2 2.6 Coasting Integration Logic

Estimates of the state vectors of two vehicles (CSM and LM) will be maintained in the computer. In various phases of the mission it will be required to extrapolate a state vector either alone or with an associated W matrix of dimension six or nine.

To accomplish all of these possible procedures, as well as to solve the computer restart problem, three state vectors will be maintained in the computer. Let  $\underline{x}_C$  and  $\underline{x}_L$  be the estimated CSM and LM state vectors, respectively, and let  $\underline{x}$  be a temporary state vector. The state vector  $\underline{x}$  is a symbolic representation of the following set of variables:

$$\begin{aligned}
 \underline{r}_0 &= \text{rectification position vector} \\
 \underline{v}_0 &= \text{rectification velocity vector} \\
 \underline{r}_{\text{con}} &= \text{conic position vector} \\
 \underline{v}_{\text{con}} &= \text{conic velocity vector} \\
 \underline{\delta} &= \text{position deviation vector} \\
 \underline{v} &= \text{velocity deviation vector} \\
 t &= \text{time associated with } \underline{r}_{\text{con}}, \underline{v}_{\text{con}}, \underline{\delta} \text{ and } \underline{v} \\
 \tau &= \text{time since rectification} \\
 x &= \text{root of Kepler's equation} \\
 P &= \text{primary body} = \begin{cases} 0 & \text{for earth} \\ 1 & \text{for moon} \end{cases}
 \end{aligned}
 \tag{2. 2. 28}$$

The state vectors  $\underline{x}_C$  and  $\underline{x}_L$  represent an analogous set of variables.

The Coasting Integration Routine is controlled by the calling program by means of the two indicators D and V. The variable D indicates the dimension of the W matrix with

$$D = 0 \tag{2. 2. 29}$$

denoting that the state vector only is to be extrapolated. The variable V indicates the appropriate vehicle as follows:

$$V = \begin{cases} 1 & \text{for CSM} \\ 0 & \text{for LM} \\ -1 & \text{for state vector specified by calling program} \end{cases} \quad (2.2.30)$$

In addition, the calling program must set the desired final time  $t_F$ ; and, for  $V$  equal to  $-1$ , the desired state vector  $\underline{x}$ .

A simplified functional diagram of the Coasting Integration Routine is shown in Fig 2.2-1. In the figure the indicated state vector is being integrated to time  $t_F$ . The value of  $\Delta t$  for each time step is  $\Delta t_{\max}$  (Eq. (2.2.27)) or the total time-to-go whichever is smaller. The integration is terminated when the computed value of  $\Delta t$  is less than  $\epsilon_t$ .

Figure 2.2-2 illustrates in more detail the logic flow of this routine. In this figure the following items, which have not been discussed fully in the text, are explicitly illustrated:

- 1) Saving of  $\underline{r}$  values for  $W$  matrix integration
- 2) Rectification procedure
- 3) Selection of disturbing acceleration

The logic flow shown in Fig. 2.2-2 is controlled by the switch  $F$  which is used to distinguish between state vector integration ( $F = 1$ ) and  $W$  matrix integration ( $F = 0$ ).

If the Coasting Integration Routine is requested to extrapolate the estimated LM state vector and the LM is on the lunar surface, then the routine will use the Planetary Inertial Orientation Subroutine (Section 5.5.2) to compute the desired LM position and velocity and the normal integration will not be performed. This procedure is not indicated in the figure.

There is a procedure for the emergency termination of the Coasting Integration Routine in order to permit correction of wrong erasable memory parameters. This emergency function is described in Section 5.6.11.

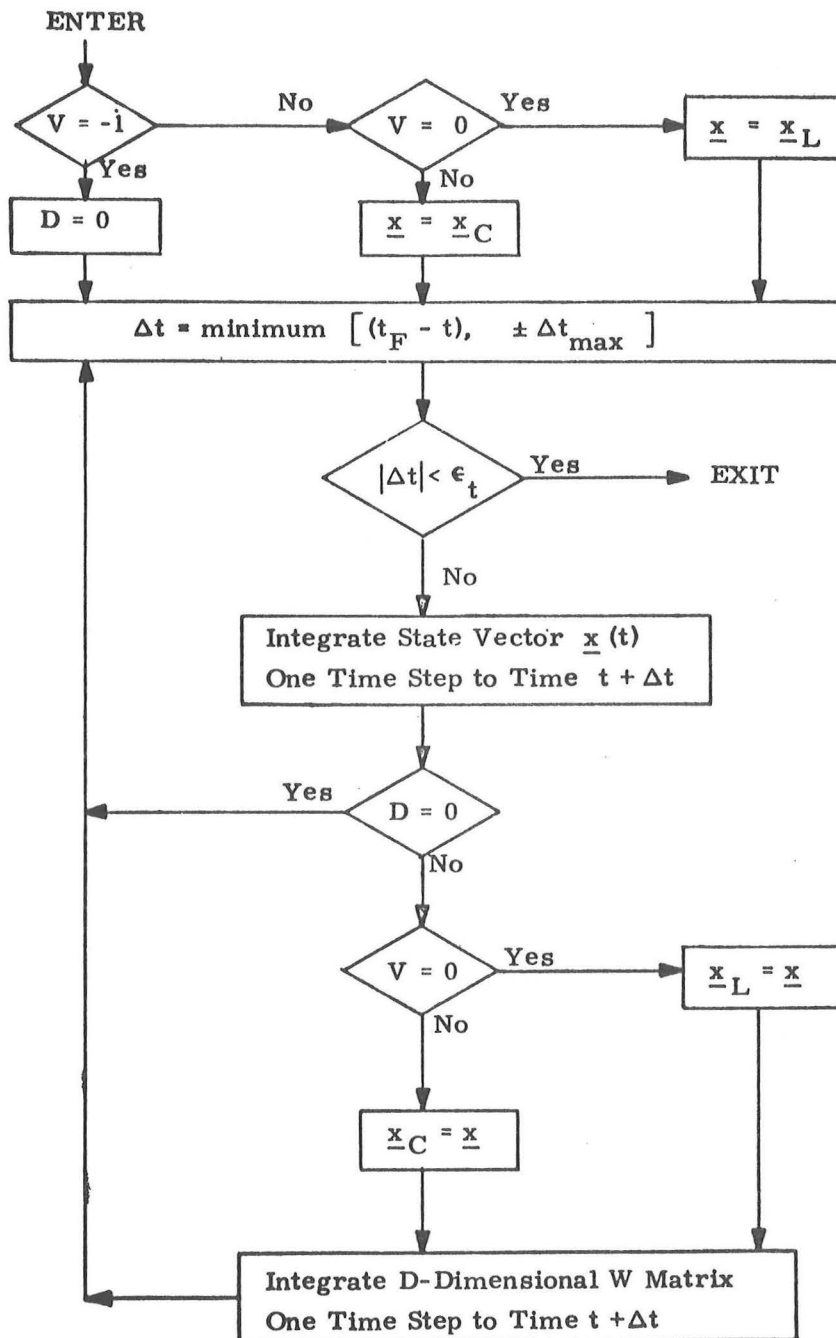


Figure 2. 2-1 Simplified Coasting Integration Routine Logic Diagram



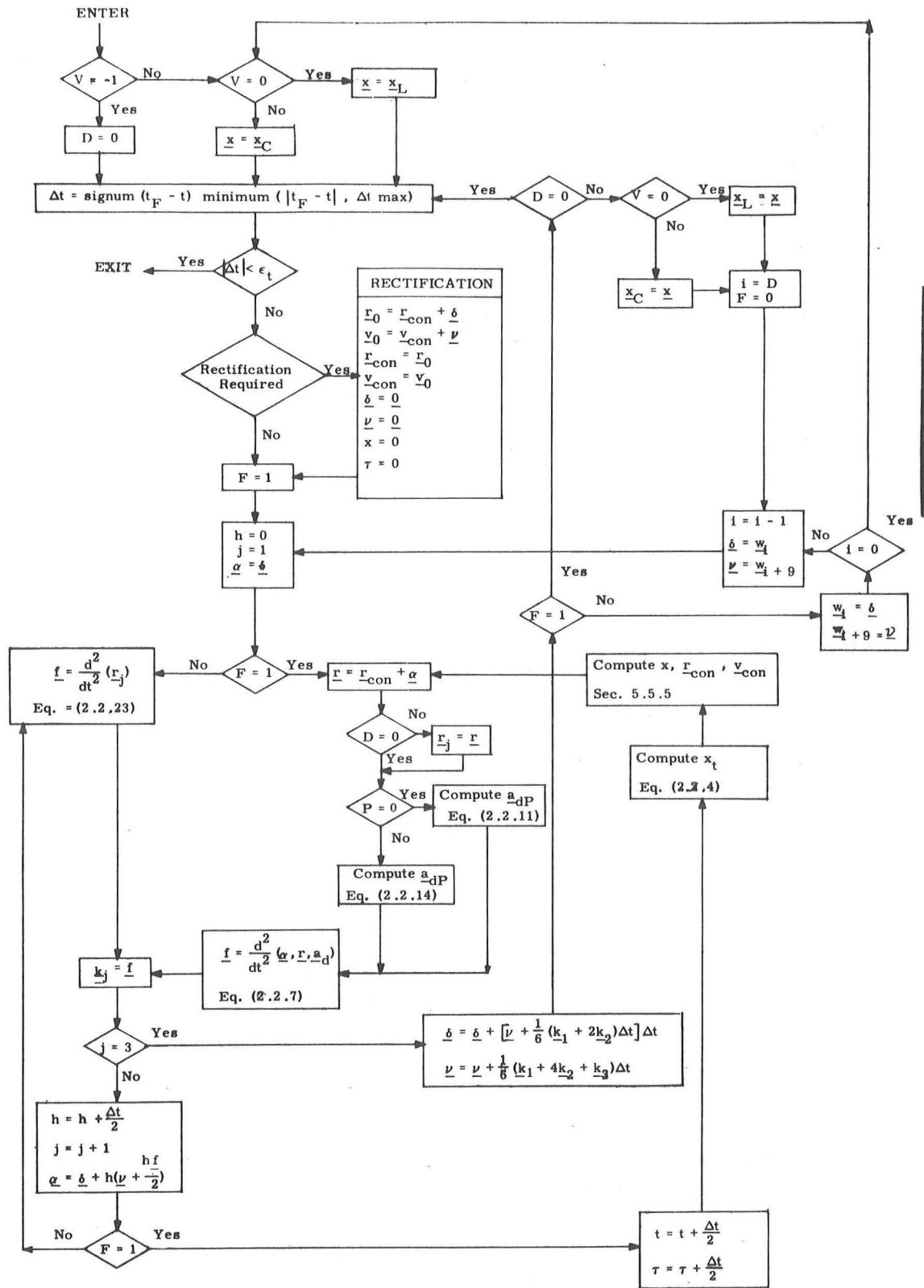


Figure 2.2-2  
Coasting Integration Routine Logic Diagram

5.2-24

In addition to the general criterion discussed in Section 5.2.2.2, the requirements for rectification (which are not shown in Fig. 2.2-2) are functions of

- 1) the computer word length,
- 2) the fact that the computations are performed in fixed-point arithmetic,
- 3) the scale factors of the variables, and
- 4) the accuracy of the Kepler Subroutine (Section 5.5.5).

If

$$\frac{\delta}{r_{\text{con}}} > 0.01$$

or if

$$\delta > \begin{cases} 0.75 \times 2^{22} \text{ m for } P = 0 \\ 0.75 \times 2^{18} \text{ m for } P = 1 \end{cases}$$

or if

$$v > \begin{cases} 0.75 \times 2^3 \text{ m/csec for } P = 0 \\ 0.75 \times 2^{-1} \text{ m/csec for } P = 1 \end{cases}$$

then rectification occurs at the point indicated in Fig. 2.2-2. Also, if the calculation of the acceleration (Eq. (2.2.7)) results in overflow (i. e. any component is equal to or greater than  $2^{-16} \text{ m/csec}^2$  for  $P = 0$ , or  $2^{-20} \text{ m/csec}^2$  for  $P = 1$ ), then the program is recycled to the beginning of the time step and rectification is performed. This procedure assumes the existence of a valid state vector. If an invalid state vector is integrated, then this overflow test can result in an infinite loop.

### 5. 2. 3 MEASUREMENT INCORPORATION ROUTINE

Periodically it is necessary to update the estimated position and velocity vectors of the vehicle (CSM or LM) by means of navigation measurements. At the time a measurement is made, the best estimate of the state vector of the spacecraft is the extrapolated estimate denoted by  $\underline{x}'$ . The first six components of  $\underline{x}'$  are the components of the estimated position and velocity vectors. In certain situations it becomes necessary to estimate more than six quantities. Then, the state vector will be of nine dimensions. From this state vector estimate it is possible to determine an estimate of the quantity measured. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to update the indicated state vector as well as its associated error transition matrix as described in Section 5.2.1. The error transition matrix,  $W$ , is defined in Section 5.2.2.4.

This routine is used to compute deviations to be added to the components of the estimated state vector, and to update the estimated state vector by these deviations provided the deviations pass a state vector update validity test as described in Section 5.2.1.

Let  $D$  be the dimension (six or nine) of the estimated state vector. Associated with each measurement are the following parameters which are to be specified by the program calling this routine:

$\underline{b}$  = Geometry vector of  $D$  dimensions

$\overline{\alpha^2}$  = A priori measurement error variance

$\delta Q$  = Measured deviation, the difference between the quantity actually measured and the expected value based on the original value of the estimated state vector  $\underline{x}'$ .

The procedure for incorporating a measurement into the estimated state vector is as follows:

- 1 Compute a D-dimensional  $\underline{z}$  vector from

$$\underline{z} = W'^T \underline{b} \quad (2.3.1)$$

where  $W'$  is the error transition matrix associated with  $\underline{x}'$ .

- 2 Compute the D-dimensional weighting vector,  $\underline{\omega}$ , from

$$\underline{\omega}^T = \frac{1}{z^2 + \alpha^2} \underline{z}^T W'^T \quad (2.3.2)$$

- 3 Compute the state vector deviation estimates from

$$\delta \underline{x} = \underline{\omega} \delta Q \quad (2.3.3)$$

- 4 If the data pass the validity test, update the state vector and the W matrix by

$$\underline{x} = \underline{x}' + \delta \underline{x} \quad (2.3.4)$$

$$W = W' - \frac{\underline{\omega} \underline{z}^T}{1 + \frac{\alpha^2}{z^2 + \alpha^2}} \quad (2.3.5)$$

In order to take full advantage of the three-dimensional vector and matrix operations provided by the interpreter in the computer, the nine-dimensional W matrix will be stored sequentially in the computer as follows :

$$\underline{w}_0, \underline{w}_1, \dots, \underline{w}_{26}$$

Refer to Section 5. 2. 2. 4 for the definition of the W matrix. Define the three-dimensional matrices

$$W_0 = \begin{pmatrix} \underline{w}_0^T \\ \underline{w}_1^T \\ \underline{w}_2^T \end{pmatrix} \quad W_1 = \begin{pmatrix} \underline{w}_3^T \\ \underline{w}_4^T \\ \underline{w}_5^T \end{pmatrix} \quad \dots \quad W_8 = \begin{pmatrix} \underline{w}_{24}^T \\ \underline{w}_{25}^T \\ \underline{w}_{26}^T \end{pmatrix} \quad (2. 3. 6)$$

so that

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \quad (2. 3. 7)$$

Let the nine-dimensional vectors  $\delta \underline{x}$ ,  $\underline{b}$ ,  $\underline{\omega}$ , and  $\underline{z}$  be partitioned as follows:

$$\delta \underline{x} = \begin{pmatrix} \delta x_0 \\ \delta x_1 \\ \delta x_2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \quad \underline{\omega} = \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \end{pmatrix} \quad \underline{z} = \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_8 \end{pmatrix} = \begin{pmatrix} z_0 \\ z_1 \\ z_2 \end{pmatrix} \quad (2. 3. 8)$$

Then, the computations shown in Eqs. (2.3.1) through (2.3.3) are performed as follows, using three-dimensional operations:

$$\underline{z}_i = \sum_{j=0}^{\frac{D}{3}-1} W'_{i+3j} \underline{b}_j$$

$$a = \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j \cdot \underline{z}_j + \alpha^2$$

(2.3.9)

$$\underline{\omega}_i^T = \frac{1}{a} \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j^T W'_{3i+j}$$

$$\delta \underline{x}_i = \delta Q \underline{\omega}_i \quad \left( i = 0, 1, \dots, \frac{D}{3} - 1 \right)$$

Equation (2.3.5) is written

$$\gamma = \frac{1}{1 + \sqrt{\alpha^2/a}}$$

(2.3.10)

$$\underline{w}_{i+9j} = \underline{w}'_{i+9j} - \gamma \underline{z}_i \underline{\omega}_j \quad \left( \begin{array}{l} i = 0, 1, \dots, D-1 \\ j = 0, 1, \dots, \frac{D}{3} - 1 \end{array} \right)$$

The Measurement Incorporation Routine is divided into two subroutines, INCORP1 and INCORP2. The subroutine INCORP1 consists of Eqs. (2.3.9), while INCORP2 is composed of Eqs. (2.3.4) and (2.3.10). The method of using these subroutines is illustrated in Fig. 2.3-1.

Since the estimated position and velocity vectors are maintained in two pieces, conic and deviation from the conic, Eq. (2.3.4) cannot be applied directly. The estimated position and velocity deviations resulting from the measurement,  $\delta \underline{x}_0$  and  $\delta \underline{x}_1$ , are added to the vectors  $\underline{\delta}$  and  $\underline{\nu}$ , the position and velocity deviations from the conics, respectively. Since  $\underline{\delta}$  and  $\underline{\nu}$  are maintained to much higher accuracy than the conic position and velocity vectors, a possible computation overflow situation exists whenever Eq. (2.3.4) is applied. If overflow does occur, then it is necessary to reinitialize the Coasting Integration Routine (Section 5.2.2) by the process of rectification as described in Section 5.2.2.2. The logic flow of the subroutine INCORP2 is illustrated in detail in Fig. 2.3-2.

Overflow occurs when

$$\delta \underline{\nu} > \begin{cases} 2^{22} & \text{m for } P = 0 \\ 2^{18} & \text{m for } P = 1 \end{cases}$$

or

$$\nu \underline{\nu} > \begin{cases} 2^3 & \text{m/csec for } P = 0 \\ 2^{-1} & \text{m/csec for } P = 1 \end{cases}$$

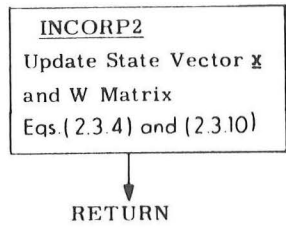
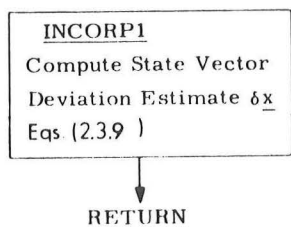
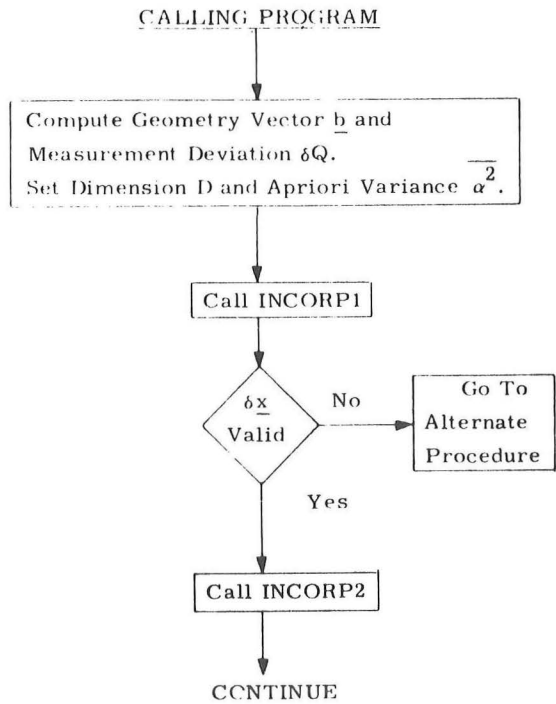


Fig. 2.3-1 Measurement Incorporation Procedure



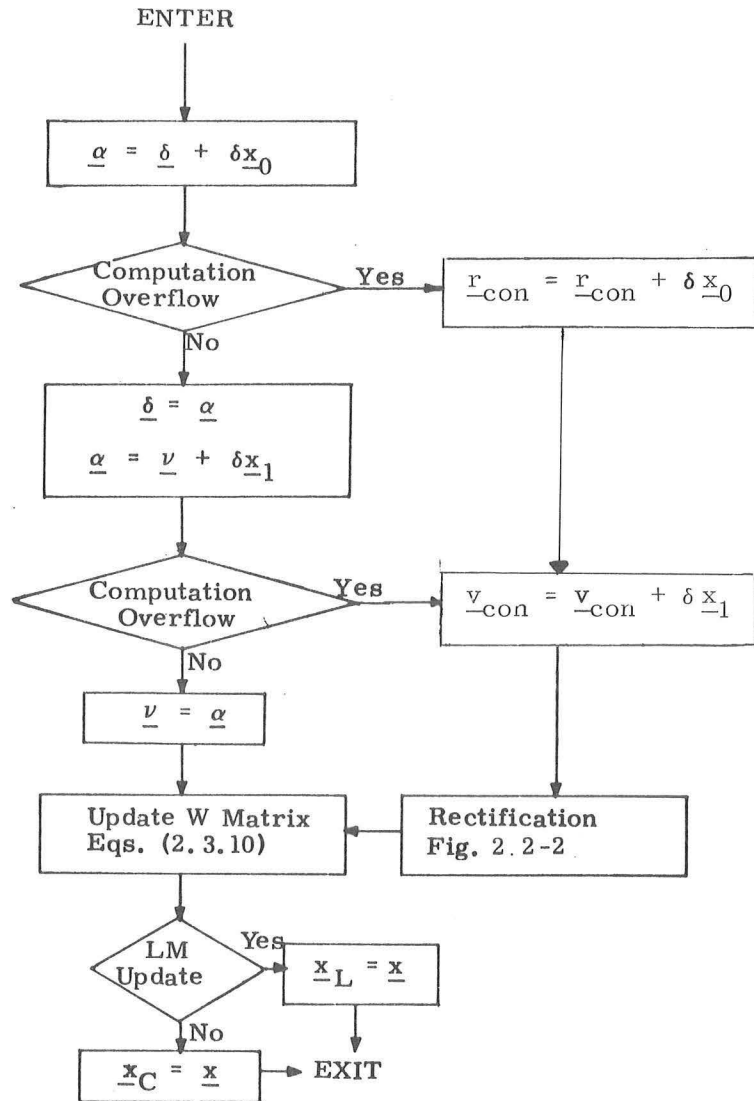


Fig. 2.3-2 INCORP2 Subroutine Logic Diagram

## 5. 2. 4 RENDEZVOUS NAVIGATION PROGRAM

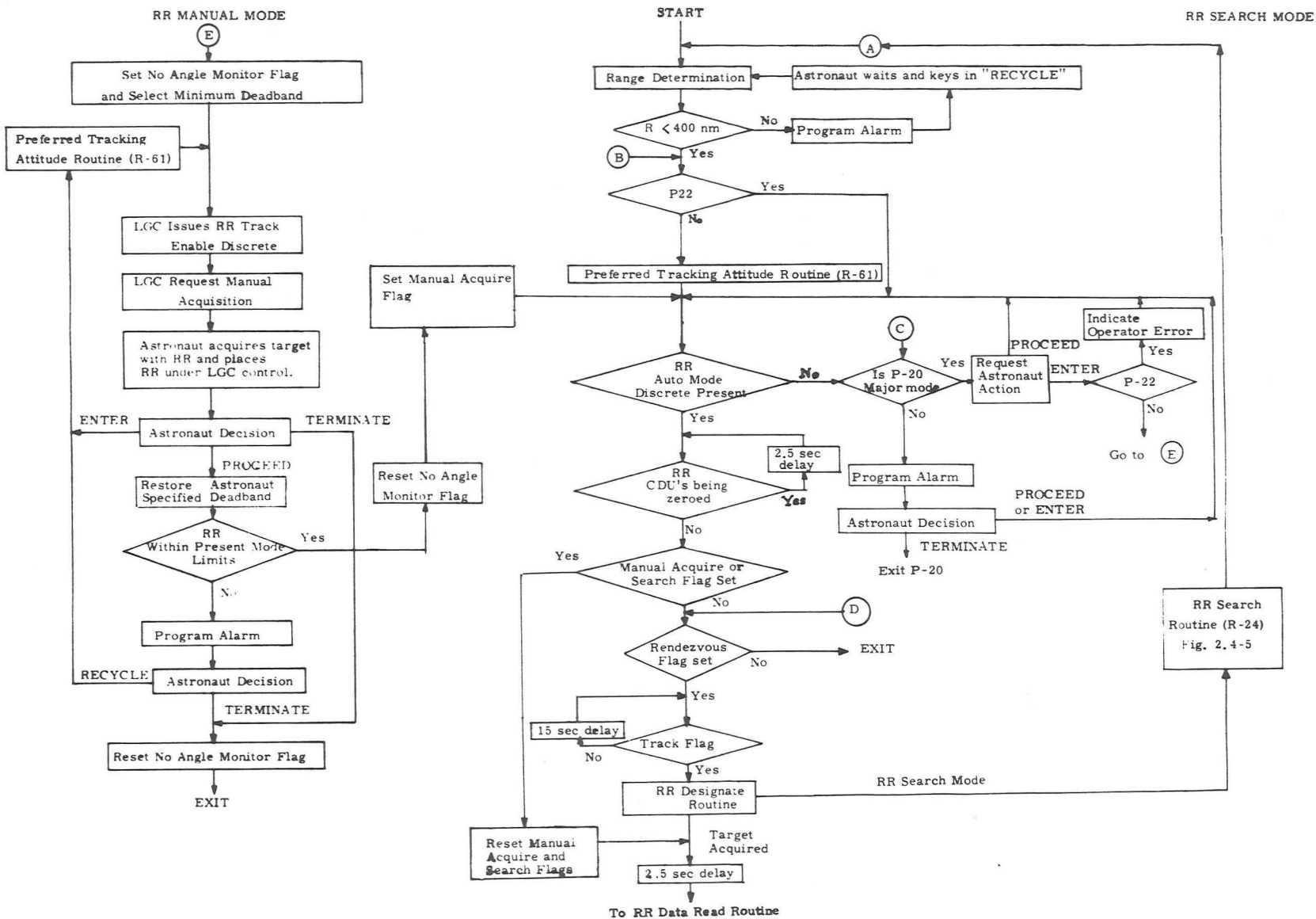
### 5. 2. 4. 1 Target Acquisition Routine

One of the first functions performed by the Rendezvous Navigation Program is to use the Target Acquisition Routine (Fig. 2. 4-1) to establish lock-on of the LM Rendezvous Radar (RR) with the transponder on the CSM. Since the problem of acquiring the CSM with the RR is essentially the same for both the Rendezvous and Lunar Surface Navigation Programs (P-20 and P-22, respectively), the same Acquisition Routine is used for both except for certain differences in operation. For the moment, however, most of the operational details presented are for the case when the Rendezvous Navigation Program is being used. In either case it is assumed at the start of the routine that the RR is on and has been permitted to warm-up to operating conditions.

There are three modes (RR LGC, RR Manual, and RR Search) for controlling the RR in target acquisition. The RR LGC and RR Manual Modes can be selected by the astronaut at the beginning of program P-20 by a procedure described later. However, the RR Search Mode can be selected only after the RR LGC Mode has failed to acquire the target. The RR Manual Mode is not used in P-22.

Prior to using the Target Acquisition Routine, program P-20 sets the Rendezvous, Track, and Update flags and resets the Surface Navigation, Manual Acquire, No Angle Monitor, and Search flags. The Rendezvous flag is set to denote that program P-20 or P-22 is being used. When this flag is reset, programs P-20 and P-22 are terminated. The purpose of the other flags is given in the following sections. In addition to initializing the above flags, the state vector update option is automatically set to the LM in program P-20; and the LM and CSM state vectors are extrapolated to current time using the Coasting Integration Routine (Section 5. 2. 2).

Figure 2.4-1 Target Acquisition Routine



5.2-34

At the start of the Target Acquisition Routine in Fig. 2. 4-1 the range is determined between the LM and CSM by taking the vector difference between the LM and CSM position vectors propagated to that time. If the range is greater than 400 nm, a program alarm is issued since the RR is unable to provide the correct range information to the LGC because of the ranging technique used in the radar.

After verifying that the range between the LM and CSM is less than 400 nm, the Preferred Tracking Attitude Routine (R-61 of Section 4) is used to align the LM +Z-axis along the LOS to the CSM. This is done to insure a sufficient period of data taking for estimation of the RR angle biases before the angle between the RR antenna and the LM Z-axis reaches 30 degrees. During the RR data taking process the LM is controlled to hold an approximate inertially fixed attitude within the astronaut selected deadband. The RR Data Read Routine, which is described in Section 5. 2. 4. 2. 1, is used to detect when the RR antenna is more than 30 degrees from the +Z-axis and causes the +Z-axis to be re-aligned with the LOS using the Preferred Tracking Attitude Routine. In addition to meeting the requirements for estimating RR angle biases, it is also essential that the above attitude constraints be used because of the limited radiation coverage of the LM optical beacon which must be directed towards the CSM so as to enable the astronaut in the CSM to optically sight upon it. The beacon is centered with respect to the LM +Z-axis and has a beamwidth of approximately 60 degrees.

After using the Preferred Tracking Attitude Routine in Fig. 2. 4-1 the LGC checks to see if the RR Auto Mode discrete is being received from the RR. This discrete signifies that the RR is on and has been placed under LGC control by a mode control switch associated with the RR. In addition, reception of this discrete at this time automatically indicates that the astronaut wishes to use the RR LGC Mode of target acquisition. If the discrete is not present, and P-20 is the major program in operation, a request is made to the astronaut to either select the RR Manual Mode by keying in "ENTER" or the RR LGC Mode by placing the RR mode control switch in the LGC position and keying in "PROCEED". If program P-20 is not the major mode at this point, a program alarm is issued to the astronaut requiring the action indicated in Fig. 2. 4-1. This latter step is taken to avoid conflict in DSKY displays between program P-20 and the program which is the major mode.

#### 5.2.4.1.1 Target Acquisition with the RR LGC Mode

If the check on the RR Auto Mode discrete in Fig. 2.4-1 indicates that the RR LGC Mode was selected, the routine next insures that the RR CDU's are not being zeroed. These CDU's are zeroed by the RR Monitor Routine (R-25) of Section 5.2.4.3 whenever the RR Auto Mode discrete is received from the RR. Afterwards, a check is made to see if either the Manual Acquire or Search flag is set, signifying that the RR Manual or RR Search Mode is being used. Since the present mode is assumed to be the RR LGC Mode, these flags should not be set. The purpose of the check on the Track flag just prior to using the RR Designate Routine is to insure that the RR is not designated by the LGC until this flag is set. The Track flag is reset by various LGC programs whenever there is no desire to designate or read data from the RR. Although the Track flag is set at the start of program P-20, it is possible for it to be temporarily reset during the operation of program P-20.

To designate the RR along the LOS to the CSM use is made of the RR Designate Routine of Fig. 2.4-2. Initially, the routine removes the RR Track Enable discrete, if it is present, to insure that the RR will respond to its designate commands. Afterwards, the LOS range vector ( $r_{\text{LOS}}$ ) and the velocity ( $v_{\text{LC}}$ ) of the CSM with respect to the LM in stable member coordinates are computed as shown in Fig. 2.4-2, and a check is made to see if the target LOS is within the angular operation limits of either RR antenna mode shown in Fig. 2.4-3. This is accomplished by first computing the equivalent RR shaft and trunnion angles for  $r_{\text{LOS}}$  with the method shown in Section 5.6.15.2 where

$$u_{\text{D}} = \left[ \text{SMNB} \right] \text{UNIT} (r_{\text{LOS}})$$

The equivalent RR shaft and trunnion angles are then compared to the electrical tracking mode limits. The electrical tracking limits within which accurate RR measurements may be made for each of the two antenna modes are shown in Fig. 2.4-3. The LGC internal mode test is based on the criteria described in Section 5.2.4.3 for the RR Monitor Routine (R-25). Routine R-25 continually updates the computer's knowledge of which mode the RR antenna is actually in, even though the antenna may not be within the tracking limits of that mode.

If the Reposition flag should be set by the RR Monitor Routine of Section 5.2.4.3 during the operation of the RR Designate Routine, a program alarm is issued as shown in Fig. 2.4-2. This flag when set denotes that the RR is being repositioned to the reference position for the present RR antenna mode.

The RR Designate Routine designates the RR antenna by issuing rate commands to the RR antenna gyros proportional to the angular difference between the indicated direction of the antenna and

$$\underline{r}'_{LOS} = \underline{r}_{LOS} + \underline{v}_{LC} (1 \text{ sec.})$$

where  $\underline{r}_{LOS}$  is the present range vector to the CSM,  $\underline{v}_{LC}$  is the relative velocity of the CSM with respect to the LM expressed as the distance covered in a one second time interval assuming a constant velocity, and  $\underline{r}'_{LOS}$  is essentially  $\underline{r}_{LOS}$  advanced one second into the future. This correction to  $\underline{r}_{LOS}$  for RR designation essentially compensates for the lag error associated with the present type of control system existing between the RR and the LGC. The method used to compute the gyro rate commands is given in Section 5.6.15.3.

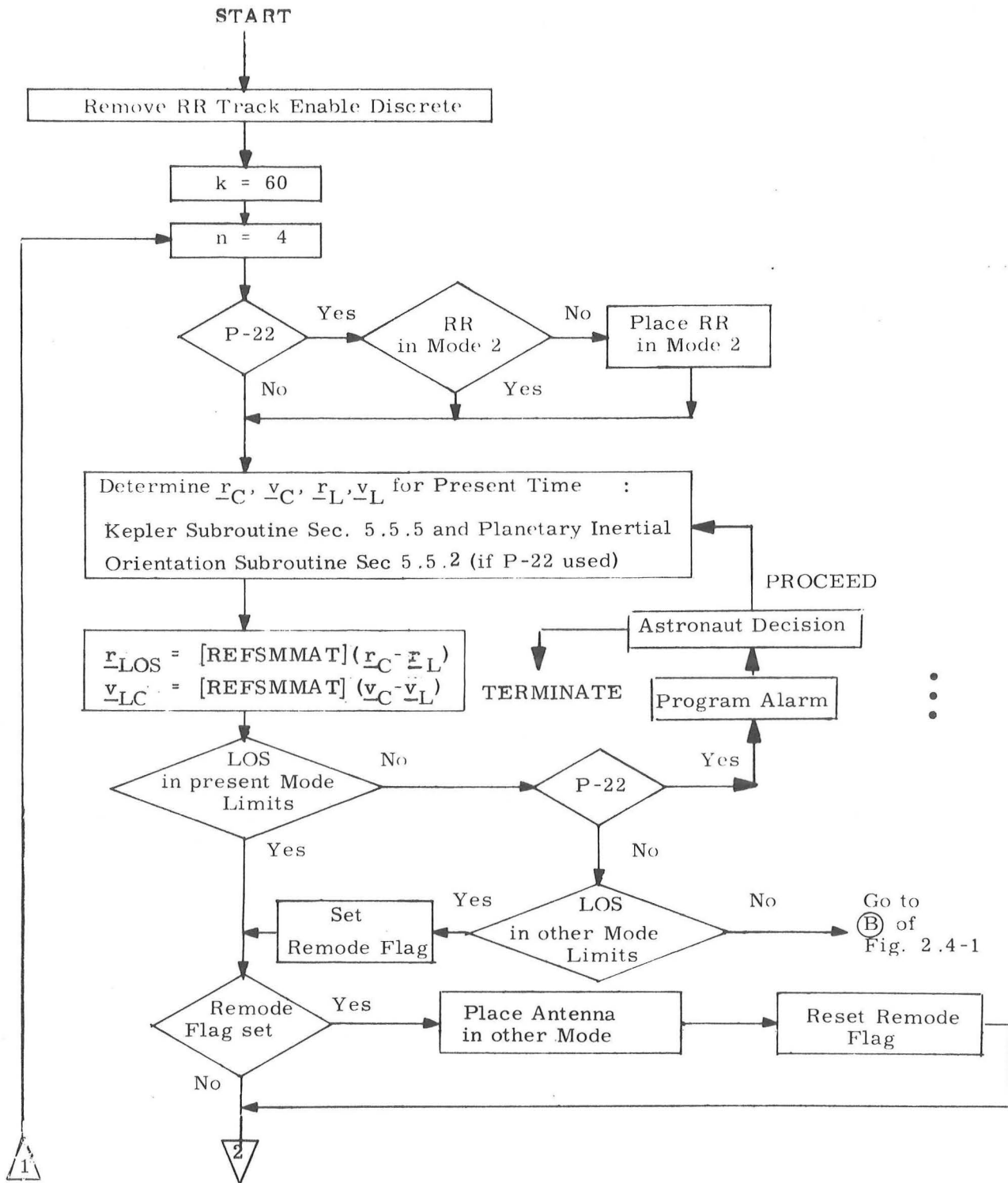


Figure 2.4-2 RR Designate Routine (Page 1 of 2)

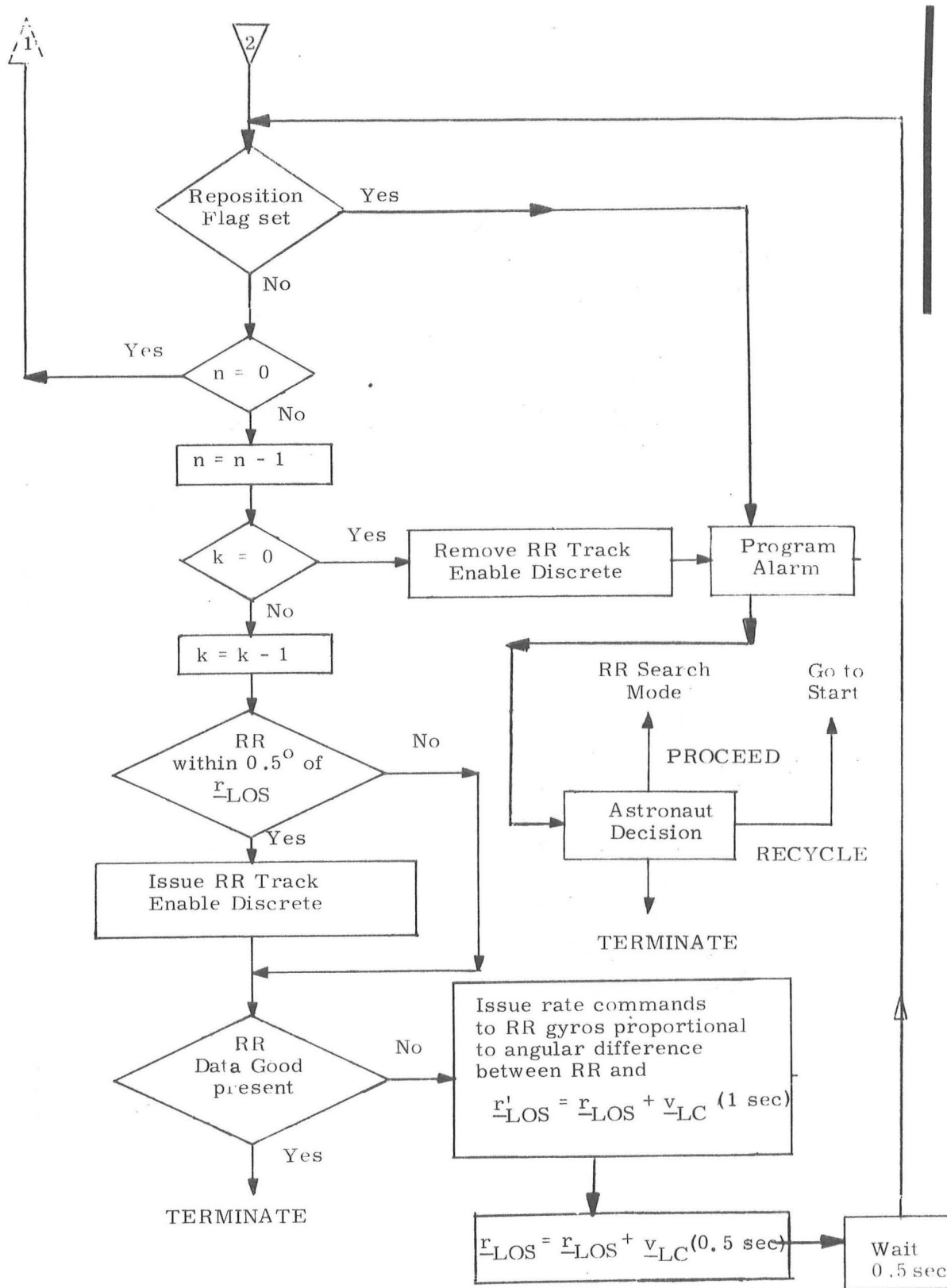


Figure 2.4-2 RR Designate Routine (Page 2 of 2)

5.2-37

Revised SUNDANCE.

Added GSOP # R-557

PCR # 414

Rev. 1

Date 3-5-68



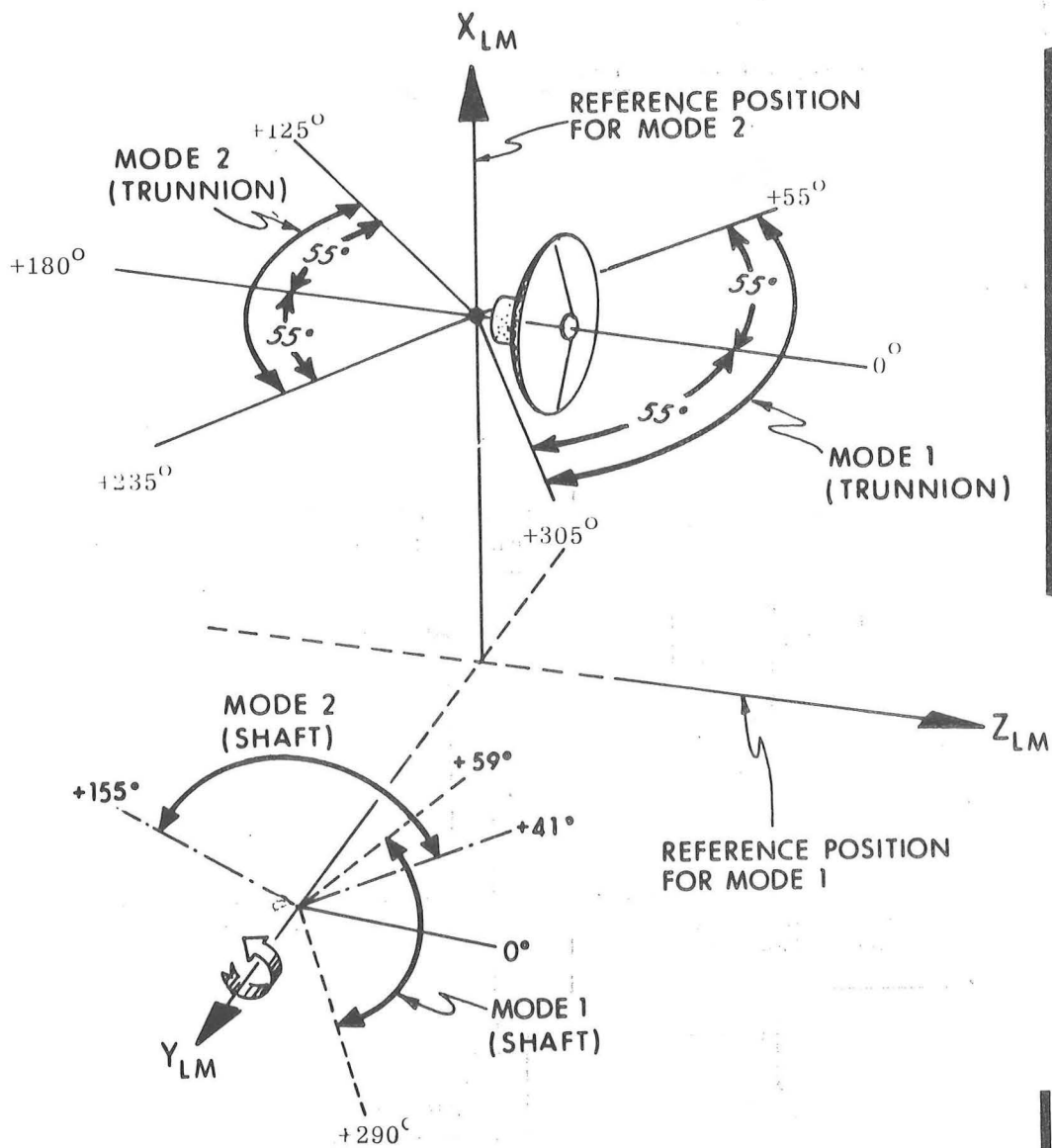


Figure 2.4-3 RR Antenna Shaft and Trunnion LOS Tracking Regions

5.2-38

Revised SUNDANCE

Added GSOP # R-557

PCR # 414

Rev. 1

Date 3-5-68

Note that new rate commands are issued to the RR gyros and an approximate update for LOS motion is made to  $\underline{r}_{LOS}$  [i. e.  $\underline{r}_{LOS} = \underline{r}_{LOS} + \underline{v}_{LC}$  (0.5 sec.)] about every 0.5 seconds until the counter n has been decremented from 4 to 0. When n = 0, the routine returns to the top of Fig. 2. 4-2 to compute new values of  $\underline{r}_{LOS}$  and  $\underline{v}_{LC}$  using the CSM and LM state vectors ( $\underline{r}_C, \underline{v}_C, \underline{r}_L, \underline{v}_L$ ); and the previously mentioned process of checking the LOS with respect to the RR antenna coverage mode is performed before issuing new commands to the RR gyros.

Near the bottom of Fig. 2. 4-2 it is seen that checks are made on the RR Data Good discrete and the pointing direction of the RR prior to issuing new rate commands to the RR gyros. When the RR is within 0.5 degrees of  $\underline{r}_{LOS}$ , the RR Track Enable discrete is issued to the RR, enabling its angle tracking servos to track the target if its range-rate tracking network has already acquired the target. Issuance of the RR Track Enable discrete also initiates the RR range tracker search. The RR Data Good discrete is sent to the LGC by the RR when lock-on has been achieved in range and range-rate and the RR Track Enable discrete has been received from the LGC.

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- 
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In the event of failure to receive the RR Data Good discrete after issuing new rate commands to the RR gyros 60 times, it is seen in Fig. 2.4-2 that the RR Track Enable discrete is removed and a program alarm is issued to the astronaut whereupon he either repeats the designate process or goes to the RR Search Mode.

If the RR Data Good discrete is received during the designate process, the routine is terminated and it is seen in Fig. 2.4-1 that a 2.5 second delay is introduced before going to the RR Data Read Routine. This delay permits any transients in the RR angle tracking servos to settle out before data is taken.

#### 5.2.4.1.2 Target Acquisition With the RR Manual Mode

In Fig. 2.4-1 it is seen that the RR Manual Mode of target acquisition can be obtained by keying in "ENTER" after the LGC discovers that the RR Auto Mode discrete is not present. Absence of this discrete at the beginning of program P-20 can be insured by not placing the mode control switch of the RR in the LGC position. The logic associated with the RR Manual Mode is shown in the left column of Fig. 2.4-1. Initially, the No Angle Monitor flag is set, the minimum deadband of the RCS DAP is selected, and the RR Track Enable discrete is issued to the RR. The No Angle Monitor flag is set during the RR Manual Mode so as to disable the angle monitor function of the RR Monitor Routine (see Section 5.2.4.3). Selection of the minimum deadband permits the astronaut to manually designate the RR more accurately. The RR Track Enable discrete, although it has no effect on manual control of the RR, is issued at this time so that there is no loss of RR angle tracking when the mode control switch of the RR is placed in the LGC position after manual target acquisition.

Afterwards, the LGC requests the astronaut to perform the manual acquisition. If manual acquisition is achieved, the astronaut places the RR under LGC control, waits until the radar panel NO TRACK light is extinguished, and keys in "PROCEED". This procedure is necessary in order to insure that the RR range tracking network has locked onto the target before entering the RR Data Read Routine. When the RR mode control switch is placed in the LGC position RR range tracking is interrupted by switching and a new range search is initiated. When the RR acquires the target in both range and range rate, the NO TRACK light is extinguished.

After the "PROCEED", the astronaut specified deadband is restored and a check is made to see if the RR is within the limits of the present coverage mode. If the RR is not within the coverage limits, a program alarm is issued and the astronaut either terminates the program or repeats the manual acquisition process as shown in Fig. 2.4-1. Note that the Preferred Tracking Attitude Routine is used to re-align the LM +Z-axis with the target LOS whenever he elects to repeat the manual acquisition process.

Once target acquisition has been achieved and the RR is found to be within the present antenna mode limits, the No Angle Monitor Flag is reset, and the Manual Acquire flag is set to denote manual acquisition. The remaining steps are indicated in Fig. 2.4-1.

### 5.2.4.1.3 Target Acquisition With the RR Search Mode

In the RR Designate Routine (Fig. 2.4-2) it is seen that the astronaut may select the RR Search Mode if the RR Designate Routine fails to acquire the target. The RR Search Mode is obtained by using the RR Search Routine (R-24 of Section 4) to designate the RR antenna in a hexagonal search pattern about the estimated LOS. The search pattern is a box with six sides where the side to side dimensions are  $5.6^\circ$  as shown in Fig. 2.4-4. At the beginning of this mode the RR is designated for six seconds along the estimated LOS to the target defined by the unit vector  $\underline{u}_{LOS}$  in Fig. 2.4-4. Afterwards, the LGC sequentially designates the RR to each corner of the hexagon for a period of six seconds. Having completed the designate to each corner, the LGC repeats the above process starting with a designate along  $\underline{u}_{LOS}$  for six seconds. The time required to generate this search pattern is approximately 48 seconds.

The logic flow associated with the RR Search Routine is shown in Fig. 2.4-5. Initially, the Search flag is set to denote that this mode is being used and a display of certain RR search parameters is instigated to which the astronaut must respond later. The RR Track Enable discrete is then issued to the RR so that the RR may acquire the target during the search pattern generation. This discrete is also re-issued at each corner of the search pattern in case it has been removed by some source such as the RR Monitor Routine.

Approximately every six seconds the position and velocity vectors of the CSM and LM are used to compute the line-of-sight unit vector ( $\underline{u}_{LOS}$ ) in basic reference coordinates and the relative velocity ( $\underline{v}_{LC}$ ) in stable member coordinates. The routine then computes the desired RR pointing direction ( $\underline{u}_D$ ) which may be

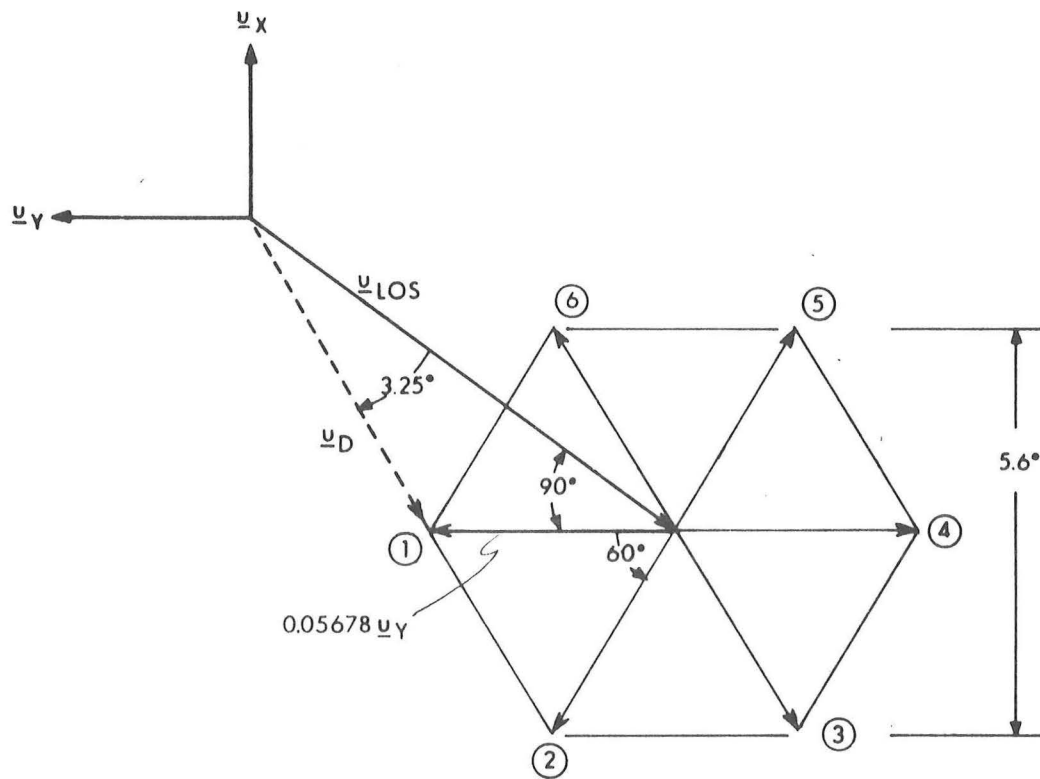


Fig. 2.4-4 RR Search Pattern

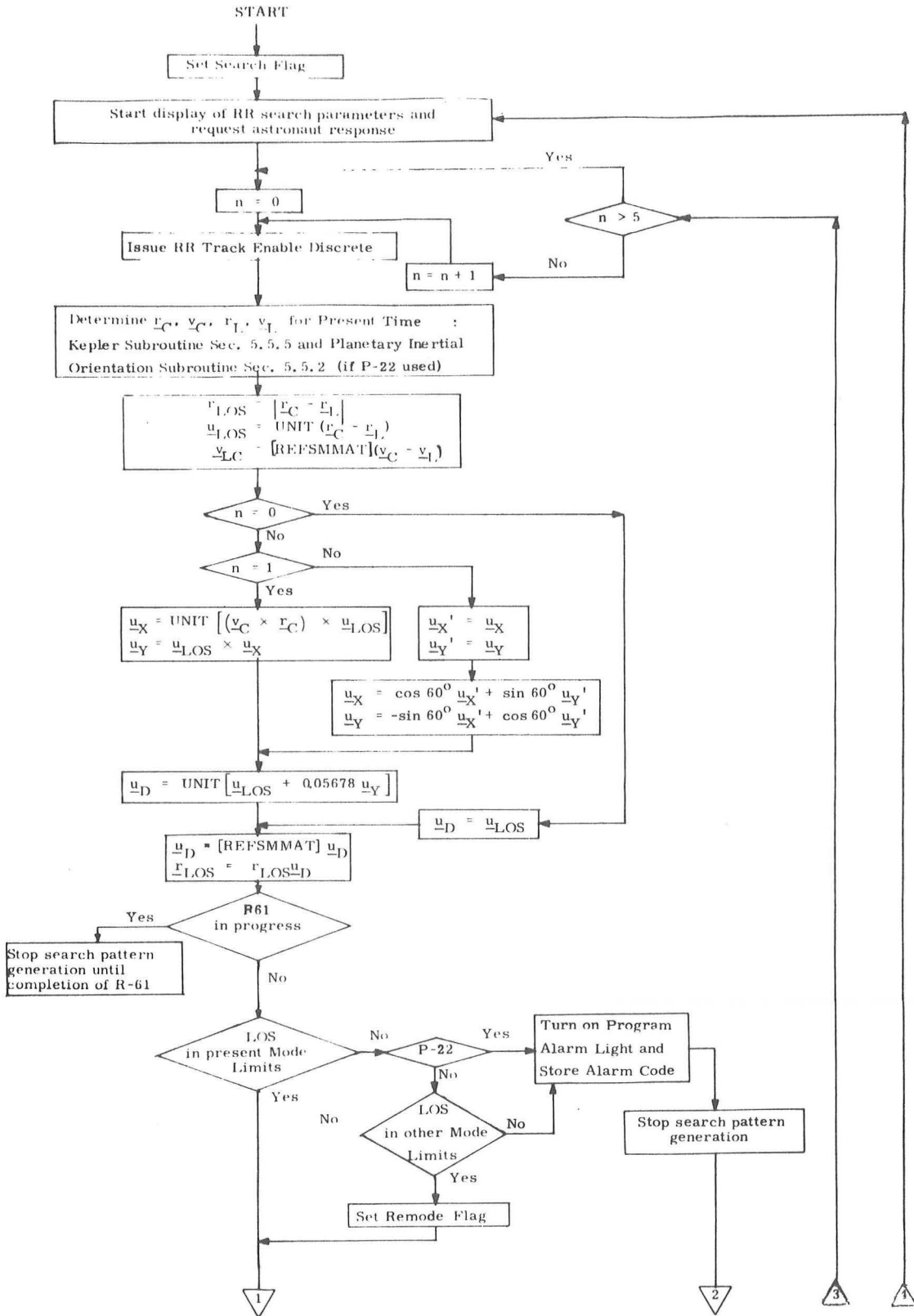


Figure 2.4-5 RR Search Routine  
(page 1 of 2)

5.2-44

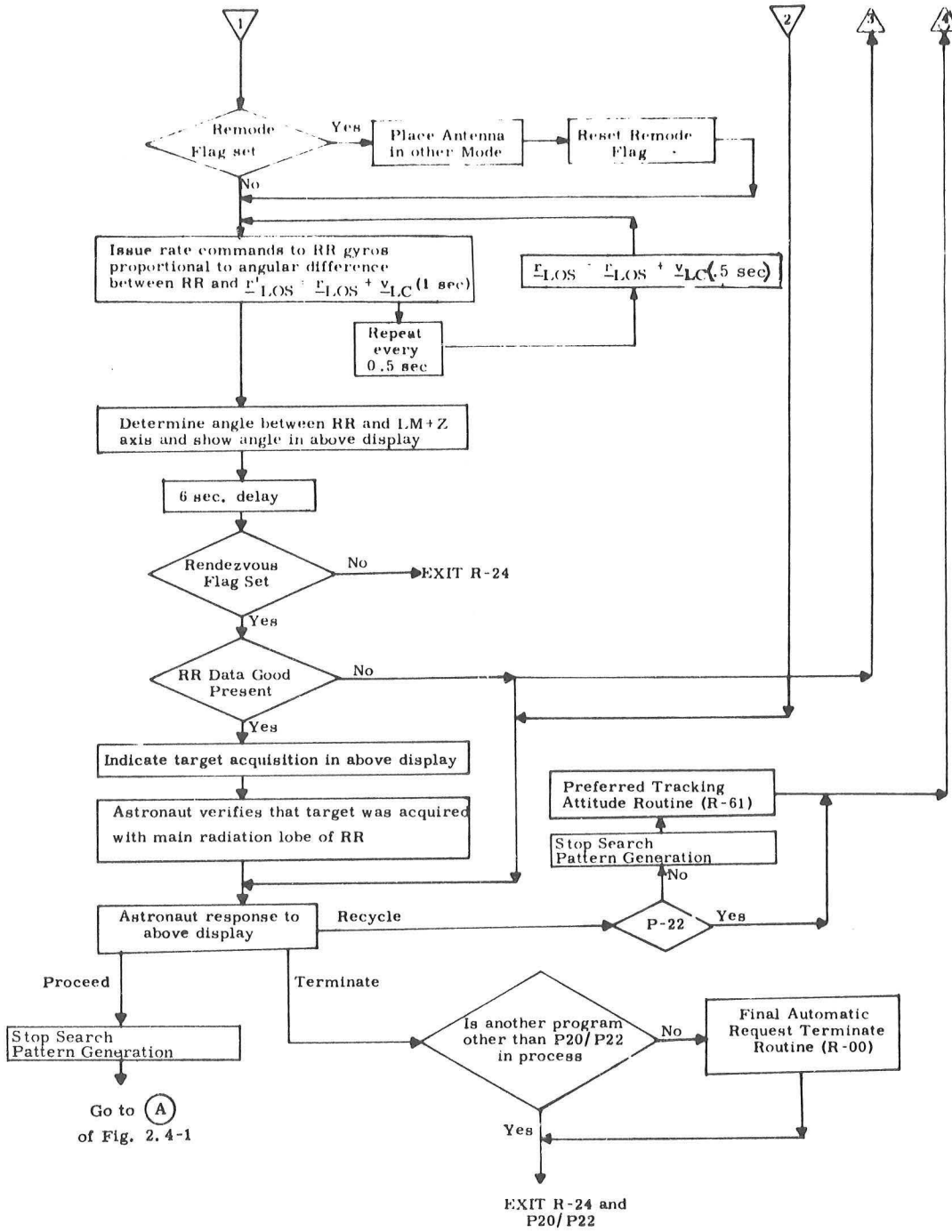


Figure 2.4-5 RR Search Routine  
(page 2 of 2)

5. 2-44A



along  $\underline{u}_{LOS}$  or to one corner of the search pattern (see Figure 2.4-4). Afterwards,  $\underline{u}_D$  is transformed to stable member coordinates and a check is made on the direction of  $\underline{u}_D$  with respect to the angular coverage modes of the RR antenna, just as was done with  $\underline{r}_{LOS}$  in the RR Designate Routine in Section 5.2.4.1.1. If  $\underline{u}_D$  is not within the angular coverage limits of either mode during program P-20, the program alarm light is turned on, an alarm code is stored, and the search pattern is stopped. If the astronaut wishes to continue the search in program P-20, he must re-established the preferred tracking attitude with the Preferred Tracking Attitude Routine (R-61) in the manner shown at the end of Fig. 2.4-5. Once  $\underline{u}_D$  is found to be within the coverage limits and the correct antenna mode has been established, the routine proceeds to designate the RR by issuing rate commands to the RR gyros about every 0.5 seconds with approximate corrections being made each time for lag error and target motion.

Note in Fig. 2.4-5 that the angle between the RR and the LM +Z-axis is periodically determined and displayed as one of the RR search parameters during the search operation. By observing the displayed angle the astronaut can determine during program P-20 when he should re-establish the preferred tracking attitude with the Preferred Tracking Attitude Routine (R-61) in the manner shown at the end of Fig. 2.4-5.

Near the end of Fig. 2.4-5 it is seen that a periodic check is made to see if the RR Data Good discrete is being received from the RR, signifying that the RR has acquired the target in range and range rate. If this discrete is present, the search pattern is stopped and the astronaut is notified. The astronaut then checks to see if acquisition was obtained with the main radiation lobe of the RR. By manually positioning the RR and observing the RR signal strength meter, he should be able to distinguish the main lobe from any side lobes. Having achieved and verified lock-on with the main radiation lobe, the astronaut places the RR mode control switch in the LGC position, waits until the radar panel NO TRACK light is extinguished, and keys in a "PROCEED". The remaining steps are indicated in Fig. 2.4-1.

## 5.2.4.2 Rendezvous Navigation Routine

### 5.2.4.2.1 RR Data Read Routine

During operation of the Rendezvous Navigation Routine use is made of the RR Data Read Routine to obtain measurement data from the RR. The logic associated with the RR Data Read Routine is given in Fig. 2.4-6. Like the Target Acquisition Routine, this routine is used by both the Rendezvous and Lunar Surface Navigation Programs, with different paths being taken at various points in the routine depending on which program (P-20 or P-22) is in operation. The RR Data Read Routine periodically obtains a complete set of data from the RR (range, range rate, shaft angle, and trunnion angle) for purposes of navigation, although only range and range rate data is used for updating during lunar surface navigation. When program P-20 is being used the maximum frequency of update of the navigation equations with a complete set of RR data is about once per minute. This frequency is based upon a rough estimate\* of 15 seconds to read the RR and perform the rendezvous navigation computations and a 45 second wait period before repeating the process.

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\*The time to complete this calculation is highly dependent on other computational priorities. No explicit control of this procedure is exercised.

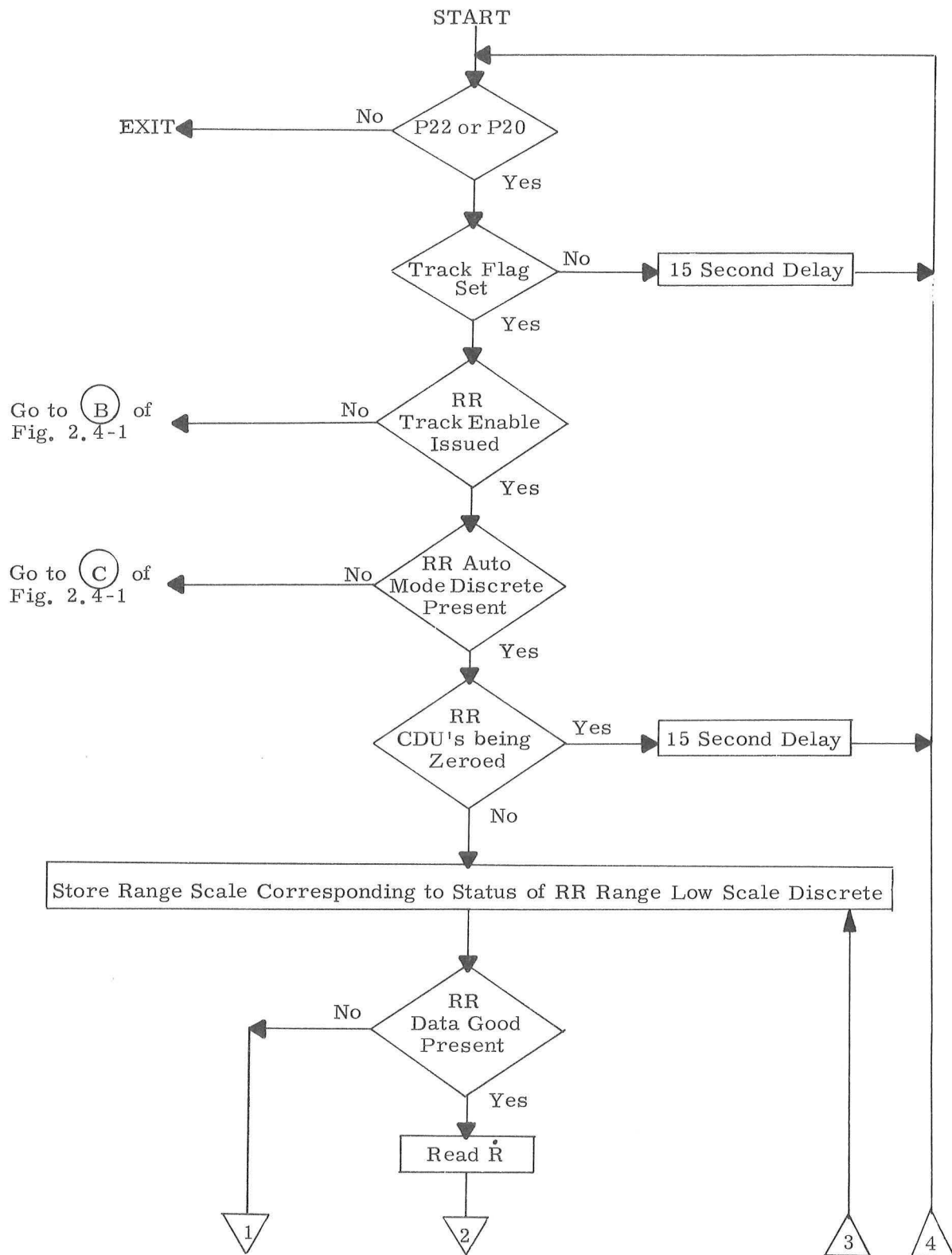


Figure 2.4-6 RR Data Read Routine ( Page 1 of 3 )

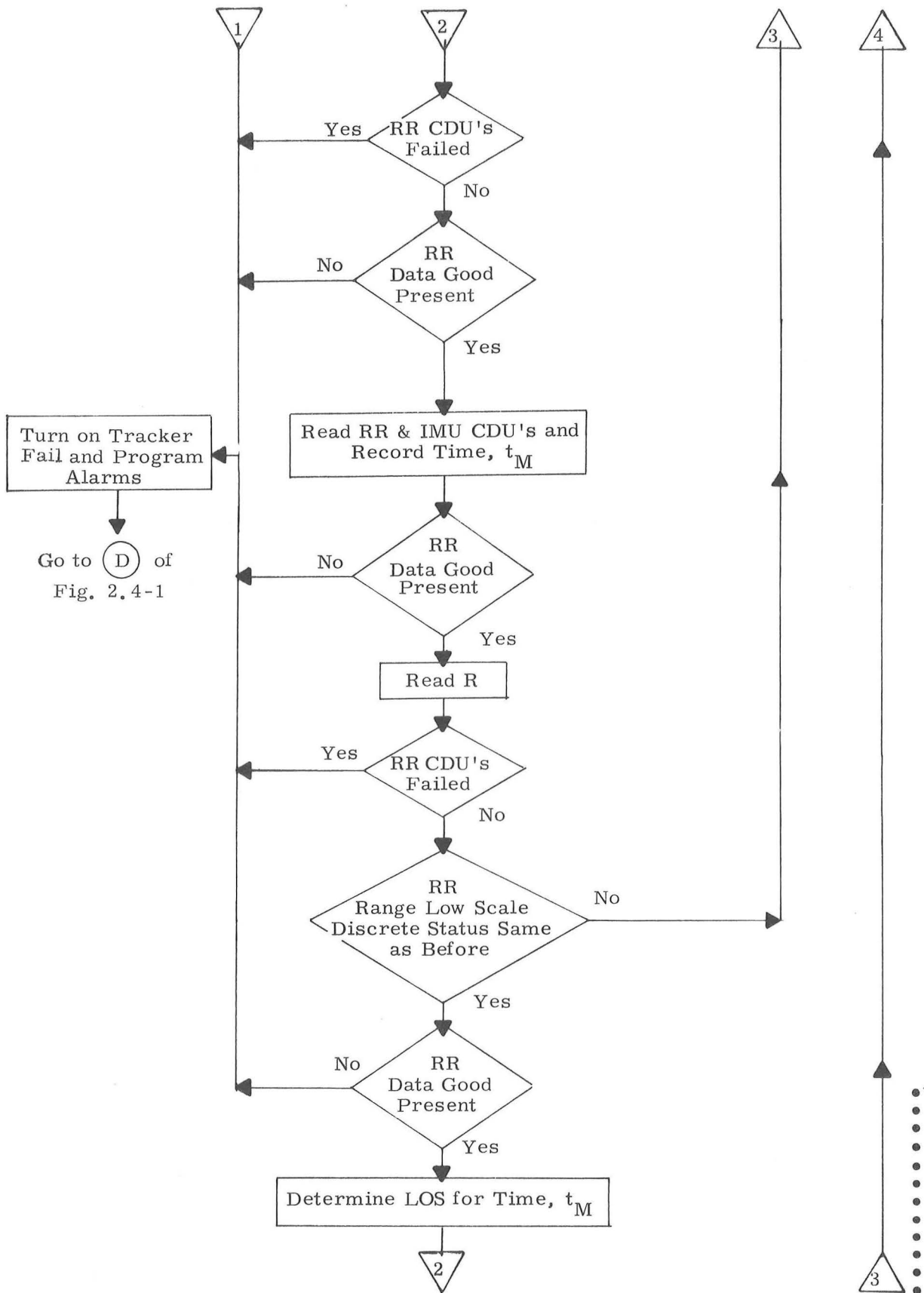


Figure 2.4-6 RR Data Read Routine ( Page 2 of 3 )

5.2-47A

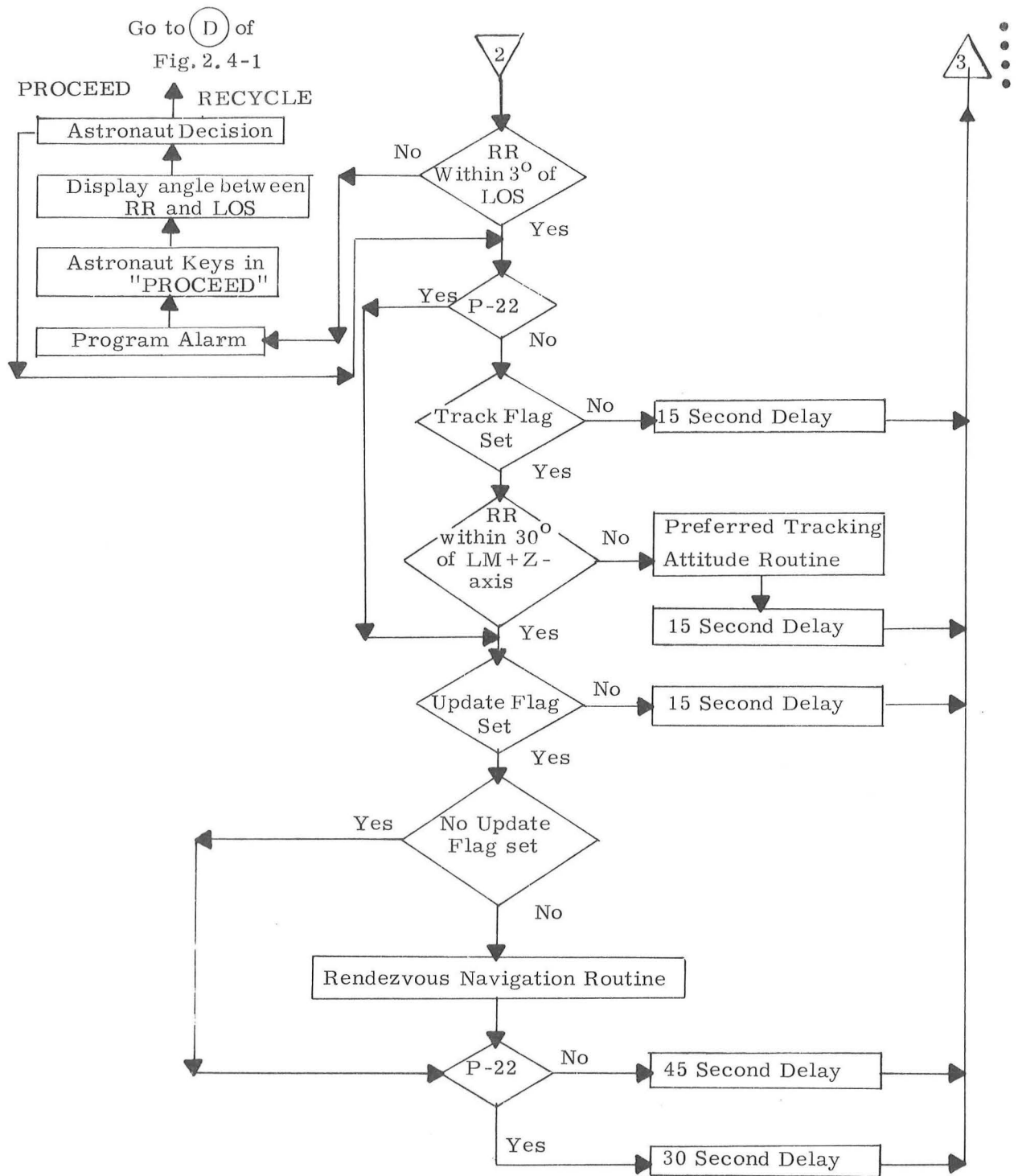


Figure 2.4-6 RR Data Read Routine ( Page 3 of 3 )

In this section an explanation will be given only for those logic steps in Fig. 2.4-6 associated with use of the RR Data Read Routine during operation of the Rendezvous Navigation Program (P-20).

In Fig. 2.4-6, the first check made by the routine, after establishing that P-20 or P-22 is being used, is to see if the Track flag is present. This flag is removed during the preparation and execution of a LM  $\Delta V$  maneuver when there is no desire to have the routine request RR data or call any other routine such as the RR Designate and Preferred Tracking Attitude Routines. Note that a new request will be made for RR data 15 seconds later if the Track flag is absent.

If the Track flag is present, the routine next checks for issuance of the RR Track Enable discrete and reception of the RR Auto Mode discrete. The former discrete is removed and not re-issued by the RR Monitor Routine whenever the RR antenna angles exceed the limits in Fig. 2.4-3. Its absence therefore indicates a need to go back to input (B) of Fig. 2.4-1 to re-establish the preferred tracking attitude and re-designate the radar. Afterwards, a check is made to insure that RR data is not taken if the RR CDU's are being zeroed.

The sequence used by the routine for reading RR data is shown in Fig. 2.4-6 where frequent checks are made on the RR Data Good discrete to insure that no RR tracking interruptions have occurred during the read-out. If this discrete is missing, the Tracker Fail and Program Alarm lights are turned on before going to point (D) of Fig. 2.4-1 to re-designate the RR. Note in Fig. 2.4-6 that checks are also made to see if the RR CDU's have failed during the read-out and, if so, the same alarms are issued as for the RR Data Good discrete. Prior to and after reading the data, the routine checks the status of the Range Low Scale discrete to insure that the proper scaling is applied to the range data. For ranges below  $9.38 \times 32,767$  feet, the RR issues the Range Low Scale discrete indicating

to the LGC that the low scale factor should be used. If the status of this discrete should change during the data reading process, it is seen in Fig. 2.4-6 that a new data request is immediately made by the routine.

Although the measurement times for the range, range rate, and CDU angles are slightly different, they are near enough to warrant using only one time tag during rendezvous navigation. This time tag is the time  $t_M$  when the CDU angles are recorded.

After the RR data is read, the LOS is determined for time  $t_M$  and compared with that indicated by the RR CDU's. This is done to insure against RR side-lobe lock-on. If the difference is more than 3 degrees, an alarm is issued to the astronaut who must then decide to go to the RR Designate Routine or request the LGC to proceed with the data.

If the Track flag is still present after the side-lobe check, the routine checks to see if the angle between the RR antenna and the LM +Z-axis is within 30 degrees. As mentioned previously, this is required in order for the LGC to perform a satisfactory determination of RR angle biases, and to also enable the LM Optical Beacon to be seen by the CSM. If the antenna is not within 30 degrees of the LM +Z-axis, the Preferred Tracking Attitude Routine is used to realign the LM +Z-axis with the target LOS before resuming the data read process.

Afterwards, the LGC checks the Update and No Update flags. The Update flag is reset by various programs and routines whenever it is desired to temporarily stop the update of the navigation equations with RR data. The No Update flag is set whenever it is desired to permanently stop the update of the navigation equations with RR data. The No Update flag is set at the beginning of the Target Delta V Routine (R-32) and may be set by the astronaut. This flag is reset whenever the astronaut indicates that he wishes to update either the LM or CSM state vector. It should be noted that it is only necessary for the Track flag to be present in order to monitor RR tracking and read RR data even though the data may not be used for navigation updates.

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The range obtained from the RR by the RR Data Read Routine is that measured by the RR between the LM and the CSM. This data is sent to the LGC from the RR as a binary data word  $R_{RR}$ . In the LGC the range  $r_{RR}$  in feet is obtained as follows:

$$r_{RR} = \begin{cases} k_{R1} & R_{RR} \\ k_{R2} & R_{RR} \end{cases}$$

where  $k_{R1}$  and  $k_{R2}$  are the bit weights respectively for the long and short range scales in order to obtain  $r_{RR}$  in feet. When the Range Low Scale discrete is being received from the RR by the LGC,  $k_{R2}$  is used.

The range rate data obtained from the RR by the RR Data Read Routine is in the form of a binary data word  $S_{RR}$  which represents the count in the RR of a frequency comprising both the doppler frequency and a bias frequency ( $f_{BRR}$ ) over a time interval  $\tau_{RR}$ . At present,  $\tau_{RR}$  is given as 80.000 milliseconds. To obtain the range rate ( $\dot{r}_{RR}$ ) in feet per second, the following computation is made:

$$\dot{r}_{RR} = k_{RR} (S_{RR} - f_{BRR} \tau_{RR})$$

where  $k_{RR}$  is the scale factor required to obtain the range rate in feet per second and is of such a polarity as to make  $\dot{r}_{RR}$  positive in the above equation for increasing range.

A summary of the processing constants required by the LGC for RR operation is given as follows:



$f_{\text{BRR}}$	Range rate bias frequency
$\tau_{\text{RR}}$	Counting interval in RR for range rate measurements
$k_{\text{RR}}$	Scale factor to convert the range rate count obtained from the RR to feet per second for the counting interval $\tau_{\text{RR}}$ . The scale factor polarity is such as to make the converted result positive for increasing range.
$k_{\text{R1}}$	Bit weight in feet for long range scale.
$k_{\text{R2}}$	Bit weight in feet for short range scale.

#### 5.2.4.2.2 Rendezvous Navigation Computations

During rendezvous phases RR navigation data are obtained by means of automatic rendezvous radar tracking of the CSM from the LM. These data are used to update the estimated six-dimensional state vector of either the LM or the CSM. The option of which state vector is to be updated by the RR tracking data is controlled by the astronaut as described in Section 5.2.1 and illustrated in Fig. 2.1-1. This decision will be based upon which vehicle's state vector is most accurately known, and upon which vehicle is performing the rendezvous. This process requires that the constant RR tracking angle biases be compensated for by estimating these biases along with the selected vehicle's state vector such that subsequent RR tracking angle data can be modified as shown in simplified form Fig. 2.1-1.

This routine is used to process the CSM-tracking RR measurement data, and is used normally during lunar-orbit rendezvous in the lunar landing mission. The routine also can be used in earth orbit during alternate missions.

After the preferred LM attitude is achieved and RR tracking acquisition and lock-on is established (Section 5.2.4.1) the following tracking data are automatically acquired by the RR Data Read Routine (Section 5.2.4.2.1) at approximately one minute intervals:

- Measured range,  $R_M$
- Measured range rate,  $\dot{R}_M$
- Measured shaft angle,  $\beta_M$
- Measured trunnion angle,  $\theta_M$

where the subscript M indicates the RR measured value. In addition to the above four measured quantities the time of the measurement and the three IMU gimbals angles are also recorded.

Although eight variables are estimated in the navigation procedure (six vehicle state-vector components and two RR angle biases), it is convenient to use the following nine-dimensional state vector:

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \\ \delta\beta \\ \delta\theta \\ 0 \end{pmatrix} \quad (2.4.1)$$

where  $\underline{r}$  and  $\underline{v}$  are the estimated position and velocity vectors, respectively, of the selected vehicle (LM or CSM) which is being updated,  $\delta\beta$  and  $\delta\theta$  are the estimates of the biases in the RR shaft and trunnion angles, respectively, and the ninth coordinate is a dummy variable. This type of RR tracking angle bias is referred to as a boresight bias and is one of two types which may be defined. With the LM attitude restriction mentioned in Section 5.2.1, either type of angle bias (tilt or boresight) can be used, and the boresight type indicated in Eq. (2.4.1) is most convenient.

Let  $\underline{r}_L$ ,  $\underline{v}_L$ ,  $\underline{r}_C$  and  $\underline{v}_C$  be the estimated position and velocity vectors of the LM and CSM, respectively, at the time of the measurement. Then, the measurement error variances,  $\overline{\alpha^2}$ , the nine-dimensional geometry vectors,  $\underline{b}$ , and the measured deviations,  $\delta Q$ , for the range and range rate measurements are computed as follows:

Measured range,  $R_M$

$$\underline{r}_{LC} = \underline{r}_C - \underline{r}_E$$

$$\underline{u}_{LC} = \text{UNIT}(\underline{r}_{LC})$$

$$\overline{\alpha^2} = \text{maximum}(r_{LC}^2 \text{var}_R, \text{var}_{Rmin})$$

(2.4.2)

$$\underline{b}_0 = \overline{\alpha} \underline{u}_{LC}$$

$$\underline{b}_1 = \underline{0}$$

$$\underline{b}_2 = \underline{0}$$

$$\delta Q = R_M - r_{LC}$$

where  $\text{var}_R$  is the RR range error variance corresponding to a percentage error, and  $\text{var}_{Rmin}$  is the minimum RR range error variance.

Measured range rate,  $\dot{R}_M$

$$\underline{r}_{LC} = \underline{r}_C - \underline{r}_L$$

$$\underline{u}_{LC} = \text{UNIT}(\underline{r}_{LC})$$

$$\underline{v}_{LC} = \underline{v}_C - \underline{v}_L$$

(2.4.3)

$$\dot{\underline{r}} = \underline{v}_{LC} \cdot \underline{u}_{LC}$$

$$\overline{\alpha^2} = r_{LC}^2 \text{maximum}(\dot{r}^2 \text{var}_V, \text{var}_{Vmin})$$

Measured range rate,  $\dot{R}_M$  (Continued)

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$$\underline{b}_0 = \mp (\underline{u}_{LC} \times \underline{v}_{LC}) \times \underline{u}_{LC}$$

$$\underline{b}_1 = \mp r_{LC}$$

$$\underline{b}_2 = 0$$

(2.4.3 Continued)

$$\delta Q = r_{LC} (\dot{R}_M - \dot{r})$$

where  $\text{var}_V$  is the RR range-rate error variance corresponding to a percentage error, and  $\text{var}_{V_{\min}}$  is the minimum RR range-rate variance.

In Eqs. (2.4.2) and (2.4.3) the negative signs are selected if it is the LM state vector that is being updated, and the positive signs if it is the CSM state vector.

In order to process the RR angle data ( $\beta_M$  and  $\theta_M$ ), it is necessary to consider the relative orientations of the various coordinate systems. If  $\underline{u}_X$ ,  $\underline{u}_Y$  and  $\underline{u}_Z$  are unit vectors along the X-, Y- and Z-axes, respectively, of the RR Measurement Coordinate System, then the measured shaft angle,  $\beta_M$ , and the measured trunnion angle,  $\theta_M$ , are defined as shown in Fig. 2.4-7. In the figure, the vector  $\underline{r}_{XZ}$  is the projection of the measured LM-to-CSM line-of-sight vector on the XZ-plane.

The RR Measurement Coordinate System is coincident with the Navigation Base Coordinate System since all RR performance specifications are referenced to the PGNC navigation base. The unit vectors  $\underline{u}_X$ ,  $\underline{u}_Y$  and  $\underline{u}_Z$  are then given in the Basic Reference Coordinate System by

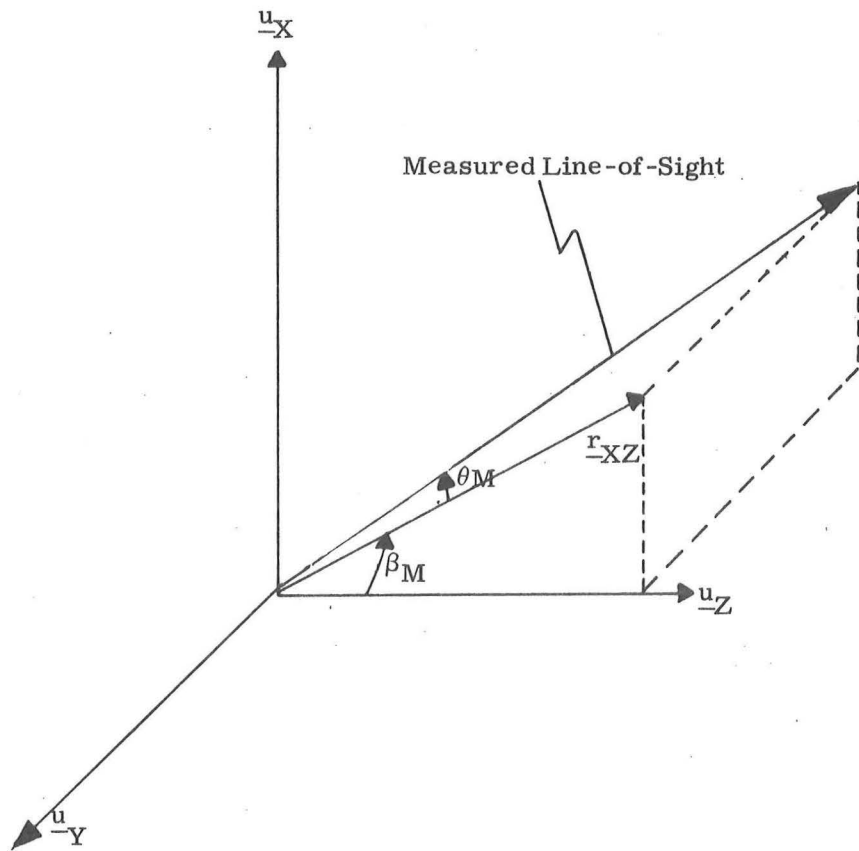


Figure 2.4-7 RR Measurement Coordinate System

$$\begin{pmatrix} \underline{u}_X^T \\ \underline{u}_Y^T \\ \underline{u}_Z^T \end{pmatrix} = [\text{SMNB}] [\text{REFSMMAT}] \quad (2.4.4)$$

where  $[\text{SMNB}]$  and  $[\text{REFSMMAT}]$  are transformation matrices as defined in Section 5.6.3 and the angles from which SMNB is determined are the values of the IMU gimbal angles which were recorded at the measurement time.

The measurement error variances,  $\overline{\alpha^2}$ , the nine-dimensional geometry vectors,  $\underline{b}$ , and the measured deviations,  $\delta Q$ , for the shaft and trunnion angle measurements are computed as follows:

Measured shaft angle,  $\beta_M$

$$\begin{aligned} \underline{r}_{LC} &= \underline{r}_C - \underline{r}_L \\ \underline{u}_{LC} &= \text{UNIT}(\underline{r}_{LC}) \\ S &= -\underline{u}_{LC} \cdot \underline{u}_Y \\ r_{XZ} &= r_{LC} \sqrt{1 - S^2} \\ \overline{\alpha^2} &= \text{var}_{\beta} + \text{var}_{\text{IMU}} \\ \underline{b}_0 &= \frac{1}{r_{XZ}} \text{UNIT}(\underline{u}_Y \times \underline{u}_{LC}) \\ \underline{b}_1 &= \underline{0} \\ \underline{b}_2 &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \delta Q &= \beta_M - \tan^{-1} \left( \frac{\underline{u}_X \cdot \underline{u}_{LC}}{\underline{u}_Z \cdot \underline{u}_{LC}} \right) - \delta\beta \end{aligned} \quad (2.4.5)$$

where  $\text{var}_\beta$  is the RR shaft-angle error variance and  $\text{var}_{\text{IMU}}$  is one-half the IMU angular error variance.

Measured trunnion angle,  $\theta_M$

$$\underline{r}_{\text{LC}} = \underline{r}_{\text{C}} - \underline{r}_{\text{L}}$$

$$\underline{u}_{\text{LC}} = \text{UNIT}(\underline{r}_{\text{LC}})$$

$$S = -\underline{u}_{\text{LC}} \cdot \underline{u}_{\text{Y}}$$

$$r_{\text{XZ}} = r_{\text{LC}} \sqrt{1 - S^2}$$

$$\overline{\alpha^2} = \text{var}_\theta + \text{var}_{\text{IMU}}$$

$$\underline{b}_0 = \mp \frac{1}{r_{\text{XZ}}} (\underline{u}_{\text{Y}} \times \underline{u}_{\text{LC}}) \times \underline{u}_{\text{LC}} \quad (2.4.6)$$

$$\underline{b}_1 = 0$$

$$\underline{b}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\delta Q = \theta_M - \sin^{-1}(S) - \delta\theta$$

where  $\text{var}_\theta$  is the RR trunnion-angle error variance.

In Eqs. (2.4.5) and (2.4.6) the negative signs are used if it is the LM state vector that is being updated, and the positive signs if it is the CSM state vector.

The data are incorporated into the state vector estimates by means of four calls to the Measurement Incorporation Routine (Section 5.2.3). The updated components of the nine-dimensional state vector resulting from each incorporation are used as initial conditions for the next update in the sequence.



Included in each use of the Measurement Incorporation Routine is the state vector update validity check, as described in Section 5.2.1 and illustrated in Fig. 2.4-8.

The results of the processing of the RR measurement data are updated values of the estimated position and velocity vectors of the CSM or the LM and estimates of the RR angle biases. The two estimated vehicle state vectors are used to compute required rendezvous targeting parameters as described in Section 5.4.4.

For convenience of calculation in the LGC, Eqs. (2.4.5) and (2.4.6) are reformulated and regrouped as follows:

Preliminary Radar Angle Calculation

$$\begin{aligned}
 \underline{r}_{LC} &= \underline{r}_C - \underline{r}_L \\
 \underline{u}_{LC} &= \text{UNIT}(\underline{r}_{LC}) \\
 S &= -\underline{u}_{LC} \cdot \underline{u}_Y \\
 r_{XZ} &= r_{LC} \sqrt{1 - S^2}
 \end{aligned}
 \tag{2.4.7}$$

Measured shaft angle,  $\beta_M$

$$\begin{aligned}
 \overline{\sigma^2} &= r_{XZ}^2 (\text{var}_{\beta} + \text{var}_{\text{IMU}}) \\
 \underline{b}_0 &= \mp \text{UNIT}(\underline{u}_Y \times \underline{u}_{LC}) \\
 \underline{b}_1 &= \underline{0} \\
 \underline{b}_2 &= \begin{pmatrix} r_{XZ} \\ 0 \\ 0 \end{pmatrix} \\
 \delta Q &= r_{XZ} \left[ \beta_M - \tan^{-1} \left( \frac{\underline{u}_X \cdot \underline{u}_{LC}}{\underline{u}_Z \cdot \underline{u}_{LC}} \right) - \delta\beta \right]
 \end{aligned}
 \tag{2.4.8}$$

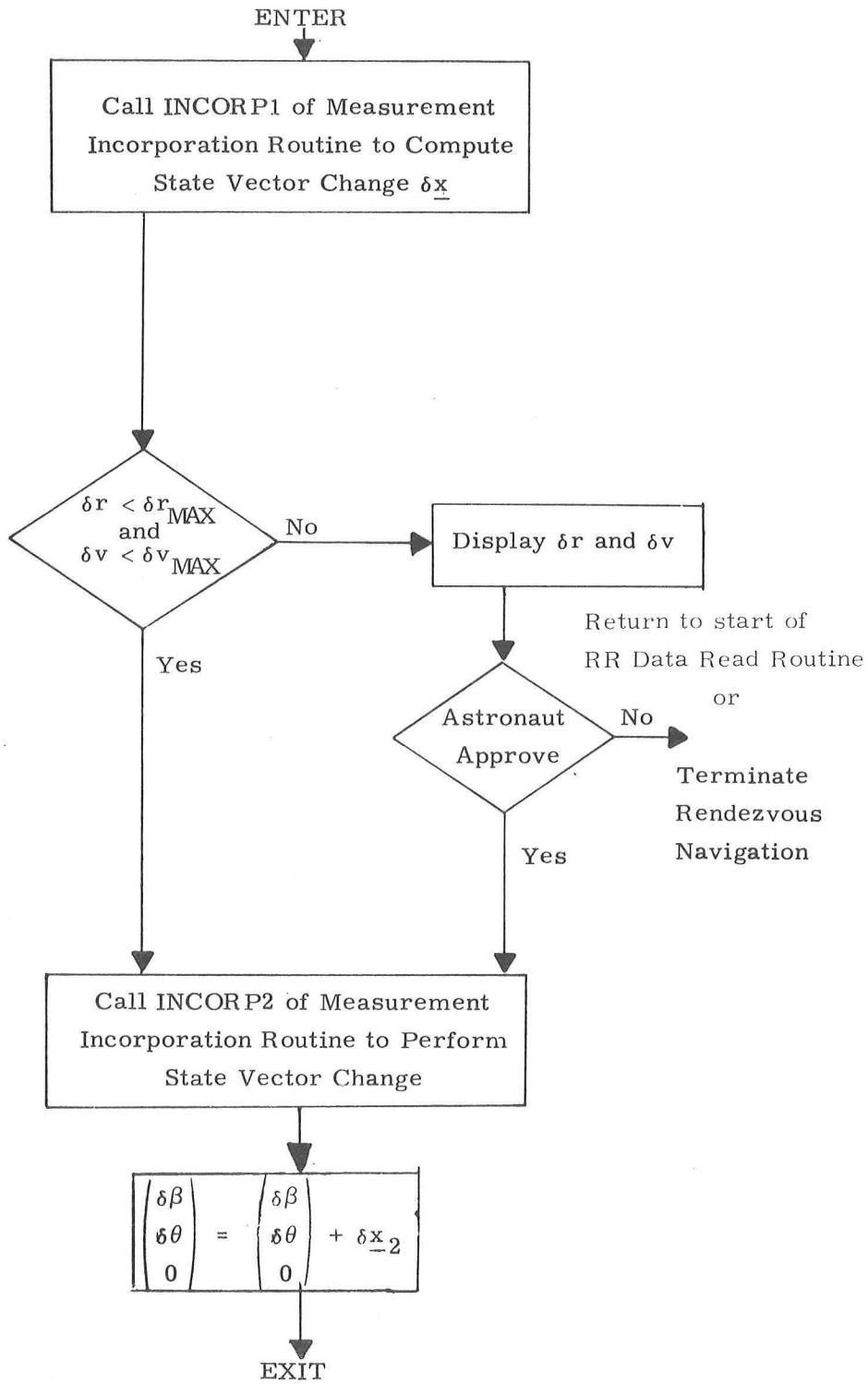


Figure 2.4-8 Rendezvous Navigation Measurement Incorporation Procedure

Measured trunnion angle,  $\theta_M$

$$\overline{\alpha^2} = r_{XZ}^2 (\text{var}_{\theta} + \text{var}_{\text{IMU}})$$

$$\underline{b}_0 = \overline{r} (\underline{u}_Y \times \underline{u}_{LC}) \times \underline{u}_{LC}$$

$$\underline{b}_1 = \underline{0}$$

$$\underline{b}_2 = \begin{pmatrix} 0 \\ r_{XZ} \\ 0 \end{pmatrix}$$

(2.4.9)

$$\delta Q = r_{XZ} \left[ \theta_M - \sin^{-1} (S) - \delta\theta \right]$$

The procedure for performing the rendezvous navigation computations is illustrated in Figs. 2.4-9 and 2.4-10. It is assumed that the following items are stored in erasable memory at the start of the computation shown in Fig. 2.4-9:

- $\underline{x}_C$  = Estimated CSM state vector as defined in Section 5.2.2.6.
- $\underline{x}_L$  = Estimated LM state vector.
- W = Six-dimensional error transition matrix associated with  $\underline{x}_C$  or  $\underline{x}_L$  as defined in Section 5.2.2.4.

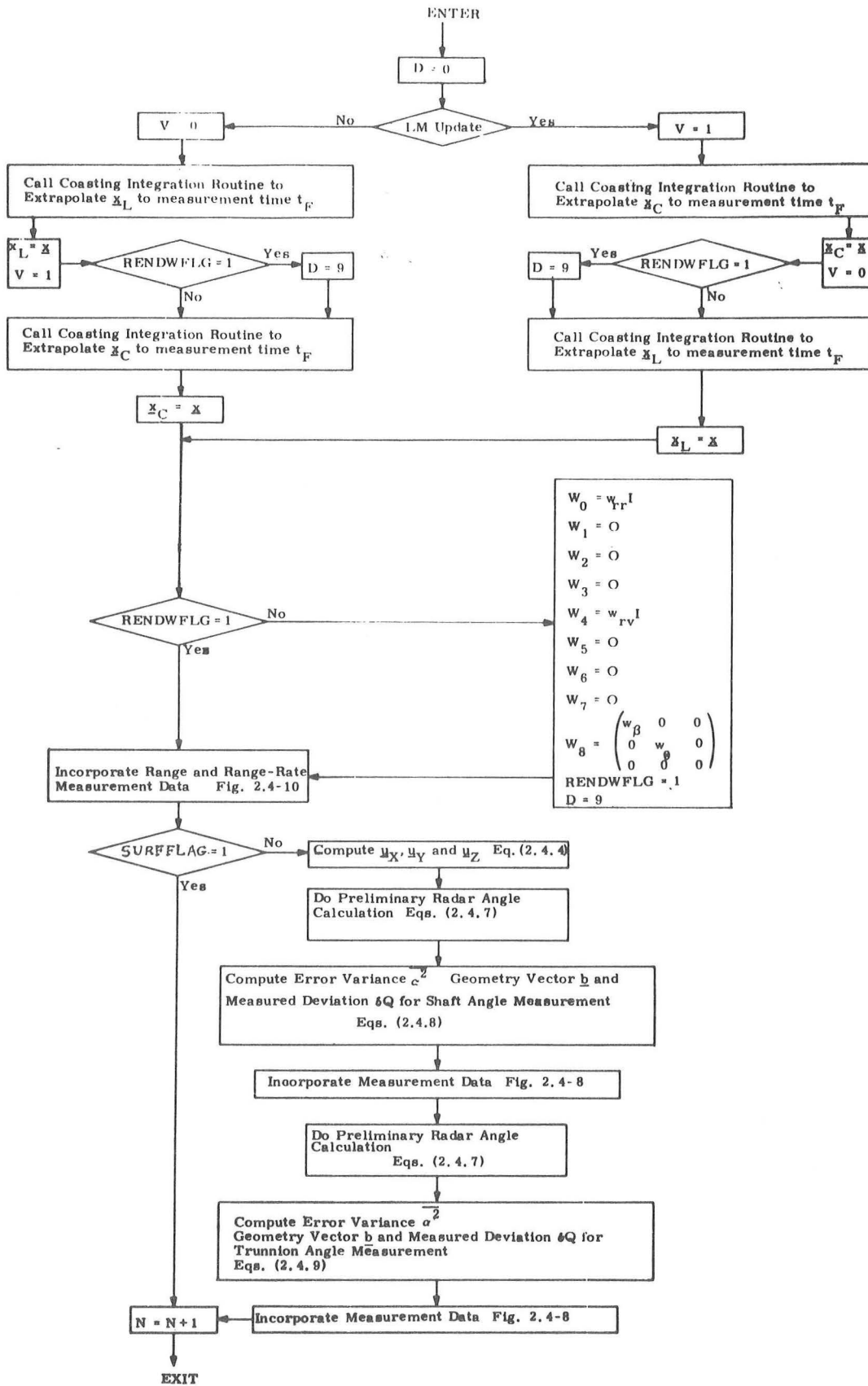


Figure 2.4-9 Rendezvous Navigation Computation Logic Diagram

5.2-62

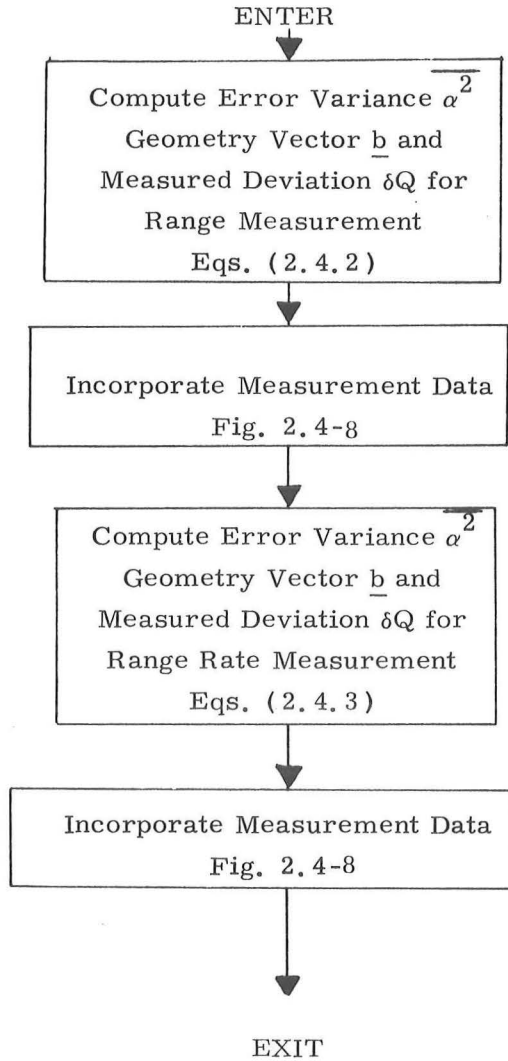


Fig. 2.4-10 RR Measured Range and Range Rate Incorporation

$\text{RENDWFLG} = \begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$

This flag or switch is maintained by programs external to the Rendezvous Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing RR tracking data. The flag is set to zero after each of the following procedures:

- 1) State vector update from ground
- 2) Astronaut command

$[\text{REFSMMAT}] =$  Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System.

$N =$  Number of sets of rendezvous navigation data already processed since the last maneuver.

$t_F =$  Measurement time.

$R_M, \dot{R}_M, \beta_M, \theta_M =$  RR measurement data

$w_{rr}, w_{rv}, w_\beta, w_\theta =$  Preselected W matrix initial diagonal elements

Three IMU gimbal angles

Vehicle update mode

SURFFLAG =  $\left\{ \begin{array}{l} 1 \text{ indicates RR angle measurements} \\ \quad (\beta_M \text{ and } \theta_M) \text{ are not to be used} \\ 0 \text{ indicates RR angle measurements} \\ \quad (\beta_M \text{ and } \theta_M) \text{ are to be used} \\ \text{The SURFFLAG should always be 0} \\ \text{in SUNDANCE since it is set to 1} \\ \text{only when on the lunar surface.} \end{array} \right.$

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6, and I and O are the three-dimensional identity and zero matrices, respectively.

At the beginning of a new rendezvous sequence, the estimates of the RR angle biases,  $\delta\beta$  and  $\delta\theta$ , may be initialized by the astronaut by direct addressing of erasable memory, and the W matrix is initialized by the program. During the remainder of the rendezvous sequence,  $\delta\beta$  and  $\delta\theta$  are never again zeroed even though there may be additional W matrix initializations.

The RR-measurement-data incorporation procedure outlined above is repeated at approximately one-minute intervals throughout the rendezvous phase except during powered maneuvers. If the LM is the passive vehicle, the CSM rendezvous maneuvers are voice-linked to the LM as an ignition time and three velocity components in a CSM local vertical coordinate system, and then entered as updates to the estimated CSM state vector in the LGC. Upon receipt of these data, RR tracking and data processing should be suspended until after the maneuver. The update is accomplished by means of the Target  $\Delta V$  Routine, R-32 (Section 5.1.16). If the LM is active vehicle, the estimated LM state vector is updated by means of the Average-G Routine (Section 5.3.2) during the maneuver.

5.2-65

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#### 5.2.4.3 RR Monitor Routine

The logic associated with the RR Monitor Routine (R-25) is given in Fig. 2.4-11. This routine is initiated every 0.48 seconds by an automatic program interrupt and monitors various items such as the Auto Mode discrete, the RR CDU Fail discrettes, and the angular excursions of the RR antenna.

Whenever the RR Auto Mode discrete changes status this routine resets various flags to insure proper initiation or termination of various radar control functions within the LGC. If the RR Auto Mode discrete has just been received from the RR as the result of placing the RR mode control switch in the LGC position, a turn-on sequence is initiated which zeroes the RR CDU's, determines the present RR antenna mode, and updates the Tracker Fail Light. The criterion used by this routine to determine the present RR antenna mode is the following:

$$\text{Mode 1: } 270^{\circ} \leq T \leq 90^{\circ}$$

$$\text{Mode 2: } 270^{\circ} > T > 90^{\circ}$$

where T is the RR antenna trunnion angle defined in Figs. 6.15-1 and 2.4-3. When the Auto Mode discrete is removed by the RR, routine R-25 removes the RR Error Counter Enable from the RR CDU's to insure that no commands are being sent to the RR gyros.

If there is an RR CDU failure when the Auto Mode discrete is present, it is seen in Fig. 2.4-11 that the Tracker Fail Light is turned on. If P-20 or P-22 is in operation (i. e. the Rendezvous Flag is set), the Program Alarm Light is also turned on and an alarm code is stored.

After checking for RR CDU failure, it is seen in Fig. 2.4-11 that a number of conditions must be met before the routine will check to see if the RR antenna angles are within the present mode limits (see Fig. 2.4-3). For example, the angles are not checked if the No Angle Monitor flag is present, signifying that the RR Manual Mode of target acquisition is being used. If all the conditions are met and the RR antenna is not within the present mode limits, the routine will remove the RR Track Enable discrete from the RR, and cause the RR to be repositioned to the reference position (see Fig. 2.4-3) for the present mode. During the repositioning of the RR this routine sets the Reposition flag to indicate to other programs and routines that this is taking place. 5.2-66



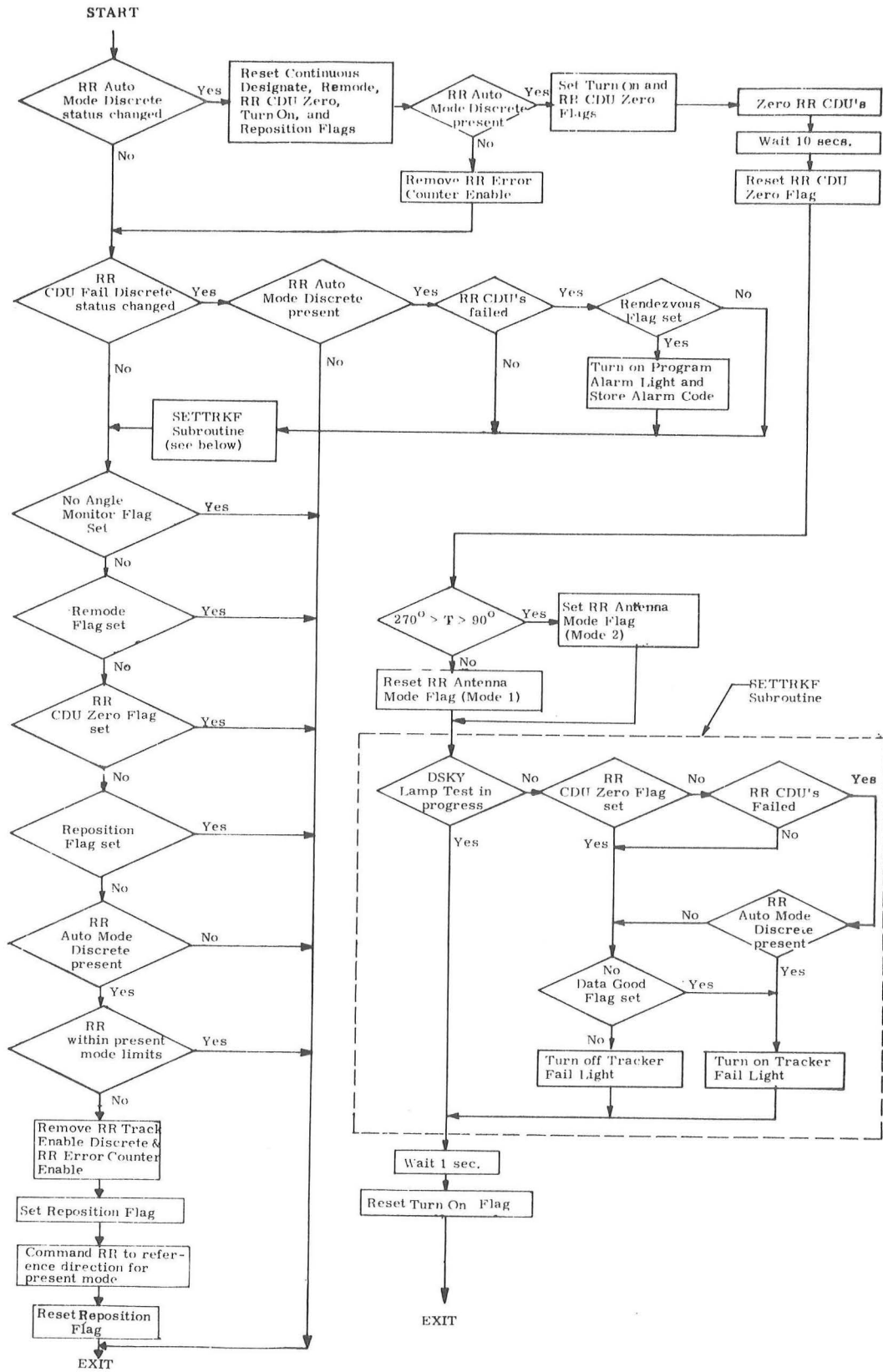


Figure 2.4-11 RR Monitor Routine

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GSOP # R-557 PCR # 657

Rev. 2

Date 12-9-68

## 5.3 POWERED FLIGHT NAVIGATION AND GUIDANCE

### 5.3.1 GENERAL COMMENTS

The objective of the powered flight guidance and navigation routines is to maintain an estimate of the LM state vector during thrusting maneuvers, and to control the thrust direction and duration such that the desired velocity cut-off conditions specified by the targeting routines of Section 5.4 are achieved. The powered flight navigation routine, used to maintain an estimate of the vehicle state vector during all thrusting conditions, is referred to as the Average-G Routine, and is presented in Section 5.3.2.

For this Earth Orbital mission the basic powered flight guidance concept used in the LGC is a velocity-to-be-gained concept with cross product steering (Section 5.3.3.4) that is used in each of the following two procedures:

1. Lambert Aim Point Maneuver Guidance  
(Section 5.3.3.3.2).
2. External  $\Delta V$  Maneuver Guidance  
(Section 5.3.3.3.1).

These two procedures, based on the cross product steering concept, differ only in the unique generation of the desired velocity vector,  $\underline{v}_R$ .

5. 3. 2     POWERED FLIGHT NAVIGATION - AVERAGE-G ROUTINE

The purpose of the Powered Flight Navigation Sub-routine is to compute the vehicle state vector during periods of powered flight steering. During such periods the effects of gravity and thrusting are taken into account. In order to achieve a short computation time the integration of the effects of gravity is achieved by simple averaging of the gravity acceleration vector. The effect of thrust acceleration is measured by the IMU Pulsed Integrating Pendulous Accelerometers (PIPA's) in the form of velocity increments ( $\Delta \underline{v}$ ) over the computation time interval ( $\Delta t$ ). The computations are, therefore, in terms of discrete increments of velocity rather than instantaneous accelerations. The repetitive computation cycle time  $\Delta t$  is set at 2 seconds to maintain accuracy and to be compatible with the basic powered flight cycle.

The Average-G Routine, in contrast to the Coasting Integration Routine, is used when a short computing time is required such as during powered flight. The Average-G Routine computations are illustrated in Figs. 3. 2-1 and 3. 2-2. In these figures the definitions of the following parameters are:

- $\underline{r}(t)$      Vehicle position vector at time  $t$ .
- $\underline{v}(t)$      Vehicle velocity vector at time  $t$ .
- $P_C$        Planet designator  $\left\{ \begin{array}{l} 0 \text{ Earth} \\ 1 \text{ Moon} \end{array} \right.$
- $\Delta t$        Computation cycle of 2 seconds.

$\Delta \underline{y}(\Delta t)$	The velocity vector sensed by the IMU PIPA's over the time interval $\Delta t$ . This velocity vector increment is initially sensed in IMU or Stable Member Coordinates and then transformed to the Basic Reference Coordinate System.
$\underline{g}_p(t)$	Previous gravity acceleration vector. This is a required initialization parameter and is supplied by the calling program.
$\underline{u}_r$	Unit vector in the direction of $\underline{r}$ .
$\underline{u}_z$	Unit vector in the direction of the polar axis of the earth.
$\mu_E$	Earth gravitational constant.
$\mu_M$	Moon gravitational constant.
$r_E$	Equatorial radius of the earth.
$J_{2E}$	Second-harmonic coefficient of the earth's potential function.
$\underline{g}_b(t)$	Component of the earth gravity acceleration vector representing earth oblateness effects.

With reference to Fig. 3. 2-2 it can be seen that a single oblateness term is included in the earth gravity subroutine computation, but none for the lunar case.

The PIPA measured velocity  $\Delta \underline{y}$  is compensated for instrument errors as described in Section 5. 6. 13 prior to being transformed into the Basic Reference Coordinate System and processed in the Average-G Subroutine of Fig. 3. 2-1.

An accelerometer threshold is used in the LGC to discard PIPA output pulses caused by accelerometer bias error during free-fall conditions. This threshold level is set at one pulse per two second  $\Delta t$  sample period, in any single accelerometer axis. The PIPA output must therefore be at least two pulses to be incorporated. The PIPA quantization level is such that one output pulse corresponds to 1 cm/sec. Over a single computation interval, this threshold is greater than the accumulated  $1 \sigma$  accelerometer bias and lower than the velocity increment due to the minimum thrust acceleration of the LM which is approximately 6 cm/sec<sup>2</sup>.

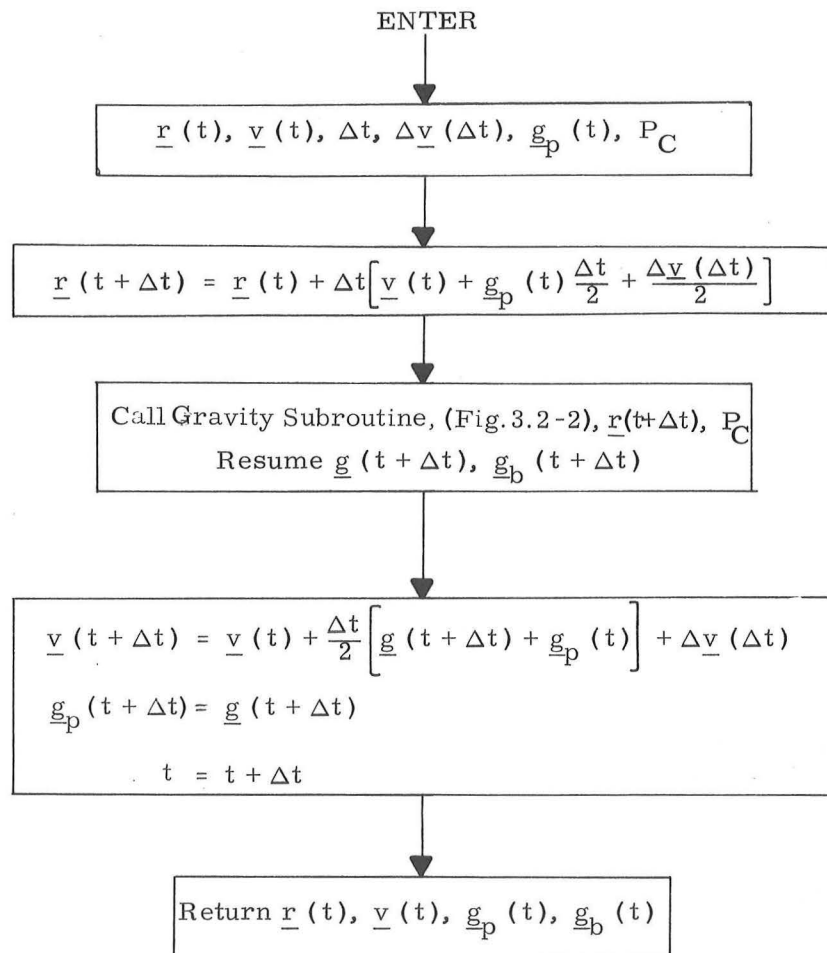


Figure 3.2-1 Average-G Subroutine

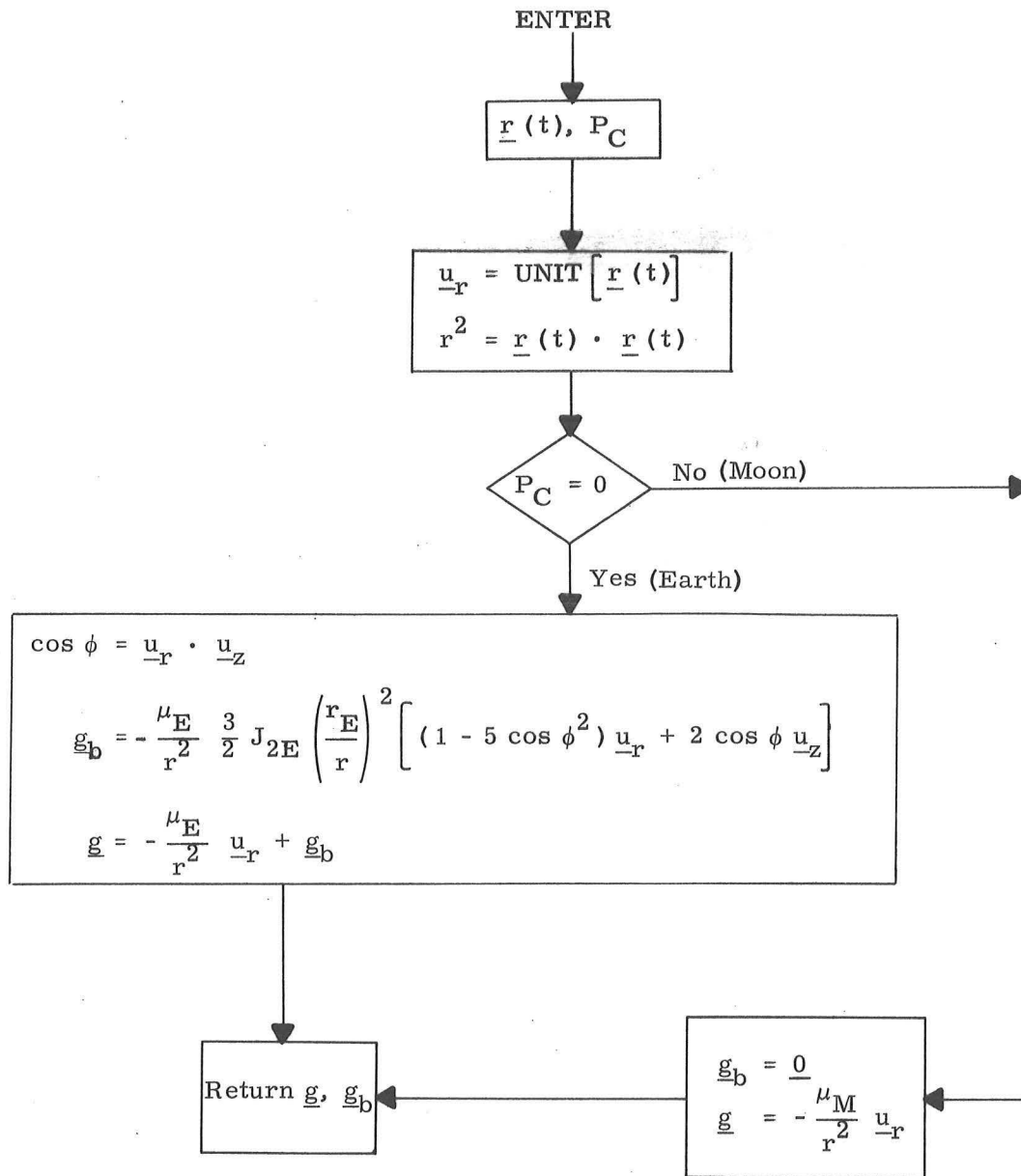


Figure 3.2-2 Gravity Subroutine

5. 3. 3     POWERED FLIGHT GUIDANCE USING CROSS  
PRODUCT STEERING

5. 3. 3. 1   Introduction

Cross product steering is used to control the following maneuvers:

- 1)  CSI            (Coelliptic Sequence Initiation)
- 2)  CDH            (Constant Differential Altitude)
- 3)  TPI            (Transfer Phase Initiation)
- 4)  MCC            (Rendezvous Midcourse Corrections)
- 5)  External  $\Delta V$  (RTCC or CMC Targeted Maneuver)

The CSI, CDH and External  $\Delta V$  maneuvers are controlled by the External  $\Delta V$  Guidance mode (Section 5.3.3.3.1) in which the required cut-off velocity,  $\underline{v}_R$ , is specified by either a pre-thrust targeting program (P-32 or P-33) or an external source (P-30). The TPI and rendezvous MCC maneuvers are controlled by the Lambert Aim Point Maneuver Guidance mode (Section 5.3.3.3.2) in which the  $\underline{v}_R$  is periodically computed by the Lambert subroutine during the maneuver to establish the desired intercept trajectory. Both External  $\Delta V$  and Lambert Aim Point guidance modes use the cross product steering concept in the LGC to control the thrust direction along the velocity-to-be-gained vector, and to terminate thrust when the desired velocity increment has been achieved.



Four subroutines are used repetitively in sequence (Section 5.3.3.2) during cross product controlled maneuvers to accomplish this function. These are:

1. The Powered Flight Navigation Average-G Routine which computes the state vector accounting for the effects of thrust acceleration and gravity.
2. The Cross-Product Steering Subroutine which has 3 functions:
  - a) generation of steering commands to the vehicle autopilot.
  - b) computation of time-to-go before engine shut-off and the issuance of engine-off commands.
  - c) updating of the incremental velocity-to-be-gained vector.
3. The Velocity-to-be-Gained Subroutine which repetitively solves the Lambert intercept problem when in the Lambert Aim Point guidance mode.
4. The  $\Delta V$  Monitor Subroutine which detects engine ignition and unexpected thrust termination.

The Average-G Routine is described in Section 5.3.2. The other subroutines listed above are described in Sections 5.3.3.4 to 5.3.3.6. The Pre-Thrust Subroutines of Section 5.3.3.3 initialize the powered maneuver programs for either the External  $\Delta V$  or Lambert Aim Point guidance modes, and for the selected engine for the maneuver.

The LGC powered flight programs which use cross product steering are:

P-40 Descent Propulsion System (DPS)

P-41 Reaction Control System (RCS)

P-42 Ascent Propulsion System (APS)

Active steering and engine-off commands are provided by programs P-40 and P-42. Maneuvers using RCS translation control (P-41) are manually controlled and terminated by the astronaut while the LGC displays the required velocity-to-be-gained in spacecraft coordinates.

The functions of the External  $\Delta V$  Pre-Thrust Program, P-30, are described in the pre-thrust subroutine description of Section 5.3.3.3.1.

### 5.3.3.2 Powered Flight Guidance Computation Sequencing

The time sequencing of the powered flight sub-routines for External  $\Delta V$  steering is shown in Figs. 3.3-1 thru 3.3-3 and for Lambert Aim Point steering in Figs. 3.3-4 thru 3.3-6. \*

Figures 3.3-1 and 3.3-4 show the sequencing during the ignition count down which starts 30 seconds before the nominal ignition time. Figures 3.3-2 and 3.3-5 show the normal sequencing for an engine-on time greater than 6 seconds as predicted by the pre-thrust subroutines. Figures 3.3-3 and 3.3-6 illustrate the sequencing during engine thrust termination.

The basic computation cycle time of the steering is 2 seconds and, as shown on the above figures, is initiated by the reading of the PIPA  $\Delta v$  registers. The various sub-routines utilized during the 2 second cycle are sequenced in time as shown.

During Lambert Aim Point steering, however, the velocity-to-be-gained ( $v_G$ ) updating is anticipated to occur nominally every four seconds. This allows sequenced parallel computing functions, such as the demands of the autopilots and telemetry, to take place without interference with the basic two second cycle time. Should the  $v_G$  computation be complete in time for use during a given 2 second cycle, it will be utilized.

---

\* These figures represent the sequence of operations for P-40 DPS, P-42 APS maneuvers lasting longer than 6 secs. RCS controlled maneuvers (P-41) require manual control, and the general timing sequence is different from that described for APS and DPS maneuvers. The general cross product steering concept is used to compute and display the required velocity-to-be-gained vector, but no  $\Delta V$  Monitor or engine-off computations are made for RCS maneuvers. The following description of sequence operations is restricted to APS and DPS maneuvers.

If the calculations conducted prior to the thrusting period (pre-thrust) indicate that the maneuver objective can be reached with a 6 second thrust period or less, provision is included (switches  $S_I$  and  $s_W$ ) to preclude the generation of steering commands and engine commands during the thrusting period. In this case at the time of ignition the engine-off signal is set to be issued at the end of the estimated thrust period.

In addition to timing information, the sequence diagram of Figs. 3.3-1 to 3.3-6 also show the basic information utilized by each subroutine and its source.

The guidance computer program which controls the various subroutines to create a powered flight sequence is called the Servicer Routine. The sequence diagrams of Figs. 3.3-1 to 3.3-6 define what the Servicer Routine does, but do not show the logic details of how these functions are accomplished.

The subroutines listed in Figs. 3.3-1 to 3.3-6 are described in Section 5.3.2 and the following Sections 5.3.3.3 to 5.3.3.6. These sections should be referenced in tracing the powered flight computation sequencing.

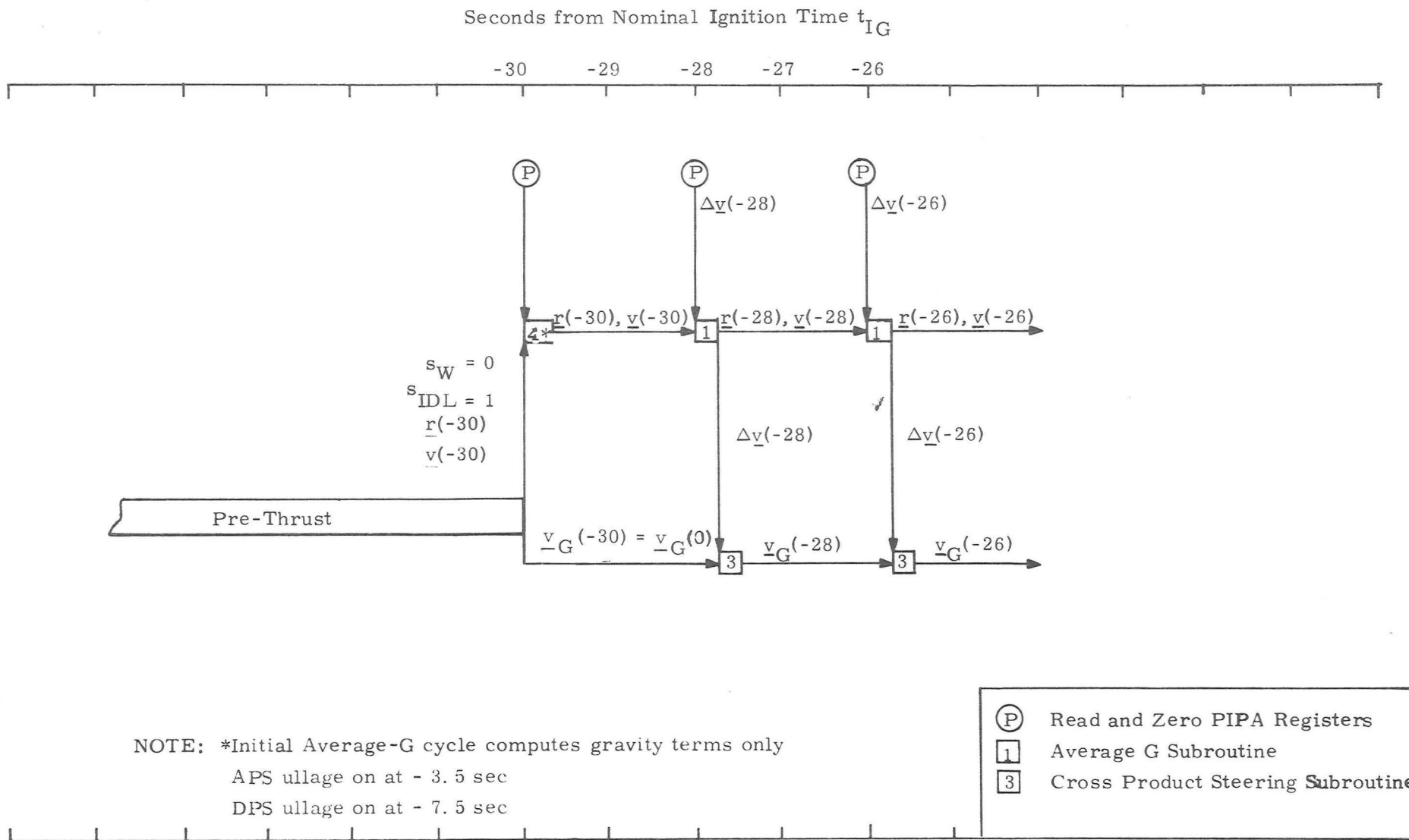
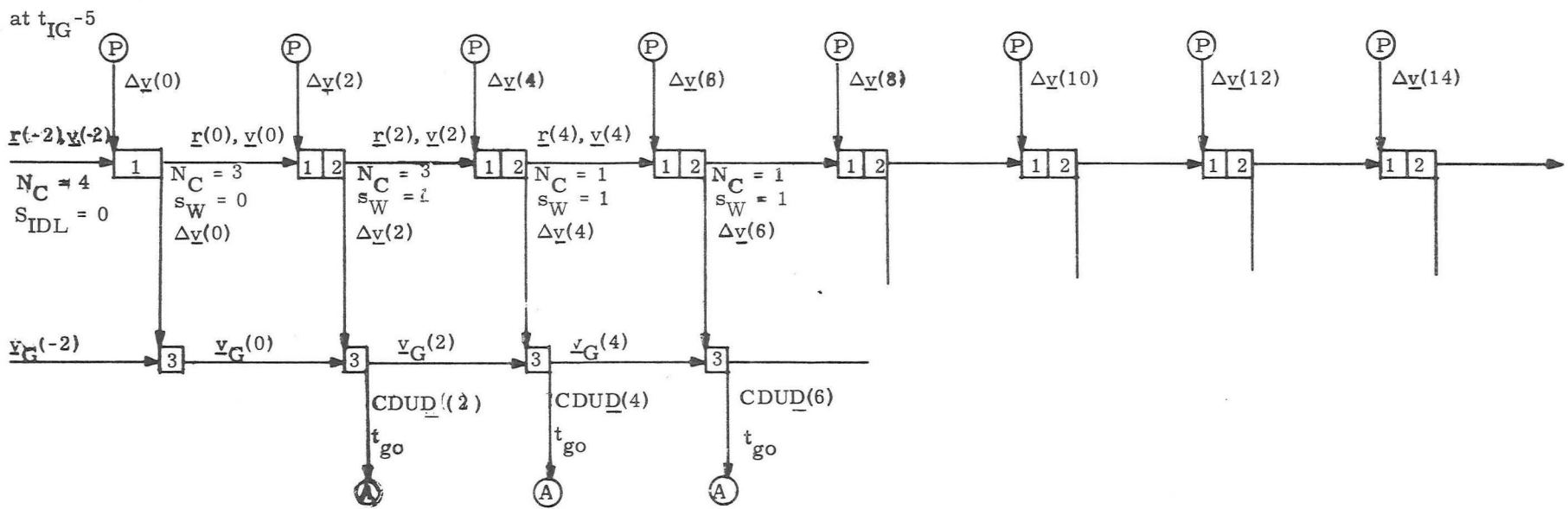
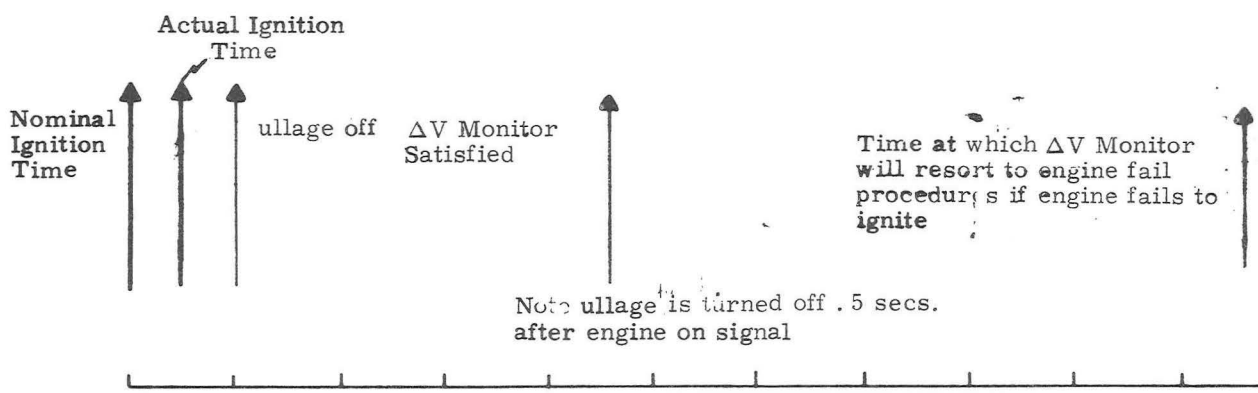


Fig. 3.3-1 Ignition Countdown - External  $\Delta V$  Subroutine Sequencing for APS and DPS Maneuvers

Seconds from Nominal Ignition time -  $t_{IG}$



5.3-13



- (P) Reading and zeroing of PIPA Registers
- (1) Average G Subroutine
- (2) ΔV Monitor Subroutine
- (3) Cross Product Steering subroutine
- (A) Transmit commands to Autopilot

Fig. 3.3-2 Normal External ΔV Subroutine Sequencing (Maneuver time greater than 6 sec) for APS and DPS

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 Added GSOP #R-557 PCR # 657  
 Rev. 2 Date 12-12-68

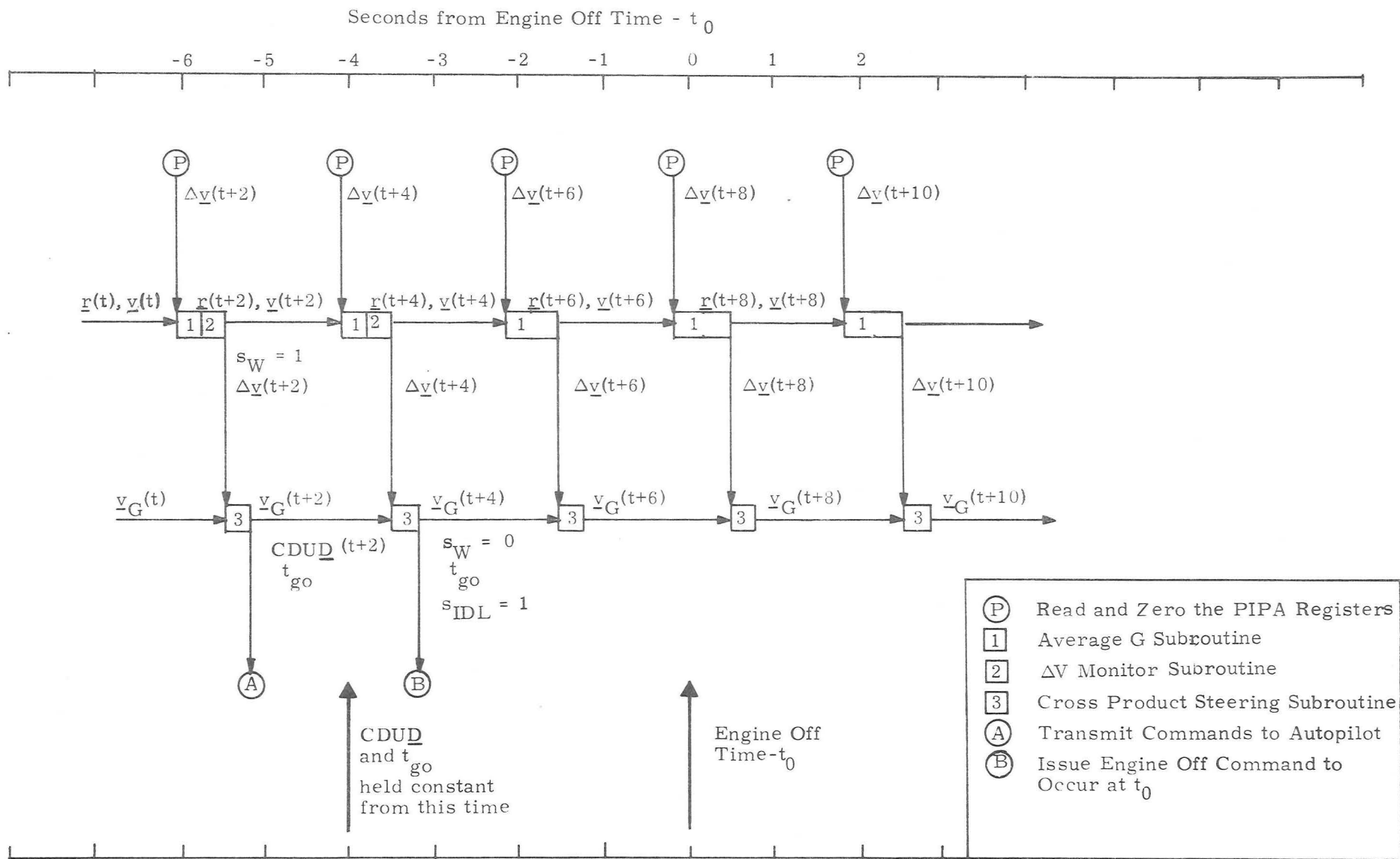


Fig. 3.3-3 Engine Off External  $\Delta V$  Subroutine Sequencing for APS and DPSS Maneuvers

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5.3-15

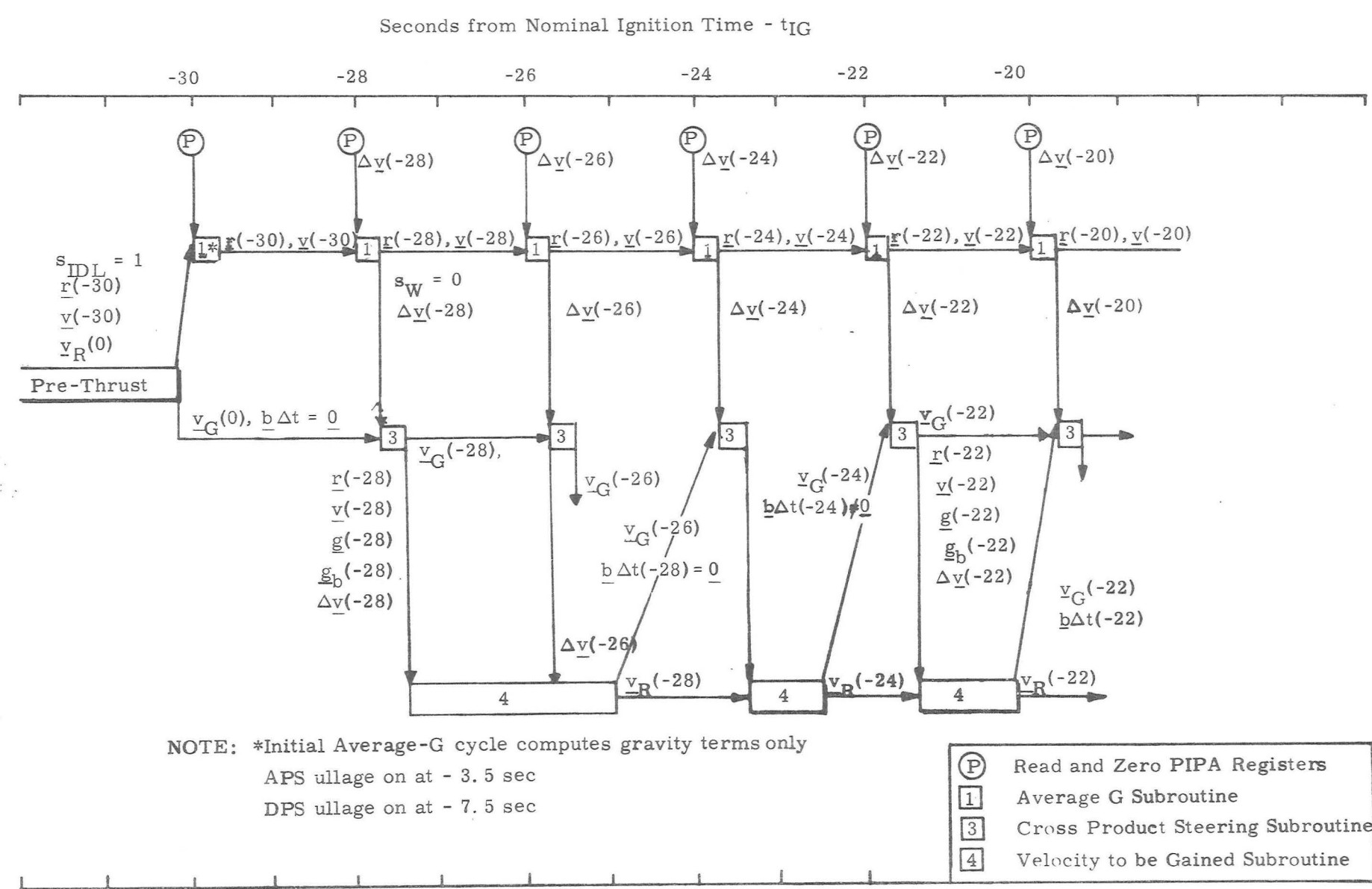


Fig. 3.3-4 Ignition Countdown - Lambert Aim Point Subroutine Sequencing for APS and DPS



Seconds from Nominal Ignition Time -  $t_{IG}$

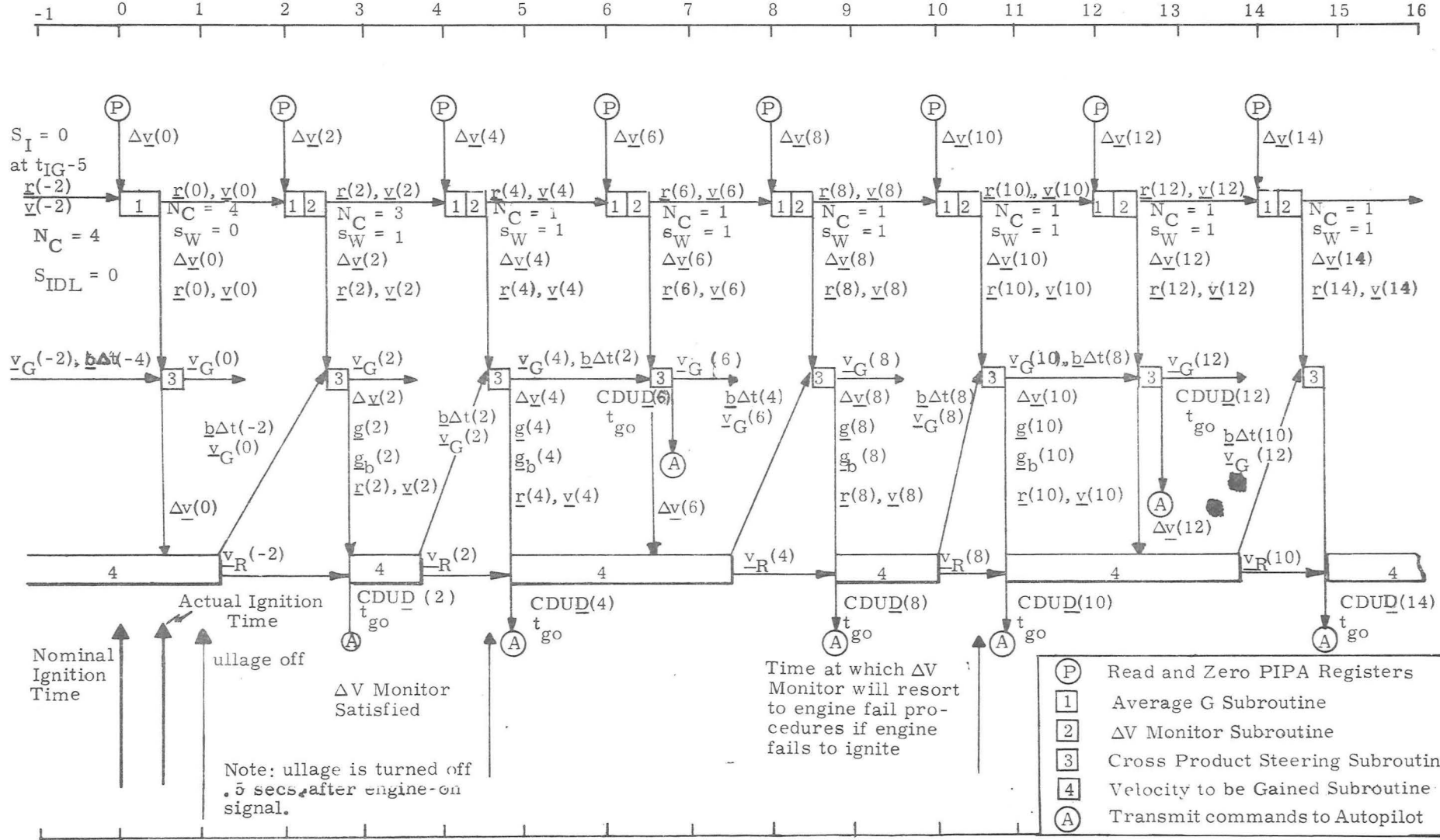
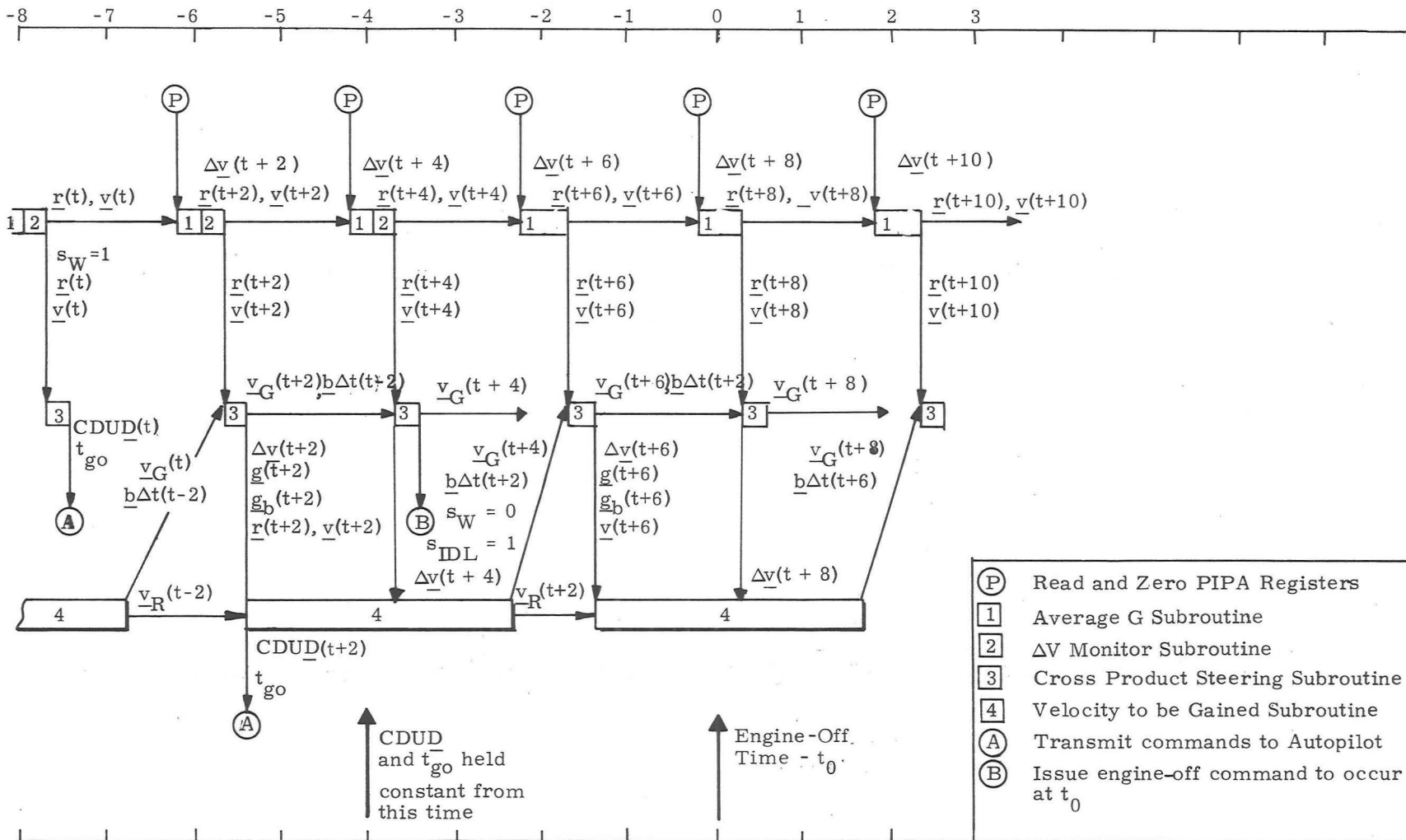


Fig. 3.3-5 Normal Lambert Aim Point Subroutine Sequencing (Maneuver time greater than 6 sec) for APS and DPS

Revised SUNDANCE  
 Added GSOP # R-657  
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 PCR # 657  
 Rev. 2  
 Date 12-12-68

Seconds from Engine Off Time -  $t_0$



- (P) Read and Zero PIPA Registers
- (1) Average G Subroutine
- (2)  $\Delta V$  Monitor Subroutine
- (3) Cross Product Steering Subroutine
- (4) Velocity to be Gained Subroutine
- (A) Transmit commands to Autopilot
- (B) Issue engine-off command to occur at  $t_0$

Fig. 3.3-6 Engine-off Lambert Aim Point Subroutine Sequencing for APS and DPS

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 Added GSOP # R-557  
 PCR #182, 414 / Rev. 1  
 Date 3-5-68

5.3-17

### 5.3.3.3 Pre-Thrust Computations

The objectives of the computations required prior to thrusting maneuvers are to determine the following factors:

- 1) The desired thrust direction at ignition.
- 2) The estimated duration of the powered maneuver to determine if there will be enough time to allow active steering, and in the case of the DPS if the throttle can be advanced to the full throttle position (FTP) after the 10% thrust start sequence.
- 3) Whether an IMU realignment is required to avoid gimbal lock.
- 4) Various parameters and variables required by subsequent powered flight routines.

The two major guidance modes using cross product steering are the External  $\Delta V$  guidance mode and the Lambert Aim Point Maneuver guidance mode. The pre-thrust computations required for the External  $\Delta V$  mode are presented in Section 5.3.3.3.1. Those required for the Lambert Aim Point Maneuver mode are described in Section 5.3.3.3.2. The initial IMU alignment computations and maneuver time logic is summarized in Section 5.3.3.3.3.

#### 5.3.3.3.1 External $\Delta V$ Maneuver Pre-Thrust Computations

External  $\Delta V$  maneuver guidance is normally used to control the CSI and CDH maneuvers of the concentric flight plan rendezvous profile, or an externally targeted maneuver in which a

constant thrust attitude is desired. The guidance program accepts input data via the DSKY (P-30) or the telemetry uplink (P-27) in the form of 3 components of an impulsive  $\Delta \underline{V}_{LV}$  expressed in a local vertical coordinate system of the active vehicle at the ignition time  $t_{IG}$ . An approximate compensation for the finite maneuver time is made within the program by rotating the  $\Delta \underline{V}_{LV}$  vector, and the guidance program issues commands to the spacecraft control system so as to apply the compensated velocity increment along an inertially fixed direction. The active vehicle state vector is normally either available or can be extrapolated to the ignition time in the LGC. If External  $\Delta V$  guidance is used in cislunar space for midcourse corrections, the state vector must be uplinked for the ignition time along with the 3 components of the desired impulsive  $\Delta \underline{V}_{LV}$ .

The pre-thrust computations required for the External  $\Delta V$  guidance mode are shown in Fig. 3.3-7. The following parameter definitions refer to this figure.

$\Delta \underline{V}_{LV}$  Specified velocity change in the local vertical coordinate system of the active vehicle at the time of ignition. This is an input parameter.

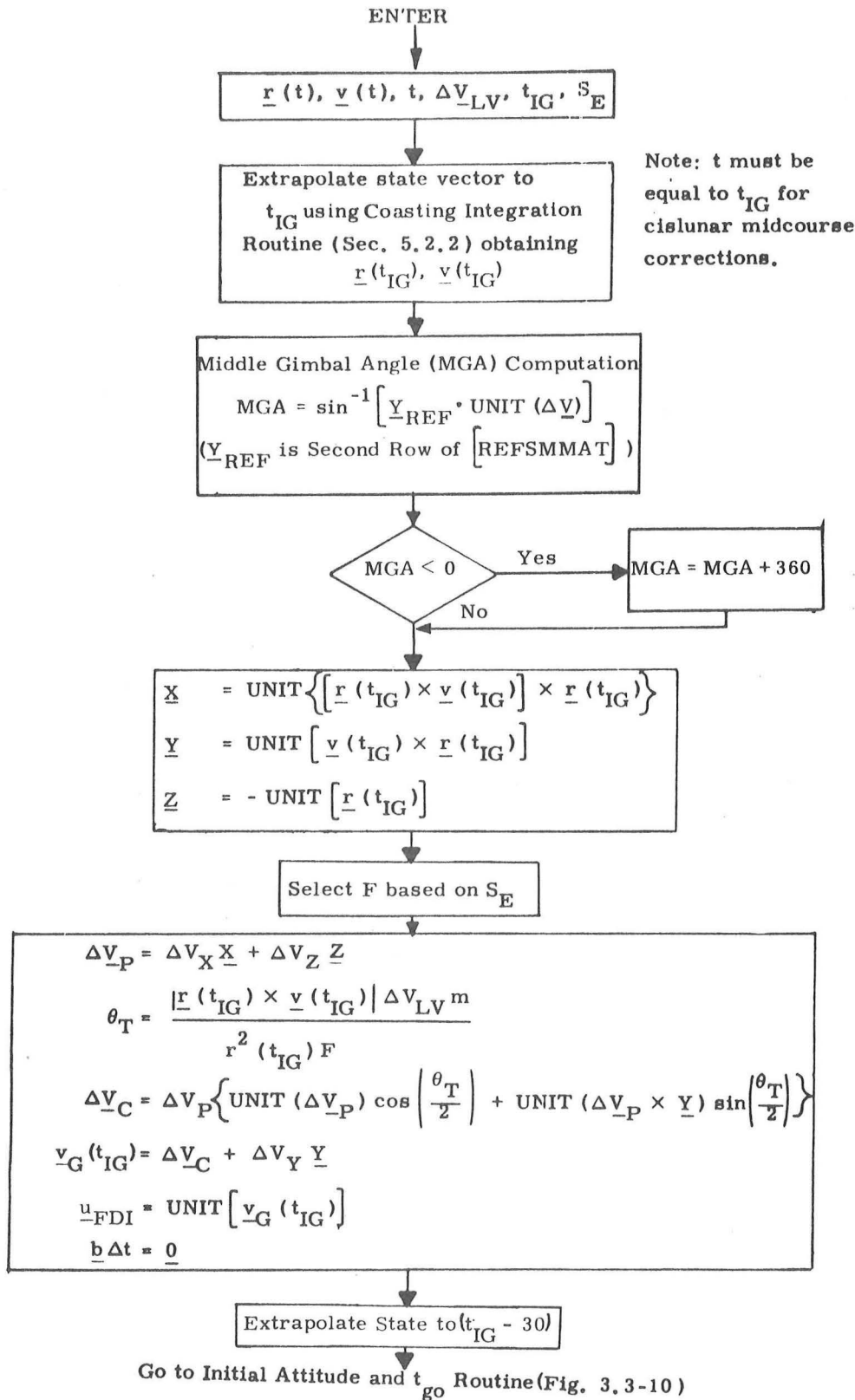
$$\Delta \underline{V}_{LV} = \begin{pmatrix} \Delta V_X \\ \Delta V_Y \\ \Delta V_Z \end{pmatrix}$$

$t_{IG}$  Ignition time, an input parameter.

$\Delta \underline{V}_{-P}$  The inplane velocity components of  $\Delta \underline{V}_{LV}$  in the Basic Reference Coordinate System.

$\theta_T$  The approximate central angle traveled during the maneuver.

$m$  Vehicle mass.



Note:  $t$  must be equal to  $t_{IG}$  for cislunar midcourse corrections.

Fig. 3.3-7 External  $\Delta V$  Prethrust Routine

Note: These calculations are performed in P30 and P40, P41, P42

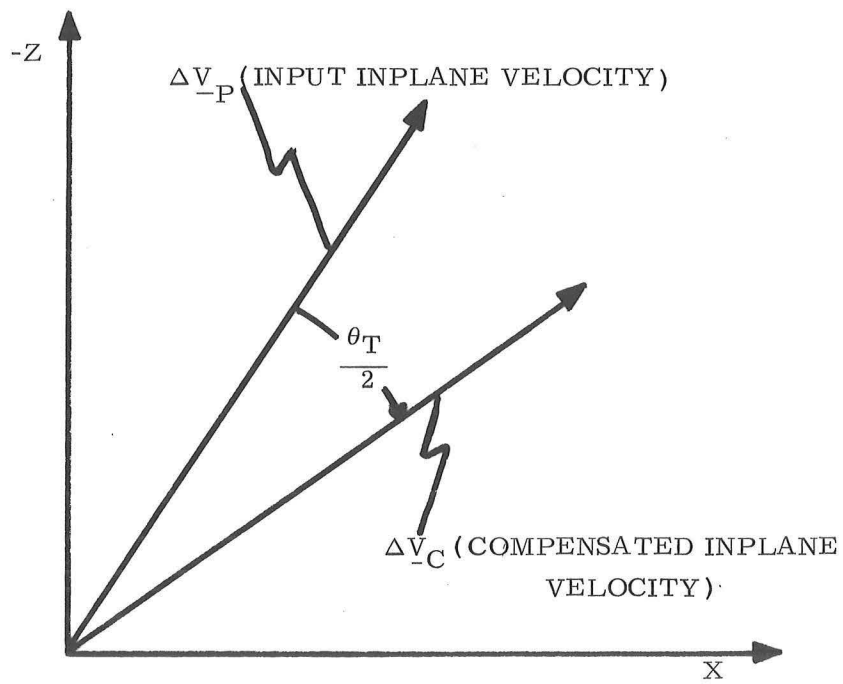


Figure 3.3-8 Inplane External  $\Delta V$  Maneuver Angle Compensation

$S_E$  Engine Select Switch  $\left\{ \begin{array}{l} -1 \text{ P-40 (DPS)} \\ 0 \text{ P-41 (RCS)} \\ 1 \text{ P-42 (APS)} \end{array} \right.$

$F$  Prestored nominal thrust based on the engine selection switch  $S_E$   
 $F_{DPS}$  for the DPS (P-40)  
 $F_{APS}$  for the APS (P-42)  
 $F_{RCS}$  for the RCS (P-41)

The values of the various engine thrust levels are listed in Sections 5.8 and 6. It should be noted that the DPS thrust level,  $F_{DPS}$ , is for the full throttle position (FTP) and the 26 second 10% thrust start sequence is not accounted for in the predicted maneuver time for the External  $\Delta V$  guidance mode. In the case of P-41, RCS, the astronaut can select either a 2 or 4 jet translation maneuver.

$\Delta \underline{V}_C$  The compensated inplane velocity-to-be-gained vector.

$\underline{v}_G(t_{IG})$  Total velocity to-be-gained at  $t_{IG}$ .

$\underline{u}_{TD}$  Unit vector in the desired initial thrust direction.

The inplane External  $\Delta V$  maneuver angle compensation involving  $\Delta \underline{V}_P$  and  $\Delta \underline{V}_C$  is illustrated in Fig. 3.3-8.

#### 5.3.3.3.2 Lambert Aim Point Maneuver Pre-Thrust Computations

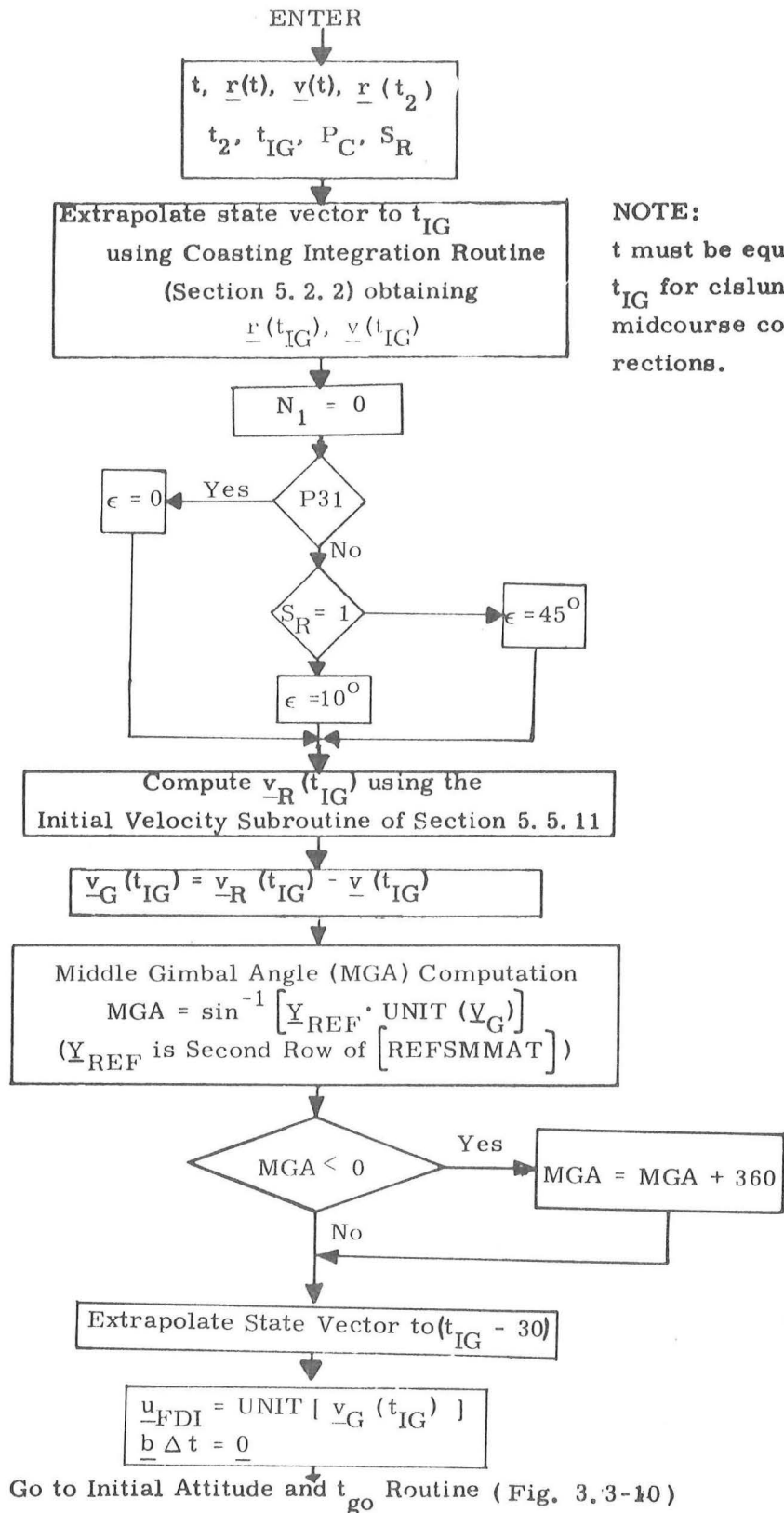
The objective of the Lambert Aim Point Maneuver Guidance program is to control the cut-off velocity vector such that the resulting trajectory intercepts a specified target position vector at a given time. This target position vector is determined from the TPI targeting program P-34, or the Rendezvous Midcourse Correction targeting program P-35.

The Lambert Aim Point Maneuver guidance mode basically uses the Lambert Subroutine of Section 5.5.6 called through the Initial Velocity Subroutine of Section 5.5.11 during

the prethrust phase to determine the required initial thrust direction,  $\underline{u}_{TD}$ . The pre-thrust computations required for the Lambert Aim Point guidance mode are shown in Fig. 3.3-9. The primary operation shown in this figure is the computation of the required intercept velocity vector  $\underline{v}_R$  at the ignition time,  $t_{IG}$ , and  $\underline{u}_{TD}$ . The following parameter definitions apply to Fig. 3.3-9.

$\underline{r}(t_2)$	Offset target intercept position vector at time $t_2$ . This parameter is determined by the preceding targeting program and is an input to the Lambert Aim Point pre-thrust subroutine.
$t_2$	Intercept time of arrival associated with the offset target vector, $\underline{r}(t_2)$ . This is an input parameter.
$S_R$	Target rotation switch set in P-34 or P-35 indicating that the target vector was rotated into the orbital plane due to proximity to $180^\circ$ transfer conditions.
$S_R$	$\begin{cases} 1 & \text{Target vector rotated} \\ 0 & \text{No rotation} \end{cases}$
$N_1$	Number of target offset iterations desired in the Initial Velocity Subroutine of Section 5.5.11.
$\epsilon$	Initial Velocity Subroutine parameter.
$\underline{v}_R(t_{IG})$	Required velocity vector at the ignition time $t_{IG}$ to establish the intercept trajectory.
$\underline{g}$	Total gravity vector associated with $\underline{r}(t_{IG})$ .
$\underline{b}\Delta t$	Incremental change of the velocity-to-be-gained vector over one sample period ( $\Delta \underline{v}_R - \underline{g} \Delta t$ ).





**NOTE:**  
 $t$  must be equal to  
 $t_{IG}$  for cislunar  
 midcourse cor-  
 rections.

**Figure 3.3-9 Lambert Aim Point Maneuver Pre-Thrust Routine**

Note: These calculations are performed in P31, P34, P35, and P40, P41, P42

The rotation switch  $S_R$  is set in the Lambert Aim Point targeting programs P-34 and P-35 to indicate whether the target aim point vector was within a specified cone angle,  $\epsilon$ , measured from the  $180^\circ$  transfer angle condition. If the initial target vector were within this cone angle, it is rotated into the active vehicle orbital plane in the Initial Velocity Subroutine of Section 5.5.11 so that excessive plane change and  $\Delta V$  requirements are avoided about the  $180^\circ$  transfer angle condition. In the intercept targeting programs P-34 and P-35 the cone angle  $\epsilon$  is set at  $15^\circ$ , and active vehicle transfer angles between  $165^\circ$  to  $195^\circ$  are normally avoided in the targeting procedure. If a transfer angle condition falling within this  $180^\circ \pm 15^\circ$  sector ( $S_R = 1$ ) is intentionally selected either during the TPI targeting (P-34), or results for a rendezvous midcourse correction maneuver (P-35) during an intercept trajectory targeted for more than  $180^\circ$ , the Lambert Aim Point Maneuver Pre-Thrust Routine of Fig. 3.3-9 increases the Initial Velocity Subroutine cone angle  $\epsilon$  to  $45^\circ$  so that active vehicle transfer angle will not change from inside to outside the cone angle during the powered maneuver. Such a condition is undesirable since the intercept trajectory would be retargeted during the powered maneuver. Likewise, if the initial transfer central angle falls outside the  $15^\circ$  cone angle  $\epsilon$  of programs P-34 and P-35 ( $S_R = 0$ ),  $\epsilon$  is decreased to  $10^\circ$  in Fig. 3.3-9 to reduce the possibility of the transfer angle changing from outside to inside the cone angle during a powered maneuver.

If Lambert Aim Point Maneuver Guidance is used in cislunar space for midcourse corrections, the state vector must be uplinked for the ignition time along with the Lambert target parameters.

5.3.3.3.3 Initial IMU Alignment and Maneuver Time-to-Go Computations

Both External  $\Delta V$  and Lambert Aim Point guidance pre-thrust computations require the determination of the IMU middle gimbal angle, MGA, and estimated maneuver time,  $t_{go}$ , prior to engine ignition. These operations are shown in Fig. 3.3-10. The input parameters required for these computations have been defined in Sections 5.3.3.3.1 and 5.3.3.3.2. With reference to Fig. 3.3-10 the initial thrust direction unit vector,  $\underline{u}_{TD}$ , and vehicle state vector are used to determine the Preferred IMU Alignment (Section 5.1.4.2). In Fig. 3.3-10 the following parameter definitions apply:

$\Delta V_K$	Required $\Delta V$ Monitor Subroutine parameter (Sec. 5.3.3.6).
$\dot{m}$	Mass flow rate for APS.
$\Delta t_{tail-off}$	Cross Product Steering Routine parameters (Sec. 5.3.3.4).
$V_e$	
$\underline{X}_{SM}$	Unit vectors in Basic Reference Coordinates (BRC) of the directions of the IMU X, Y, and Z axes for preferred alignment.
$\underline{Y}_{SM}$	
$\underline{Z}_{SM}$	
$N_C$	Required $\Delta V$ Monitor Subroutine counter (Sec. 5.3.3.6).
[REFSMMAT]	Transformation matrix from the BRC System to the IMU or Stable Member Coordinate System. (Sec. 5.6.3.2).
[SMNB]	Transformation matrix from the Stable Member Coordinate System to the Navigation Base Coordinate System. (Sec. 5.6.3.2).
$\underline{u}_A$	Unit vector of the assumed thrust acceleration vector along the Navigation Base X axis.

In Lambert Aim Point guidance pre-thrust computations the angle between  $\underline{u}_{TD}$  and the IMU inner axis ( $\underline{Y}_{REF}$ ) is computed as described in Section 5.4.4.2 where  $\underline{u}_{TD}$  is set equal to  $\Delta \underline{V}$ . This angle is equivalent to the IMU middle gimbal angle (MGA) when the vehicle X axis is aligned along  $\underline{u}_{TD}$ , and is used to check for gimbal lock tolerance. In External  $\Delta V$  guidance pre-thrust computations, the MGA display is computed as the angle between the input velocity vector  $\Delta \underline{V}_{LV}$  transformed to the Basic Reference Coordinate System and  $\underline{Y}_{REF}$  since  $\underline{u}_{TD}$  computations involving the inplane maneuver angle compensation can not be made until the engine has been selected by the appropriate P40, 41, or 42 program.

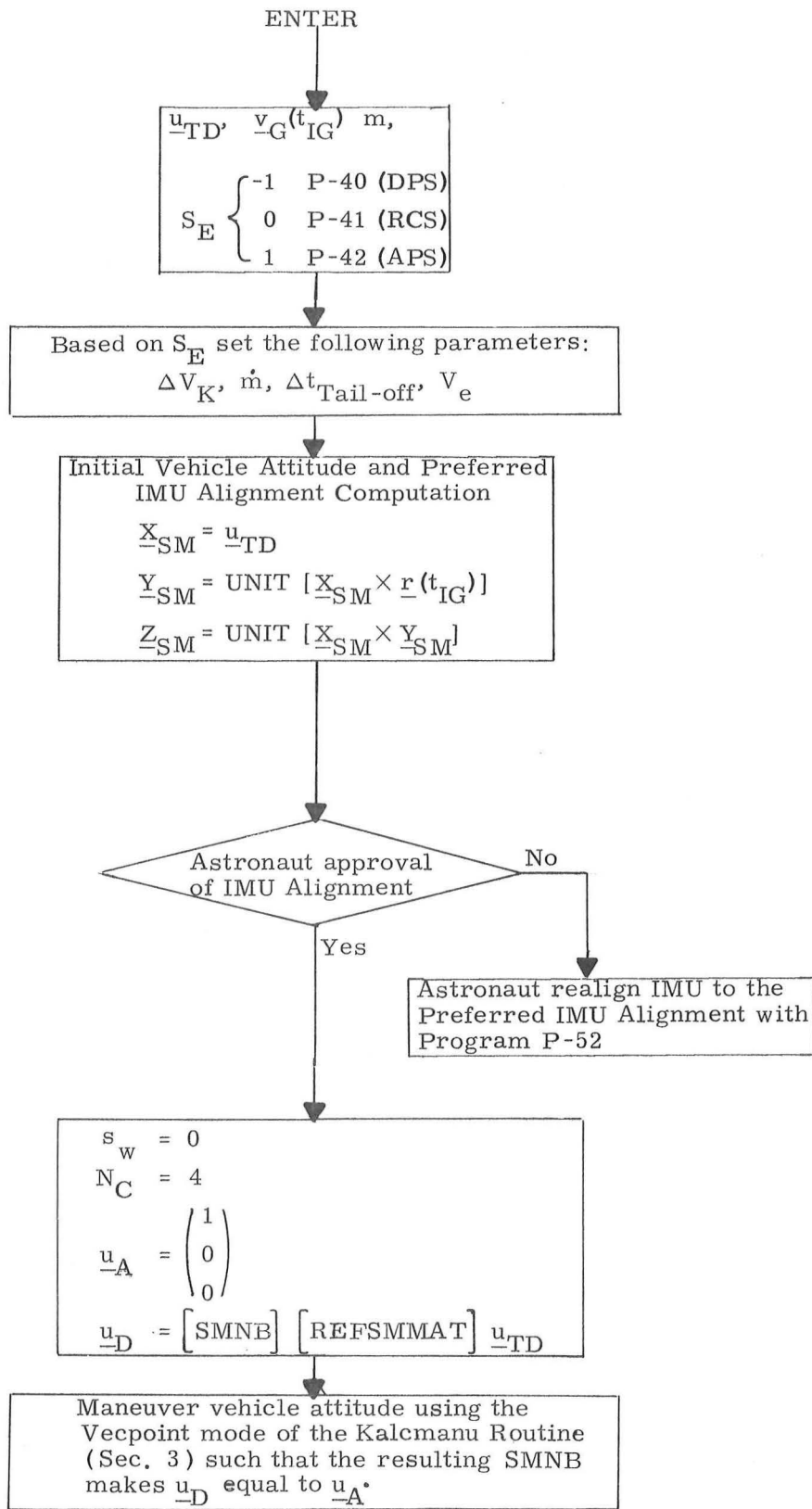


Figure 3.3-10 Initial Attitude and  $t_{go}$  Routine  
(Page 1 of 2)

5.3-27

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Rev. 2

Date 12-12-68

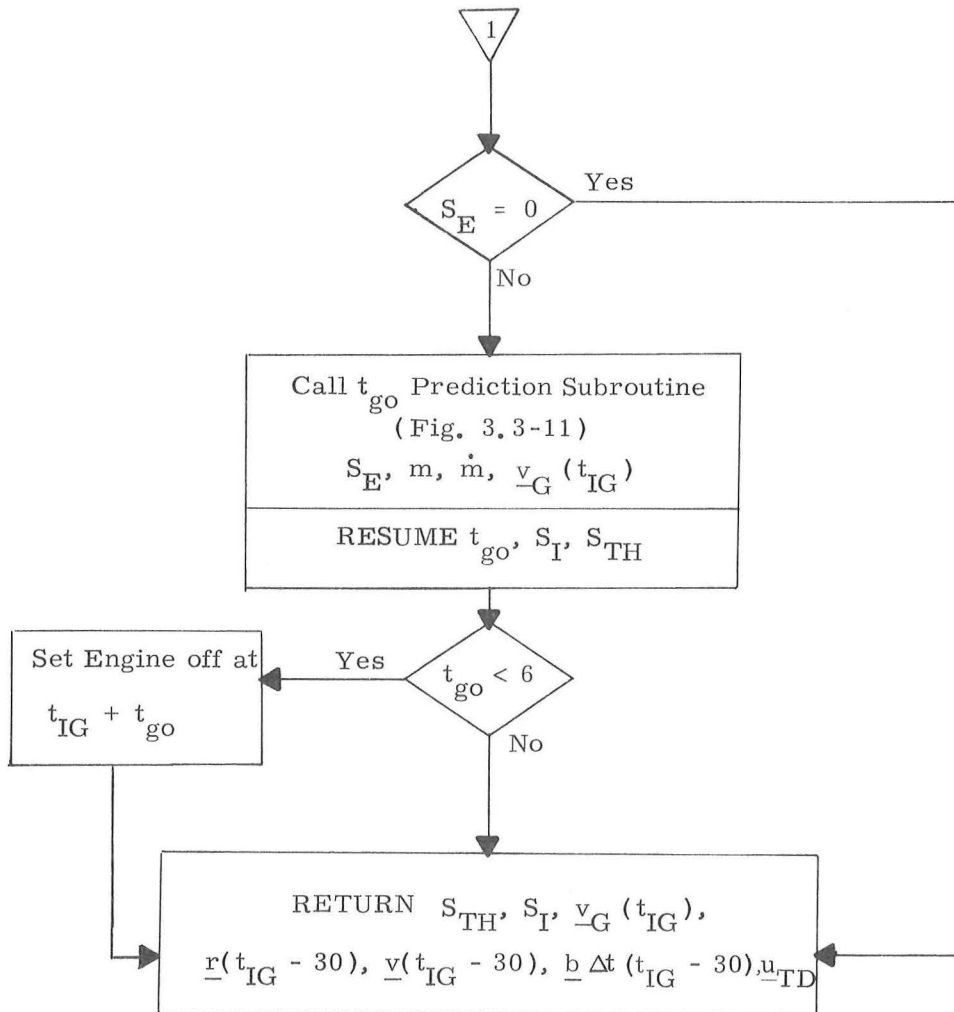


Figure 3.3-10 Initial Attitude and  $t_{go}$  Routine  
(Page 2 of 2)

If either the DPS or APS were selected for the maneuver, the Time-to-Go Prediction Subroutine of Fig. 3.3-11 is then processed. With reference to Fig. 3.3-11:

$S_{TH}$	DPS throttle inhibit switch			
$S_{TH}$	<table> <tr> <td rowspan="2">}</td> <td>0 Allow DPS throttle up</td> </tr> <tr> <td>1 Inhibit DPS throttle up</td> </tr> </table>	}	0 Allow DPS throttle up	1 Inhibit DPS throttle up
}	0 Allow DPS throttle up			
	1 Inhibit DPS throttle up			
$S_I$	Impulse Switch			
$S_I$	<table> <tr> <td rowspan="2">}</td> <td>0 Allow steering at the proper time via <math>\Delta V</math> Monitor Routine (Section 5.3.3.6)</td> </tr> <tr> <td>1 Indicates short maneuver and no steering is required</td> </tr> </table>	}	0 Allow steering at the proper time via $\Delta V$ Monitor Routine (Section 5.3.3.6)	1 Indicates short maneuver and no steering is required
}	0 Allow steering at the proper time via $\Delta V$ Monitor Routine (Section 5.3.3.6)			
	1 Indicates short maneuver and no steering is required			
$F_L$	Ullage thrust			
$K_1$	APS impulse velocity acquired in a one second maneuver for a unit mass vehicle			
$K_2$	APS minimum impulse constant			
$K_3$	APS minimum impulse constant equal to the slope of minimum impulse curve.			
$K_4$	10% DPS engine thrust			
$F_{APS}$	APS Thrust			

The initial computation in Fig. 3.3-11 estimates the velocity-to-be-gained after 7 seconds of ullage. If the DPS were selected (P-40), the maneuver time,  $t_{go}$ , is then computed on the basis of 10% thrust. If this time is less than 6 seconds no active guidance steering is attempted ( $S_I = 1$ ) and the vehicle attitude is maintained at the pre-thrust alignment throughout the maneuver. If the DPS  $t_{go}$  is greater than 95 seconds normal steering will

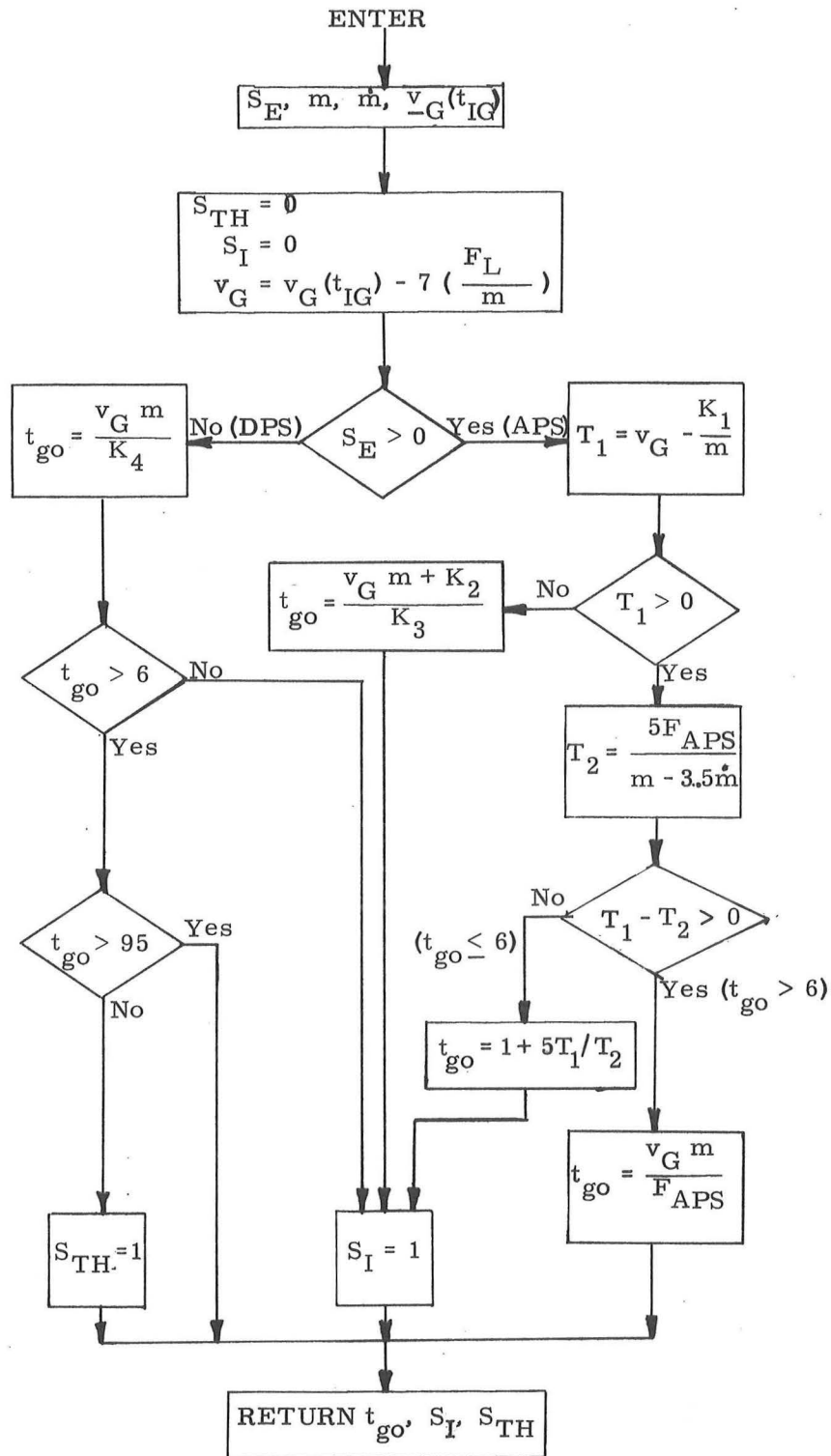


Figure 3.3-11 Time-to-go Predictor Routine

be allowed to throttle the DPS to FTP after the nominal DPS trim phase at 10% thrust. If the estimated DPS  $t_{go}$  is less than 95 seconds, the DPS will be inhibited from throttling up to FTP after the 10% thrust trim phase ( $S_{TH} = 1$ ), and will remain at 10% thrust for the duration of the maneuver. It should be noted that the DPS start trim phase duration (nominally 26 seconds) is an erasable memory parameter and can be modified.

If the APS were chosen (P-42) in Fig. 3.3-11, a check is first made to determine if the maneuver time is less than one second. If the maneuver time is less than one second, the  $t_{go}$  estimate is made on APS minimum impulse test data represented by the constants  $K_2$  and  $K_3$  (See Section 5.8). In this case no active steering is attempted ( $S_I = 1$ ). If the maneuver time is greater than one second but less than 6 seconds,  $t_{go}$  is computed as shown in Fig. 3.3-11 and again no active steering is attempted. If the estimated maneuver time is greater than 6 seconds active steering is used and  $t_{go}$  computations are performed during the maneuver.

With reference to Fig. 3.3-10, if the RCS were chosen for the maneuver, (P-41), no  $t_{go}$  prediction is made.

If the estimated maneuver time,  $t_{go}$ , for either the APS or DPS is less than 6 seconds, the Engine Off signal is set for  $t_{IG} + t_{go}$  as shown in Fig. 3.3-10.



#### 5.3.3.4 Cross Product Steering Routine

The cross product steering concept is the basic control concept for both External  $\Delta V$  and Lambert Aim Point guidance modes. The cross product steering logic is shown in Fig. 3.3-12. The following parameter definitions not previously described apply to this figure.

$\Delta v_{-SM}$	PIPA measured velocity vector over the computation cycle $\Delta t$ in IMU coordinates.
$\Delta v$	PIPA measured velocity vector over the computation cycle $\Delta t$ transformed to the Basic Reference Coordinate System.
$s_w$	A logic switch in the cross product steering routine which when set to 1 allows steering commands and $t_{go}$ calculations to be made. When set to 0, the velocity-to-be-gained, $\underline{v}_G$ , is computed by $\underline{b}\Delta t$ and $\Delta v$ only (Section 5.3.3.5). $s_w$ is set to 1 in the $\Delta V$ Monitor Subroutine of Section 5.3.3.6 after engine thrust has been detected, and set to 0 when the computed time-to-go first becomes less than four seconds.
$\Delta t_{Tail-off}$	A constant representing the duration of a burn at full or maximum thrust equivalent to the tail-off impulse after the engine-off signal is issued.
$V_e$	Engine exhaust velocity = $g I_{SP}$

<u>CDUM</u>	The three measured IMU CDU angles at a given computation cycle.
	$\left. \begin{array}{l} \text{CDUX} = \text{Outer} \\ \text{CDUY} = \text{Inner} \\ \text{CDUZ} = \text{Middle} \end{array} \right\} \text{IMU gimbal angles}$
$(\text{CDUX})_0$	Measured IMU outer gimbal angle at the desired pre-thrust vehicle orientation.
[REFSMMAT]	Transformation matrix between the BRC system and the IMU or Stable Member Coordinate system (Section 5.6.3.2).
[SMNB]	Transformation matrix between the Stable Member Coordinate system and the Navigation Base Coordinate System (Section 5.6.3.2).
<u>E</u>	Cross product steering command vector
	$\left. \begin{array}{l} E_X \\ E_Y \\ E_Z \end{array} \right\} \text{steering commands in vehicle axis}$
<u>u<sub>A</sub></u>	Unit vector of the actual thrust acceleration vector in Navigation Base Coordinates. This parameter is initialized in the Pre-thrust computations of Fig. 3.3-10.
<u>u<sub>D</sub></u>	Unit vector of the desired thrust acceleration vector in Navigation Base Coordinates.
<u>CDUD</u>	Desired IMU CDU angles based on steering commands. This is the primary output signal to the LM digital autopilot (DAP)
<u>CDUC</u>	Cross Product Steering command vector in terms of IMU CDU angles.

With reference to Fig. 3.3-12, the logic switch  $S_w$  is set to zero by the sequencing routine. When  $S_w$  is zero the only function of the cross product steering routine is to update the velocity-to-be-gained vector  $\underline{v}_G$  with  $\Delta \underline{v}$  and  $\underline{b} \Delta t$ .

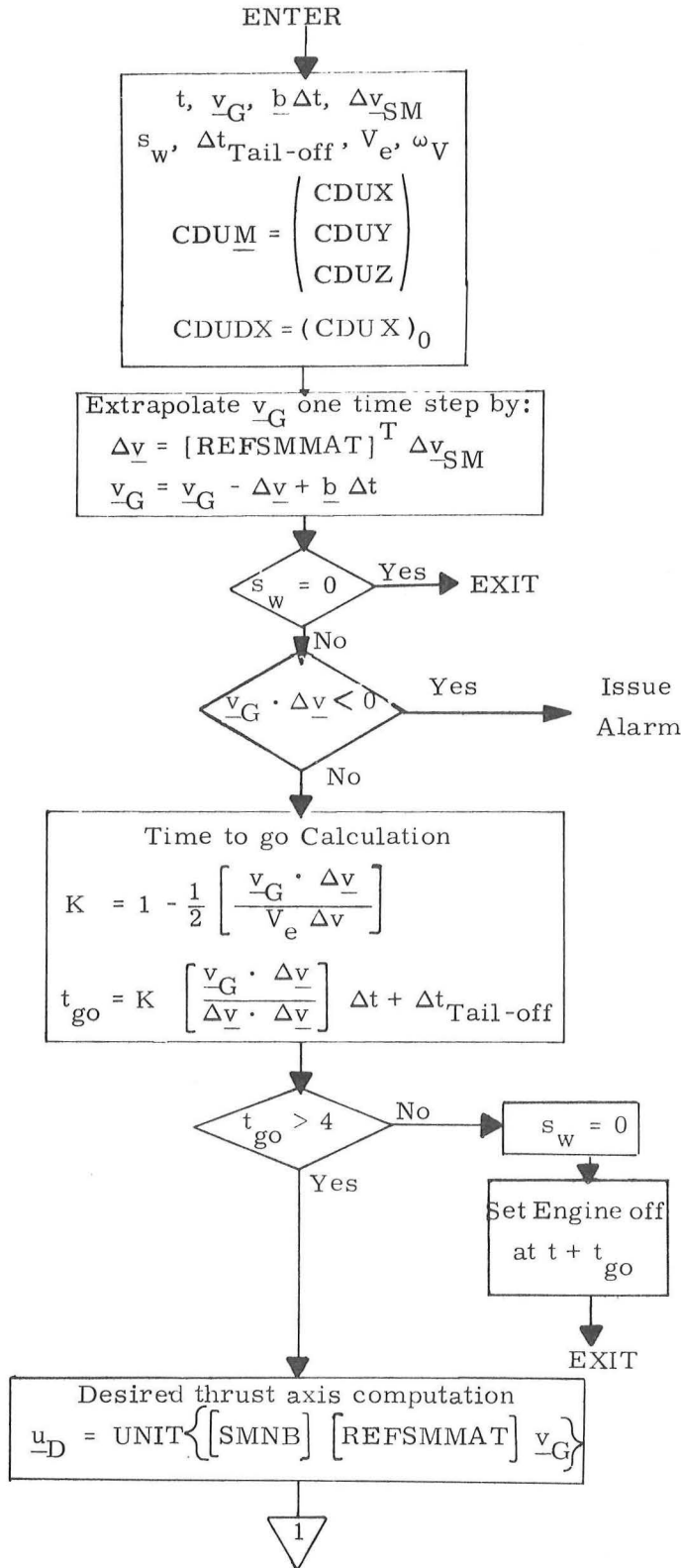


Figure 3.3-12 Cross Product Steering Subroutine  
(page 1 of 2)

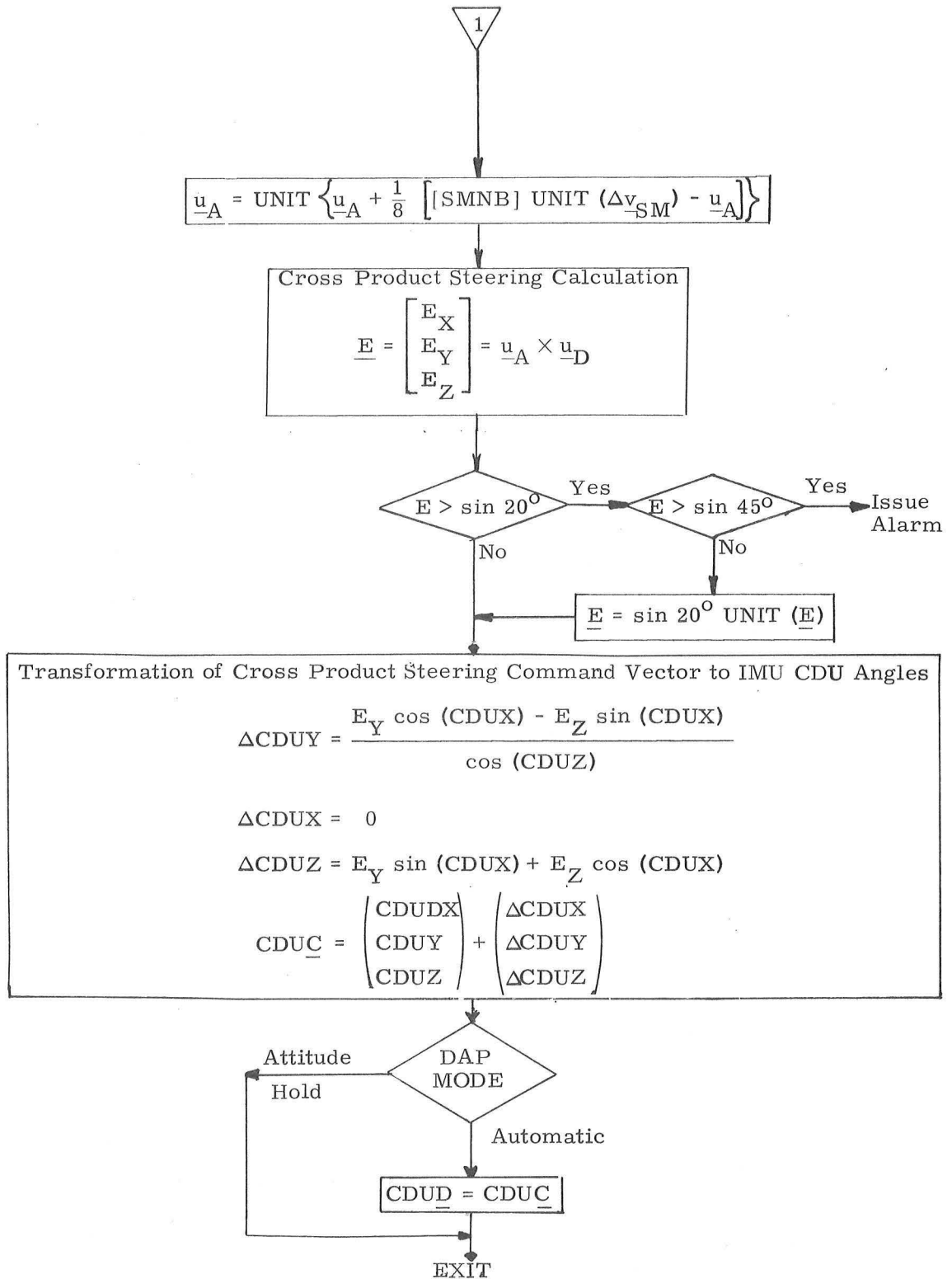


Figure 3.3-12 Cross Product Steering Subroutine (page 2 of 2)

Switch  $s_w$  remains zero for short duration thrust periods or during the initiation of long duration thrust periods before the thrust has increased above the threshold level. In both these cases there is no active steering and the vehicle attitude is held at the pre-thrust alignment. When  $s_w$  is set to 1, active steering is performed. The time-to-go,  $t_{go}$ , computation and steering commands,  $CDUD$ , are performed as shown in Fig. 3.3-12. When the computed  $t_{go}$  becomes less than 4 seconds, then the engine-off signal is set and switch  $s_w$  is reset to zero. For the remainder of the maneuver, no further computations are made except for  $v_G$  updating.

The steering command generated by this routine is the cross product of the desired thrust direction vector,  $\underline{u}_D$ , and the actual thrust acceleration vector,  $\underline{u}_A$ . Both of these vectors are expressed in Navigation Base Coordinates, and the thrust acceleration vector,  $\underline{u}_A$ , is generated by the smoothing computation shown in Fig. 3.3-12. The objective of the cross product steering concept is to control the thrust acceleration vector such that:

$$\underline{E} = \underline{u}_A \times \underline{u}_D = \underline{0}$$

The cross product steering command vector,  $\underline{E}$ , is transformed to IMU CDU angles,  $CDUC$ , required to align the actual thrust acceleration vector with the desired thrust direction. The calculation of the CDU angles,  $CDUC$ , of Fig. 3.3-12 results in attitude hold about the vehicle X axis for a zero IMU middle gimbal angle condition. This is accomplished by setting  $CDUDX$  equal to the measured IMU outer gimbal angle,  $CDUX$ , at the desired pre-thrust vehicle orientation. If the DAP mode switch is in automatic, the desired CDU angles,  $CDUD$ , are set equal to  $CDUC$ . The primary output signal to the LM digital autopilot (DAP) is  $CDUD$ . If the DAP mode switch is in attitude hold, the DAP maintains the attitude defined by the last  $CDUD$ , and the IMU CDU angles  $CDUC$  of Fig. 3.3-12 are used to determine attitude errors for display.

The following control options are provided by the cross product steering subroutine:

- 1) An alarm is issued if the actual thrust direction deviates more than  $45^{\circ}$  from the desired thrust direction ( $E > \sin 45^{\circ}$ ).
- 2) The commanded gimbal angle change is limited to a value of  $20^{\circ}$  difference between the actual and desired thrust directions if  $E > \sin 20^{\circ}$ .

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### 5.3.3.5 Velocity-to-be-Gained Routine

The velocity-to-be-gained computations shown in Fig. 3.3-13 are those carried out during the Lambert Aim Point powered flight guidance. The velocity-to-be-gained computation for the External  $\Delta V$  guidance mode is simpler than that for the Lambert Aim Point guidance mode. The External  $\Delta V$  velocity-to-be-gained computation is that shown in the cross product steering routine of Fig. 3.3-12 and is equivalent to

$$\underline{v}_G = \underline{v}_G - \underline{\Delta v}$$

since  $\underline{b}\Delta t = \underline{0}$  for the External  $\Delta V$  mode as shown in Fig. 3.3-7.

The velocity-to-be-gained computations for the Lambert Aim Point guidance mode involve the determination of a new  $\underline{v}_G$  by processing the Lambert Subroutine via the Initial Velocity Subroutine. A second objective is the computation of a new  $\underline{b}\Delta t$  parameter for use by the cross product steering routine. A new required velocity,  $\underline{v}_R$ , is also determined for use in the next computation cycle of the velocity-to-be-gained subroutine.

The following parameter definitions refer to Fig. 3.3-13.

$\left. \begin{array}{l} \underline{r}(t) \\ \underline{v}(t) \\ t \end{array} \right\}$  active vehicle state vector

$\underline{r}(t_2)$  Offset target intercept position vector at time  $t_2$ . This parameter is determined by the preceding targeting program (P-34 or P-35).

$t_2$	Intercept time of arrival associated with the offset target vector $\underline{r}(t_2)$ . This is an input target parameter.
$t_{IG}$	Nominal ignition time.
$\underline{v}_R(t-\tau)$	Required velocity vector at the preceding computation cycle.
$\tau$	Time interval between the current and previous computation cycles.
$S_R$	Target rotation switch set in P-34 or P-35 indicating that the target vector was rotated into the orbital plane due to proximity to $180^\circ$ transfer. $S_R \begin{cases} 1 & \text{Target vector rotated} \\ 0 & \text{No rotation} \end{cases}$
$\underline{g}$	Total gravity acceleration vector (Section 5.3.2).
$\underline{g}_b$	Component of the earth gravity acceleration vector representing earth oblateness effects (Section 5.3.2).
$\Delta \underline{v}(t)$	Measured PIPA velocity change in the BRC system.
$N_1$	Number of target offset iterations desired in the Initial Velocity Subroutine (Section 5.5.11).
$\epsilon$	Initial Velocity Subroutine parameter.
$\underline{v}_R(t)$	Required velocity vector at time t.
$\underline{v}_G(t)$	Velocity-to-be-gained vector at time t.
$\underline{v}_G(t+\Delta t)$	Extrapolated $\underline{v}_G$ one computation time step.



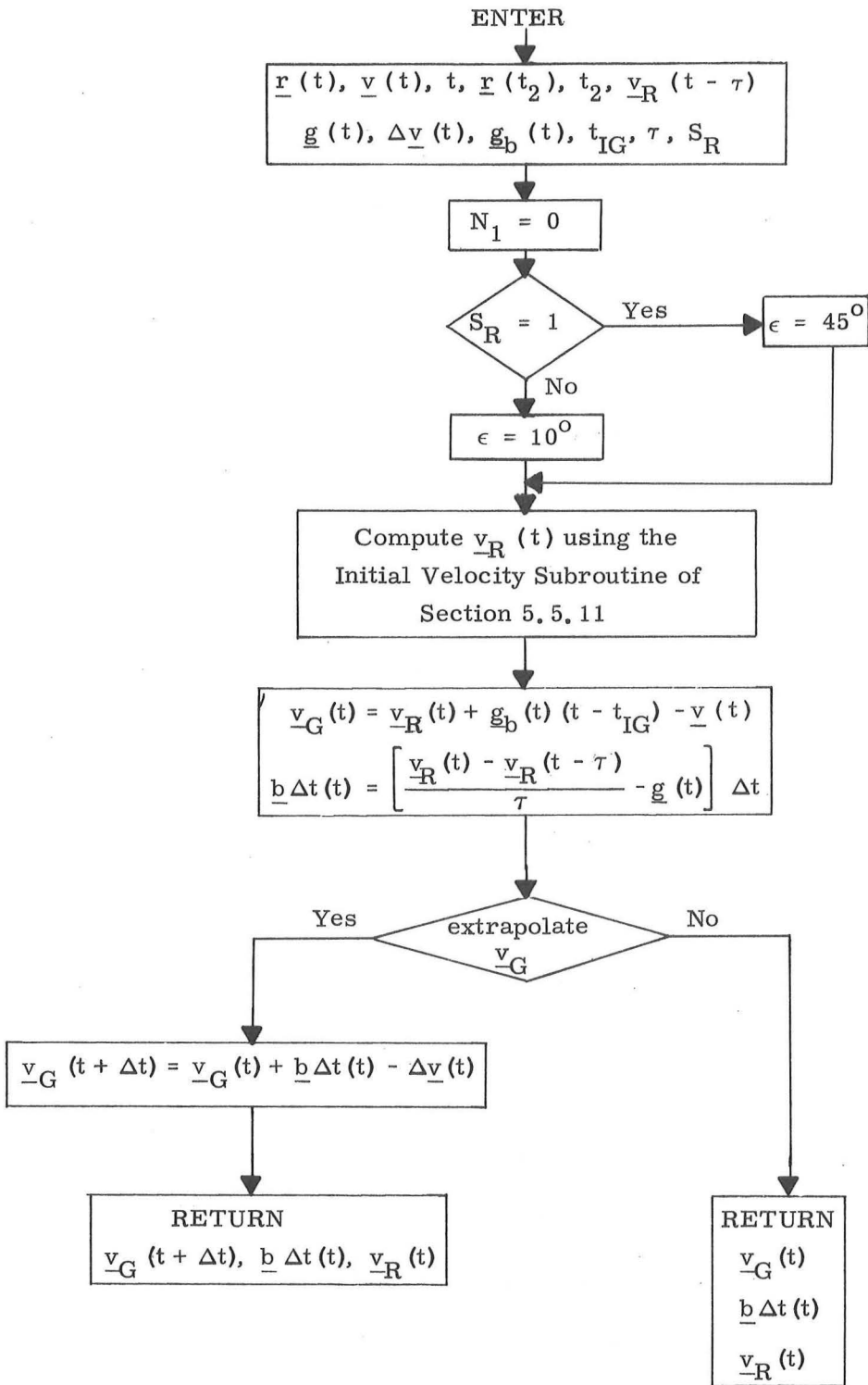


Figure 3.3-13 Velocity-to-be-Gained Subroutine

It should be noted that in Fig. 3.3-13 the velocity-to-be-gained,  $\underline{v}_G$ , derived from the Lambert solution using an offset target vector is modified by the term  $\underline{g}_b(t) [t - t_{IG}]$ . This term is an approximation to the velocity change contributed by the earth oblateness effect. The compensation used in this subroutine is computed as the current oblateness acceleration,  $\underline{g}_b(t)$ , multiplied by the period of thrust application  $(t - t_{IG})$  where  $t_{IG}$  is the nominal ignition time. This correction is zero for lunar orbits. The objective of this correction is to reduce cut-off errors due to finite maneuver time effects, and to minimize commanded thrust attitude variations during the maneuver. These two effects occur during long maneuvers because in accounting for earth oblateness effects in the initial targeting programs (P-34 and P-35), it is assumed that an impulsive maneuver will be applied at ignition time. Since a finite maneuver time is required, the precomputed target aim point becomes less accurate as the maneuver progresses. The  $\underline{g}_b(t - t_{IG})$  correction is an approximate substitute for a retargeting procedure which can not be performed during a powered maneuver.

The velocity-to-be-gained subroutine of Fig. 3.3-13 accounts for the possibility that the  $\underline{v}_G$  computation will not be completed for use during the next 2 second computation cycle. In this case the  $\underline{v}_G$  is extrapolated forward over a 2 second  $\Delta t$  interval by use of the velocity-to-be-gained increment  $\underline{b} \Delta t$ , which was just computed, and the last PIPA velocity measurement,  $\underline{\Delta v}$ .

### 5.3.3.6 $\Delta V$ Monitor Subroutine

The purpose of the  $\Delta V$  Monitor Subroutine is to detect and monitor the presence of engine thrust every 2 seconds commencing at the nominal ignition time. The subroutine also turns on the cross product steering routine if the maneuver is not a short burn. If subsequent to the nominal ignition time or during the thrusting period, the engine thrust is detected to be below a given level for 2 cycles of this routine, control is transferred to the engine fail procedure (Section 4).

The  $\Delta V$  Monitor Subroutine logic diagram is illustrated in Fig. 3.3-14. The following parameter definitions apply to this figure.

$\Delta v$	PIPA measured velocity change over the last computation cycle.
$N_C$	A counter in the $\Delta V$ Monitor Subroutine which indicates an engine failure when $\leq 0$ . This counter is initialized during pre-thrust computations to a value of 4.
$S_I$	Impulse Switch
$S_I$	$\left\{ \begin{array}{l} 0 \text{ Allows steering at the proper time via } \Delta V \text{ Monitor Subroutine} \\ 1 \text{ Indicates short maneuver and no steering is required} \end{array} \right.$
$\Delta V_K$	A constant set by the pre-thrust routine which establishes the $\Delta V$ which must be sensed in a 2 second computation interval if the engine is to be considered on.

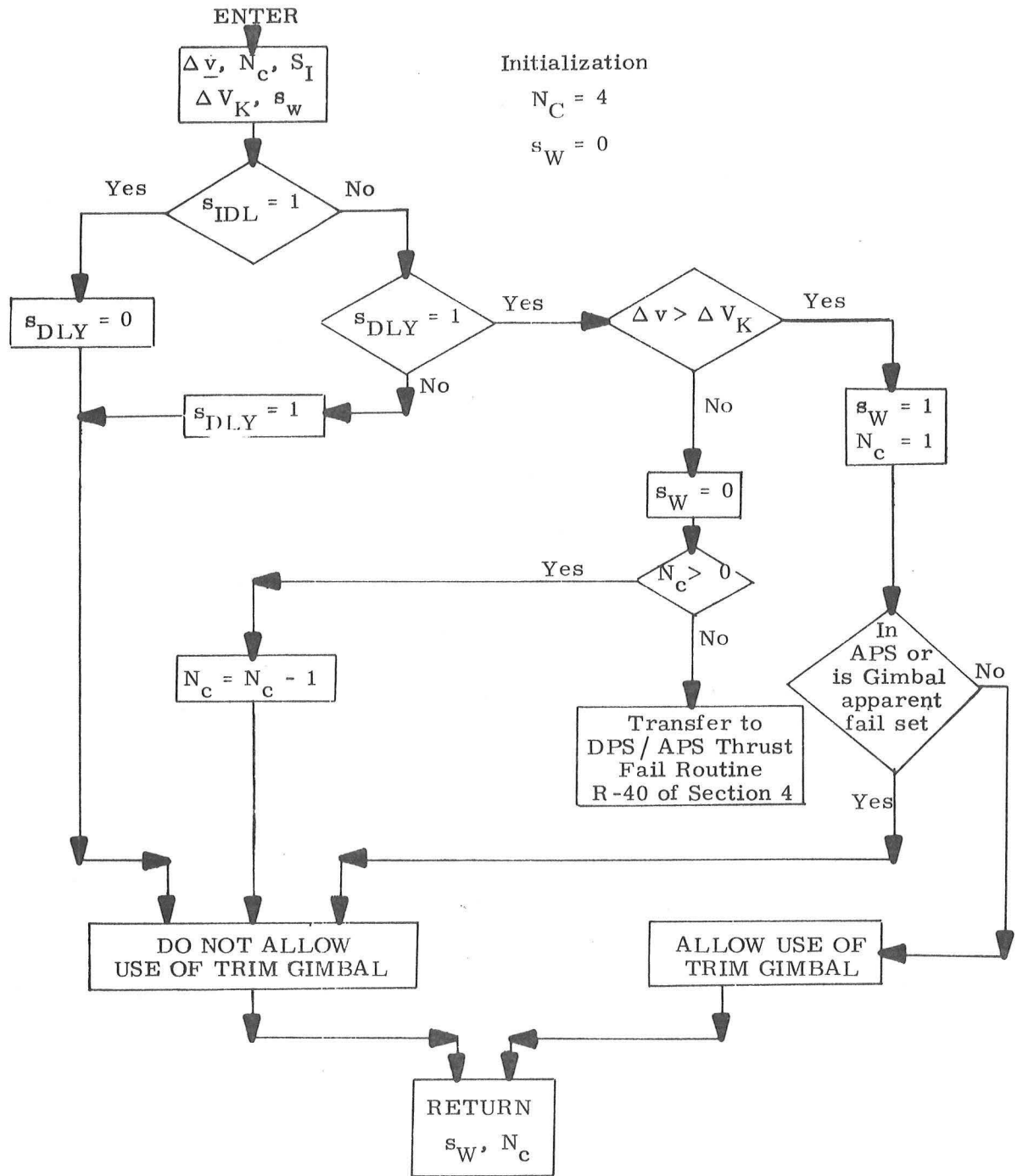


Figure 3.3-14  $\Delta V$  Monitor Subroutine

$s_w$  A logic switch in the cross product steering routine which when set to 1 allows steering commands and  $t_{go}$  calculations to be made. When set to 0, the velocity-to-be-gained,  $\underline{v}_G$ , is computed by  $\underline{b} \Delta t$  and  $\Delta \underline{v}$  only. (Section 5.3.3.4).

$S_{IDL}$   $\Delta V$  Monitor Idle Switch

$S_{IDL} \begin{cases} 1 & \text{By pass } \Delta V \text{ Monitor Operation} \\ 0 & \text{Activates } \Delta V \text{ Monitor Operation} \end{cases}$

$S_{IDL}$  is set to 1 prior to ignition for all maneuvers. For all RCS maneuvers (P-41), APS maneuvers (P-42) and DPS maneuvers (P-40) of less than 6 seconds estimated duration ( $S_I = 1$ )  $S_{IDL}$  is maintained at 1. For DPS and APS maneuvers of greater than 6 seconds,  $S_{IDL}$  is set to zero at ignition.  $S_{IDL}$  is set to 1 when  $t_{go} < 4$  seconds.

$S_{DLY}$   $\Delta V$  Monitor Delay Switch

$S_{DLY} \begin{cases} 0 & \text{By pass } \Delta V \text{ test in } \Delta V \text{ Monitor} \\ 1 & \text{Activate } \Delta V \text{ test in } \Delta V \text{ Monitor} \end{cases}$

The function of  $S_{DLY}$  is to prevent the  $\Delta V$  Monitor from checking  $\Delta V$  until  $(t_{IG} + 2)$ , since  $S_{IDL}$  is set to 0 at  $t_{IG}$ .

It should be noted that the  $\Delta V$  Monitor Subroutine is activated by the Servicer Routine during all PGNCS controlled thrust maneuvers. The  $\Delta V$  Monitor Subroutine is activated at  $(t_{IG} + 2)$  for DPS and APS maneuvers exceeding 6 seconds. The engine fail procedures activated by this subroutine differ for various mission phase maneuvers as described in R-40 of Section 4.

### 5.3.6 THRUST MONITOR PROGRAM

The Thrust Monitor Program, P-47, is used during manual or non-PGNCS controlled maneuvers to monitor and display the velocity change applied to the vehicle. The program should be called at least one minute before the start of such a maneuver. The selection of this program suspends state vector updating by the RR (resets the update and track flags), and P-47 then advances the vehicle state vector to the current time plus one minute ( $t + 60$ ) by the Coasting Integration Routine of Sec. 5.2.2. At  $t + 60$  the Average-G Routine of Sec. 5.3.2 is called and left on until the program is terminated after the maneuver. The primary output of the P-47 is the measured maneuver  $\Delta \underline{V}$  in vehicle coordinates as described in Section 4.

The Thrust Monitor Program is normally used during the manually controlled terminal rendezvous phase. During this mission phase the Rendezvous Display Routine R-31 is normally called to display range, range rate and the line-of-sight angle  $\theta$ . The operation of R-31 with the Average-G Routine of P-47 is described in Section 5.6.7.

5.4 TARGETING ROUTINES

5.4.1 GENERAL COMMENTS

The objectives of the targeting routines presented in this section are to provide the LGC capability to determine the required input target parameters and set control modes for the various powered flight guidance routines of Section 5.3. The LGC targeting capability represented by the programs in the following subsections is sufficient to complete the nominal LM rendezvous phases of the mission. These LGC targeting routines are summarized in the following subsection:

5.4.4 Rendezvous Targeting

These routines include the following rendezvous maneuver phases:

1. Pre-CSI
2. Pre-CDH
3. Pre-TPI
4. Rendezvous Midcourse Corrections

All LGC targeting programs use the Basic Reference Coordinate System defined in Section 5.1.4.1. The Rendezvous Targeting Routines of Section 5.4.4 can be used in either earth or lunar orbits. The basic input parameters required by the targeting routines of this section are the vehicle state vector estimates determined by the navigation programs of Section 5.2.

The above four routines are discussed in the following sections as if they could be used with either vehicle being the active vehicle. This is no longer the case. The routines associated with allowing the CSM to be the active vehicle (P-72, P-73, P-74 and P-75) have not been tested and are not available for use, although their coding remains in the LGC computer.

#### 5. 4. 4 RENDEZVOUS TARGETING

##### 5. 4. 4. 1 General

The rendezvous Targeting Programs P-32, P-33, P-34 and P-35 in the LGC are based on the Concentric Flight Plan rendezvous scheme. This scheme illustrated in Fig. 4. 4-1 employs an active vehicle to ideally make three impulsive maneuvers to establish a rendezvous intercept trajectory with a passive vehicle in an approximately coplanar orbit. Sufficient constraints are imposed to allow only one degree of freedom in choosing the final rendezvous profile. These constraints include the timing of both the first and third maneuvers, the direction of the first maneuver, a unique formulation for the second maneuver and the relative geometry of the two vehicles at the final maneuver point. The second maneuver results in the active vehicle orbit becoming "coelliptic" or "concentric" with the passive vehicle orbit, resulting in the differential altitude between the orbits being approximately constant. The following names have been given to the basic maneuvers of the Concentric Flight Plan rendezvous profile:

- 1) Coelliptic Sequence Initiation (CSI),
- 2) Constant Differential Altitude (CDH),
- 3) Transfer Phase Initiation (TPI).

A rendezvous midcourse correction program is used in addition to the above three primary rendezvous programs to maintain an intercept trajectory from the TPI maneuver until the manual terminal rendezvous is started.



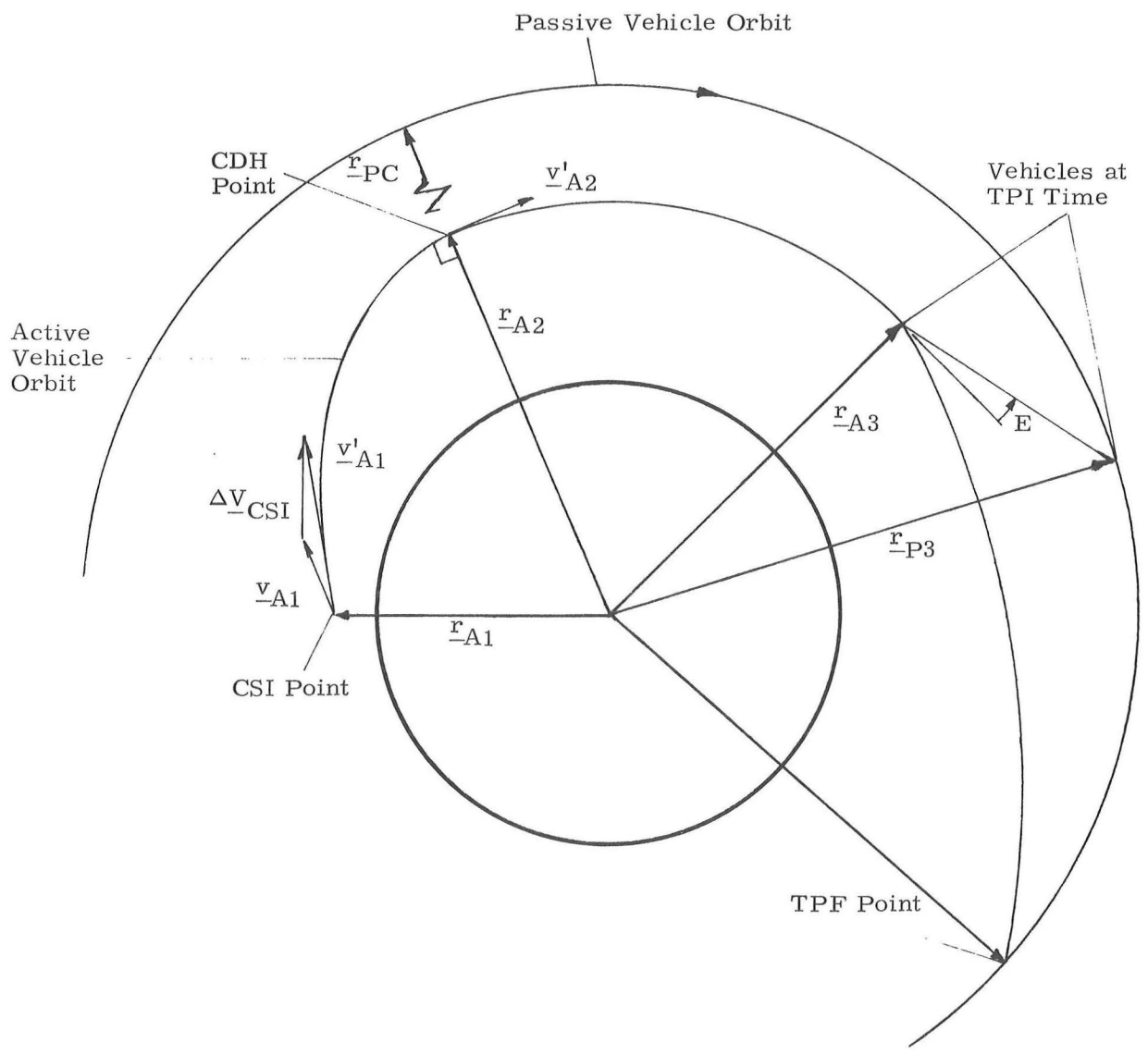


Figure 4.4-1 Concentric Rendezvous Profile

The LGC Rendezvous Targeting Programs consist of the following programs:

- 1) P-32 Pre-CSI Maneuver Program. This program is called by the astronaut any time before the Co-elliptic Sequence Initiation (CSI) maneuver is performed.
- 2) P-33 Pre-CDH Maneuver Program. This program is called between the CSI and Constant Differential Altitude (CDH) maneuvers.
- 3) P-34 Pre-TPI Maneuver Program. This program is called between the CDH and Transfer Phase Initiation (TPI) maneuvers.
- 4) P-35 Rendezvous Midcourse Maneuver Program. This program is called after the TPI maneuver has been performed.

Each program is designed to select either the LM or CSM as the active vehicle depending on the program number selected by the astronaut. All program operations in Section 5.4.4 are independent of which vehicle is chosen as the active vehicle.

A LM direct rendezvous capability is possible using the last two maneuvers in the above list.

All computations performed in the Concentric Rendezvous Program are in the Basic Reference Coordinate System (Section 5.1.4.1) and use the latest state vectors obtained from the Navigation Routines (Sections 5.2 and 5.3.2).

The pre-thrust programs P-32 and P-33 use External  $\Delta V$  guidance (Section 5.3.7) and P-34 and P-35 use the Lambert targeting concept employing two precision offsets (Section 5.3.6). All four of these programs use the Cross-Product Steering of Section 5.3.3.

An alarm code is displayed if the initial conditions along with the astronaut inputs fail to result in a satisfactory solution.

#### 5. 4. 4. 2 Pre-CSI Maneuver

This program, corresponding to Program P-32 (LM active) or P-72 (CSM active) of Section 4, computes the parameters associated with the CSI and CDH maneuvers. The astronaut inputs are:

1. Choice of active vehicle (P-32 LM, P-72 CSM).
2. Time  $t_1$  of the CSI maneuver.
3. Number N of the apsidal crossing.  
(If  $N = 1$ , the CDH maneuver occurs when the active vehicle reaches its first apsidal point following the CSI maneuver, etc.)
4. Desired line of sight LOS angle E at the time of the TPI maneuver. (See Fig. 4. 4-2 and Fig. 4. 4-3).
5. Time  $t_3$  of the TPI maneuver.

The active vehicle state vector  $\underline{r}_A$ ,  $\underline{v}_A$  and passive vehicle state vector  $\underline{r}_P$ ,  $\underline{v}_P$  are available in the guidance computer.

The following constraints must be satisfied (see Fig. 4. 4-1 and Fig. 4. 4-2).

1. The CSI  $\Delta V$  is applied in the horizontal plane of the active vehicle at the CSI time.

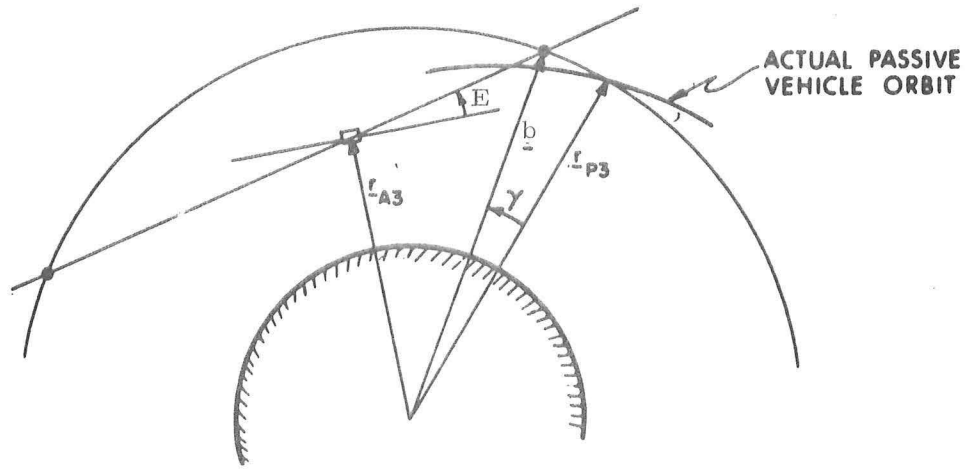


Fig. 4.4-2a TPI geometry, active vehicle below

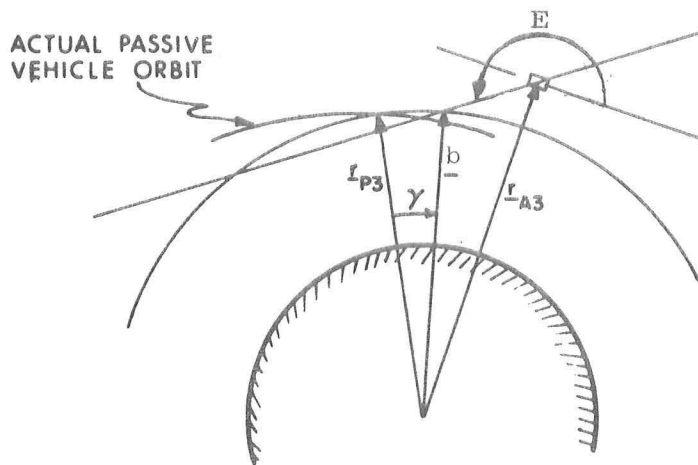
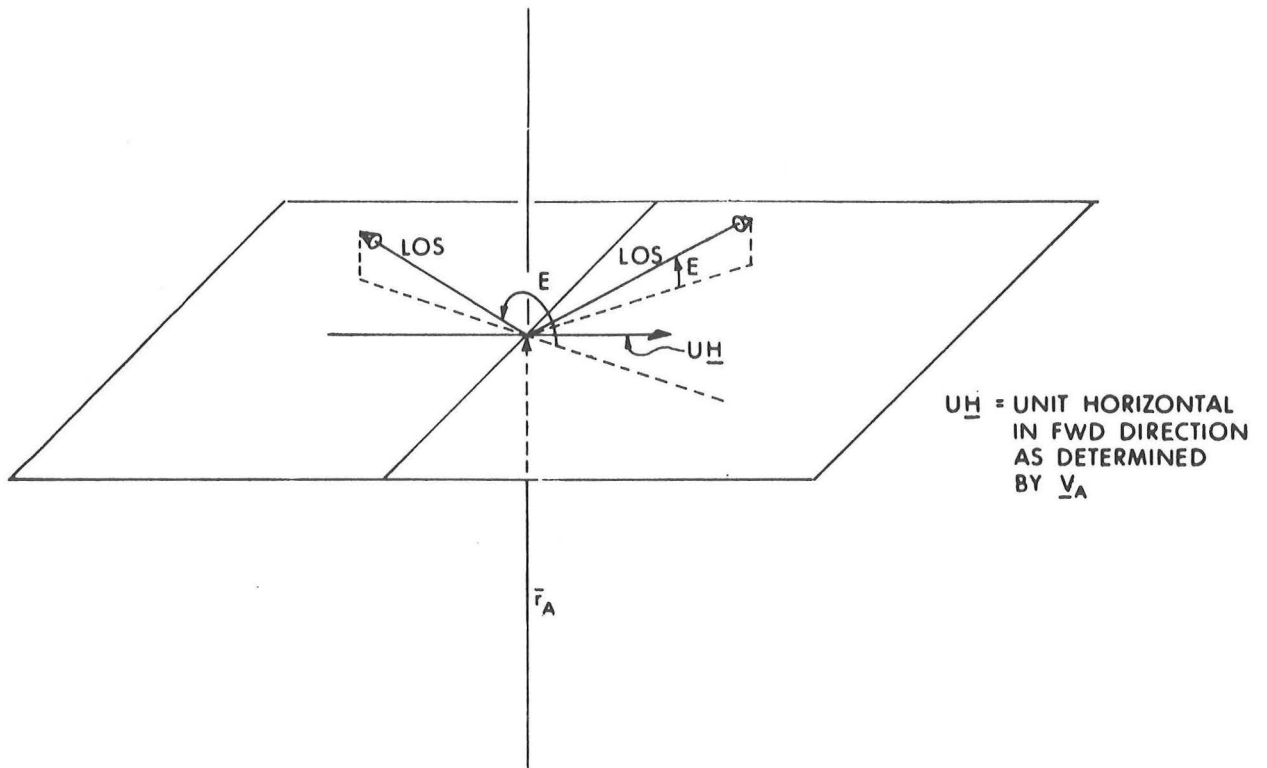


Fig. 4.4-2b TPI geometry, active vehicle above



- 1) IF THE LOS PROJECTION ON  $\underline{U}_H$  IS POSITIVE:
  - a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE  $0 < E < \pi/2$
  - b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE  $3\pi/2 < E < 2\pi$
- 2) IF THE LOS PROJECTION ON  $\underline{U}_H$  IS NEGATIVE
  - a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE  $\pi/2 < E < \pi$
  - b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE  $\pi < E < 3\pi/2$

Fig. 4. 4-3 Definition of Elevation Angle, E

2. The CDH maneuver occurs when the active vehicle is at a specified apsidal crossing.
3. The semimajor axis and radial component of velocity of the active vehicle are such that the active and passive vehicles are in coelliptic orbits following the CDH maneuver.
4. The line of sight connecting the active vehicle and the passive vehicle at the TPI time forms the angle  $E$  with the horizontal plane of the active vehicle.
5. The time intervals between the CSI-CDH and CDH-TPI maneuvers are 10 minutes or greater.
6. After both the CSI and CDH maneuvers, the perigee altitude of the active vehicle orbit is greater than 35,000 ft. for lunar orbits and 85 nm. for earth orbits.

The program solution, as illustrated in the logic flow diagram in Fig. 4.4-4, is based on conic trajectories and contains an iteration loop to select the CSI maneuver magnitude  $v_1$ .

After updating of the active and passive vehicle state vectors to the CSI time, the active vehicle state vector is rotated into the passive vehicle orbital plane resulting in coplanar orbits. The passive vehicle is then updated through the time  $t_3 - t_1$  to the TPI point obtaining  $r_{P3}$ ,  $v_{P3}$ . The initial guess of  $v_1$ , when added to the velocity of the active vehicle, results in the active vehicle attaining a radius equal to  $r_{P3}$  180 degrees from the CSI point.

If the eccentricity  $e$  of the active vehicle orbit is now less than .0001 or the vertical velocity at the CSI point is less than .05 ft/sec., the time  $t_2$  of the CDH maneuver is set equal to the CSI time plus  $N$  times half the period  $t_P$  of the active vehicle. Otherwise, the angle  $\psi$  to the nearest perigee is computed and used in the Time-Theta Subroutine to find the corresponding time of flight  $\Delta t$ . This is then used to calculate  $t_2$  based on the value of  $N$ . After verifying that  $t_3$  is greater than  $t_2$ , both vehicles are updated to the CDH point.

The angle  $\phi$  between the active and passive vehicles at this CDH point is calculated and used in the Time-Theta Routine to update the passive vehicle to a point radially above or below the active vehicle, obtaining  $r_{PC}$ ,  $v_{PC}$ .

The semimajor axis  $a_A$  and the radial component of velocity  $v_{AV}$ , along with a radius, uniquely define the orbit of the active vehicle immediately following the CDH maneuver. The equations for  $a_A$  (based on the semimajor axis  $a_P$  of the passive vehicle) and  $v_{AV}$ , contained in Fig. 4.4-4, therefore specify the "coelliptic" CDH maneuver  $\Delta v_2$ .

The active vehicle state vector is next updated to the TPI time. The unit vector which passes through the active vehicle position and is coincident with the desired TPI line of sight is given by  $\underline{u}_L$ . The position vector of the two points of intersection between the line of sight and a circle (with origin at the center of the attracting body) as shown in Fig. 4.4-2 is defined by:

$$\underline{b} = r_{A3} + k \underline{u}_L \quad (4.4.1)$$

where  $k$  normally assumes two values (see Fig. 4.4-4). Equating the magnitude of  $\underline{b}$  to  $r_{P3}$  results in a quadratic expression for  $k$

$$k^2 + 2k \frac{r_{A3}}{r_{L}} + r_{A3}^2 - r_{P3}^2 = 0 \quad (4.4.2)$$

If there are no real solutions to the above equation ( $c_2 < 0$  in Fig. 4.4-4), the line of sight does not intersect the circle. If there is a real solution to the above equation, the  $k$  of minimum absolute value is chosen as the desirable TPI solution. The error  $\gamma$  is defined as the central angle between the position vector of the passive vehicle at the TPI point and the position vector  $\underline{b}$  representing where the passive vehicle should be based on the active vehicle location. This error is driven to zero using a Newton-Raphson iteration loop with  $v_1$  as the independent variable. The two initial values of  $v_1$  are 10 ft/sec apart with the succeeding step size  $\Delta v_1$  restricted to 200 ft/sec.

A second computational attempt is made if the first iteration attempt fails.  $v_1$  is incremented in steps of 50 ft/sec. in the direction opposite to that initially taken in the first attempt until the error  $\gamma$  has undergone a sign change. The second attempt then starts with the latest value of  $v_1$  and an adjacent point in the direction of the 50 ft/sec. steps.

Included in the Pre-CSI program are the following seven program checks and the corresponding Alarm Codes AC (see Fig. 4.4-4).

1. If on the first iteration there is no solution to the TPI geometry, the iteration cannot be established. The alarm code AC is set equal to A in this case. The alarm code is listed in Section 4.



2. The iteration counter  $n$  exceeds 15. (AC = C)
3. Two succeeding iterations resulting in  $v_1$  greater than 1000 ft/sec. (AC = B)
4. The altitude of perigee  $h_{P1}$  following the CSI maneuver is less than a minimum acceptable amount. (AC = D)
5. The altitude of perigee  $h_{P2}$  following the CDH maneuver is less than a minimum acceptable amount. (AC = E)
6. The time  $\Delta T_2$  between the CDH and CSI maneuver is less than 10 minutes. (AC = F)
7. The time  $\Delta T_3$  between the TPI and CDH maneuvers is less than 10 minutes. (AC = G)

For the first check above, a check failure results in an immediate program exit. The last four checks are made only if the iteration has succeeded. The alarm codes are set (excluding the first check) only if the second iteration attempt has been made.

The outputs for the Pre-CSI maneuver program are:

1. Differential altitude  $\Delta H_{CDH}$  at the CDH point.  $\Delta H_{CDH}$  is positive when the active vehicle's altitude is less than the passive vehicle's altitude when measured at the CDH point.
2. Time  $\Delta T_2$  between the CDH and CSI maneuvers displayed in minutes and seconds with hours deleted.
3. Time  $\Delta T_3$  between the TPI and CDH maneuvers displayed in minutes and seconds with hours deleted.
4.  $\Delta V_{CSI}(LV)$  in local vertical coordinates at CSI time
5.  $\Delta V_{CDH}(LV)$  in local vertical coordinates at CDH time
6. The alarm codes

In the Pre-CSI program P-32, the astronaut can change the CSI computed maneuver by over writing the displayed maneuver velocity vector in local vertical coordinates,  $\Delta \underline{V}_{\text{CSI}}$  (LV). In such a case the astronaut input then becomes the defined velocity-to-be-gained in the External  $\Delta V$  Maneuver Guidance (Section 5. 3. 3. 3. 1) used to control the CSI maneuver. The over writing procedure is normally used to correct or minimize out-of-plane conditions which are not directly controlled by the CSI or CDH maneuvers computations such that minimum out of plane conditions exist at the TPI maneuver.

In addition, M the number of navigation measurements processed since the last maneuver, MGA the middle gimbal angle at the start of the last maneuver assuming plus X acceleration and TFI the time between the CSI maneuver and the present time are displayed. The middle gimbal angle is computed with the following equation.

$$\text{MGA} = \sin^{-1} \left[ \underline{Y}_{\text{REF}} \cdot \text{UNIT} (\Delta \underline{V}) \right]$$

if  $-90^\circ \leq \text{MGA} < 0^\circ$ , MGA is redefined such that  $270^\circ \leq \text{MGA} < 360^\circ$ .  $\Delta \underline{V}$  is the maneuver  $\Delta \underline{V}$  in reference coordinates and  $\underline{Y}_{\text{REF}}$  is the second row of the REFSMMAT matrix defined in Section 5. 6. 3.

#### 5. 4. 4. 3 Pre-CDH Maneuver

This program, corresponding to Program P-33 (LM active) or P-73 (CSM active) of Section 4, computes the parameters associated with the CDH maneuver.

The astronaut inputs to this program are:

1. Choice of active vehicle (P-33 LM, P-73 CSM)
2. Time  $t_2$  of CDH maneuver

The active and passive vehicle's state vectors  $\underline{r}_A$ ,  $\underline{v}_A$ ,  $\underline{r}_P$ ,  $\underline{v}_P$  are available in the guidance computer. The TPI elevation angle E and the TPI maneuver time  $t_3$  are also available from the previous Pre-CSI program computations.

Figure 4.4-4 illustrates the program logic. The state vectors of both the active and passive vehicles are advanced to the CDH time using the Coasting Integration Routine. After rotating the active vehicle state vector into the plane of the passive vehicle, the CDH maneuver is calculated as in the Pre-CSI program. Following a precision update of the state vectors to time  $t_3$ , the Pre-TPI program is used to calculate the time  $t$  at which the specified elevation angle is attained. If the iteration was not successful an alarm code is displayed. At this point, the astronaut can elect to recycle the program or to proceed with the calculation of the displays, which are:

1.  $\Delta V_{CDH}$  (LV)
2. Differential altitude  $\Delta H_{CDH}$  at the CDH point.
3. Time  $\Delta T_3$  between the calculated TPI time and the CDH time displayed in minutes and seconds with hours deleted.
4. Time  $\Delta T_{TPI}$  between the calculated TPI time and the specified TPI time. (This number is positive if the new TPI time is later than that previously used).

If the iteration did not converge, the last two displays reflect an unchanged TPI time. In addition, M, MGA and TFI are displayed as in the Pre-CSI program displays.

The astronaut can modify the CDH maneuver to minimize out-of-plane conditions by overwriting the displayed maneuver velocity vector,  $\Delta V_{CDH}$  (LV), as in the Pre-CSI program. The out-of-plane information used in this process is determined by the astronaut with the aid of the Out of Plane Rendezvous Display Routine, R-36 of Section 5.6.7.3.

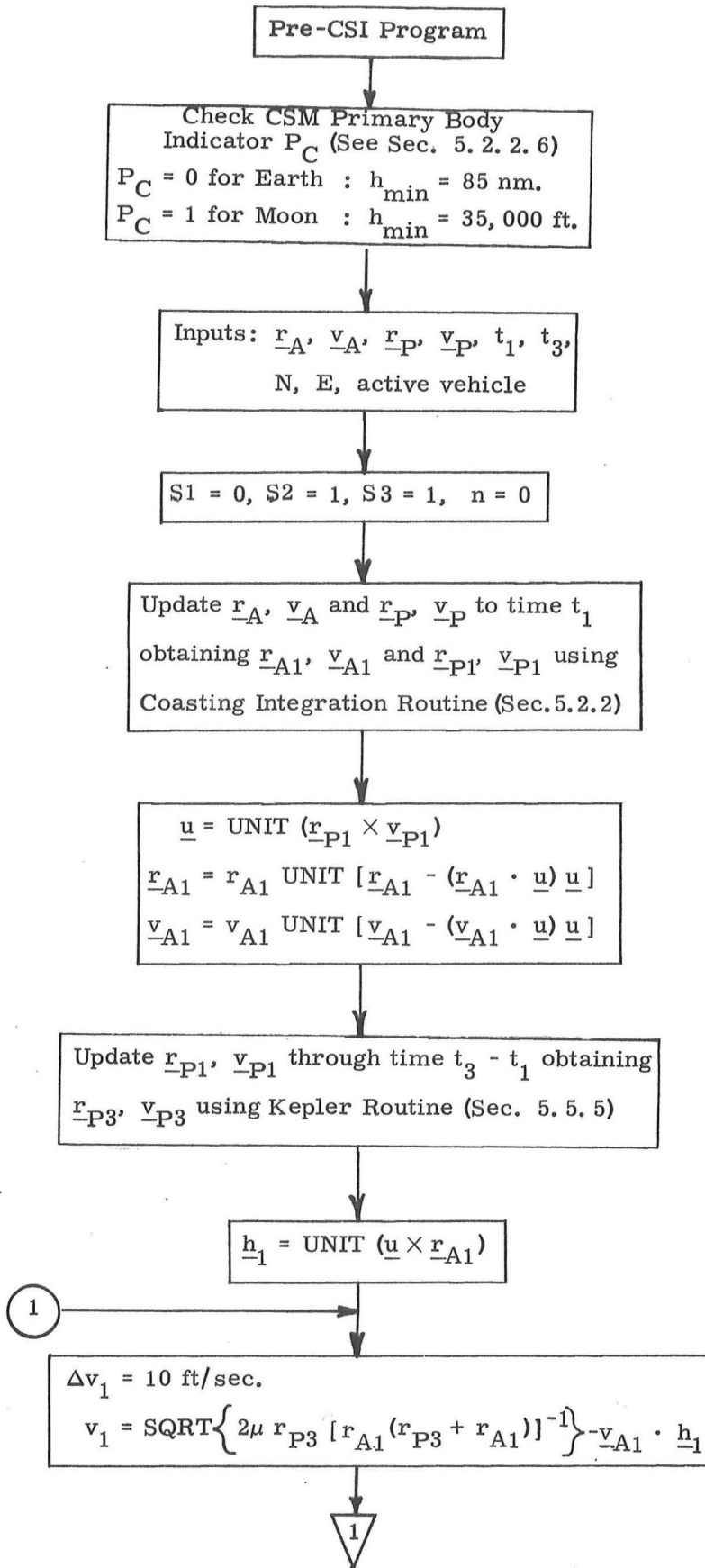


Fig. 4.4-4 Pre-CSI-CDH Maneuver Program

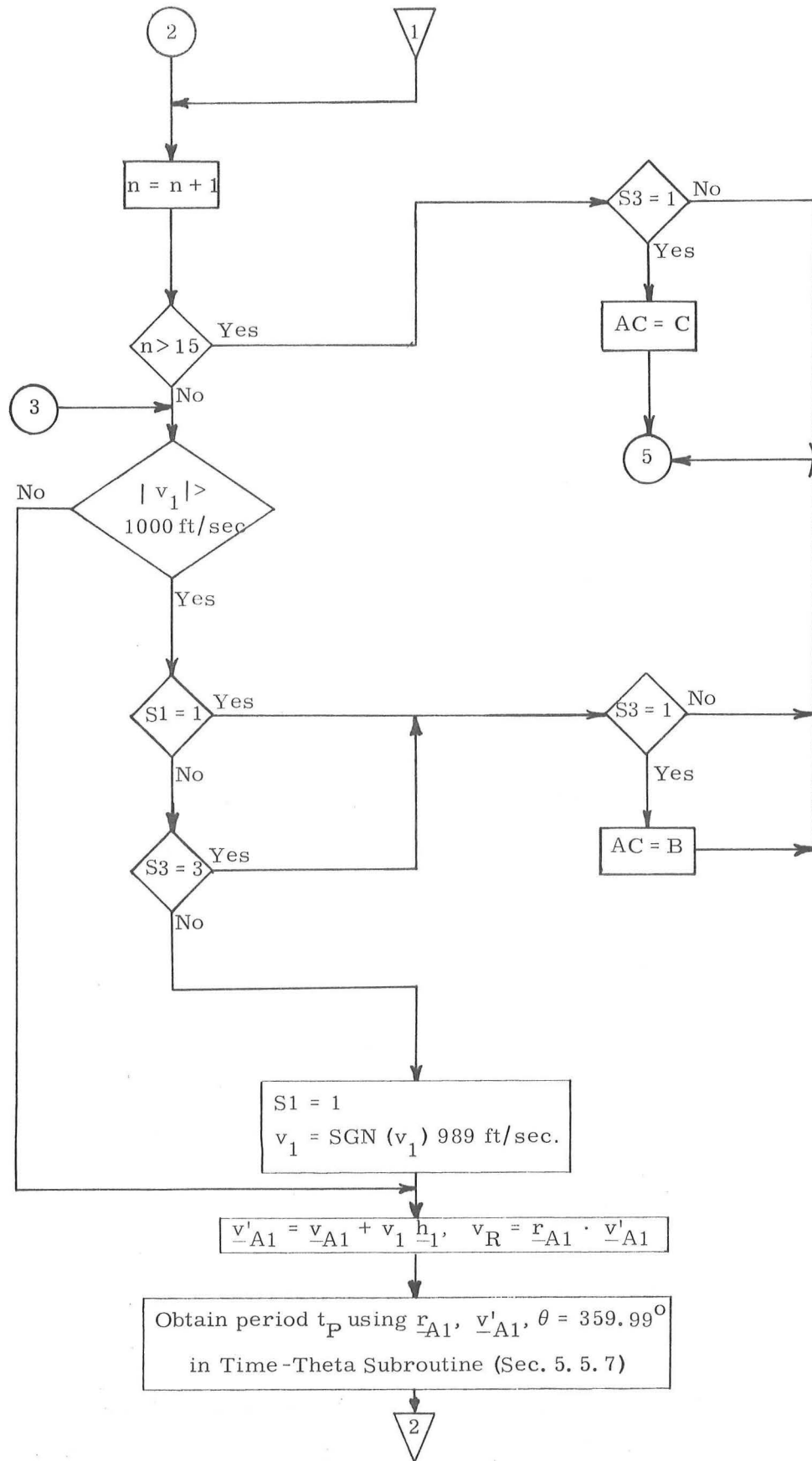


Fig. 4.4-4 Pre-CSI-CDH Maneuver Program

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5.4-15

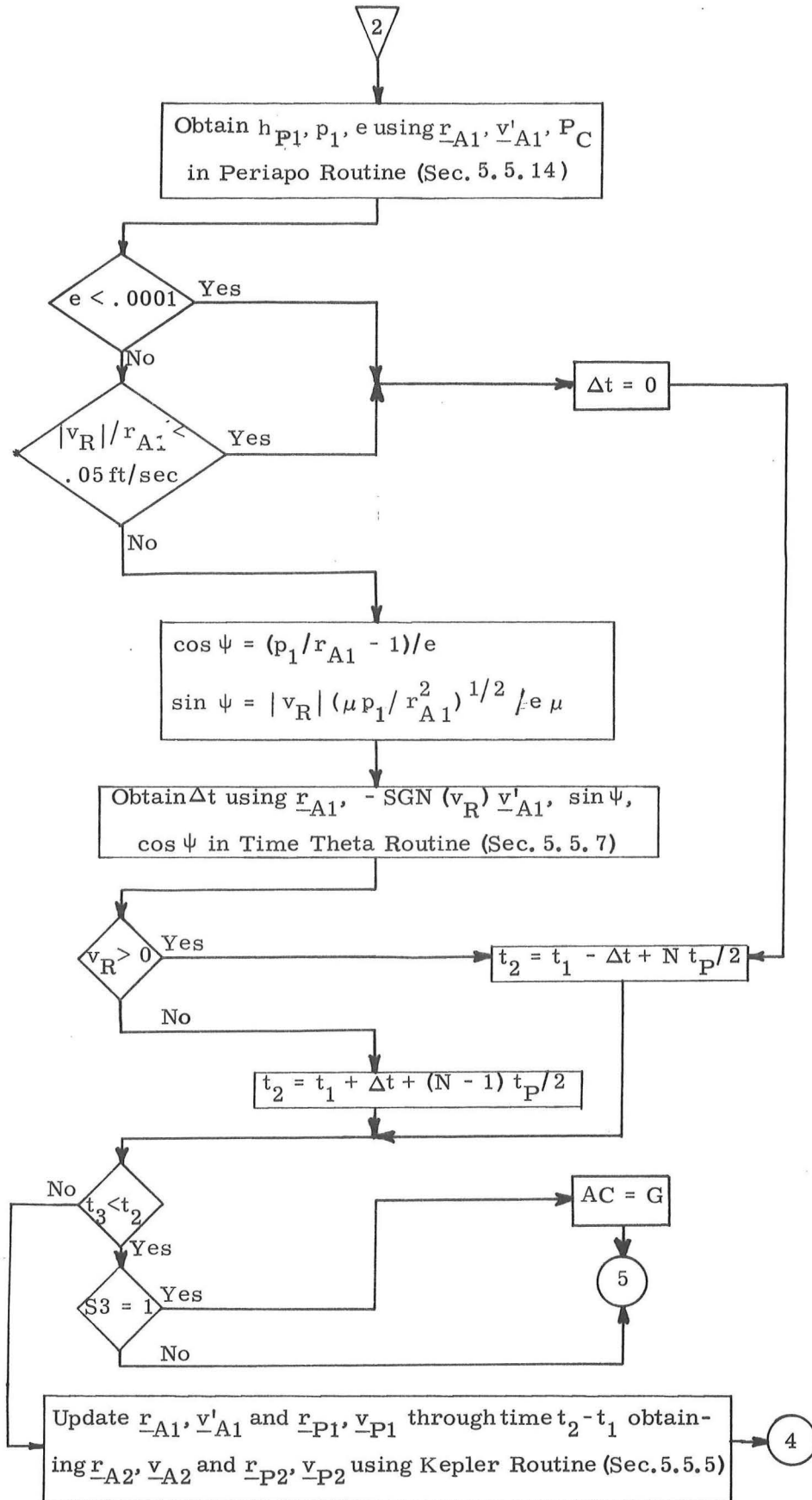


Fig. 4.4-4 Pre-CSI-CDH Maneuver Program

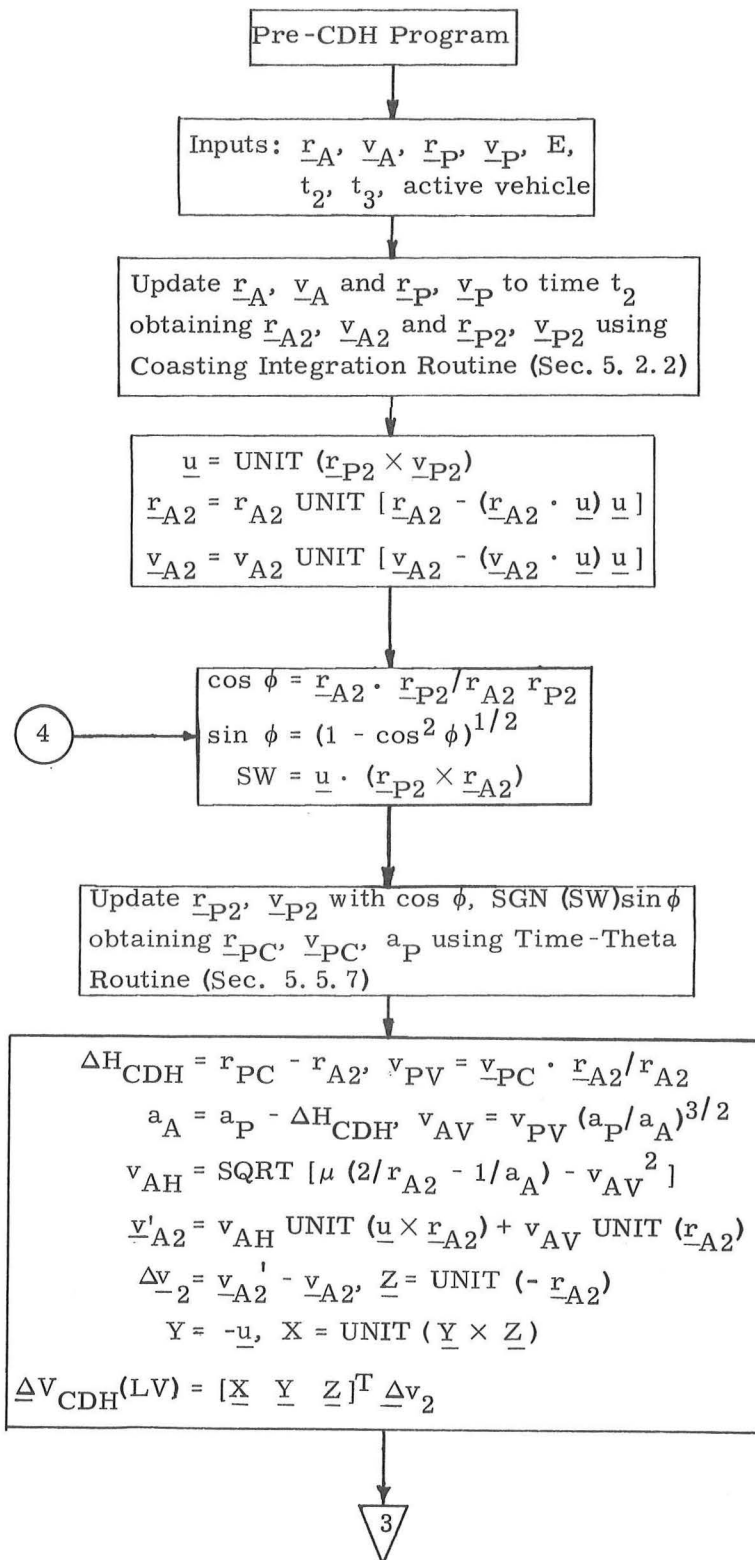


Fig. 4. 4-4 Pre-CSI-CDH Maneuver Program  
(Page 4 of 7)

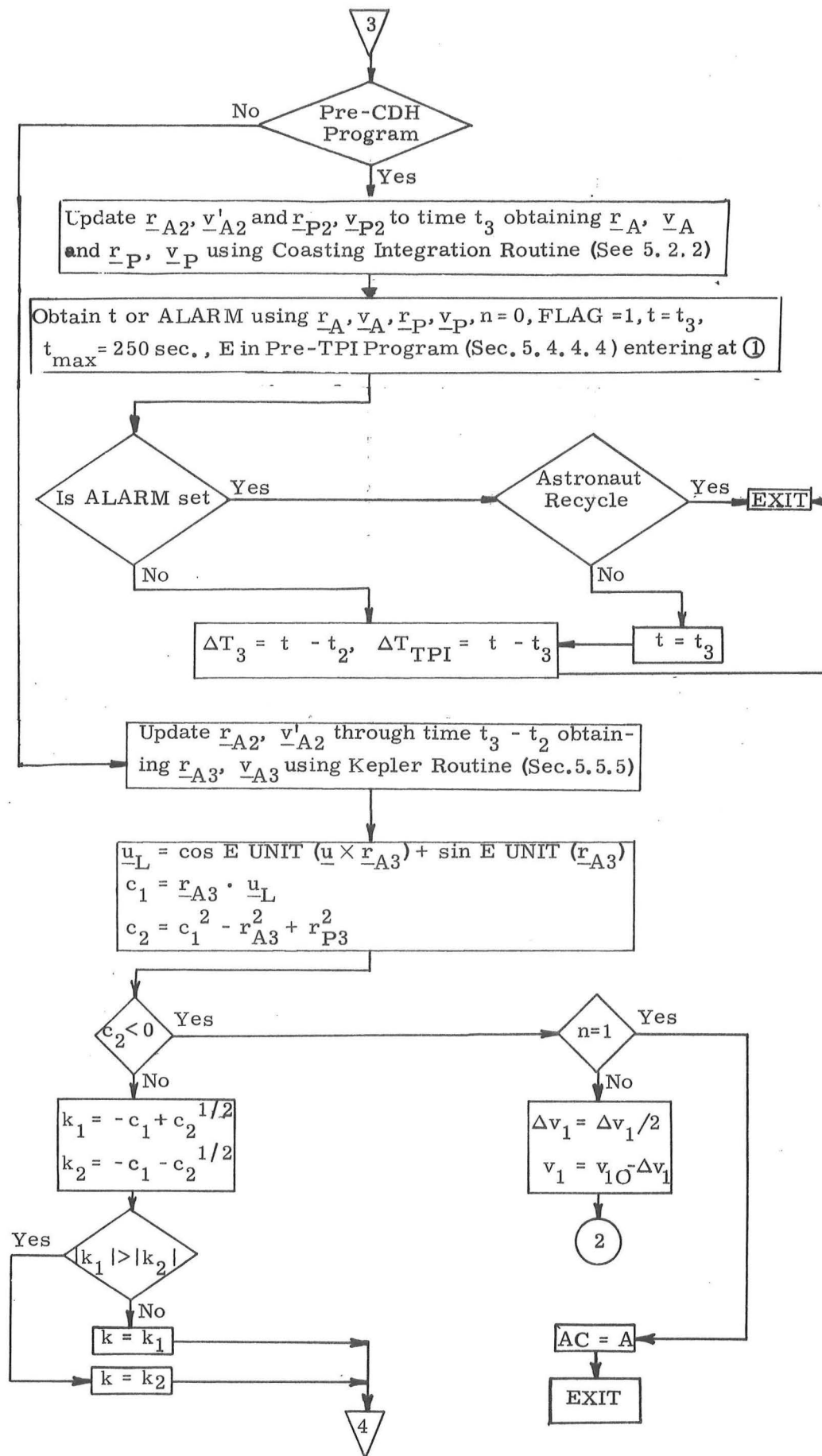


Fig. 4.4-4 Pre-CSI-CDH Maneuver Program

( Page 5 of 7 )

5.4-18

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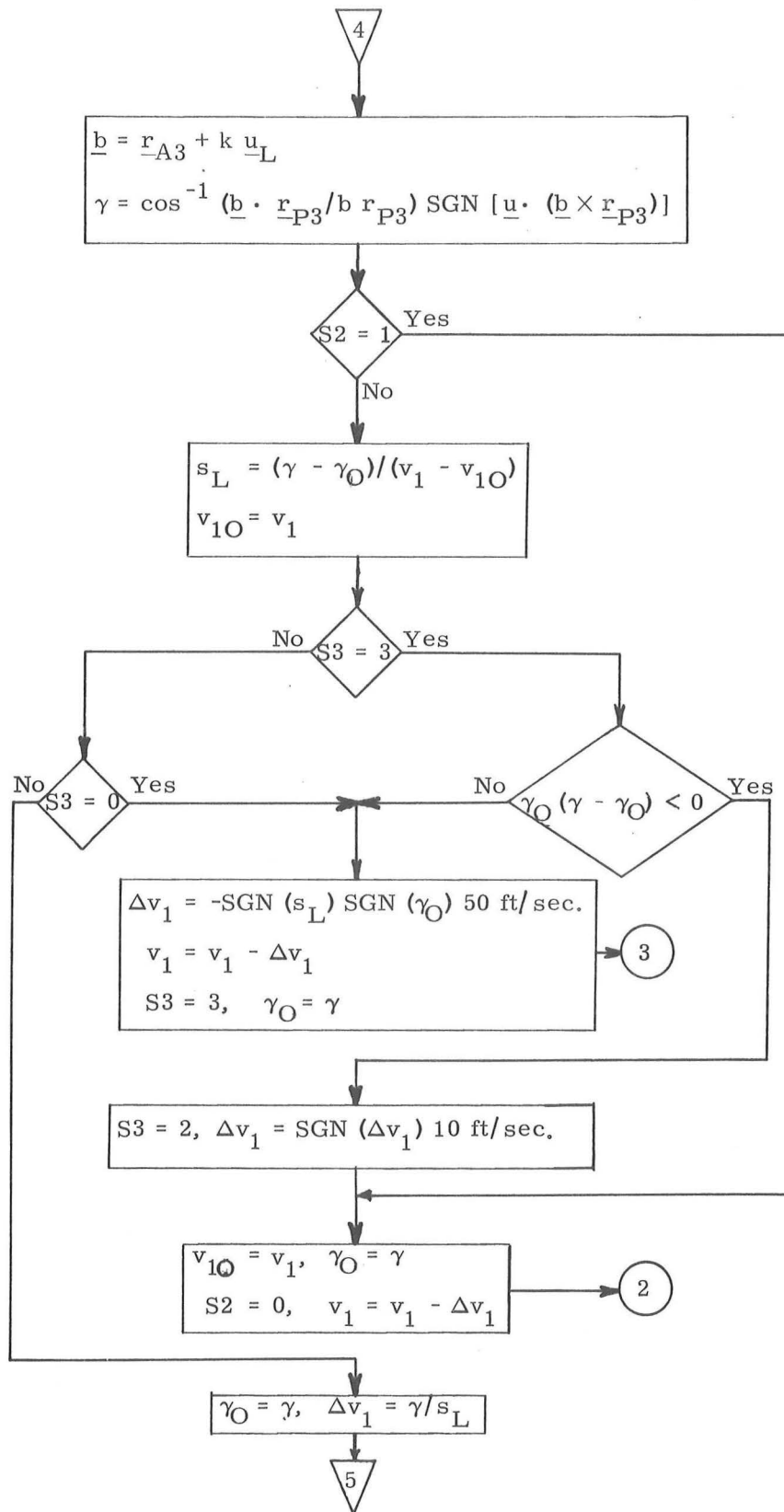


Fig. 4. 4-4 Pre-CSI-CDH Maneuver Program  
( Page 6 of 7)

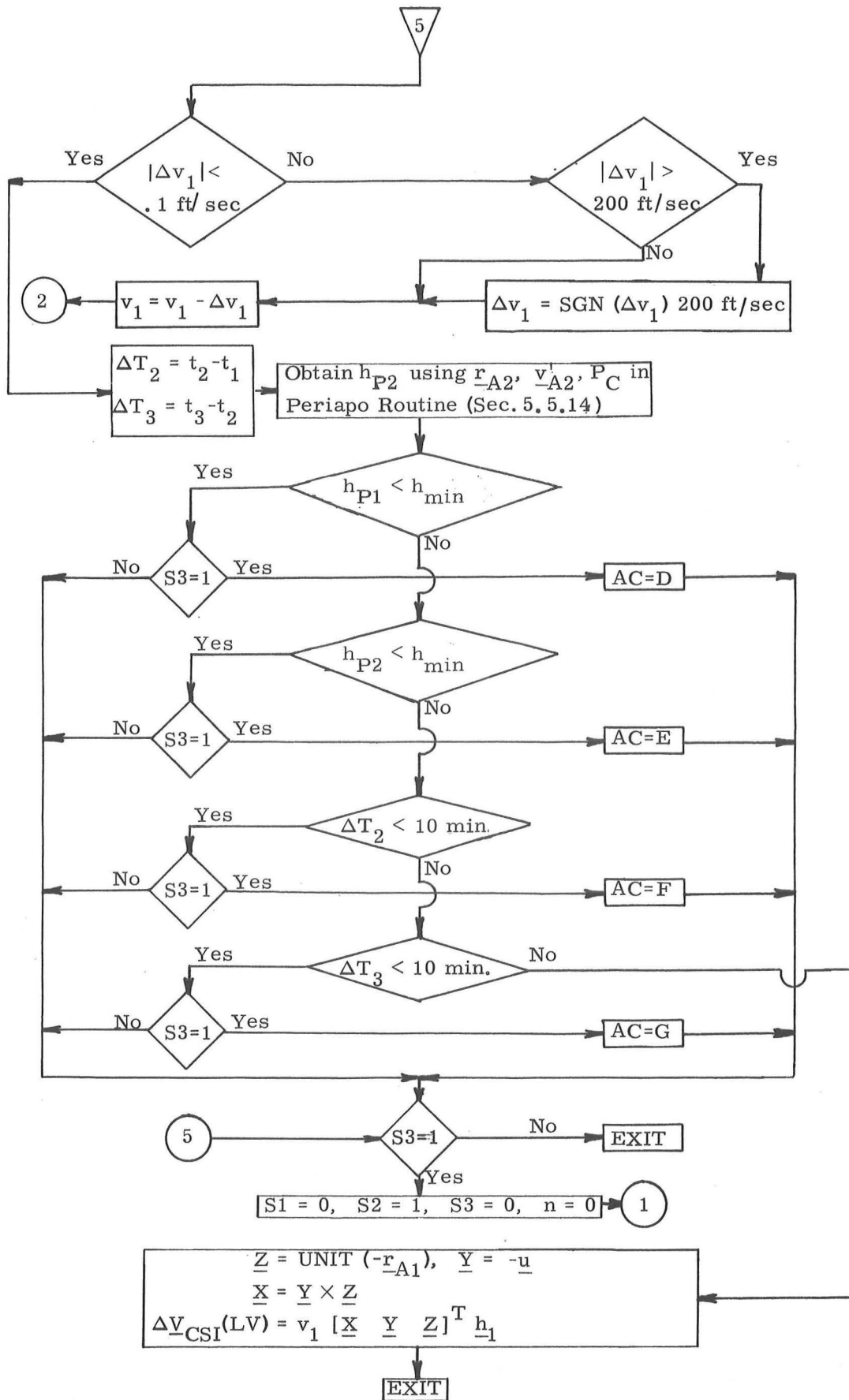


Fig. 4.4-4 Pre-CSI-CDH Maneuver Program

(Page 7 of 7)

5.4-20

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#### 5.4.4.4 Pre-TPI Maneuver

This program corresponds to Program P-34 (LM active) or P-74 (CSM active) of Section 4. Its objective is to establish the terminal phase initiation TPI maneuver. The position of the TPI maneuver is determined by specifying either the TPI time or the elevation angle which specifies the relative geometry of the vehicles at the TPI point. The astronaut inputs are:

1. Choice of active vehicle (P-34 LM, P-74 CSM)
2. Time  $t$  of the TPI maneuver
3. Elevation angle  $E$  (set equal to zero if  $t$  is specified)
4. Central angle  $\omega t$  between the passive vehicle at the TPF and TPI points.

The active  $\underline{r}_{AI}$   $\underline{v}_{AI}$  and passive vehicle  $\underline{r}_{PI}$   $\underline{v}_{PI}$  state vectors are available in the guidance computer. The program starts with a precision update of these vectors to the TPI time.

If the elevation angle is not specified, it is computed after the precision updating of both vehicles to the TPI maneuver time. If  $E$  is specified, an iteration procedure is initiated to find the TPI time at which  $E$  is attained. This procedure uses the input TPI time as the initial guess and is based on conic trajectories (except for the original precision update). The time correction  $\delta t$  is based on (1) the angular distance between  $\underline{r}_P$  and the desired position of the passive vehicle, obtained by assuming the passive vehicle is in a circular orbit, and (2) assuming the vehicles are moving at a constant angular rate. The flow diagram for this procedure is shown in Fig. 4.4-5.

The iteration is successful when the computed elevation angle  $E_A$  is sufficiently close to  $E$ .

To help insure convergence, the following steps are taken:

- a. The step size  $\delta t$  is restricted to 250 secs.
- b. If the solution has been passed ( $\delta E \delta E_0 < 0$ ), the step size is halved and forced in the opposite direction of the last step.
- c. If the iteration is converging ( $|\delta E_0| - |\delta E| > 0$ ), the sign of  $\delta t$  is maintained.
- d. If the iteration is proceeding in the wrong direction, the step direction is reversed.

The iteration is terminated for any of the following reasons, and a single alarm described in Section 4 is set:

1. The iteration counter  $n$  has exceeded its maximum value of 15.
2. The line of sight emanating from the active vehicle does not intersect the circular orbit with radius equal to that of the passive vehicle.
3. The elevation angle is inconsistent with the relative altitudes of the two vehicles (e. g. if the elevation angle is less than  $180^\circ$  when the active vehicle is above the passive vehicle).

Upon convergence, the state vectors are precision updated to the TPI time.

The TPI-TPF phase of the program starts with the use of the angle  $\omega t$  in the Time-Theta Subroutine (Section 5.5.7) to determine the corresponding transfer time  $t_F$ . After precision

updating of the passive vehicle through  $t_F$ , the Initvel Routine (Section 5.5.11) is then called to compute a transfer trajectory between the TPI and TPF points with the switch  $N_1$  set equal to two and the cone angle  $\epsilon$  set equal to  $15^\circ$ . This results in the precision offsetting procedure to be used twice in succession. The rotation projection switch  $f_2$  is obtained from the Initvel Routine for use in the powered flight steering program as  $S_R$ .

The central angle  $\phi$  traversed by the active vehicle from the TPI time to intercept is computed as shown in Fig. 4.4-5 for display if requested by the astronaut. This display is used to avoid  $180^\circ$  transfer angle problems.

The displays for the Pre-TPI program are:

1.  $\Delta V_{TPI}$
2.  $\Delta V_{TPF}$
3.  $\Delta V_{TPI}(\text{LOS})$  : Line of Sight Coordinates
4.  $\Delta V_{TPI}(\text{LV})$  : Local Vertical Coordinates
5. Perigee altitude ( $H_P$ ) following the TPI maneuver

The display of  $\Delta V_{TPI}(\text{LV})$  may be overwritten by the astronaut. If so, a new target vector is generated as shown in Fig. 4.4-5. This new aimpoint is used for powered flight steering.

In addition, M, MGA, and TFI are displayed as in the Pre-CSI program displays.

#### 5.4.4.5 Rendezvous Midcourse Maneuver

This program, corresponding to Program P-35 (LM active) or P-75 (CSM active) of Section 4, computes a midcourse correction maneuver. This maneuver insures that the active vehicle will intercept the passive vehicle at the time established in the previous Pre-TPI program. The astronaut may call this program any time after the TPI maneuver but in general no later than 10 minutes before the intercept time. The flow diagram for this program is shown in Fig. 4.4-6.

There is one astronaut input: choice of the active vehicle (P-35 LM, P-75 CSM). The active  $\underline{r}_A$ ,  $\underline{v}_A$  and passive  $\underline{r}_P$ ,  $\underline{v}_P$  vehicle state vectors, the intercept time  $t_{TPI}$  (from the Pre-TPI program) and a time delay are available in the guidance computer. The time delay ( $\delta\tau_3$  for P-35 and  $\delta\tau_7$  for P-75) is the time required to prepare for the thrust maneuver and is stored in either of two erasable locations.

When the program is initiated the number of navigation measurements since the last maneuver is displayed. In addition, on the first pass through the program the time from the previous TIG, if P41 was performed, or time from previous cutoff if P40 or P42 was performed, is displayed. On the second pass through the program, the time from the previously computed TIG is displayed. Based on this information and additional displays discussed in Section 5.6.7, the astronaut may elect to proceed with the midcourse maneuver at some time. When he does so, the program updates the state vectors to the present time plus the time delay using the Coasting Integration Routine. The Initvel Routine is then called to obtain the velocity vector on the transfer ellipse. The cone angle  $\epsilon$  is set equal to  $15^\circ$  and the  $N_1$  switch is set equal to two, resulting in a double use of the offsetting procedure. The rotation projection switch  $f_2$  is obtained from the Initvel Routine for use in the powered flight steering program as  $S_R$ .

The central angle  $\phi$  traversed by the active vehicle from the maneuver time to intercept is computed as shown in Fig. 4.4-6 for display if requested by the astronaut. This display is used to avoid  $180^\circ$  transfer angle problems.

After obtaining the maneuver  $\Delta\underline{V}$ , it is rotated into a local vertical (LV) and a line of sight (LOS) coordinate system obtaining  $\Delta\underline{V}(LV)$  and  $\Delta\underline{V}(LOS)$ , and both are displayed.

The display of  $\Delta\underline{V}(LV)$  may be overwritten by the astronaut. If so, a new target vector is generated as shown in Fig. 4.4-6. This new aimpoint is used for powered flight steering.

In addition, M, MGA and TFI are displayed as in the Pre-CSI program displays.

5.4-24

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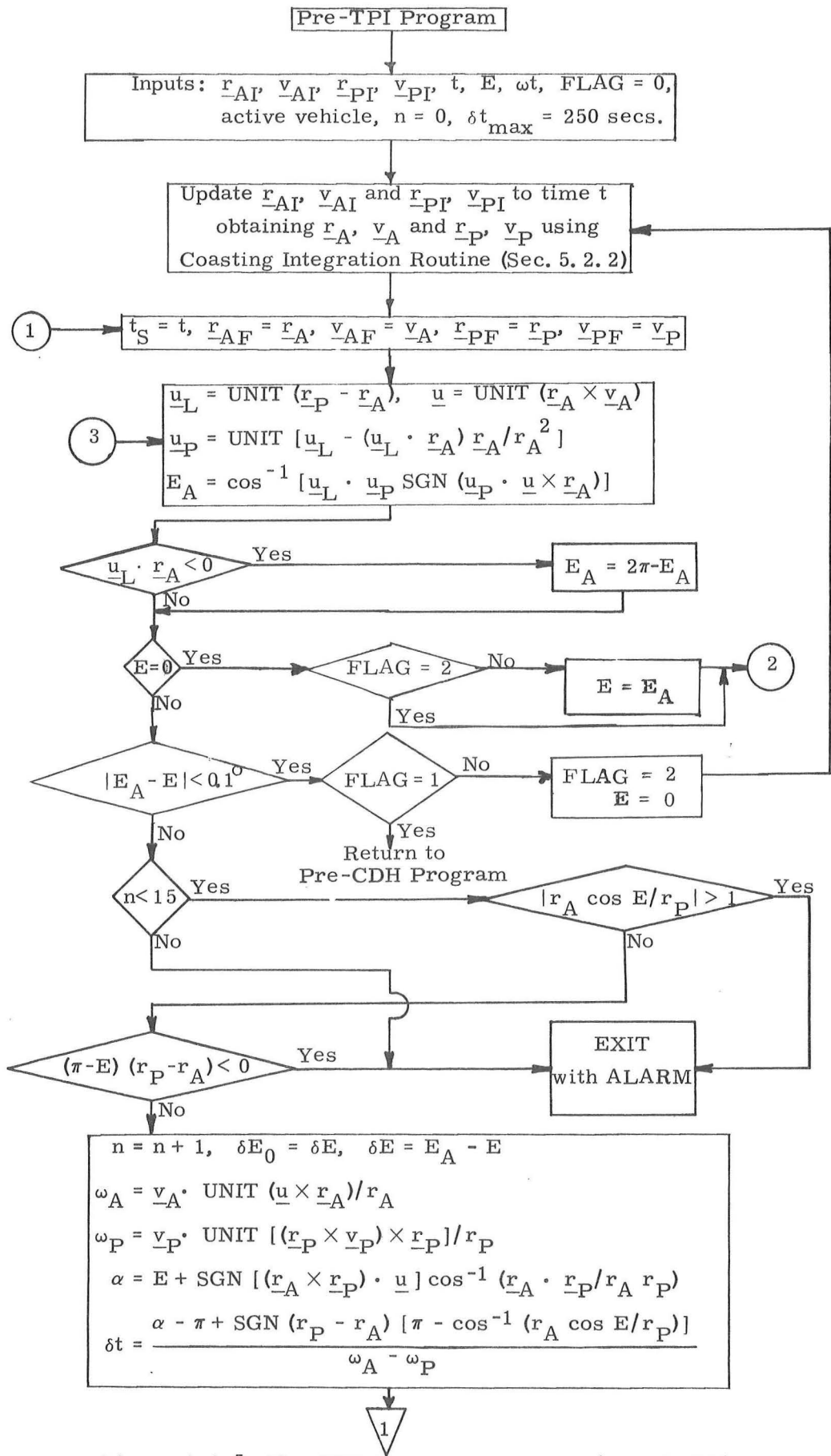


Figure 4. 4-5 Pre-TPI Maneuver Program (page 1 of 3)

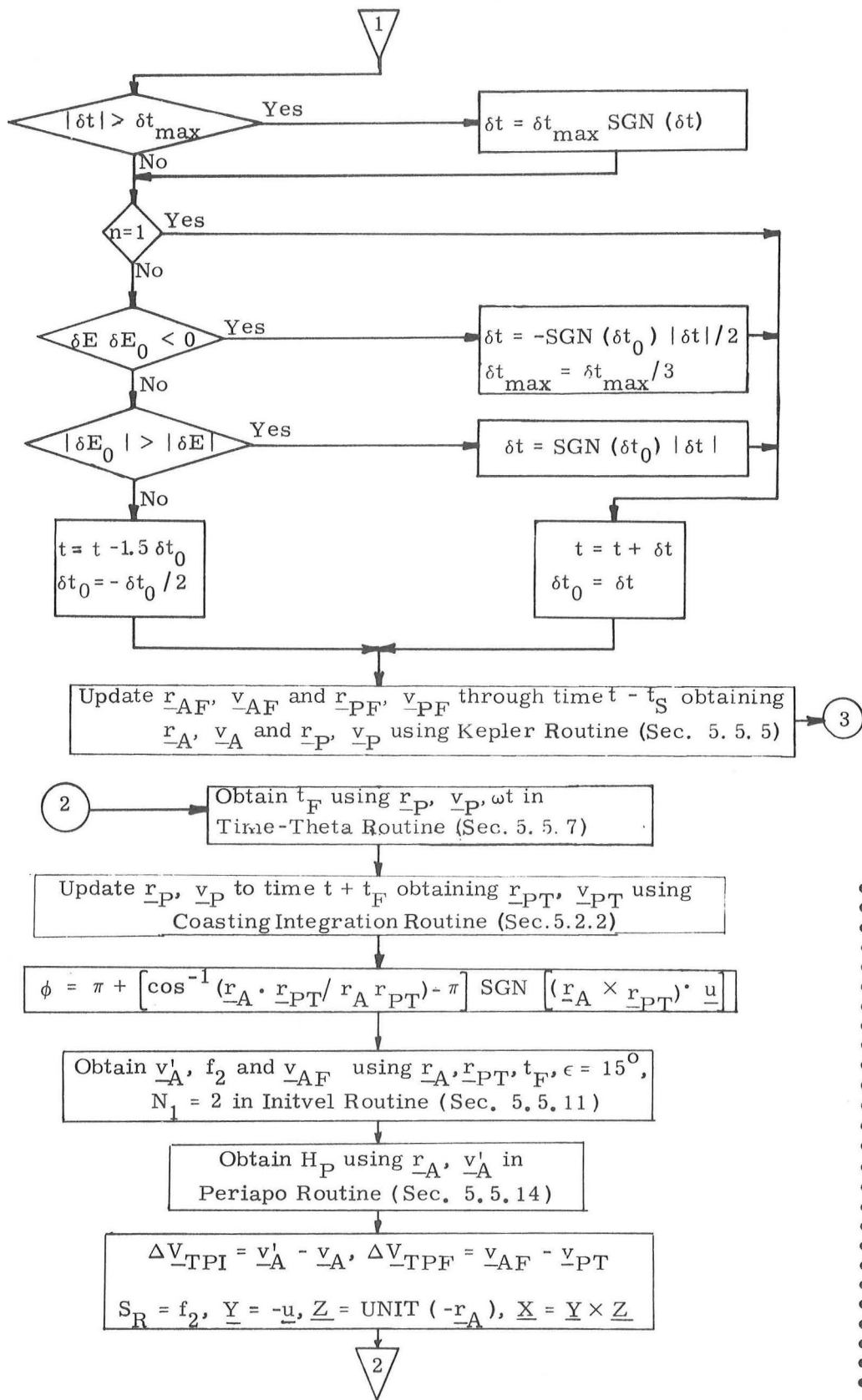


Figure 4.4-5 Pre-TPI Maneuver Program (page 2 of 3)



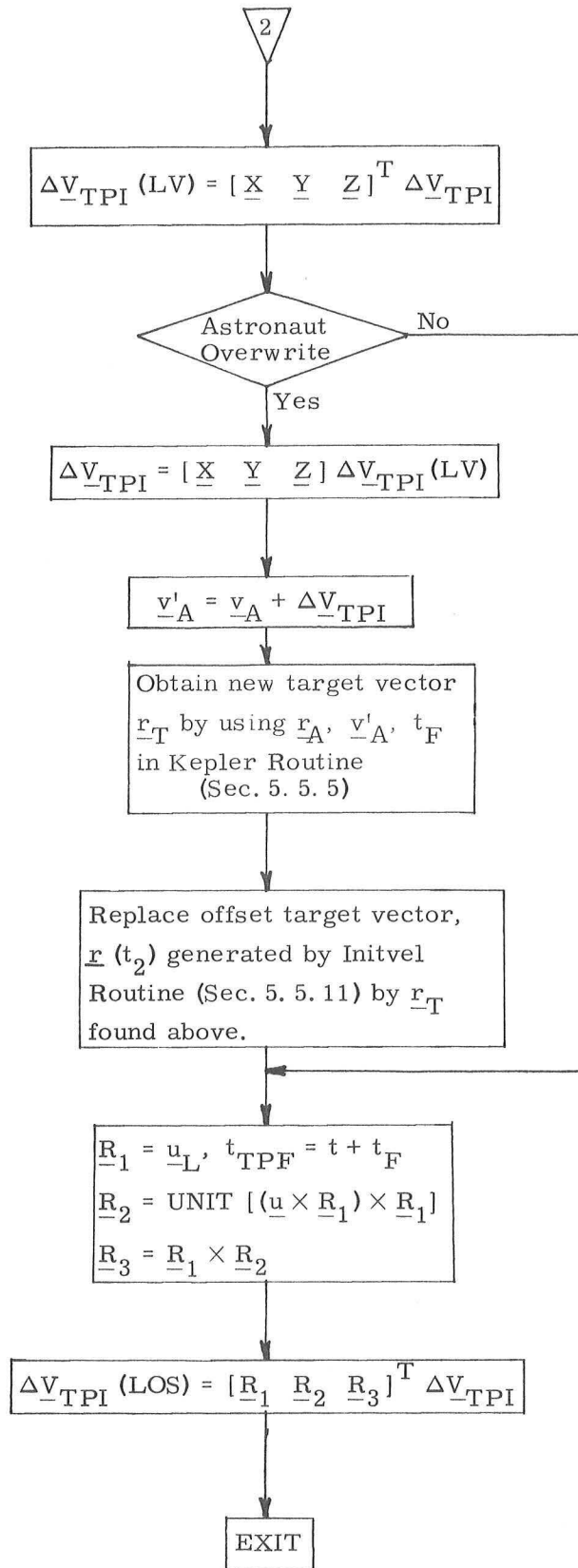


Figure 4. 4-5 Pre-TPI Maneuver Program (page 3 of 3)

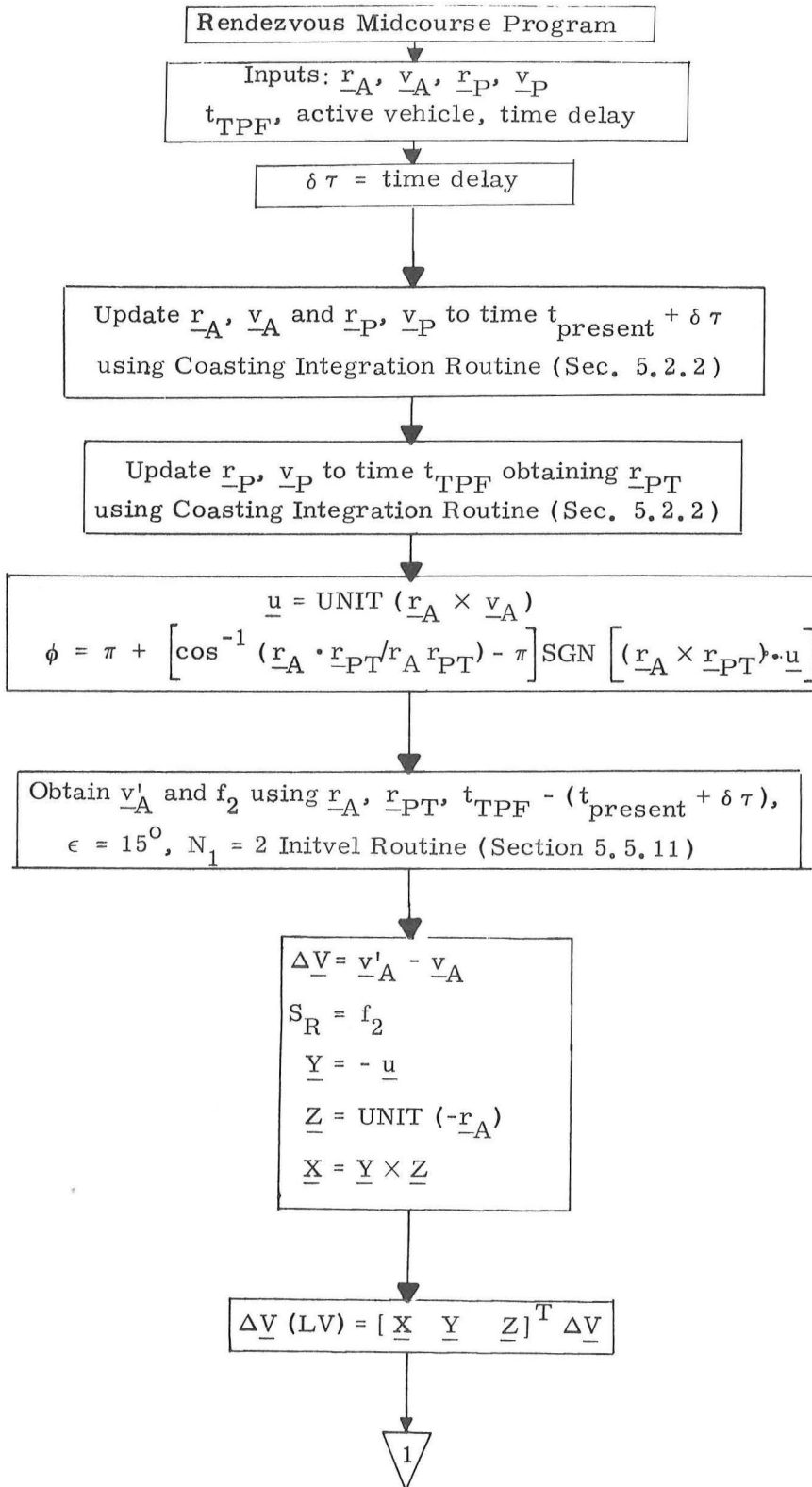


Figure 4.4-6 Rendezvous Midcourse Program  
(page 1 of 2)

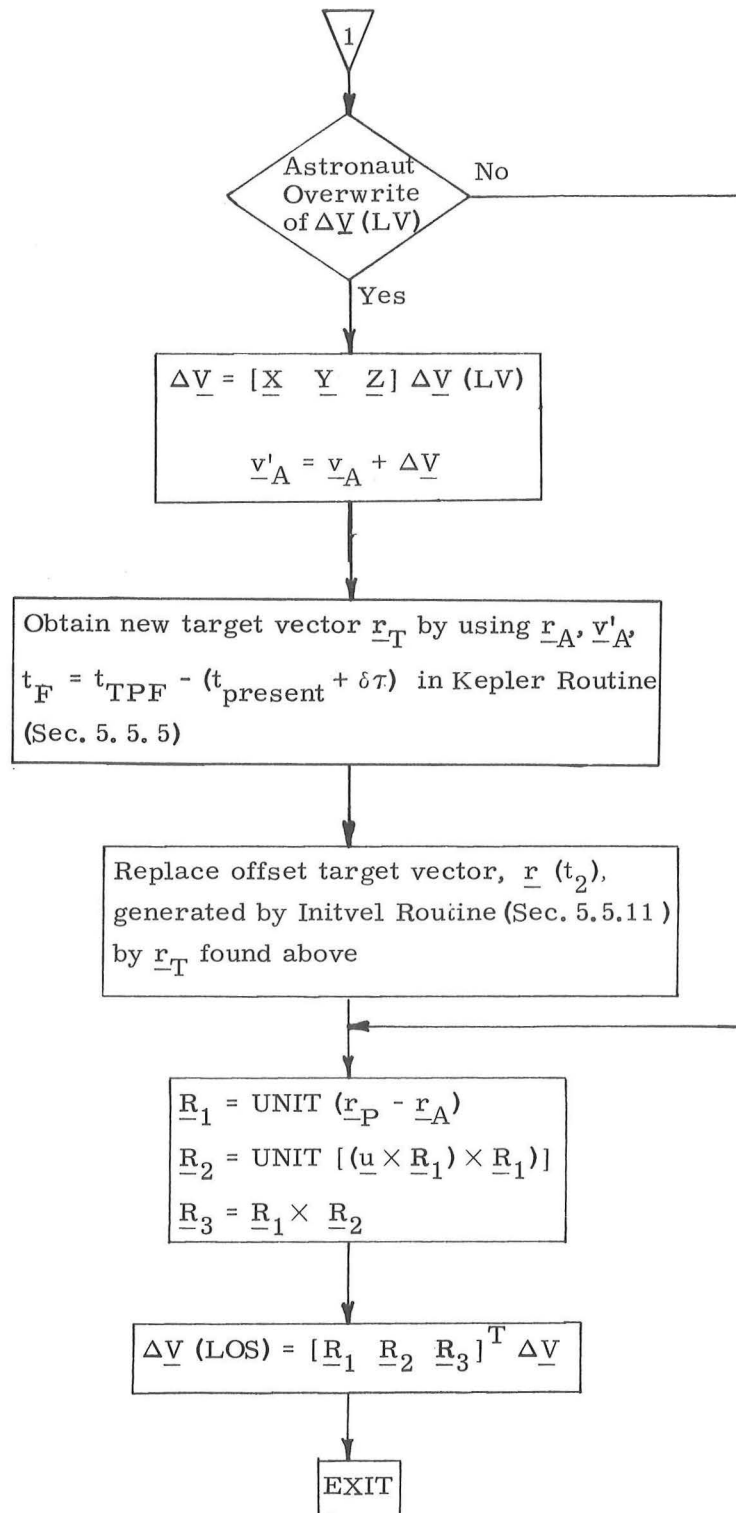


Figure 4.4-6 Rendezvous Midcourse Program  
(page 2 of 2)

## 5.5 BASIC SUBROUTINES

### 5.5.1 GENERAL COMMENTS

The basic solar system and conic trajectory sub-routines which are used by the various guidance and navigation routines are described in this section.

#### 5.5.1.1 Solar System Subroutines

The subroutines used to determine the translation and rotation of the relevant solar system bodies (earth, moon and sun) are designed specifically for a fourteen day lunar landing mission. The method of computing the moon and the sun lines-of-sight relative to the earth is given in Section 5.5.4. The transformations between the Basic Reference Coordinate System and the Earth- and Moon-fixed Coordinate Systems are described in Section 5.5.2. The procedure for transforming between vectors in the Basic Reference Coordinate System and latitude, longitude, altitude coordinates is given in Section 5.5.3. Although these subroutines are normally used in the lunar landing mission, they are valid for use in any mission of not more than fourteen days duration in earth-moon space.

### 5.5.1.2 Conic Trajectory Subroutines

This is a description of a group of conic trajectory subroutines which are frequently used by higher level routines and programs in both the Command Module and the Lunar Module computers.

These subroutines, whose block diagrams are presented in Sections 5.5.5 to 5.5.10, provide solutions to the following conic problems. (See nomenclature which follows)

- (1) Given  $\underline{r}(t_1)$ ,  $\underline{v}(t_1)$ ,  $t_D$ ; solve for  $\underline{r}(t_2)$ ,  $\underline{v}(t_2)$   
(Kepler Subroutine)
- (2) Given  $\underline{r}(t_1)$ ,  $\underline{r}(t_2)$ ,  $t_{D21}, s_G$ ; solve for  $\underline{v}(t_1)$   
(Lambert Subroutine)
- (3) Given  $\underline{r}(t_1)$ ,  $\underline{v}(t_1)$ ,  $\theta$ ; solve for  $t_{21}$ ,  $\underline{r}(t_2)$ ,  $\underline{v}(t_2)$   
(Time-Theta Subroutine)
- (4) Given  $\underline{r}(t_1)$ ,  $\underline{v}(t_1)$ ,  $r(t_2), s_r$ ; solve for  $t_{21}$ ,  $\underline{r}(t_2)$ ,  $\underline{v}(t_2)$   
(Time-Radius Subroutine)
- (5) Given  $\underline{r}(t)$ ,  $\underline{v}(t)$ ; solve for  $r_P$ ,  $r_A$ ,  $e$   
(Apsides Subroutine)

In addition, the following useful subroutines are provided.

- (6) Conic Parameters Subroutine (See Fig. 5.10-1).
- (7) Geometric Parameters Subroutine (See Fig. 5.10-2).
- (8) Iterator Subroutine (See Fig. 5.10-3).

The solutions to the above set of conic problems have stringent accuracy requirements. Programming the fixed-point Apollo computer introduces two constraints which determine accuracy limitations: the 28 bit double precision word length, and the range of variables which is several orders of magnitude for the Apollo mission.

In order to maintain numerical accuracy when these subroutines are programmed into the Apollo computer, floating point programming techniques must be exercised. The effect is for even a simple equation to require a large number of computer instructions. The alternative to this is to separate the problem into phases, each with a different variable range. This, however, requires an even larger number of instructions. These considerations provide the incentive for efficiently organizing the conic equations as shown in the block diagrams.

In addition to the requirement for accuracy, the solution to the Kepler and Lambert Problems must be accomplished in a minimum of computation time in order that the guidance system operate satisfactorily in real time. This additional constraint dictates that a minimum of computer instructions be performed when solving the problem.

#### Method of Solution

To minimize the total number of computer instructions, the problems are solved in the "universal" form; i. e. only equations which are equally valid for the ellipse, parabola and hyperbola are used. Also these subroutines can be used with either the earth or the moon as the attracting body.

Kepler's equation, in the universal form, is utilized to relate transfer time to the conic parameters. All other necessary equations are also universal. The Kepler and Lambert problems are solved with a single iteration loop utilizing a simple first-order slope iterator. In the case of the Kepler problem a third order approximation is available to produce the initial guess for the independent variable (See Eq. (2.2.4) of Section 5.2.2.2).

Sections 5.5.5 thru 5.5.10 provide block diagrams of the detailed computational procedures for solving the various problems. The equations are presented in block diagram form with nomenclature below.

#### Range of Variables

As indicated previously, the programming of the conic subroutines requires a careful balance between accuracy, computational speed and number of instructions. This balance, in the Apollo Guidance Computer, leaves very little margin in any of these areas.

Since the values of problem variables are determined by the solution of the problem being solved and since the problem may originate from the ground system, it is essential that the variable range limitations be defined. The conic routines are incapable of handling problems when the solution lies outside of the range.

The following is a list of the maximum allowable numeric values of the variables. Note that, in addition to fundamental quantities such as position and velocity, there are limitations on intermediate variables and combinations of variables.

Scaling for Conic Subroutines (Sections 5.5.5 to 5.5.10)

<u>Parameter</u>	<u>Maximum Value*</u>	
	<u>Earth</u> <u>Primary Body</u>	<u>Moon</u> <u>Primary Body</u>
r	$2^{29}$	$2^{27}$
v	$2^7$	$2^5$
t	$2^{28}$	$2^{28}$
$\alpha^{**}$	$2^{-22}$	$2^{-20}$
$\alpha_N^{**}$	$2^6$	$2^6$
$P_N$	$2^4$	$2^4$
$\cot \gamma$	$2^5$	$2^5$
$\cot \frac{\theta}{2}$	$2^5$	$2^5$
x	$2^{17}$	$2^{16}$
$\xi = \alpha x^2^{***}$	- 50 $+ 4\pi^2$	- 50 $+ 4\pi^2$
$c_1 = \underline{r} \cdot \underline{v} / \sqrt{\mu}$	$2^{17}$	$2^{16}$
$c_2 = r v^2 / \mu - 1$	$2^6$	$2^6$
$\lambda = r(t_1) / r(t_2)$	$2^7$	$2^7$
$\cos \theta - \lambda$	$2^7$	$2^7$

\* All dimensional values are in units of meters and centiseconds.

\*\* The maximum absolute value occurs for negative values of this parameter.

\*\*\* Both the maximum and minimum values are listed since neither may be exceeded.



Parameter	Maximum Value*	
	Earth	Moon
$e$	$2^3$	$2^3$
$x^2$	$2^{34}$	$2^{32}$
$x^2 c(\xi)$	$2^{33}$	$2^{31}$
$x^3 s(\xi)/\sqrt{\mu}$	$2^{28}$	$2^{28}$
$c_1 x^2 c(\xi)$	$2^{49}$	$2^{46}$
$c_2 x^2 s(\xi)$	$2^{35}$	$2^{33}$
$x [c_2 x^2 s(\xi) + r(t_1)]$	$2^{49}$	$2^{46}$
$\xi s(\xi)$	$2^7$	$2^7$
$x^2 c(\xi)/r$	$2^8$	$2^8$
$\sqrt{\mu} x (\xi s(\xi) - 1)/r(t_2)$	$2^{15}$	$2^{13}$
$c(\xi)$	$2^4$	$2^4$
$s(\xi)$	$2^1$	$2^1$

\* All dimensional values are in units of meters and centisecond

Nomenclature for Conic Subroutines (Sections 5.5.5 to 5.5.10)

$\underline{r}(t_1)$	initial position vector
$\underline{v}(t_1)$	initial velocity vector
$\underline{r}(t_2)$	terminal position vector
$\underline{v}(t_2)$	terminal velocity vector
$\underline{u}_N$	unit normal in the direction of the angular momentum vector
$\alpha$	reciprocal of semi-major axis (negative for hyperbolas)
$r_P$	radius of pericenter
$r_A$	radius of apocenter
$e$	eccentricity
$\alpha_N$	ratio of magnitude of initial position vector to semi-major axis
$p_N$	ratio of semi-latus rectum to initial position vector magnitude
$\gamma$	inertial flight path angle as measured from vertical
$\theta$	true anomaly difference between $\underline{r}(t_1)$ and $\underline{r}(t_2)$
$f$	true anomaly of $r(t_2)$

$x$	a universal conic parameter equal to the ratio of eccentric anomaly difference to $\sqrt{+\alpha}$ for the ellipse, or the ratio of the hyperbolic analog of eccentric anomaly difference to $\sqrt{-\alpha}$ for the hyperbola
$x'$	value of $x$ from the previous Kepler solution
$t_{21}$	computed transfer time from Kepler's equation ( $t_2 - t_1$ )
$t_{21}'$	transfer time corresponding to the previous solution of Kepler's equation
$t_D$	desired transfer time through which the conic update of the state vector is to be made
$t_{D21}$	desired transfer time to traverse from $\underline{r}(t_1)$ to $\underline{r}(t_2)$
$t_{ERR}$	error in transfer time
$\epsilon_t$	fraction of desired transfer time to which $t_{ERR}$ must converge
$\Delta x$	increment in $x$ which will produce a smaller $t_{ERR}$
$\epsilon_x$	value of $\Delta x$ which will produce no significant change in $t_{21}$
$\Delta \cot \gamma$	increment in $\cot \gamma$ which will decrease the magnitude of $t_{ERR}$
$\epsilon_c$	value of $\Delta \cot \gamma$ which will produce no significant change in $t_{21}$

$\mu$	product of universal gravitational constant and mass of the primary attracting body
$x_{MAX}$	maximum value of x
$x_{MIN}$	minimum value of x
$\cot_{MAX}$	maximum value of $\cot \gamma$
$\cot_{MIN}$	minimum value of $\cot \gamma$
$\ell_{MAX}$	upper bound of general independent variable
$\ell_{MIN}$	lower bound of general independent variable
$x_{MAX 1}$	absolute upper bound on x with respect to the moon
$x_{MAX 0}$	absolute upper bound on x with respect to the earth
k	a fraction of the full range of the independent variable which determines the increment of the independent variable on the first pass through the iterator
y	general dependent variable
y'	previous value of y
$y_{ERR}$	error in y
z	general independent variable

- $\Delta z$  increment in z which will produce a smaller  $y_{ERR}$
- $s_G$  a sign which is plus or minus according to whether the true anomaly difference between  $\underline{r}(t_1)$  and  $\underline{r}(t_2)$  is to be less than or greater than 180 degrees
- $s_r$  a sign which is plus or minus according to whether the desired radial velocity at  $r(t_2)$  is plus or minus
- $\eta_1$  general vector # 1
- $\eta_2$  general vector # 2
- $\phi$  angle between  $\eta_1$  and  $\eta_2$
- $f_1$  a switch set to 0 or 1 according to whether a guess of  $\cot \gamma$  is available or not
- $f_2$  a switch set to 0 or 1 according to whether Lambert should determine  $\underline{u}_N$  from  $\underline{r}(t_1)$  and  $\underline{r}(t_2)$  or  $\underline{u}_N$  is an input
- $f_3$  a tag set to 0 or 1 according to whether the iterator should use the "Regula Falsi" or bias method
- $f_4$  a flag set to 0 or 1 according to whether the iterator is to act as a first order or a second order iterator
- $f_5$  a flag set to 0 or 1 according to whether Lambert converges to a solution or not

$f_6$  a switch set to 0 or 1 according to whether or not the new state vector is to be an additional output requirement of the Time-Theta or Time-Radius problems.

$f_7$  a flag set to 1 if the inputs require that the conic trajectory must close through infinity

$f_8$  a flag set to 1 if the Time-Radius problem was solved for pericenter or apocenter instead of  $r(t_2)$

$f_9$  a flag set to 1 if the input to the Time-Radius Subroutine produced an  $e$  less than  $2^{-18}$ .

$t_p$  Period of the orbit

$t_R$  integral periods subtracted from  $t_D$  to produce a  $t_D$  less than  $t_p$

$x_R$  value of  $x$  corresponding to  $t_R$

$k_1$  the minimal acceptance fraction of  $t_{D21}$  to which  $t_{ERR}$  must converge

$n_1$  a flag set to 0 or 1 according to whether or not the velocity vector at the terminal position is to be an additional output requirement of the Lambert Routine.

$i$  the maximum number of iterations to be allowed in the Lambert Subroutine.

### 5. 5. 2 PLANETARY INERTIAL ORIENTATION SUBROUTINE

This subroutine is used to transform vectors between the Basic Reference Coordinate System and a Planetary (Earth-fixed or Moon-fixed) Coordinate System at a specified time. These three coordinate systems are defined in Section 5. 1. 4.

Let  $\underline{r}$  be a vector in the Basic Reference Coordinate System,  $\underline{r}_P$  the same vector expressed in the Planetary Coordinate System, and  $t$  the specified ground elapsed time (GET). Then,

$$\underline{r}_P = M(t) (\underline{r} - \underline{l} \times \underline{r}) \quad (5. 2. 1)$$

and

$$\underline{r} = M^T(t) (\underline{r}_P + \underline{l}_P \times \underline{r}_P) \quad (5. 2. 2)$$

where  $M(t)$  is a time dependent orthogonal transformation matrix,  $\underline{l}$  is a small rotation vector in the Basic Reference Coordinate System, and  $\underline{l}_P$  is the same vector  $\underline{l}$  expressed in the Planetary Coordinate System. The vector  $\underline{l}$  is considered constant in one coordinate system for the duration of the mission. The method of computing  $M(t)$  and  $\underline{l}$  depends on whether the relevant planet is the earth or the moon.

#### Case I - Earth

For the earth, the matrix  $M(t)$  describes a rotation about the polar axis of the earth (the Z-axis of the Earth-fixed Coordinate

System), and the vector  $\underline{\ell}$  accounts for the precession and nutation of the polar axis (the deviation of the true pole from the mean pole).

Let  $A_X$  and  $A_Y$  be the small angles about the X- and Y-axes of the Basic Reference Coordinate System, respectively, that describe the precession and nutation of the earth's polar axis. The values of these two angles at the midpoint of the mission are included in the pre-launch erasable data load and are considered constant throughout the flight. Then,

$$\underline{\ell} = \begin{pmatrix} A_X \\ A_Y \\ 0 \end{pmatrix}$$

$$A_Z = A_{Z0} + \omega_E (t + t_0) \quad (5.2.3)$$

$$M(t) = \begin{pmatrix} \cos A_Z & \sin A_Z & 0 \\ -\sin A_Z & \cos A_Z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{\ell}_P = M(t) \underline{\ell}$$

where  $A_{Z0}$  is the angle between the X-axis of the Basic Reference Coordinate System and the X-axis of the Earth-fixed Coordinate System (the intersection of the Greenwich meridian and the equatorial plane of the earth) at July 1.0, 1968 universal time (i. e., midnight at Greenwich just prior to July 1, 1968),  $t_0$  is the elapsed time between July 1.0, 1968 universal time and the time that the computer clock was zeroed, and  $\omega_E$  is the angular velocity of the earth.



## Case II - Moon

For the moon, the matrix  $M(t)$  accounts for the difference in orientation of the Basic Reference and Moon-fixed Coordinate Systems in exact accordance with Cassini's laws, and the rotation vector  $\underline{l}$  corrects for deviations from the above orientation because of physical libration.

Define the following three angles which are functions of time:

- B = the obliquity, the angle between the mean earth equatorial plane and the plane of the ecliptic.
- $\Omega_1$  = the longitude of the node of the moon's orbit measured from the X-axis of the Basic Reference Coordinate System.
- F = the angle from the mean ascending node of the moon's orbit to the mean moon.

Let  $I$  be the constant angle between the mean lunar equatorial plane and the plane of the ecliptic ( $1^\circ 32.1'$ ). Then, the sequence of rotations which brings the Basic Reference Coordinate System into coincidence with the Moon-fixed Coordinate System (neglecting libration) is as follows:

<u>Rotation</u>	<u>Axis of Rotation</u>	<u>Angle of Rotation</u>
1	X	B
2	Z	$\Omega_I$
3	X	-I
4	Z	$\pi + F$

The transformation matrices for these rotations are, respectively,

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos B & \sin B \\ 0 & -\sin B & \cos B \end{pmatrix}$$

$$M_2 = \begin{pmatrix} \cos \Omega_I & \sin \Omega_I & 0 \\ -\sin \Omega_I & \cos \Omega_I & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos I & -\sin I \\ 0 & \sin I & \cos I \end{pmatrix}$$

$$M_4 = \begin{pmatrix} -\cos F & -\sin F & 0 \\ \sin F & -\cos F & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(5. 2. 4)

The matrix  $M(t)$  is then given by

$$M(t) = M_4 M_3 M_2 M_1 \quad (5.2.5)$$

The following approximate method is used to determine the transformation between the Basic Reference and Moon-fixed Coordinate Systems.

The angles  $B$ ,  $\Omega_I$  and  $F$  are computed as linear functions of time. Let  $\underline{l}_M$  be the value of the vector libration  $\underline{l}_P$  (expressed in the Moon-fixed Coordinate System) at the midpoint of the mission. The vector  $\underline{l}_M$  is included in the pre-launch erasable data load and is considered constant throughout the flight. Then,

$$\begin{aligned} \underline{l}_P &= \underline{l}_M \\ t_M &= t + t_0 \\ B &= B_0 + \dot{B} t_M \\ \Omega_I &= \Omega_{I0} + \dot{\Omega}_I t_M \\ F &= F_0 + \dot{F} t_M \end{aligned}$$

$$\underline{a} = \begin{pmatrix} \cos \Omega_I \\ \cos B \sin \Omega_I \\ \sin B \sin \Omega_I \end{pmatrix} \quad (5.2.6)$$

$$\underline{b} = \begin{pmatrix} -\sin \Omega_I \\ \cos B \cos \Omega_I \\ \sin B \cos \Omega_I \end{pmatrix}$$

$$\underline{c} = \begin{pmatrix} 0 \\ -\sin B \\ \cos B \end{pmatrix}$$

$$\underline{d} = \underline{b} C_I - \underline{c} S_I$$

$$\underline{m}_2 = \underline{b} S_I + \underline{c} C_I$$

$$\underline{m}_0 = -\underline{a} \cos F - \underline{d} \sin F \quad (5.2.6)$$

(cont.)

$$\underline{m}_1 = \underline{a} \sin F - \underline{d} \cos F$$

$$M(t) = \begin{pmatrix} \underline{m}_0^T \\ \underline{m}_1^T \\ \underline{m}_2^T \end{pmatrix}$$

$$\underline{l} = M^T(t) \underline{l}_P$$

where  $B_0$ ,  $\Omega_{I0}$ , and  $F_0$  are the values of the angles  $B$ ,  $\Omega_I$  and  $F$ , respectively, at July 1.0, 1968 universal time;  $\dot{B}$ ,  $\dot{\Omega}_I$  and  $\dot{F}$  are the rates of change of these angles; and  $C_I$  and  $S_I$  are the cosine and sine, respectively, of the angle  $I$ .

### 5. 5. 3 LATITUDE-LONGITUDE SUBROUTINE

For display and data load purposes, the latitude, longitude, and altitude of a point near the surface of the earth or the moon are more meaningful and more convenient to use than the components of a position vector. This subroutine is used to transform position vectors between the Basic Reference Coordinate System and Geographic or Selenographic latitude, longitude, altitude at a specified time.

In the case of the moon, the altitude is computed above either the landing site radius,  $r_{LS}$ , or the mean lunar radius,  $r_M$ . For the earth, the altitude is defined with respect to either the launchpad radius,  $r_{LP}$ , or the radius of the Fisher ellipsoid,  $r_F$ , which is computed from

$$r_F^2 = \frac{b^2}{1 - (1 - \frac{b^2}{a^2}) (1 - \text{SINL}^2)} \quad (5.3.1)$$

where  $a$  and  $b$  are the semi-major and semi-minor axes of the Fischer ellipsoid, respectively, and  $\text{SINL}$  is the sine of the geocentric latitude.

The computational procedures are illustrated in Figs. 5.3-1, 5.3-2, and 5.3-3. The calling program must specify either a vector  $\underline{r}$  or latitude (Lat), longitude (Long), and altitude (Alt). In addition, the program must set the time  $t$  and the two indicators  $P$  and  $F$  where

$$P = \begin{cases} 0 & \text{for earth} \\ 1 & \text{for moon} \end{cases}$$

$$F = \begin{cases} 1 & \text{for Fischer ellipsoid or mean lunar radius} \\ 0 & \text{for launch pad or landing site radius} \end{cases}$$

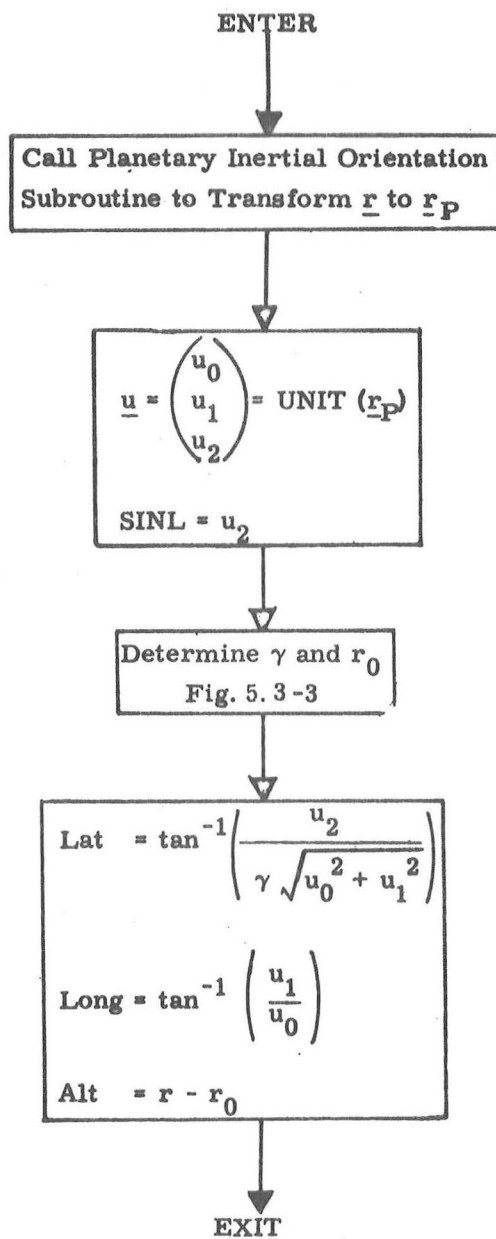


Fig. 5.3-1 Vector to Latitude, Longitude, Altitude  
Computation Logic Diagram

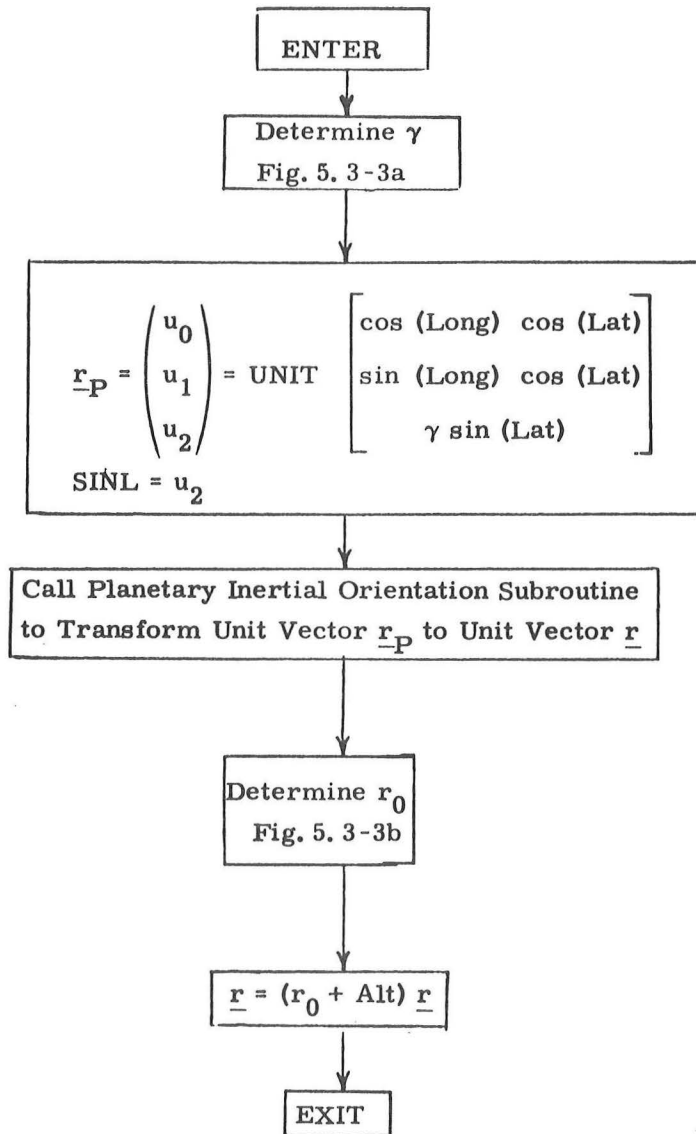


Fig. 5.3-2 Latitude, Longitude, Altitude to Vector Computation Logic Diagram



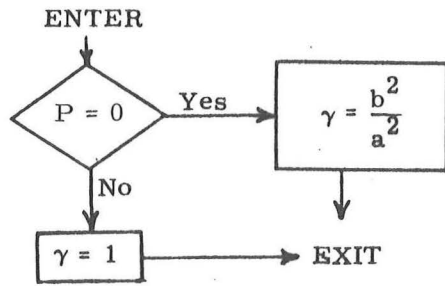


Figure 5.3-3a Determination of  $\gamma$

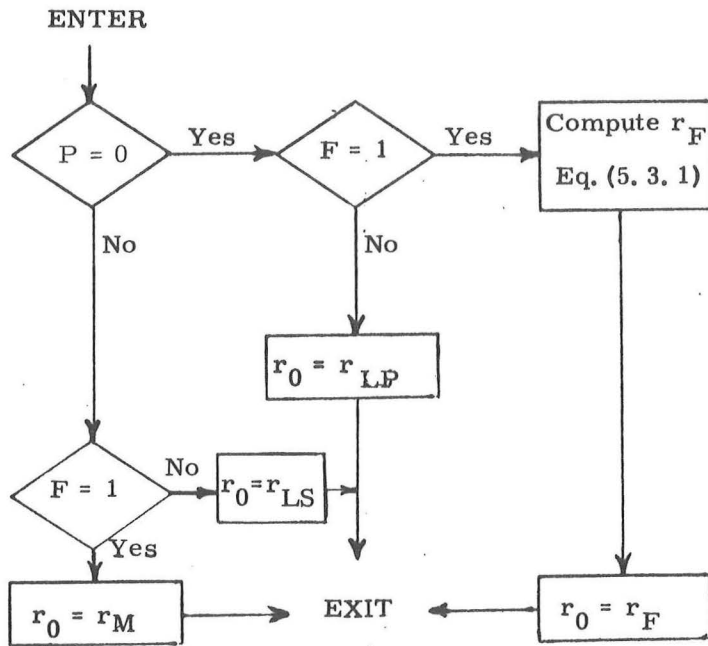


Figure 5.3-3b Determination of  $r_0$

#### 5.5.4 LUNAR AND SOLAR EPHEMERIDES

This subroutine is called by LOSSEM (Section 5.5.13) in order to obtain the line-of-sight unit vectors of the moon and sun, with respect to the LM, which may be used for IMU alignment.

The positions of the moon and sun are obtained from simple approximation schemes which are stored in the computer's fixed memory.

If the obliquity of the ecliptic and the inclination of the lunar orbit are considered constant, the unit position vector of the moon with respect to the earth in the Basic Reference Coordinate System defined in Section (5.1.4), is given by

$$\underline{u}_{EM} = \begin{bmatrix} \cos (LOM) \\ K_1 \sin (LOM) - K_2 \sin (LOM - LON) \\ K_3 \sin (LOM) + K_4 \sin (LOM - LON) \end{bmatrix}$$

where the longitude of the mean moon (LOM) is approximated using its value and rate at July 1.0, 1968, and using empirically determined periodic terms:

$$\begin{aligned} LOM = LOM_O + LOM_R t - [A \sin (OMEGA_A t + PHASE_A) \\ + B \sin (OMEGA_B t + PHASE_B)] \end{aligned}$$

and where the longitude of the moon's node (LON) is similarly approximated:

$$\text{LON} = \text{LON}_O + \text{LON}_R t$$

and where  $t$  is measured from July 1.0, 1968.

The approximate unit position vector of the sun with respect to the earth is given by:

$$\underline{u}_{ES} = \begin{bmatrix} \cos(\text{LOS}) \\ K_1 \sin(\text{LOS}) \\ K_3 \sin(\text{LOS}) \end{bmatrix}$$

where the longitude of the mean sun is approximated by:

$$\text{LOS} = \text{LOS}_O + \text{LOS}_R t - C \sin(\text{OMEGA}_C t + \text{PHASE}_C)$$

and where  $K_1$  and  $K_3$  are the same as for the moon.

### 5.5.5 KEPLER SUBROUTINE

The Kepler Subroutine solves for the two body position and velocity vectors at the terminal position given the initial position and velocity vectors and a transfer time to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Kepler Subroutine block diagram depicted in Figs. 5.5-1 thru 5.5-3. The subroutines referred to in these figures are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

Prior to entering the Kepler Subroutine an initial estimate of  $x$  can be generated via Eq. (2.2.4) of Section 5.2.2.2 with  $\frac{\Delta t}{2} = t_D - t_{21}$ , and  $\tau = t_D$ . However,  $x'$  and  $t_{21}$  are non-zero only if the subroutine is being used repetitively.

Although, theoretically, there is no upper bound on  $x$ , the practical bound is set to  $x_{MAX0}$  or  $x_{MAX1}$  to eliminate non-feasible trajectories and increase the accuracy to which  $x$  can be computed. In addition,  $\alpha x^2$  has a practical range of  $-50 < \alpha x^2 < (2\pi)^2$  which determines an independent upper bound on  $x$ . The  $x_{MAX}$  used, then, corresponds to the smaller of the two values.

The transfer time convergence criterion is approximately the same as the granularity of the time input. Since, for some of the problems to be solved, the sensitivity of time to  $x$  is so large that the granularity in  $x$ ,  $\epsilon_x$ , produces a change in time which exceeds the granularity in time, it is necessary to introduce  $\epsilon_x$  as a redundant convergence criterion.

The Kepler Subroutine, provided the parameter range constraints are satisfied, will always produce a solution.

A negative value of  $t_D$  will cause the subroutine to update the state vector backward in time. If  $t_D$  is negative but its absolute value is greater than one period, the solution returned will correspond to an update backward of exactly one period. If  $t_D$  is positive but greater than one period the update will correspond to the true, total value of  $t_D$ , just as it does for  $t_D$  less than one period.

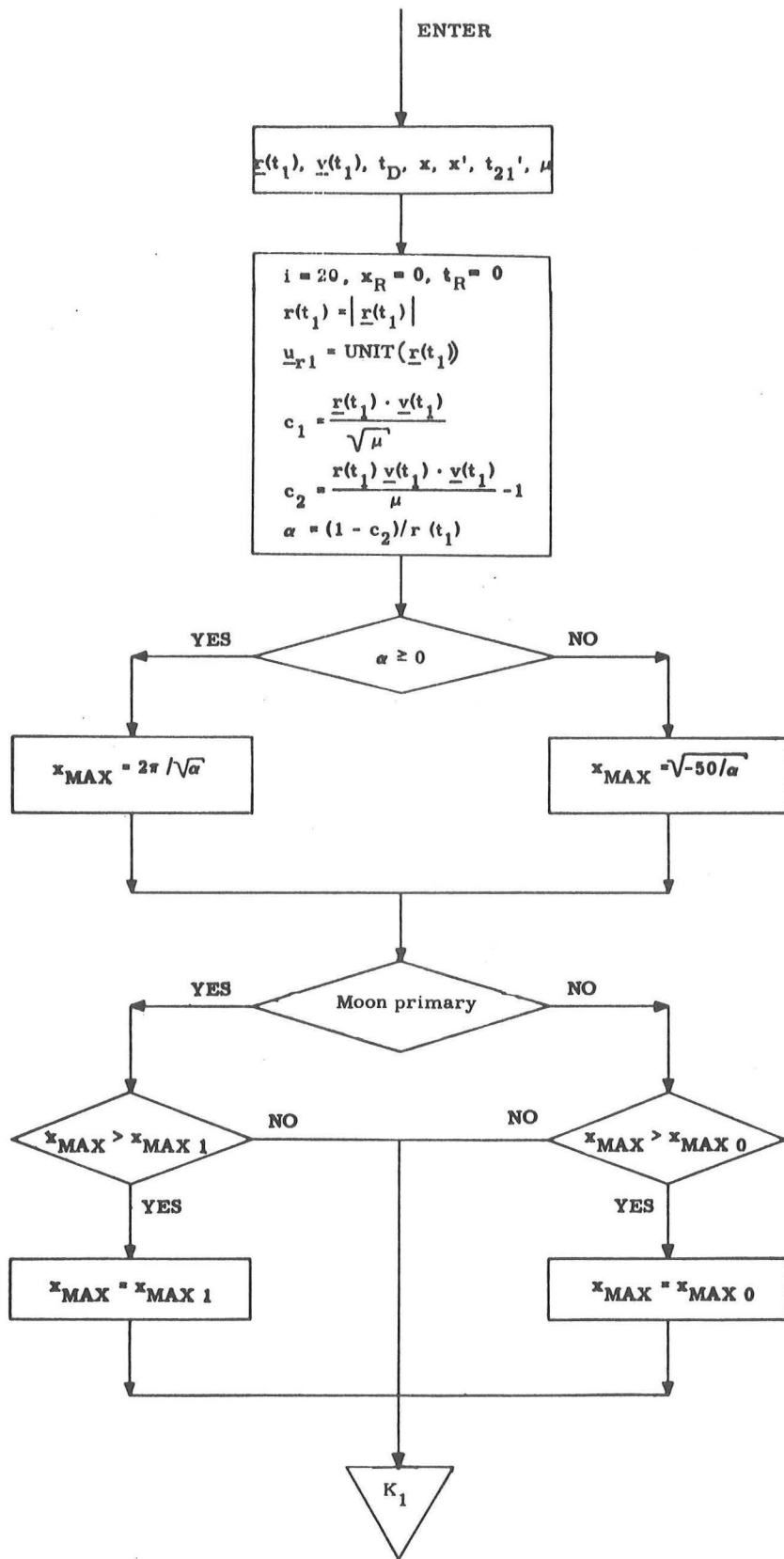


Figure 5.5-1 Kepler Subroutine

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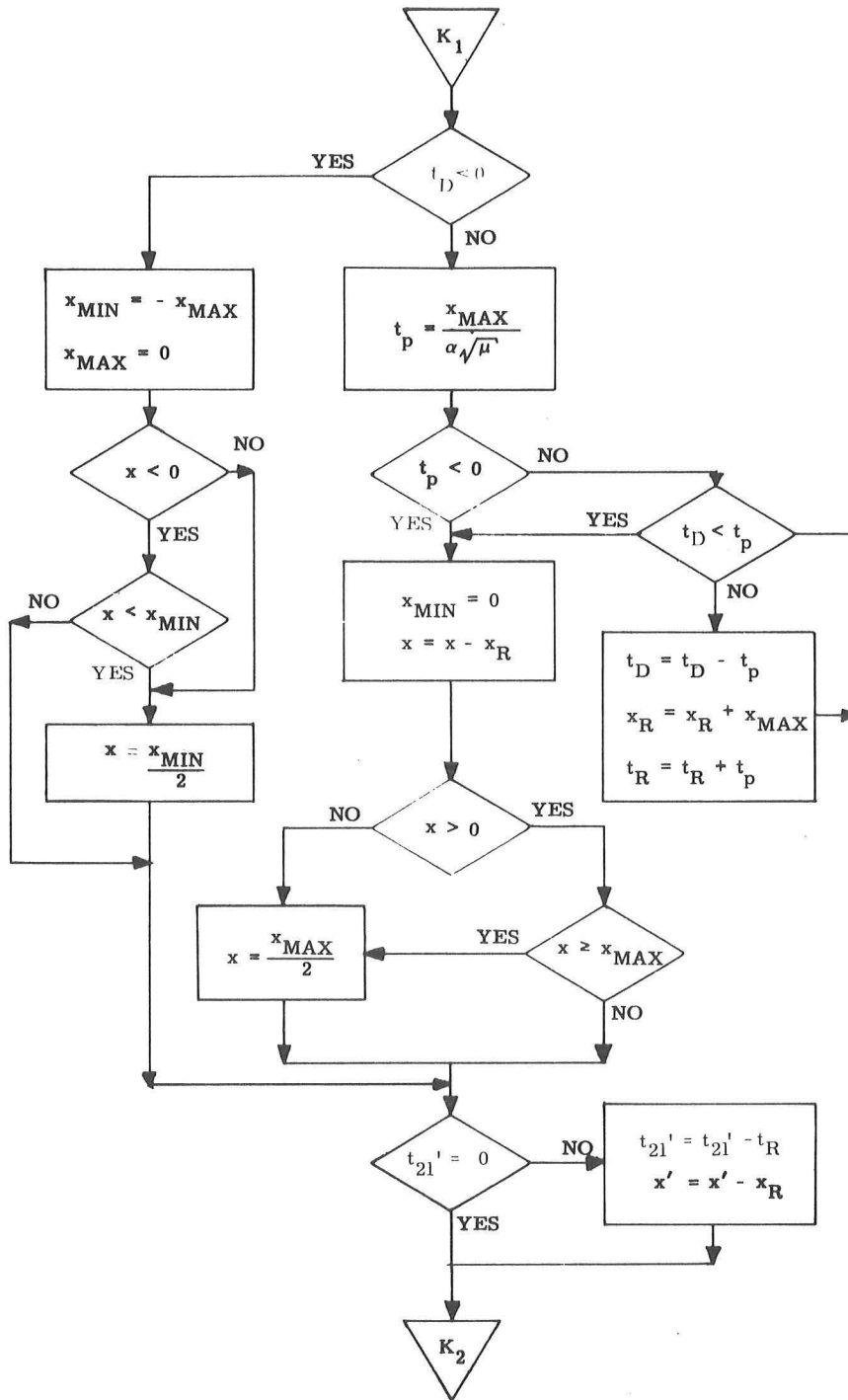


Figure 5.5-2 Kepler Subroutine

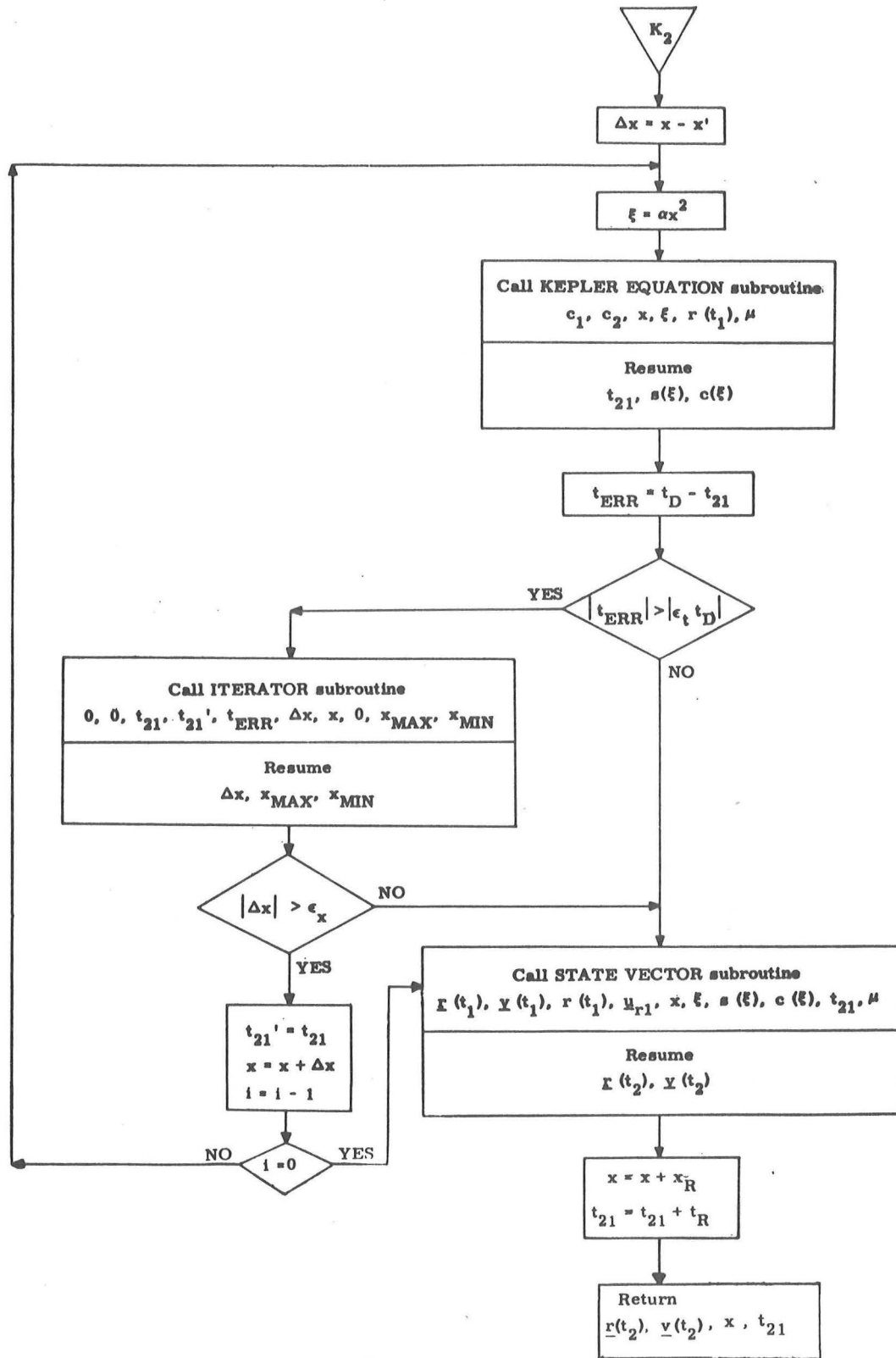


Fig. 5.5-3 Kepler Subroutine

5.5-29



### 5.5.6 LAMBERT SUBROUTINE

The Lambert Subroutine solves for the two body initial velocity vector given the initial and terminal position vectors and a transfer time between the two.

This section contains information to aid the reader in understanding the less obvious aspects of the Lambert Subroutine block diagrams depicted in Figs. 5.6-1 and 5.6-2. The subroutines referred to in these figures are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

If the Lambert Subroutine is used repetitively and rapid computation is required, the previous value of the independent variable,  $\cot \gamma$ , can be used as a starting point for the new iteration. Flag  $f_1$  provides this option.

The Lambert Subroutine computes the normal to the trajectory,  $\underline{u}_N$ , using the two input position vectors. If these vectors are nearly colinear, it is desirable to specify the normal as an input rather than rely on the ill-defined normal based on the two input position vectors. Flag  $f_2$  provides this option. The presence of the inputs in parentheses, therefore, is contingent upon the setting of these flags.

The theoretical bounds on the independent variable,  $\cot \gamma$ , correspond to the infinite energy hyperbolic path and the parabolic path which closes through infinity. These bounds are dynamically reset by the iterator to provide a more efficient iteration scheme. In addition, if during the course of the iteration,  $\cot \gamma$  causes a parameter of the problem to exceed its maximum as determined by its allowable range, the appropriate bound is reset and the iterator continues trying to find an acceptable solution. (This logic does not appear in Figs. 5.6-1 and 2

as it is pertinent only to fixed-point programming). If no acceptable solution is reached, the transfer time input was too small to produce a practical trajectory between the input position vectors. When this happens,  $\Delta \cot \gamma$  approaches its granularity limit  $\epsilon_c$  before time converges to within a fraction  $\epsilon_t$  of the desired time. However, this same granularity condition exists when the sensitivity problem described in the Kepler Subroutine, Section 5.5.5, occurs. In this case an acceptable solution does exist. This dual situation is resolved via a third convergence criterion. If the error in transfer time is greater than the usual fraction  $\epsilon_t$  of the desired transfer time, but still less than a slightly larger fraction  $k_1$  of the desired transfer time and  $\Delta \cot \gamma$  is less than  $\epsilon_c$ , then the solution is deemed acceptable and the required velocity is computed. If, on the other hand, the error in the transfer time is greater than the fraction  $k_1$  of the desired transfer time and  $\Delta \cot \gamma$  is less than  $\epsilon_c$ , then the flag  $f_5$  is set to indicate that the required velocity will have degraded accuracy.

The maximum number of iterations allowable in the Lambert Subroutine has been made an input to the routine, in order to give the user direct control over the maximum computation time to be allowed. If no initial guess of  $\cot \gamma_0$  is supplied, this number should be set to 20. If an initial guess of  $\cot \gamma_0$  is supplied, which is sufficiently close to the true  $\cot \gamma_0$  [such as the  $\cot \gamma_0$  from the previous computation cycle during powered-flight guidance computation sequencing (see Section 5.3.3.2)], then this number may be set as low as 5.

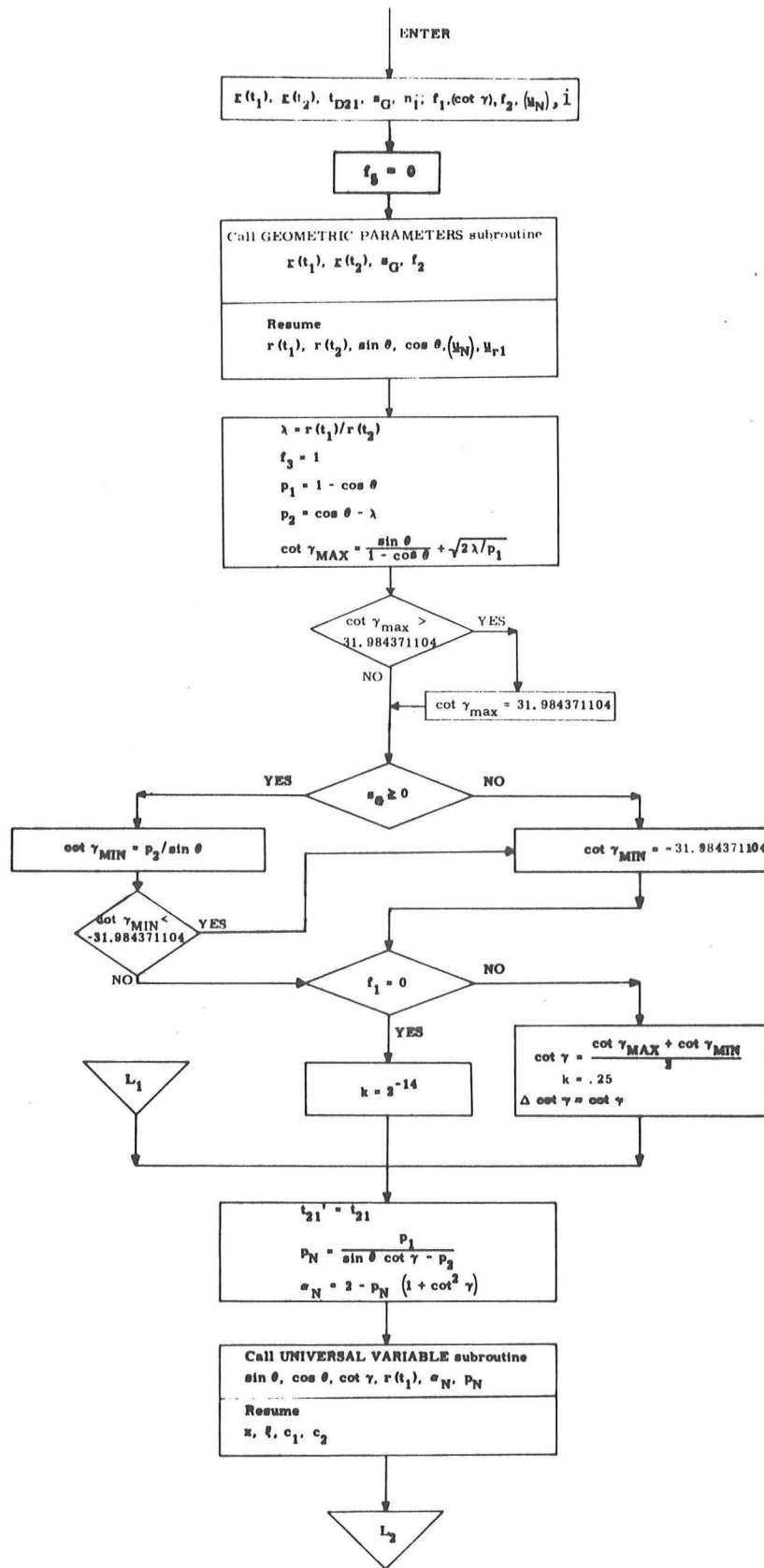


Figure 5.6-1 Lambert Subroutine

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Rev. 1

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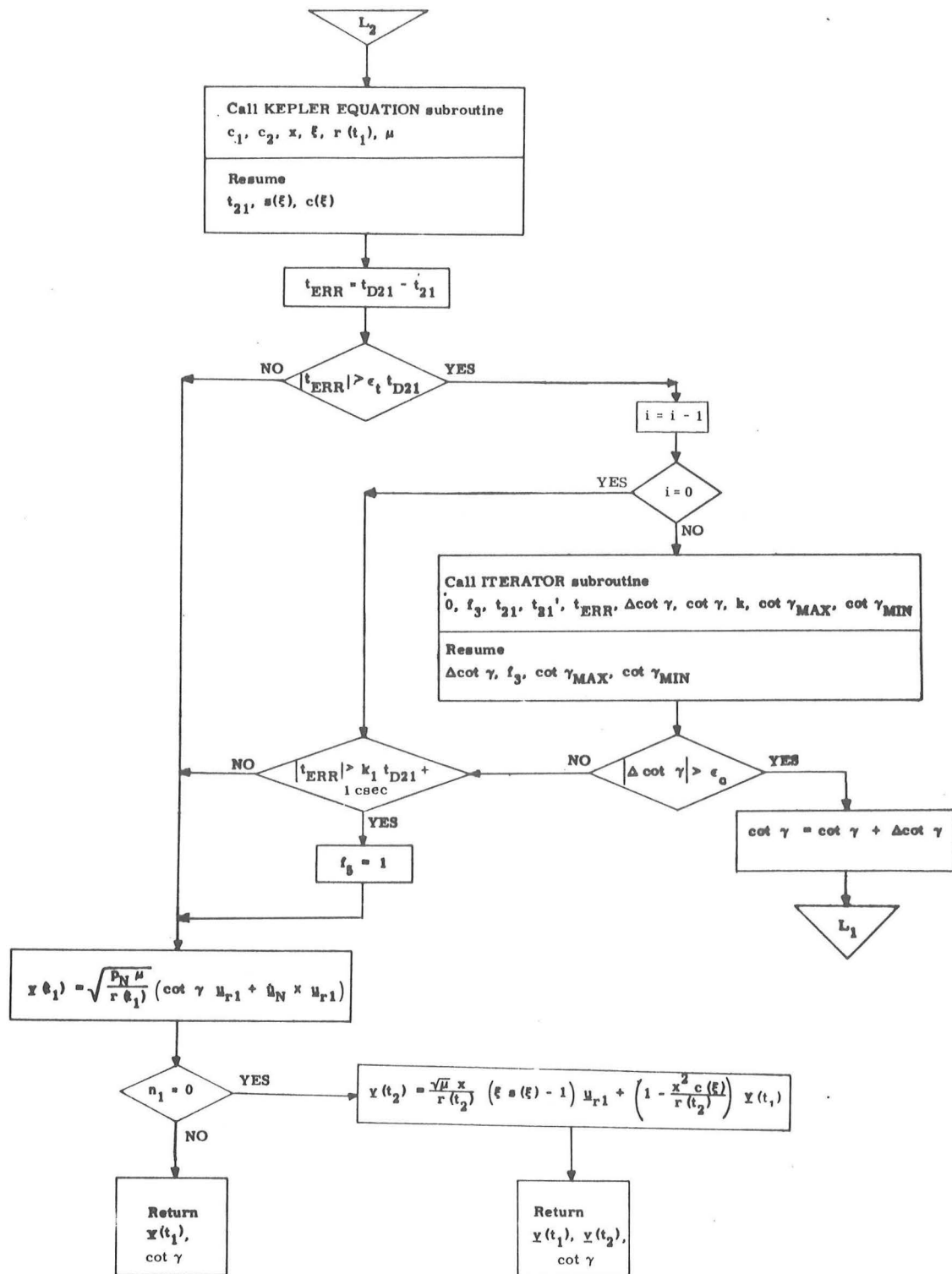


Figure 5.6-2 Lambert Subroutine

### 5.5.7 TIME-THETA SUBROUTINE

The Time-Theta Subroutine solves for the two body transfer time given the initial position and velocity vectors and the true anomaly difference (transfer angle) to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Theta Subroutine block diagram depicted in Fig. 5.7-1. The subroutines referred to in this figure are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

The flag  $f_6$  must be zero if the user desires computation of the terminal state vector in addition to the transfer time.

If the conic trajectory is a parabola or hyperbola and the desired transfer angle,  $\theta$ , lies beyond the asymptote of the conic,  $f_7$  will be set indicating that no solution is possible.

In addition to the parameter range constraints imposed on Kepler's equation, the additional restriction on Time-Theta that the trajectory must not be near rectilinear is indicated by the range of  $\cot \gamma$ .

The Time-Theta problem is not well defined for near rectilinear trajectories, i. e. the transfer angle  $\theta$  is no longer a meaningful problem parameter. This will not cause difficulties provided the input variables are within the specified ranges.

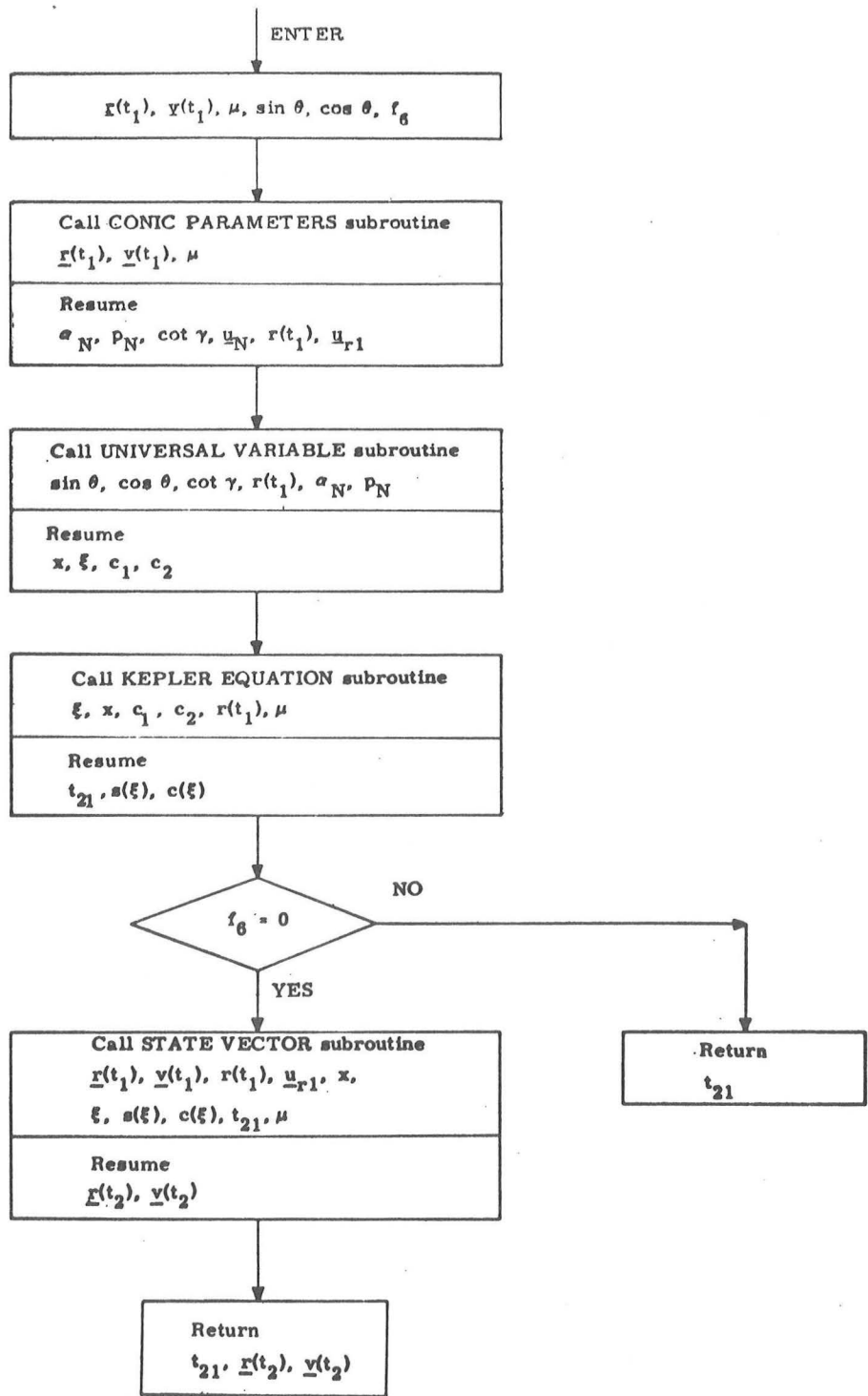


Figure 5.7-1 Time-Theta Subroutine

#### 5.5.8 TIME-RADIUS SUBROUTINE

The Time Radius Subroutine solves for the two body transfer time to a specified radius given the initial position and velocity vectors and the radius magnitude.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Radius Subroutine block diagrams depicted in Figs. 5.8-1 and 5.8-2. The subroutines referred to in this figure are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2

Paragraphs 3, 4, and 5 of Section 5.5.7 apply to the Time-Radius Subroutine as well.

Since an inherent singularity is present for the circular orbit case, near-circular orbits result in a loss of accuracy in computing both the transfer time,  $t_{21}$ , and the final state vector. This is caused by the increasing sensitivity of  $t_{21}$  to  $r(t_2)$  as the circular orbit is approached. In the extreme case that eccentricity is less than  $2^{-18}$ , the problem is undefined and the subroutine will exit without a solution, setting flag  $f_9$  to indicate this.

If  $r(t_2)$  is less than the radius of pericenter or greater than the radius of apocenter, then  $r(t_2)$  will be ignored and the pericenter or apocenter solution, respectively, will be computed. A flag,  $f_8$ , will be set to indicate this.





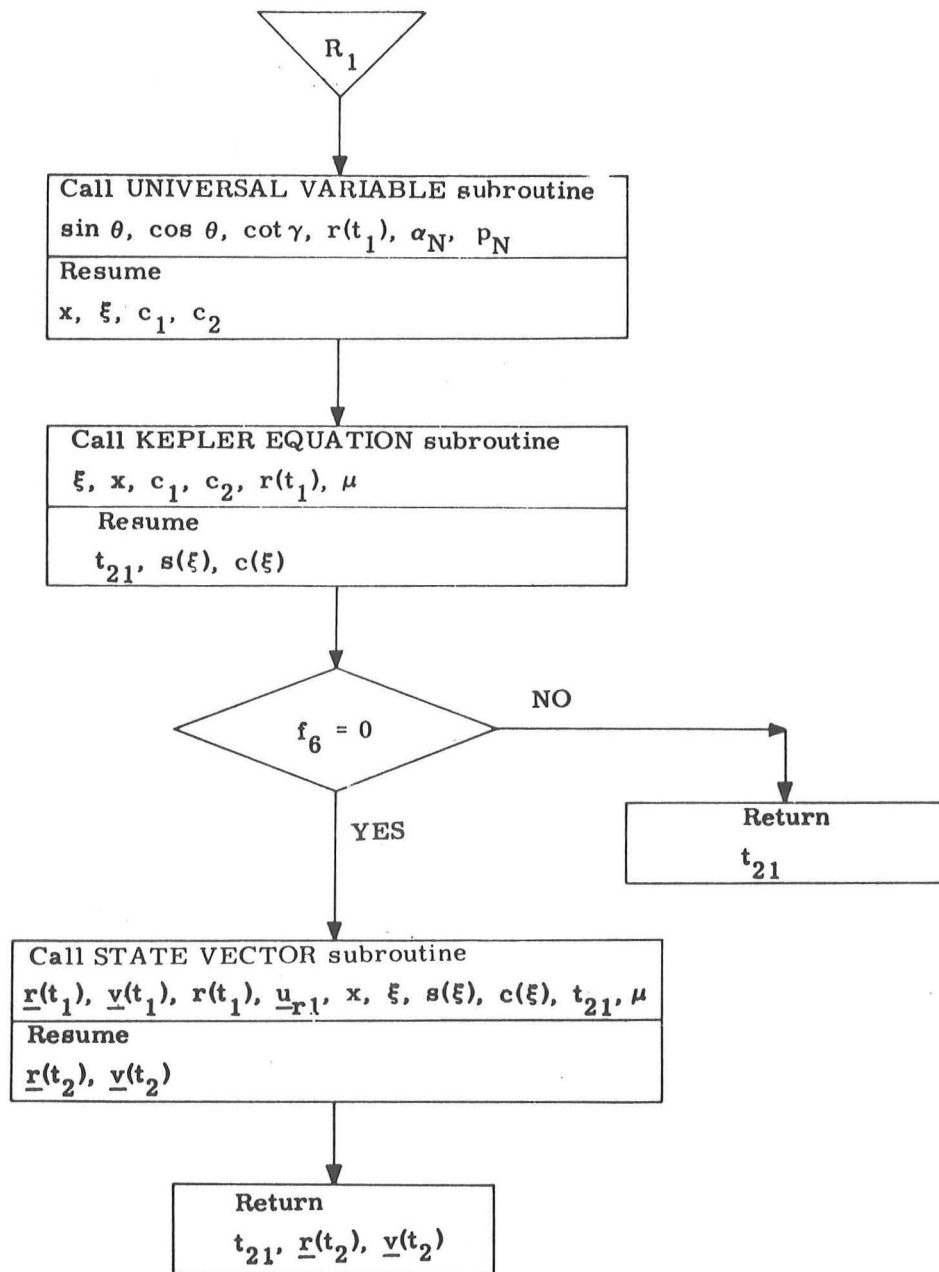


Figure 5.8-2 Time-Radius Subroutine

#### 5.5.9 APSIDES SUBROUTINE

The Apsides Subroutine solves for the two body radii of apocenter and pericenter and the eccentricity of the trajectory given the position and velocity vectors for a point on the trajectory.

This subroutine is depicted in Fig. 5.9-1. The subroutines referred to in this figure are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

It is characteristic of this computation that the apsides become undefined as the conic approaches a circle. This is manifested by decreasing accuracy. When the conic is nearly parabolic, or hyperbolic, the radius of apocenter is not defined. In this event the radius of apocenter will be set to the maximum positive value allowed by the computer.

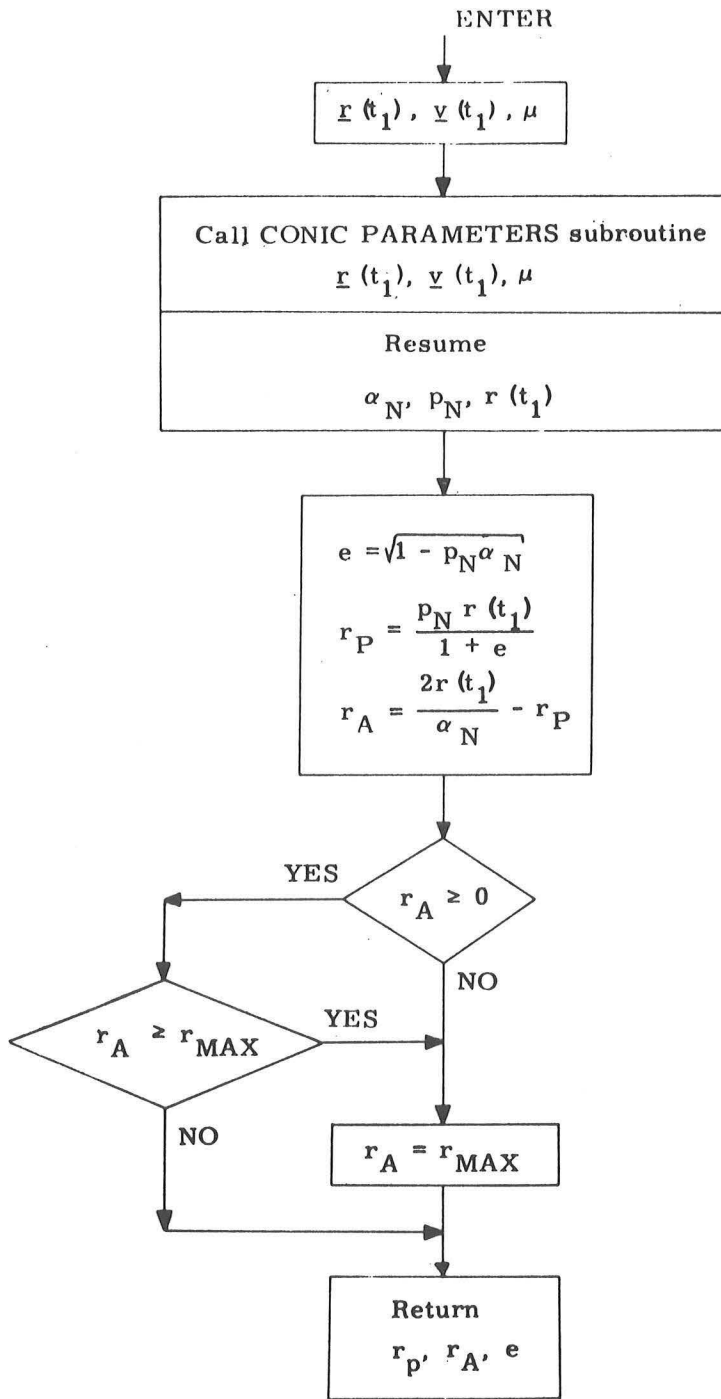


Figure 5.9-1 Apsides Subroutine

#### 5. 5. 10 MISCELLANEOUS SUBROUTINES

There are, as part of the Conic Trajectory Subroutines, three subroutines which are useful in their own right. These are the Conic Parameters, the Geometric Parameters and the Iterator Subroutines which are depicted in Figs. 5. 10-1, 5. 10-2 and 5. 10-3, respectively.

The Conic Parameters and Geometric Parameters Subroutines are self explanatory.

The Iterator Subroutine serves several purposes. It is used when flag  $f_4$  is set to zero to solve for the value of the independent variable which drives the error in the dependent variable to zero, provided the function is monotonically increasing. To improve convergence for functions whose derivative changes rapidly, the limits are reset as shown in the block diagram.

With  $f_4$  set to 1, the Iterator seeks a minimum of the function, provided the first derivative is single-valued between the limits. The inputs are redefined so that "y" is the derivative of the independent variable with respect to the dependent variable, and "x" is the value at which the derivative was computed or approximated. Since the desired value of y is zero,  $y_{ERR} = -y$ .

Since the Iterator uses the "Regula Falsi" technique, it requires two sets of variables to begin iteration. If only one set is available, flag  $f_3$  must be set to 1, causing the iterator to generate the independent variable increment from a fraction of the full range.

In addition to the above subroutines there are three other subroutines of primary interest to the five basic conic subroutines described in Sections 5.5.5 to 5.5.9. These are the Universal Variable Subroutine, the Kepler Equation Subroutine, and the State Vector Subroutine shown in Figs. 5.10-4, 5.10-5 and 5.10-6, respectively.

The Universal Variable Subroutine is utilized by the Lambert, the Time-Theta and the Time-Radius Subroutines to compute the universal parameter  $x$  required for the time equation. There are two different formulations required according to the size of the parameter  $w$ .

If the input to the subroutine requires the physically impossible solution that the trajectory "close through infinity", the problem will be aborted, setting flag  $f_7$ .

The Kepler Equation Subroutine computes the transfer time given the variable  $x$  and the conic parameters.

The State Vector subroutine computes the position and velocity vectors at a point along the trajectory given an initial state vector, the variable  $x$  and the transfer time.

The final miscellaneous subroutine, the SETMU Subroutine, is depicted in Fig. 5.10-7. It sets  $\mu$  to the appropriate primary body gravitational constant consistent with the estimated CSM or LM state vector as defined in Section 5.2.2.6.

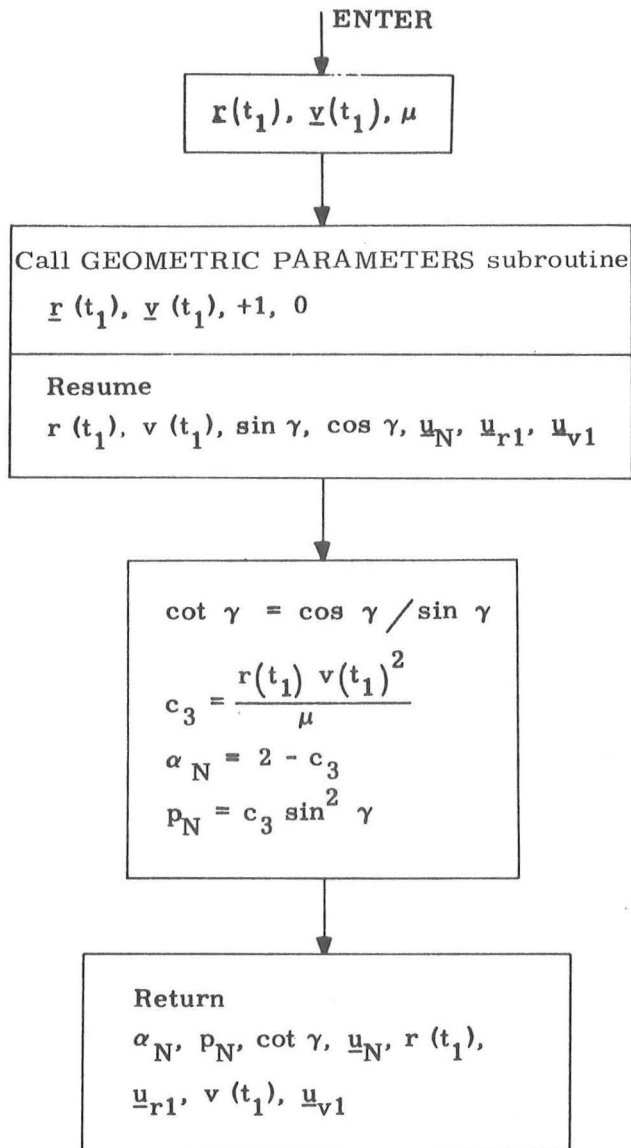


Figure 5.10-1 Conic Parameters Subroutine

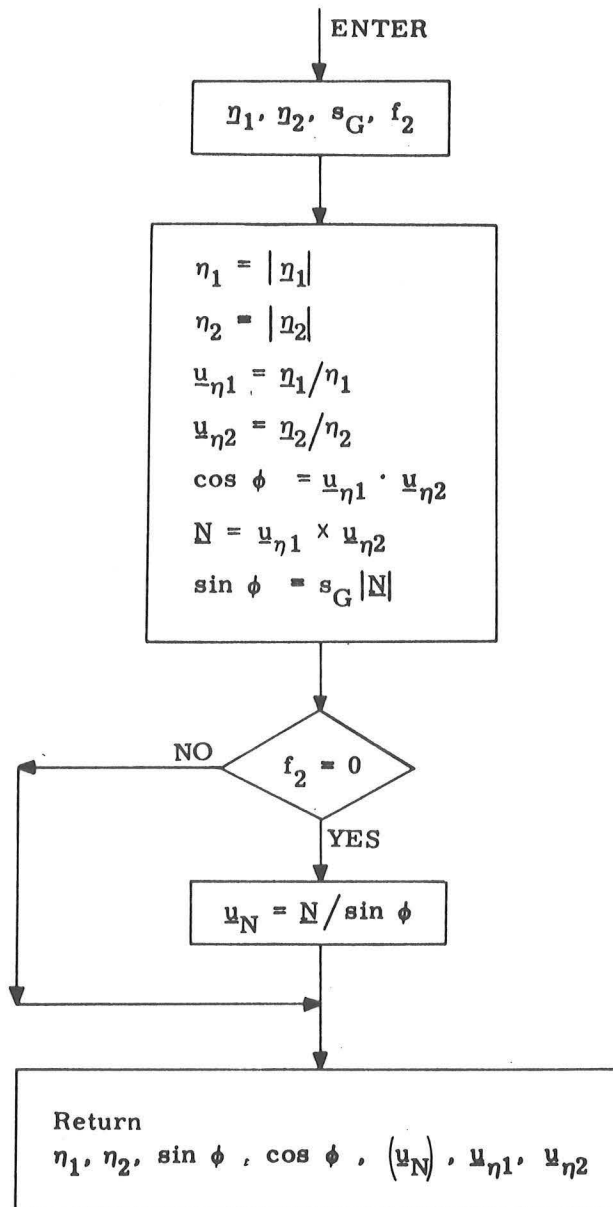


Figure 5.10-2 Geometric Parameters Subroutine

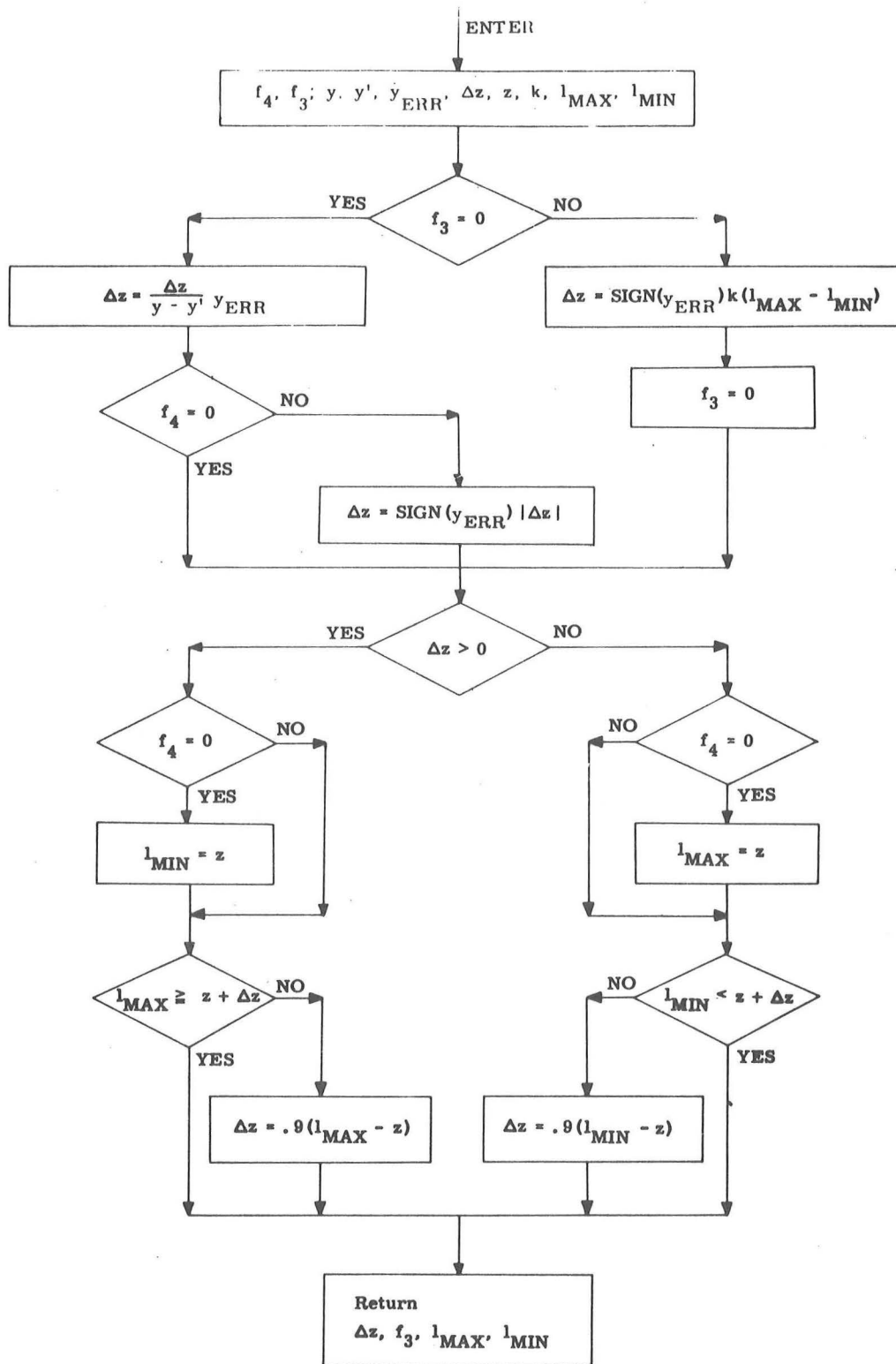


Figure 5.10-3 Iterator Subroutine



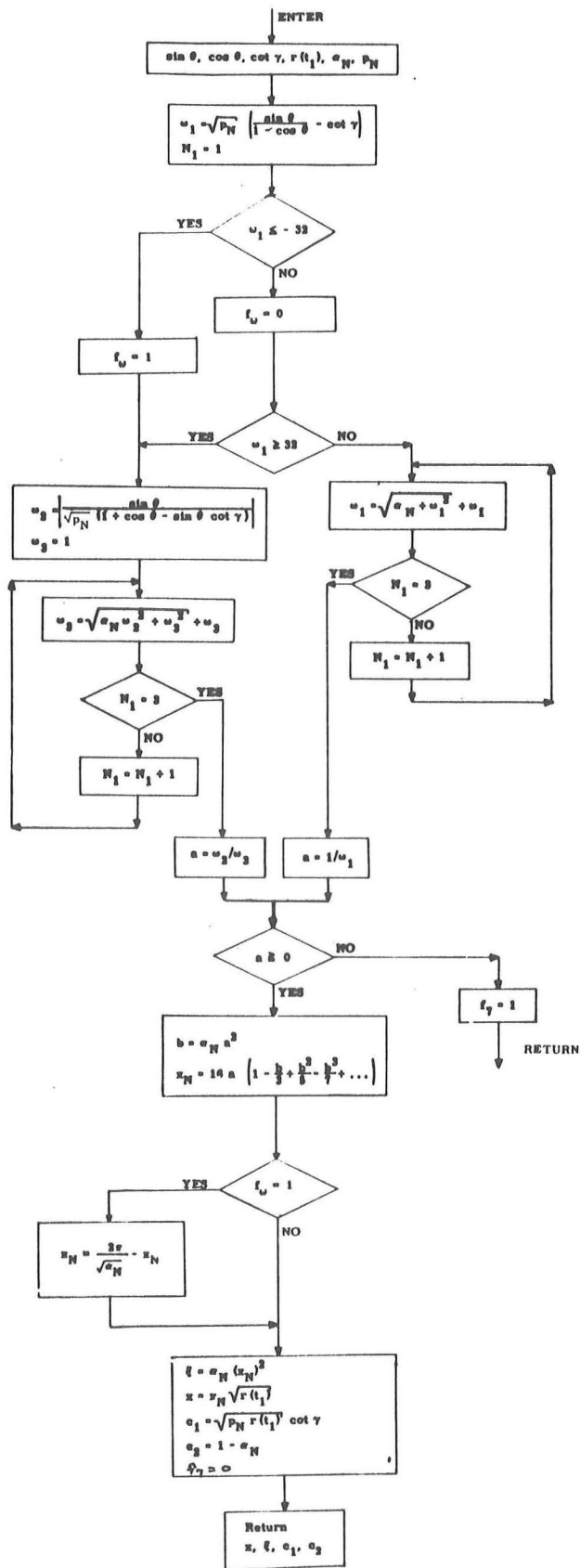


Figure 5.10-4 Universal Variable Subroutine  
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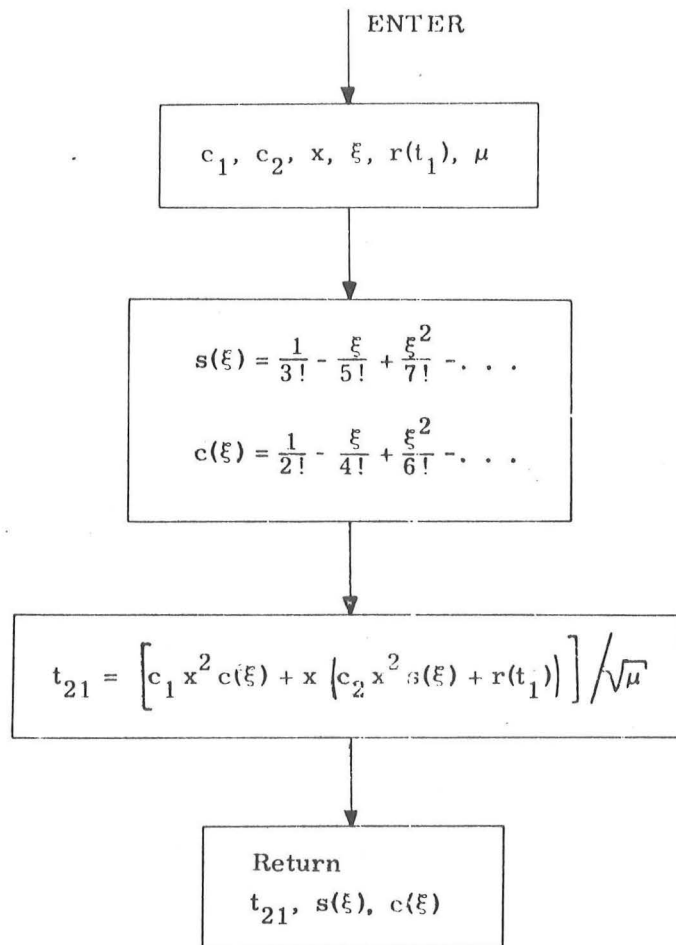


Figure 5.10-5 Kepler Equation Subroutine

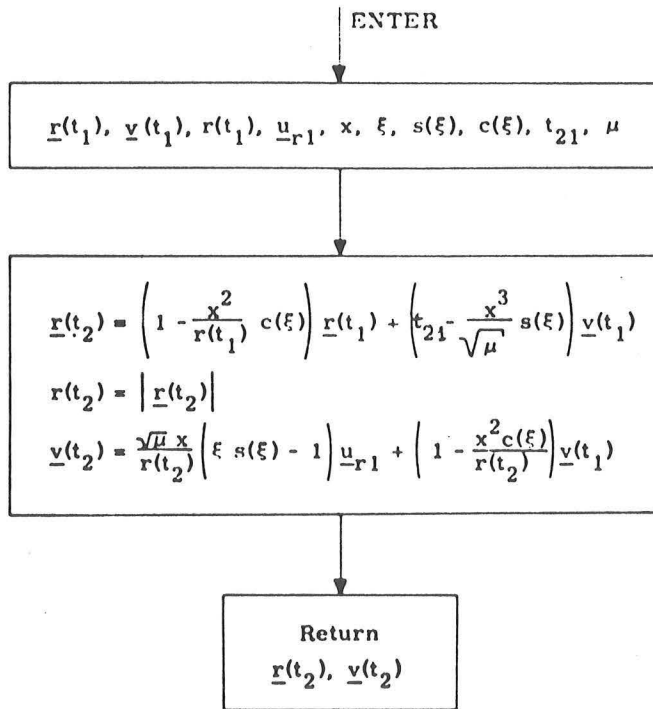


Figure 5.10-6 State Vector Subroutine

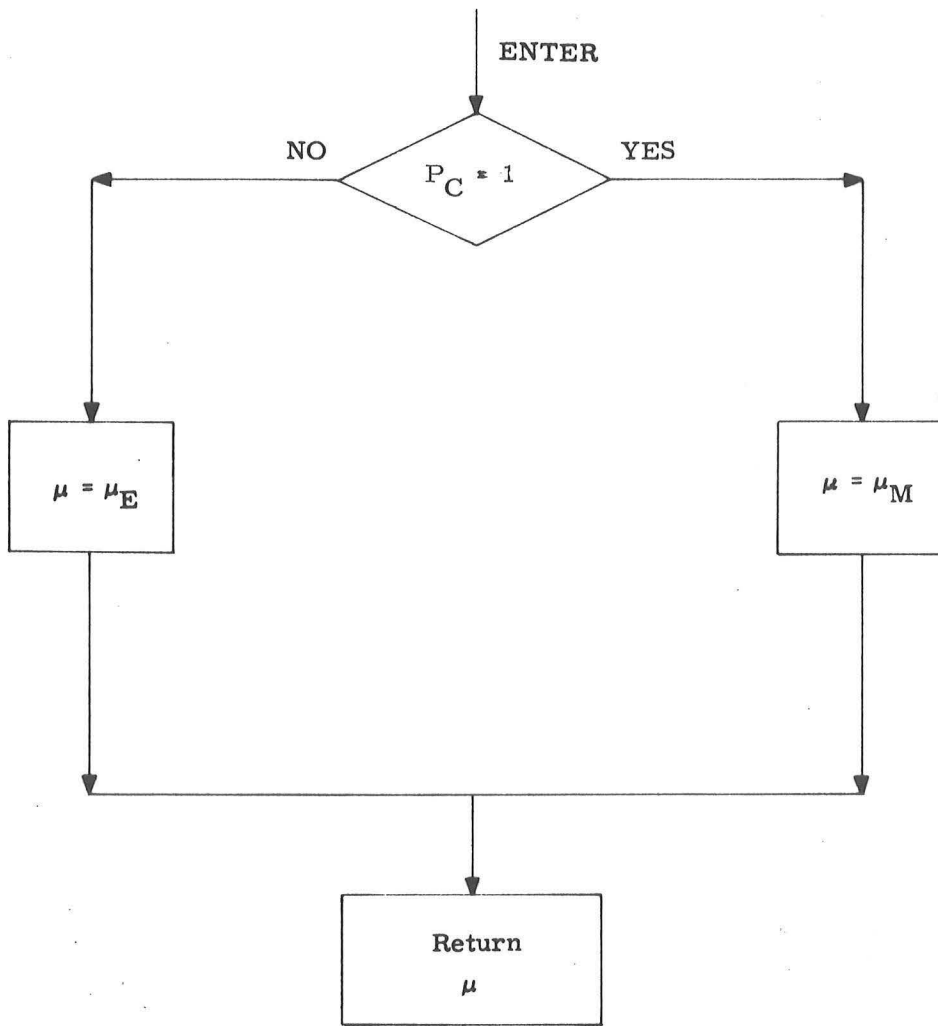


Figure 5.10-7 SETMU Subroutine

5. 5. 11 INITIAL VELOCITY SUBROUTINE

The Initial Velocity Subroutine computes the required initial velocity vector for a trajectory of specified transfer time between specified initial and target position vectors. The trajectory may be either conic or precision depending on an input parameter (namely, number of offsets). In addition, in the precision trajectory case, the subroutine also computes an "offset target vector", to be used during pure-conic cross-product steering. The offset target vector is the terminal position vector of a conic trajectory which has the same initial state as a precision trajectory whose terminal position vector is the specified target vector.

In order to avoid the inherent singularities in the  $180^\circ$  transfer case when the (true or offset) target vector may be slightly out of the orbital plane, the Initial Velocity Subroutine rotates this vector into a plane defined by the input initial position vector and another input vector (usually the initial velocity vector), whenever the input target vector lies inside a cone whose vertex is the origin of coordinates, whose axis is the  $180^\circ$  transfer direction, and whose cone angle is specified by the user.

The Initial Velocity Subroutine is depicted in Fig. 5. 11-1. The Lambert Subroutine, Section 5. 5. 6, is utilized for the conic computations; and the Coasting Integration Subroutine, Section 5. 2. 2, is utilized for the precision trajectory computations.

The Initial Velocity Subroutine sets the maximum number of iterations allowable inside the Lambert Subroutine to 20 in all cases, except when the powered-flight guidance computation sequencing is being performed (see Section 5. 3. 3. 2), in which case it is set to 5.

### Nomenclature for the Initial Velocity Subroutine

$\underline{r}(t_1)$	Initial position vector.
$\underline{v}(t_1)$	Vector (usually the actual initial velocity vector) used to determine whether the transfer from the initial position vector to the target vector is through a central angle of less or greater than $180^\circ$ , and also used in certain cases to specify the transfer plane (see text).
$\underline{r}_T(t_2)$	Target Vector (True target vector if $N_1 > 0$ , or Offset target vector if $N_1 = 0$ ).
$t_D$	Desired transfer time from initial position vector to target vector.
$N_1$	Number of offsets to be used in calculating the offset target vector from the true target vector. ( $N_1 = 0$ implies conic calculations only with offset target vector input).
$\epsilon$	Cone Angle of a cone whose vertex is the coordinate origin and whose axis is the $180^\circ$ transfer direction (i. e., the negative initial position direction). The cone angle $\epsilon$ is measured from the axis to the side of the cone.
$f_1$	Switch set to 0 or 1 according to whether a guess of $\cot \gamma$ is input or not.
[ $\cot \gamma$ ]	Guess of $\cot \gamma$ .
$\underline{v}_T(t_1)$	Required initial velocity vector of a precision [a conic] trajectory which passes through the true [or offset] target vector, or the rotated true [or offset] target vector if the original target vector was in the cone, at the end of the desired transfer time, if $N_1 > 0$ [or $N_1 = 0$ ].
$\underline{r}(t_2)$	Computed offset target vector.
$\underline{v}_T(t_2)$	Final precision [conic] velocity vector resulting from a precision [conic] update of the initial position vector and the required initial velocity vector $\underline{v}_T(t_1)$ , if $N_1 > 0$ , [or $N_1 = 0$ , respectively].
$\underline{r}_T(t_2)$	Final precision position vector.
$\cot \gamma$	Value to which the Lambert Subroutine converged (for later use as guess to minimize computation time).
$f_2$	Switch set to 0 or 1 according to whether the input (true or offset) target vector was not or was in the cone, and consequently was not or was rotated into the plane.

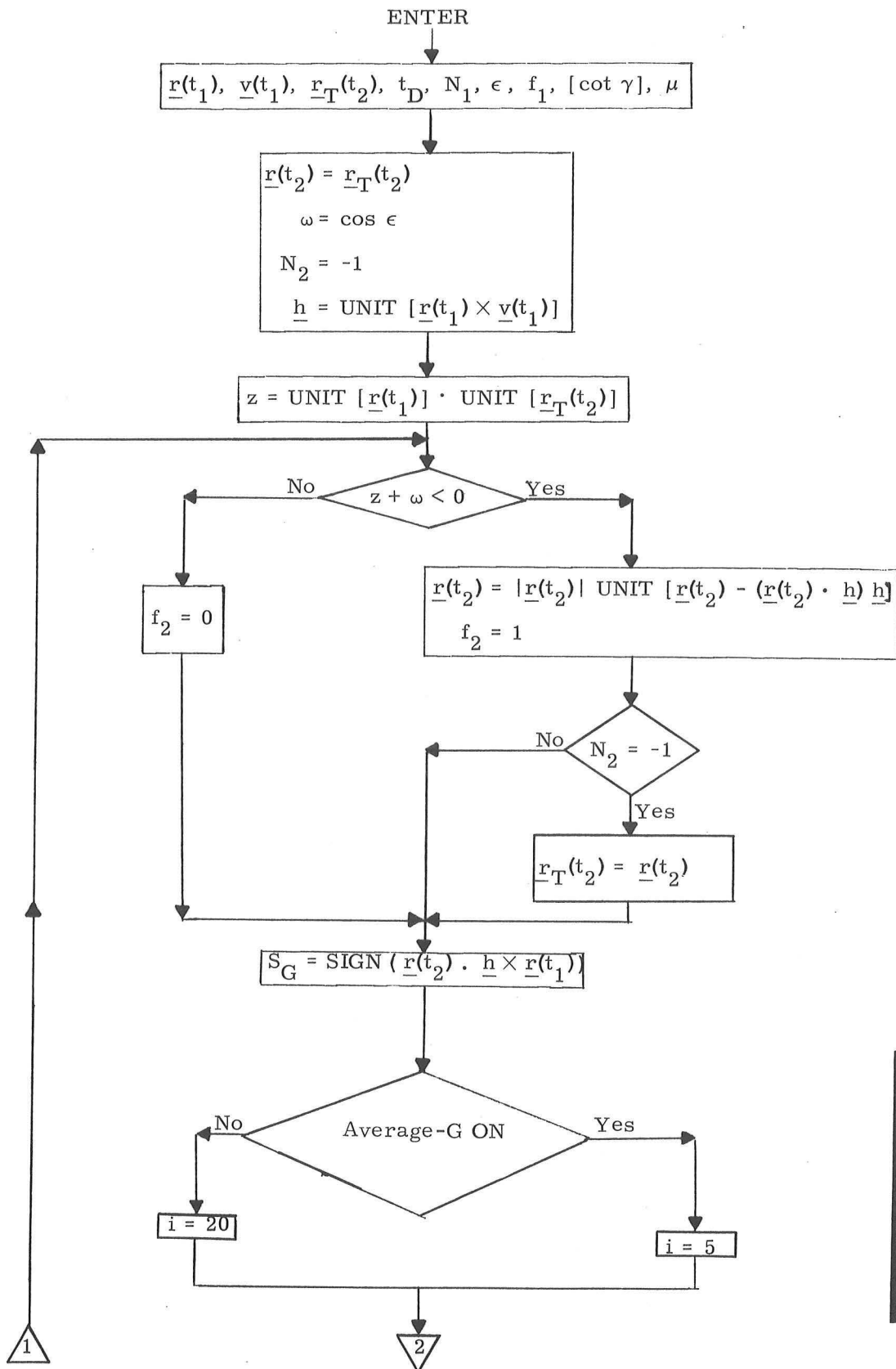


Figure 5.11-1 Initial Velocity Subroutine  
(page 1 of 2)

5.5-52

Revised  
 Added

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GSOP #R-557

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Rev. 1

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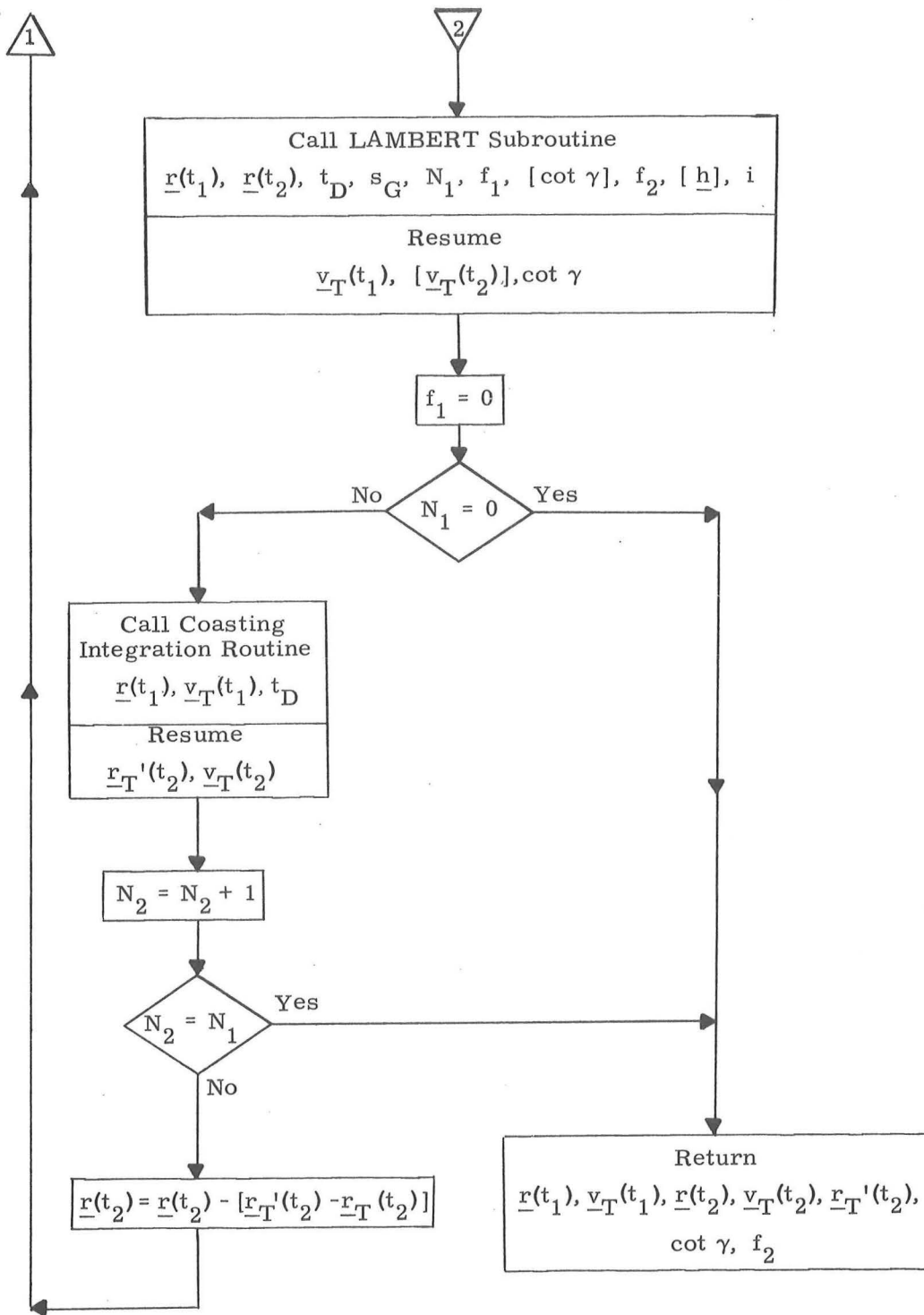


Figure 5.11-1 Initial Velocity Subroutine  
(page 2 of 2)

5.5-52A

Revised  
 Added

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PCR # 188,414/

Rev. 1  
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### 5. 5. 13 LOSSEM SUBROUTINE

The LOSSEM Subroutine computes the lines-of-sight of the Sun, Earth, and Moon with respect to the spacecraft in the Basic Reference Coordinate System. This data is used by the IMU alignment programs whenever the astronaut elects to sight on the Sun, Earth, or Moon instead of a star for purposes of IMU alignment. The data is also used by the Star Selection Routine (Section 5. 6. 4) when testing for star occultation. In addition, this subroutine computes the sizes of the occultation cones used in the Star Selection Routine.

The unit vectors  $\underline{u}_S$ ,  $\underline{u}_E$ , and  $\underline{u}_M$  specifying the lines-of-sight to the Sun, Earth, and Moon, respectively, in the Basic Reference Coordinate System are computed as follows:

$$\underline{u}_S = \begin{cases} \underline{u}_{ES} & \text{if } P = E \\ \text{UNIT} (\underline{u}_{ES} - \rho \underline{u}_{EM}) & \text{if } P = M \end{cases} \quad (5. 13. 1)$$

$$\underline{u}_E = \begin{cases} -\text{UNIT} (\underline{r}_L) & \text{if } P = E \\ -\text{UNIT} (\underline{r}_{EM} \underline{u}_{EM} + \underline{r}_L) & \text{if } P = M \end{cases} \quad (5. 13. 2)$$

$$\underline{u}_M = \begin{cases} \text{UNIT} (\underline{r}_{EM} \underline{u}_{EM} - \underline{r}_L) & \text{if } P = E \\ -\text{UNIT} (\underline{r}_L) & \text{if } P = M \end{cases} \quad (5. 13. 3)$$

where P, E, M, S, and L respectively denote the primary body, Earth, Moon, Sun, and Lunar Module,  $\underline{r}_L$  is the position vector of the LM with respect to the primary body,  $\underline{u}_{EM}$  and  $\underline{u}_{ES}$  are the unit position vectors of the Moon and Sun with respect to the Earth obtained from the Lunar and Solar Ephemerides Subroutine of Section 5. 5. 4,  $r_{EM}$  is the mean distance (384402 km) between the Earth and Moon, and  $\rho$  is the ratio (0. 00257125) of  $r_{EM}$  over the mean distance between the Sun and Earth. The line-of-sight vectors are determined for a time specified by the calling program or routine. When LOSSEM is used by the sighting mark routines it is not called until just after the astronaut finishes his optical marks on the Sun, Earth, or Moon, and the time used in this case is that recorded for the last mark. When LOSSEM is used by the Auto Optics Positioning

Routine (R-52) it is called just before positioning the spacecraft and the specified time is the present time. When LOSSEM is called by the Fine Alignment Routine (R-51) just prior to calling the Star Selection Routine, the specified time is the present time plus an additional amount ( $T_{SS}$ ) in order to project it into the middle of the sighting mark process. This is done to insure that the line-of-sight vector of the primary body used in the occultation test of the Star Selection Routine is that which occurs during the sighting mark process.

The occultation cones used in the Star Selection Routine for the Sun, Earth, and Moon are computed as follows:

$$c_S = \cos 60^\circ \quad (5.13.4)$$

$$c_E = \begin{cases} \cos \left[ 5^\circ + \sin^{-1} \left( \frac{R_E}{r_L} \right) \right] & \text{if } P = E \\ \cos 5^\circ & \text{if } P = M \end{cases} \quad (5.13.5)$$

$$c_M = \begin{cases} \cos 5^\circ & \text{if } P = E \\ \cos \left[ 5^\circ + \sin^{-1} \left( \frac{R_M}{r_L} \right) \right] & \text{if } P = M \end{cases} \quad (5.13.6)$$

where  $c$  is the cosine of one half the total angular dimension of a cone and represents a more convenient way of treating the dimension of a cone in the Star Selection Routine,  $r_L$  is the magnitude of the LM position vector,  $R_E$  is the equatorial radius (6378.166 km) of the Earth, and  $R_M$  is the mean radius (1738.09 km) of the Moon.

5.5.14 PERICENTER - APOCENTER (PERIAPO) SUBROUTINE

The Pericenter - Apocenter Subroutine computes the two body apocenter and pericenter altitudes given the position and velocity vectors for a point on the trajectory and the primary body.

This subroutine is depicted in Fig. 5.14-1.

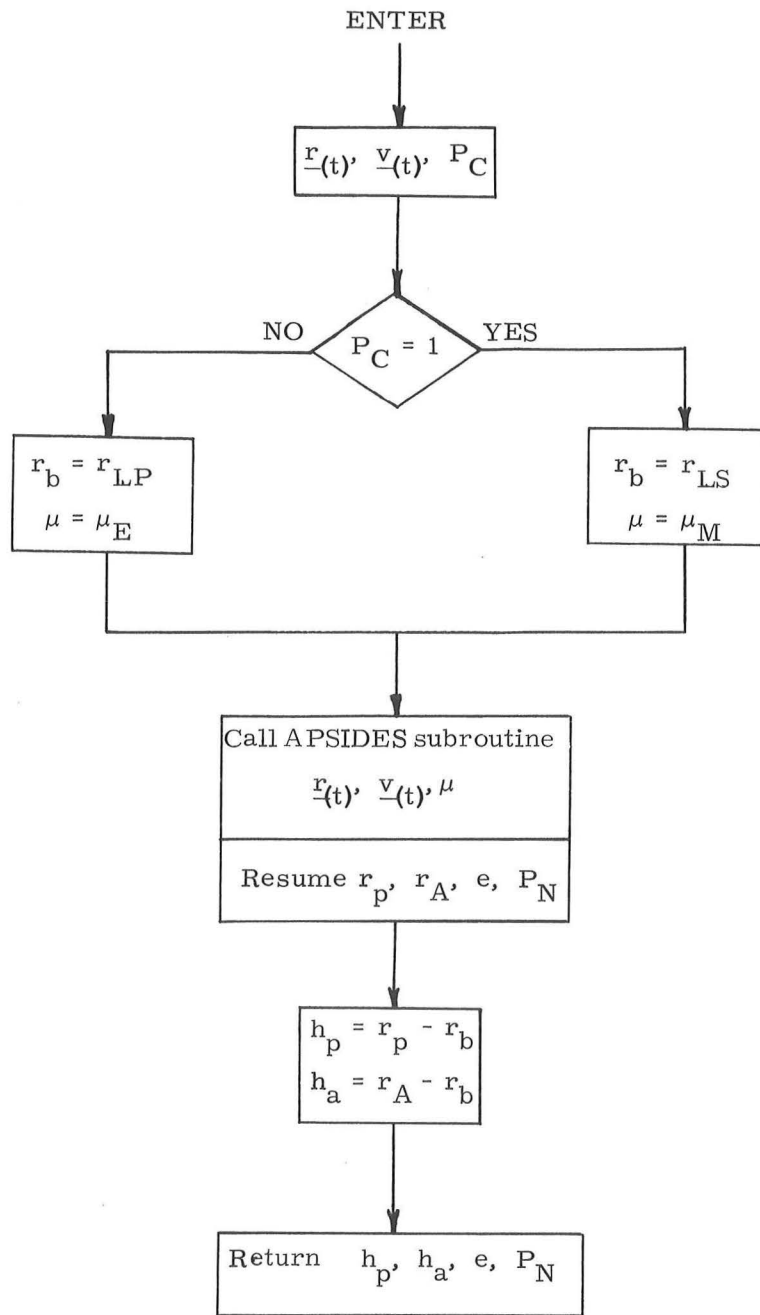


Figure 5.14-1 Pericenter - Apocenter Subroutine

5.6 GENERAL SERVICE ROUTINES

5.6.1 GENERAL COMMENTS

The routines presented in this section are used for the following general service functions:

- 1) IMU alignment modes
- 2) Basic Coordinate Transformations
- 3) Computer initialization procedures
- 4) Special display routines which can be called by the astronaut

## 5.6.2 IMU ALIGNMENT MODES

### 5.6.2.1 Orbital Alignment

#### 5.6.2.1.1 IMU Orientation Determination Program

The IMU Orientation Determination Program (P-51) is used during free-fall to determine the present IMU stable member orientation with respect to the Basic Reference Coordinate System by sighting on two navigation stars or known celestial bodies with the Alignment Optical Telescope (AOT) or the Crew Optical Alignment Sight (COAS). At the start of program P-51 the astronaut acquires the desired celestial bodies by maneuvering the spacecraft until they are visible in the optical device which he intends to use. During the above operation he checks to insure that gimbal lock does not occur. Afterwards, the Inflight Sighting Mark Routine (R-53) is used twice to sight upon each of two celestial bodies. Prior to sighting on each body, the astronaut indicates to the computer which optical device he is going to use and which celestial body he is going to sight on. The method of selection and operation of the AOT and COAS is given in Section 5.6.3.1.1 and in the Inflight Sighting Mark Routine (R-53) of Section 4. The identity of the celestial body is indicated by the astronaut at the beginning of the Inflight Sighting Mark Routine by use of a star (or celestial body) code. Separate codes are provided for the 37 navigation stars, Sun, Earth, Moon, and a general celestial body. The latter code is referred to as the Planet code. If the astronaut selects the Sun, Earth, or Moon code, the subroutine LOSSEM of Section 5.5.13 will be called after optical sightings have been made on the body to compute the line-of-sight vector to that body in the Basic Reference Coordinate System. If he selects the Planet code, the computer will request after the optical sightings that he load the coordinates of the desired celestial body in the Basic Reference Coordinate System.

After optical sightings have been made on both celestial bodies, the Sighting Data Display Routine (R-54 of Section 4) computes the angle between the line-of-sight vectors obtained for the two bodies in stable member coordinates and the angle between the corresponding line-of-sight vectors for the two bodies stored in basic reference coordinates. The difference between these two angles is displayed to the astronaut and he either accepts or rejects the vectors obtained in stable member coordinates. If he accepts the vectors, they are used with the corresponding vectors stored in basic reference coordinates to determine the present stable member orientation and REFSMMAT, using the procedure given in Section 5.6.3.4.1.

#### 5.6.2.1.2 IMU Realignment Program

The IMU Realignment Program (P-52) is used during free-fall to align the IMU from a known orientation to one of the desired orientations given in Section 5.6.3.4 and in P-52 of Section 4. This alignment is made by sighting on two navigation stars or known celestial bodies with the AOT or the COAS. Initially, the astronaut selects the desired stable member orientation, and the program computes and displays the IMU gimbal angles for the desired stable member orientation using the present vehicle attitude. If the computed gimbal angles are unsatisfactory, the astronaut maneuvers the vehicle to a more suitable attitude and has the program re-compute and display the new gimbal angles. Once satisfactory angles have been obtained, the Coarse Alignment Routine (R-50 of Section 4) is used to coarse align the IMU to the desired orientation. If all of the required gimbal angle changes are less than or equal to one degree, routine R-50 will leave the IMU at its present orientation. Afterwards, the astronaut maneuvers the vehicle to a desired attitude for celestial body acquisition and the Fine Alignment Routine (R-51) is used to obtain the necessary sightings and to align the IMU to the desired orientation. This routine accomplishes the above tasks by using various other routines. Initially, the astronaut decides whether or not to use the Star Selection Routine of Section 5.6.4. If he selects the Star Selection Routine, routine

R-51 will call the subroutine LOSSEM of Section 5. 5. 13 prior to calling the Star Selection Routine since LOSSEM computes certain parameters which are used in the occultation tests of the Star Selection Routine. If the Star Selection Routine is unable to find two satisfactory stars at the present vehicle attitude, the astronaut either repeats the above process of changing the vehicle attitude and using the Star Selection Routine or selects his own celestial bodies. It should be noted that the Star Selection Routine only selects stars for the forward viewing position of the AOT. However, the astronaut may select stars or celestial bodies with the intention of using one of the other viewing positions described in Section 5. 6. 3. 1. 1.

Prior to performing the optical sightings on each celestial body, the astronaut is given the option of either maneuvering the vehicle himself to place the celestial body at the desired sighting position or having the Auto Optics Positioning Routine (R-52 of Section 4) command the vehicle attitude so as to place the celestial body at the desired sighting position. The sighting position is established by the astronaut at the beginning of the Auto Optics Positioning Routine in the same manner as is done at the start of the Inflight Sighting Mark Routine. By selecting the appropriate detent code as described in Section 5. 6. 3. 1. 1 and in routines R-52 and R-53 of Section 4, the sighting position can be the center of the field-of-view of the COAS or any of the six fields-of-view of the AOT. In addition to establishing the sighting position, the astronaut can also specify the celestial body (star, etc.) which is to be used in the Auto Optics Positioning Routine just as is described for the Inflight Sighting Mark Routine in Section 5. 6. 2. 1. 1. If he selects



the Sun, Earth, or Moon code, the subroutine LOSSEM of Section 5.5.13 will be called just prior to positioning the spacecraft to compute the line-of-sight vector to that body. If he selects the Planet code, the computer will request that he load the coordinates of the desired celestial body just before positioning the spacecraft. Once the celestial body is properly positioned in the field-of-view, the Inflight Sighting Mark Routine is used to perform the sightings on the body.

**After sightings have been made on two celestial bodies, the Sighting Data Display Routine is used to check the angle between the two measured directions, and the Gyro Torquing Routine (R-55 of Section 4) is used to torque the IMU stable member to the desired orientation.**

It should be noted that the ground can indicate to the LGC via uplink the present stable member orientation or a desired stable member orientation. If the present orientation is being indicated, this is done by transmitting a REFSMMAT to the LGC. Under normal circumstances, however, it would not be desirable for the ground to indicate the present stable member orientation since this orientation should be determined by the LGC PGNCS. However, if an orientation different from the present orientation is desired by the ground, this desired orientation can be transmitted as a Preferred orientation. By treating a desired stable member orientation in this manner, program P-52 will be able to correct for any large differences between the present and desired orientations by coarse alignment. In addition, this approach avoids the introduction of orientation errors which affect celestial body acquisition. Whenever a Preferred orientation is being transmitted, the ground or the astronaut must also set the Preferred Orientation flag so that program P-52 will know what is desired.

### 5. 6. 3 IMU ROUTINES

#### 5. 6. 3. 1 AOT and COAS Transformations

##### 5. 6. 3. 1. 1 Determination of the Star Line-of-Sight During Free-Fall

To perform IMU alignment during free-fall it is necessary to determine the lines-of-sight to two separate navigation stars or known celestial bodies in stable member coordinates. As a matter of convenience all celestial bodies used for this purpose shall be referred to as stars in this section. The line-of-sight to a star is determined during free-fall by making optical sightings on the star with the Alignment Optical Telescope (AOT) or the Crew Optical Alignment Sight (COAS). These optical sightings are made by varying the LM attitude so that the star crosses the X and Y reticle lines of the AOT or the equivalent X and Y lines of the COAS. The astronaut depresses the X or Y mark button whenever the star crosses the corresponding reticle line. When the star coincides with one of the reticle lines, this defines a plane containing the star. Once the location of the star has been established in two separate planes, the line-of-sight to the star can be obtained by solving for the intersection of these two planes.

The AOT is a unity power telescope with a field-of-view (FOV) of 60 degrees and can be rotated to six distinct positions about an axis parallel to the navigation base X-axis. These positions are accurately obtained by the use of detents. The center of the FOV for each of the viewing positions is defined by the azimuth ( $AZ_N$ ) and elevation ( $EL_N$ ) angles shown in Fig. 6. 3-1 where the subscript N denotes the viewing position being used. The approximate values of these angles for the six viewing positions are:

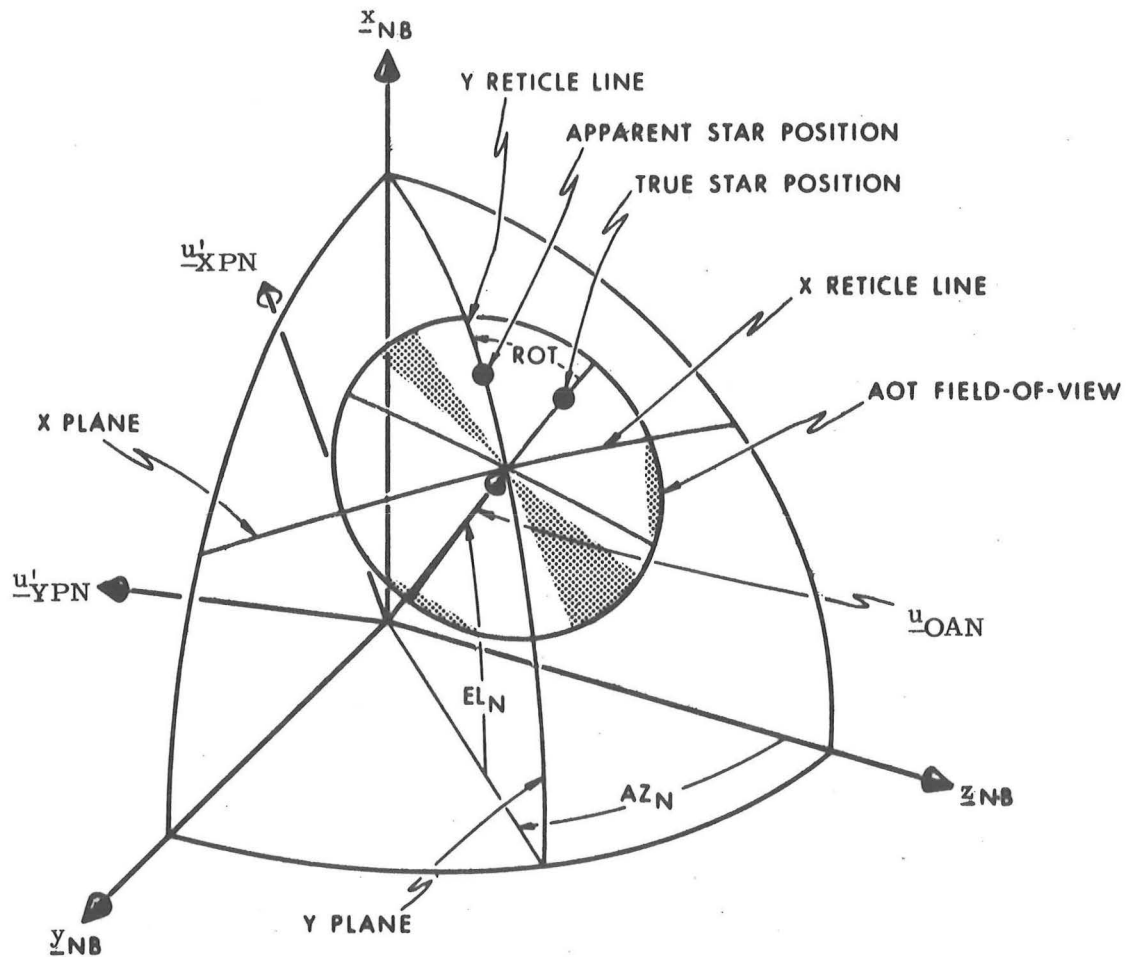


Fig. 6. 3-1 AOT Alignment Geometry

Position 1 (left forward)	$AZ_1 = -60^\circ$ , $EL_1 = 45^\circ$
Position 2 (forward)	$AZ_2 = 0^\circ$ , $EL_2 = 45^\circ$
Position 3 (right forward)	$AZ_3 = 60^\circ$ , $EL_3 = 45^\circ$
Position 4 (right rear)	$AZ_4 = 120^\circ$ , $EL_4 = 45^\circ$
Position 5 (rear)	$AZ_5 = 180^\circ$ , $EL_5 = 45^\circ$
Position 6 (left rear)	$AZ_6 = -120^\circ$ , $EL_6 = 45^\circ$

Under normal circumstances only the three forward positions (1, 2, and 3) are used since the fields of view of the three rear positions are either completely or partially blocked by a prism shield. However, since the shield can be removed by the astronaut for backup purposes, provisions have been made in the computer for using the rear positions.

At the beginning of the Inflight Sighting Mark Routine (R-53) and the Auto Optics Positioning Routine (R-52) the astronaut indicates whether he is going to use one of the viewing positions of the AOT or the COAS by selecting one of the following detent codes:

<u>Detent Code</u>	<u>Use</u>
1	AOT Position 1
2	AOT Position 2
3	AOT Position 3
4	Any rear AOT Position
5	COAS

If detent code 1, 2, or 3 is selected the computer uses the angles  $AZ_N$  and  $EL_N$  associated with the AOT position which have been pre-stored in erasable memory. These angles must be stored in erasable memory since the exact values vary slightly from one AOT to another and are not known until the final AOT installation. If detent code 4 is selected the astronaut must load-in the correct angles  $AZ$  and  $EL$  for the desired rear AOT position. These angles completely specify the desired position to the computer. If detent code 5 is selected the astronaut must load-in the correct angles  $AZ$  and  $EL$  for the COAS. If the COAS is pointed along the LM +Z axis when used in this application, its  $AZ$  and  $EL$  angles are approximately zero.

The direction of the center of the FOV (or optical axis) for each AOT viewing position and the COAS can be expressed in navigation base coordinates by the following unit vector:

$$\underline{u}_{OAN} = \begin{bmatrix} \sin (EL_N) \\ \cos (EL_N) \sin (AZ_N) \\ \cos (EL_N) \cos (AZ_N) \end{bmatrix} \quad (6.3.1)$$

where  $N$  denotes the viewing position of the AOT ( $N = 1$  through  $6$ ) or the COAS ( $N = 7$ ).

The X and Y reticle lines of the AOT (shown in Fig. 6.3-1) and those of the COAS define two planes perpendicular to each other. The orientation of these planes with respect to the navigation base for each viewing position of the AOT and the COAS are given by the following unit vectors:

$$\underline{u}'_{YPN} = \begin{bmatrix} 0 \\ \sin (90^\circ + AZ_N) \\ \cos (90^\circ + AZ_N) \end{bmatrix} \quad (6.3.2)$$

$$\underline{u}'_{XPN} = \underline{u}'_{YPN} \times \underline{u}_{OAN}$$

where N denotes the viewing position of the AOT or the COAS (N = 7). Note that the X and Y reticle lines of the COAS are assumed to be in the planes defined by the respective vectors  $\underline{u}'_{XPN}$  and  $\underline{u}'_{YPN}$  in Eq. (6.3.2). Since the AOT reticle can be rotated about the AOT optical axis by means of a knob near the eyepiece, it is necessary for the astronaut to set the reticle rotation angle to zero when performing optical sightings during free-fall in order for the vectors in Eq. (6.3.2) to correctly define the X and Y reticle planes.

At this point it is necessary to consider one important difference between the AOT and the COAS which is the following. When the AOT is rotated from one viewing position to another, the apparent star field observed through the instrument rotates about the optical axis. This effect is due to the optical design of the AOT and does not occur for the AOT reticle lines. In the forward position (Position 2) the apparent star field is the same as the true star field. As the AOT is rotated through a given azimuth angle from Position 2 to 3, the apparent star field rotates with respect to the true star field by an equal amount in a clockwise direction about the AOT optical axis. For Position 3 the relationship between the apparent and true positions of a star would be as shown in Fig. 6.3-1 where the amount and sense of rotation is shown as ROT. Consequently, when an apparent star coincides with the Y reticle line, as shown in Fig. 6.3-1, the true location of this star is in a plane obtained by rotating the Y plane about the optical axis by an amount ROT in the reverse direction. If it is

assumed that the azimuth angle,  $AZ_N$ , is positive as shown in Fig. 6.3-1, then the correct sense and magnitude of the rotation ( $R_N$ ) which is applied to the X and Y planes for a given AOT viewing position is

$$R_N = AZ_2 - AZ_N \quad (N = 1 \text{ thru } 6) \quad (6.3.3)$$

Therefore, the correct orientation in navigation base coordinates of the planes represented by the AOT X and Y reticle lines is

$$\underline{u}_{\text{XPN}} = \cos(R_N) \underline{u}'_{\text{XPN}} + \sin(R_N) \underline{u}'_{\text{YPN}} \quad (6.3.4)$$

$$\underline{u}_{\text{YPN}} = -\sin(R_N) \underline{u}'_{\text{XPN}} + \cos(R_N) \underline{u}'_{\text{YPN}}$$

Since the above AOT planes ( $\underline{u}_{\text{XPN}}$  and  $\underline{u}_{\text{YPN}}$ ) and those for the COAS in Eq. (6.3.2) are fixed with respect to the navigation base coordinate system it is only necessary to compute them once when using a given AOT viewing position or the COAS; regardless of the number of marks made on a star.

The orientations of the above AOT planes for a given viewing position are obtained in stable member coordinates by

$$\begin{aligned} \underline{u}_{XP} &= \left[ \text{NBSM} \right]_X \underline{u}_{XPN} \\ \underline{u}_{YP} &= \left[ \text{NBSM} \right]_Y \underline{u}_{YPN} \end{aligned} \quad (6.3.5)$$

where  $\left[ \text{NBSM} \right]_X$  and  $\left[ \text{NBSM} \right]_Y$  are the transformation matrices based upon the IMU CDU readings stored during the X and Y marks and are defined in Section 5.6.3.2.1.

The orientations of the corresponding COAS reticle planes are obtained in stable member coordinates by

$$\begin{aligned} \underline{u}_{XP} &= \left[ \text{NBSM} \right]_X \underline{u}'_{XPN} \\ \underline{u}_{YP} &= \left[ \text{NBSM} \right]_Y \underline{u}'_{YPN} \end{aligned} \quad (6.3.6)$$

The vector describing the line-of-sight to the star in stable member coordinates is

$$\underline{s}_{SM} = \text{UNIT} (\underline{u}_{XP} \times \underline{u}_{YP}) \quad (6.3.7)$$

To achieve greater accuracy in determining the line-of-sight to a star, a multiple mark capability is provided, whereby the astronaut can make up to five pairs of marks. This is accomplished by storing the IMU CDU readings for each pair of



X and Y marks until the marking process is completed for the star and then using the CDU readings for each pair of marks to obtain a star vector  $\underline{s}_{SM}$  as indicated in Eqs. (6.3.5) and (6.3.6). As each star vector  $\underline{s}_{SM}$  is computed it is averaged with the previously computed star vectors to obtain an average star vector  $\underline{s}_n$  as follows:

$$\underline{s}_n = \frac{1}{n} [(n - 1) \underline{s}_{n-1} + \underline{s}_{SM}]$$

where n is the number of  $\underline{s}_{SM}$  accumulated.

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### 5.6.3.2 IMU Transformations

#### 5.6.3.2.1 Stable Member-Navigation Base

Let IGA, MGA, OGA be the IMU inner, middle and outer gimbal angles, respectively. Define the following matrices:

$$Q_1 = \begin{pmatrix} \cos \text{IGA} & 0 & -\sin \text{IGA} \\ 0 & 1 & 0 \\ \sin \text{IGA} & 0 & \cos \text{IGA} \end{pmatrix} \quad (6.3.14)$$

$$Q_2 = \begin{pmatrix} \cos \text{MGA} & \sin \text{MGA} & 0 \\ -\sin \text{MGA} & \cos \text{MGA} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (6.3.15)$$

$$Q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \text{OGA} & \sin \text{OGA} \\ 0 & -\sin \text{OGA} & \cos \text{OGA} \end{pmatrix} \quad (6.3.16)$$

Stable Member to Navigation Base Transformation

$$\underline{u}_{\text{NB}} = Q_3 Q_2 Q_1 \underline{u}_{\text{SM}} \quad (6.3.17)$$

$$[\text{SMNB}] = Q_3 Q_2 Q_1$$

### Navigation Base to Stable Member Transformation

$$\underline{u}_{SM} = Q_1^T Q_2^T Q_3^T \underline{u}_{NB}, \quad [NBSM] = Q_1^T Q_2^T Q_3^T \quad (6.3.18)$$

#### 5.6.3.2.2 Calculation of Gimbal Angles (CALCGA)

Given a stable member orientation and a navigation base orientation both referred to the same coordinate system, the following procedure is used to compute the corresponding gimbal angles.

$$\begin{aligned} \underline{a}_{MG} &= \text{UNIT} (\underline{x}_{NB} \times \underline{y}_{SM}) \\ \cos \text{OGA} &= \underline{a}_{MG} \cdot \underline{z}_{NB} \\ \sin \text{OGA} &= \underline{a}_{MG} \cdot \underline{y}_{NB} \\ \text{OGA} &= \text{ARCTRIG} (\sin \text{OGA}, \cos \text{OGA}) \\ \cos \text{MGA} &= \underline{y}_{SM} \cdot (\underline{a}_{MG} \times \underline{x}_{NB}) \\ \sin \text{MGA} &= \underline{y}_{SM} \cdot \underline{x}_{NB} \\ \text{MGA} &= \text{ARCTRIG} (\sin \text{MGA}, \cos \text{MGA}) \\ \cos \text{IGA} &= \underline{a}_{MG} \cdot \underline{z}_{SM} \\ \sin \text{IGA} &= \underline{a}_{MG} \cdot \underline{x}_{SM} \\ \text{IGA} &= \text{ARCTRIG} (\sin \text{IGA}, \cos \text{IGA}) \end{aligned} \quad (6.3.19)$$

where the inputs are three vectors along the stable member axes and three vectors along the navigation base axes. In the above equations ARCTRIG implies computing the angle, choosing either  $\sin^{-1}$  or  $\cos^{-1}$  so as to yield maximum accuracy.

### 5.6.3.2.3 Calculation of Gyro Torquing Angles (CALCGTA)

In the fine align procedure, after the present platform orientation is determined, the torquing angles required to move the platform into the desired orientation must be computed. This is achieved as follows:

Let  $\underline{x}_D$ ,  $\underline{y}_D$ , and  $\underline{z}_D$  be the desired stable member axes referred to present stable member orientation. The rotations are performed in three steps: (1) rotating through  $\theta_y$  about the y axis, yielding  $\underline{x}'_D$ ,  $\underline{y}'_D$ ,  $\underline{z}'_D$ ; (2) rotating through  $\theta_z$  about the z' axis, yielding  $\underline{x}''_D$ ,  $\underline{y}''_D$ ,  $\underline{z}''_D$ ; (3) and finally rotating through  $\theta_x$  about the x'' axis, yielding  $\underline{x}'''_D$ ,  $\underline{y}'''_D$ ,  $\underline{z}'''_D$ . The relevant equations are as follows:

$$\underline{z}'_D = \text{UNIT}(-x_{D,3}, 0, x_{D,1})$$

$$\sin \theta_y = z'_{D,1}$$

$$\cos \theta_y = z'_{D,3}$$

$$\theta_y = \text{ARCTRIG}(\sin \theta_y, \cos \theta_y)$$

$$\sin \theta_z = x_{D,2}$$

(6.3.20)

$$\cos \theta_z = z'_{D,3} x_{D,1} - z'_{D,1} x_{D,3}$$

$$\theta_z = \text{ARCTRIG}(\sin \theta_z, \cos \theta_z)$$

$$\cos \theta_x = \underline{z}'_D \cdot \underline{z}_D$$

$$\sin \theta_x = \underline{z}'_D \cdot \underline{y}_D$$

$$\theta_x = \text{ARCTRIG}(\sin \theta_x, \cos \theta_x)$$

The required inputs are the three coordinate axes of the desired stable member orientation referred to the present stable member orientation.

#### 5.6.3.2.4 Coordinate Axes Generator (AXISGEN)

Given two unit vectors (usually star vectors),  $\underline{s}_A$  and  $\underline{s}_B$ , expressed in two coordinate systems, denoted by primed and unprimed characters, i. e.,  $\underline{s}'_A$ ,  $\underline{s}'_B$ ,  $\underline{s}_A$ ,  $\underline{s}_B$ , this routine computes the unit vectors  $\underline{x}$ ,  $\underline{y}$ ,  $\underline{z}$  which are the primed coordinate system axes referred to the unprimed coordinate system. This is accomplished by defining two ortho-normal coordinate sets, one in each system, in the following manner:

$$\begin{aligned}
 \underline{u}'_X &= \underline{s}'_A \\
 \underline{u}'_Y &= \text{UNIT}(\underline{s}'_A \times \underline{s}'_B) \\
 \underline{u}'_Z &= \underline{u}'_X \times \underline{u}'_Y \\
 \underline{u}_X &= \underline{s}_A \\
 \underline{u}_Y &= \text{UNIT}(\underline{s}_A \times \underline{s}_B) \\
 \underline{u}_Z &= \underline{u}_X \times \underline{u}_Y
 \end{aligned}
 \tag{6.3.21}$$

The primed coordinate system axes expressed in terms of the unprimed coordinate system axes are:

$$\begin{aligned}
 \underline{x} &= u'_{X1} \underline{u}_X + u'_{Y1} \underline{u}_Y + u'_{Z1} \underline{u}_Z \\
 \underline{y} &= u'_{X2} \underline{u}_X + u'_{Y2} \underline{u}_Y + u'_{Z2} \underline{u}_Z \\
 \underline{z} &= u'_{X3} \underline{u}_X + u'_{Y3} \underline{u}_Y + u'_{Z3} \underline{u}_Z
 \end{aligned}
 \tag{6.3.22}$$

It should be noted that vectors can be transformed from the unprimed to the primed coordinate systems by using the following matrix constructed with the output (Eq. (6.3.22)) of AXISGEN:

$$\begin{bmatrix} \underline{x}^T \\ \underline{y}^T \\ \underline{z}^T \end{bmatrix}
 \tag{6.3.23}$$

#### 5.6.3.4 REFSMMAT Transformations

The matrix required to transform a vector from the Basic Reference Coordinate System to the IMU Stable Member Coordinate System is referred to as REFSMMAT. This matrix can be constructed as follows with the unit vectors  $\underline{u}_{XSM}$ ,  $\underline{u}_{YSM}$ , and  $\underline{u}_{ZSM}$  defining the orientations of the stable member axes with respect to the Basic Reference Coordinate System:

$$\text{REFSMMAT} = \begin{bmatrix} \underline{u}_{XSM}^T \\ \underline{u}_{YSM}^T \\ \underline{u}_{ZSM}^T \end{bmatrix} \quad (6.3.29)$$

##### 5.6.3.4.1 Present REFSMMAT From Star Sightings

The present IMU stable member orientation with respect to the reference coordinate system, and the associated REFSMMAT, can be determined by sighting on two navigation stars with the AOT. If  $\underline{s}'_A$  and  $\underline{s}'_B$  are the unit vectors defining the measured directions of the two stars in the present stable member coordinate system, and  $\underline{s}_A$  and  $\underline{s}_B$  are the unit vectors to the corresponding stars as known in the reference coordinate system, then these vectors can be used as the input to the routine AXISGEN (Section 5.6.3.2.4) to obtain the present IMU orientation and REFSMMAT (Eqs. (6.3.22) and (6.3.23)).

#### 5.6.3.4.2 Alignment for Thrusting Maneuvers ( Preferred Orientation )

During certain thrusting maneuvers the IMU will be aligned according to the following equations.

$$\begin{aligned}\underline{u}_{XSM} &= \underline{u}_{TD} \\ \underline{u}_{YSM} &= \text{UNIT} (\underline{u}_{XSM} \times \underline{r}) \\ \underline{u}_{ZSM} &= \underline{u}_{XSM} \times \underline{u}_{YSM}\end{aligned}\tag{6.3.30}$$

where  $\underline{u}_{TD}$  is the unit vector in the desired thrust direction at ignition and  $\underline{r}$  is the LM position vector.

The associated transformation matrix (REFSMMAT) is given by Eq. (6.3.29).

#### 5.6.3.4.3 Alignment to Local Vertical in Orbit ( Nominal Orientation )

The IMU stable member may be aligned to the local vertical at a specified time. For this type of orientation the stable member axes are found from the following.

$$\begin{aligned}\underline{u}_{XSM} &= \text{UNIT} (\underline{r}) \\ \underline{u}_{YSM} &= \text{UNIT} (\underline{v} \times \underline{r}) \\ \underline{u}_{ZSM} &= \text{UNIT} (\underline{u}_{XSM} \times \underline{u}_{YSM})\end{aligned}\tag{6.3.31}$$

where  $\underline{r}$  and  $\underline{v}$  are the position and velocity vectors of the LM at the specified time.

The REFSMMAT associated with this IMU orientation is found from Eq. (6.3.29).



#### 5. 6. 4 STAR SELECTION ROUTINE

The Star Selection Routine is used by the Fine Alignment Routine (R-51) to select the best pair of stars in or near the forward AOT field of view for fine alignment of the IMU. The logic diagram for this routine is shown in Fig. 6. 4-1.

Each pair from the computer catalog of 37 stars is tested to see if both stars are within a 100 degree viewing cone centered with respect to the optical axis of the Alignment Optical Telescope (AOT) when in the forward viewing position (Detent 2). Although this viewing cone is larger than the actual field of view (60 degrees) of the AOT, it is used to enhance the probability that a star pair will be found with sufficient angular separation so as to insure accurate IMU alignment.

Afterwards, the routine checks to see if the angle of separation between the stars is at least 50 degrees.

If a pair passes the above tests, a check is then made to see if either star is occulted by the Sun, Earth, or Moon. The sizes of the occultation cones about each of the three bodies are such as to not only account for true occultation but to also prevent the selection of stars too near the bodies because of visibility problems. The directions and the associated occultation cone sizes of the three bodies are actually computed by the subroutine LOSSEM (Section 5. 5. 13) which is called by the Fine Alignment Routine (R-51) just prior to calling the Star Selection Routine.

The pair of stars passing the above tests and having the largest angular separation is chosen by this routine. If the routine is unable to find a satisfactory pair of stars after testing all combinations, it is seen in Routine R-51 of Section 4 that an Alarm Code is displayed, whereupon the astronaut may either repeat the star selection process at a different spacecraft attitude or select his own stars later in the Fine Alignment Routine (R-51).

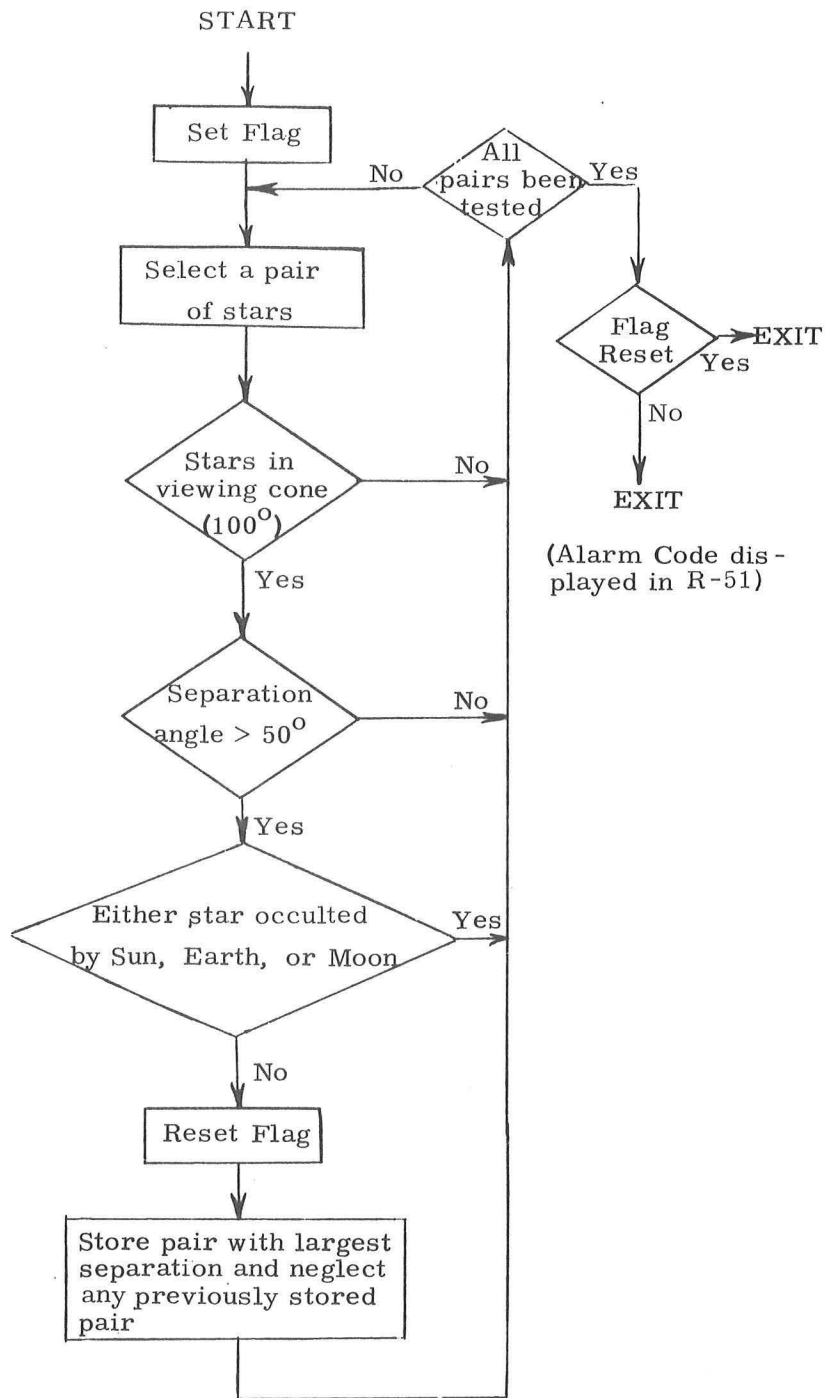


Figure 6. 4-1 Star Selection Routine

#### 5.6.5 GROUND TRACK ROUTINE

This routine is used by the astronaut in near-earth or near-moon orbit to obtain CSM or LM trajectory information. The astronaut specifies a time (GET) and a vehicle (CSM or LM). The routine uses the Coasting Integration Routine (Section 5.2.2) to extrapolate the desired vehicle's state vector to the specified time. The resulting estimated position vector is converted to latitude, longitude, altitude coordinates by means of the Latitude-Longitude Subroutine (Section 5.5.3) and these data are displayed. Altitude is defined with respect to the landing site radius for lunar orbit, and the launch pad radius for earth orbit. The astronaut can request the state vector extrapolation to continue in ten minute steps, or to another specified time, and obtain additional displays of the coordinates of points in the spacecraft's orbit.

### 5. 6. 7 ADDITIONAL RENDEZVOUS DISPLAYS

During the rendezvous sequence the following routines may be called by the astronaut for the purpose of computing and displaying special quantities related to the rendezvous geometry.

#### 5. 6. 7. 1 Range, Range Rate, Theta Display

Routine R-31 may be called in order to display the computed range and range rate between the two vehicles and an angle  $\theta$  shown in Fig. 6. 7-1.

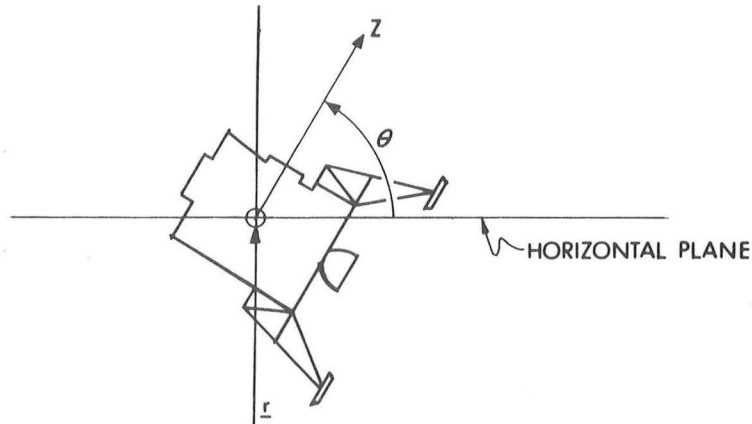


Figure 6. 7-1 Definition of Theta

The angle  $\theta$  represents the angle between the LM Z-body axis and the local horizontal plane of the LM. It is defined in a manner completely analogous to the definition of E in Section 5. 4. 4. 2 and, therefore, has limits of 0 to 360 degrees.

The equations used to compute the display parameters for R-31 are given below.

$$\underline{R} = \underline{r}_C - \underline{r}_L$$

$$\underline{u}_R = \text{UNIT}(\underline{R})$$

$$\text{RANGE} = |\underline{R}| \quad (6. 7. 1)$$

$$\text{RANGE RATE} = (\underline{v}_C - \underline{v}_L) \cdot \underline{u}_R \quad (6. 7. 2)$$

To compute  $\theta$  the following vector is defined

$$\underline{u}_Z = [\text{REFSMMAT}]^T [\text{NBSM}] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

where NBSM and REFSMMAT are defined in Section 5.6.3 and  $\underline{u}_Z$  is a unit vector along the Z-body axis expressed in basic reference coordinates. The angle  $\theta$  is then found as follows:

$$\begin{aligned} \underline{u} &= \text{UNIT} (\underline{r}_L \times \underline{v}_L) \\ \underline{u}_P &= \text{UNIT} \left[ \underline{u}_Z - \left( \frac{\underline{u}_Z \cdot \underline{r}_L}{r_L^2} \right) \underline{r}_L \right] \\ \theta &= \cos^{-1} [\underline{u}_Z \cdot \underline{u}_P \text{SGN} (\underline{u}_P \cdot \underline{u} \times \underline{r}_L)] \quad (6.7.3) \\ \text{If } \underline{u}_Z \cdot \underline{r}_L < 0 &; \theta = 2\pi - \theta \end{aligned}$$

The three displays of R-31 are automatically updated until R-31 is terminated by the astronaut. The logic flow required to accomplish this update is shown in Fig. 6.7-2.

#### 5.6.7.2 Final Attitude Display

Routine R-63 may be used to compute and display the FDAI angles required to point either the LM Z-axis or LM X-axis at the CSM. The choice of axis is made by the astronaut at the beginning of the routine as described in Section 4.

After initiation of this routine the state vectors of both vehicles are extrapolated to the present time plus one minute using the Coasting Integration Routine (Section 5.2.2). Based on these new state vectors the required gimbal angles are computed. These angles are converted to FDAI angles using the transformation described in Section 5.6.12 and the result is displayed.

There is no automatic display update; however, R-63 may easily be recycled manually.

#### 5.6.7.3 Out-of-Plane Rendezvous Display

Routine R-36 may be used during any phase of the rendezvous sequence to provide information about the out-of-plane geometry. Three quantities ( $Y$ ,  $\dot{Y}$ , and  $\psi$ ) are computed for a given time which is determined by the astronaut. The first two,  $Y$  and  $\dot{Y}$ , represent the out-of-plane position and velocity in some sense. The third display,  $\psi$ , is the angle between the

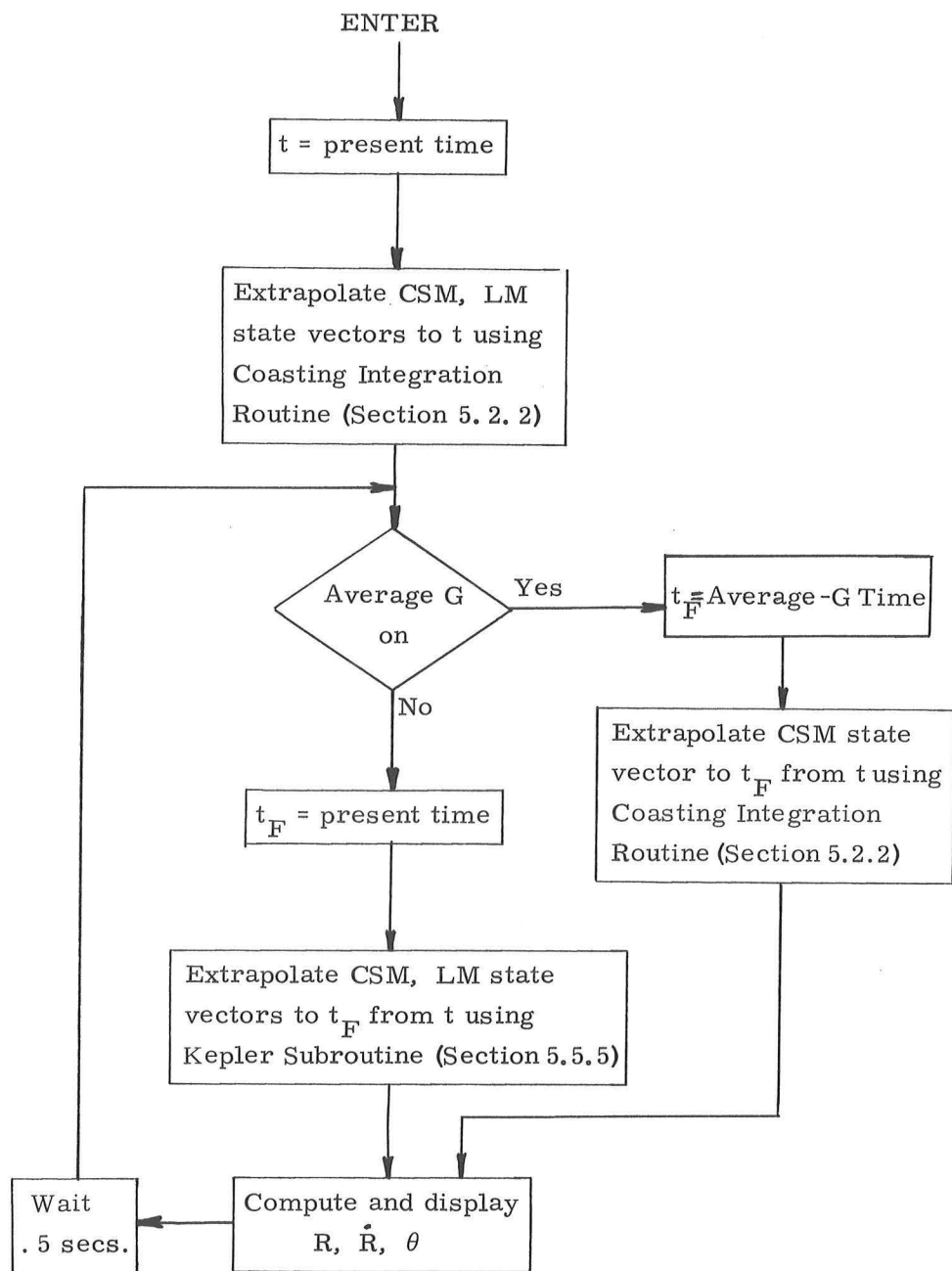


Figure 6.7-2 Range, Range Rate,  $\theta$

line of sight and the forward direction, measured in the local horizontal plane. It is equivalent to the yaw angle on the FDAI ball if the ball were aligned to an inplane, local horizontal attitude and the vehicle were rotated such that the Z-axis were pointed along the line of sight.

The exact definition of  $Y$ ,  $\dot{Y}$  and  $\psi$  is demonstrated by the following set of equations.

$$\underline{u} = \text{UNIT} (\underline{v}_C \times \underline{r}_C) \quad (6.7.4)$$

where  $\underline{r}_C$  and  $\underline{v}_C$  are the position and velocity vectors of the CSM, respectively.

$$\begin{aligned} Y &= \underline{r}_L \cdot \underline{u} \\ \dot{Y} &= \underline{v}_L \cdot \underline{u} \end{aligned} \quad (6.7.5)$$

where  $\underline{r}_L$  and  $\underline{v}_L$  are the position and velocity vectors of the LM, respectively.

$$\begin{aligned} \underline{u}_F &= \text{UNIT} \left[ (\underline{r}_L \times \underline{v}_L) \times \underline{r}_L \right] \\ \underline{u}_R &= \text{UNIT} (\underline{r}_L) \\ \underline{LOS} &= \underline{r}_C - \underline{r}_L \end{aligned} \quad (6.7.6)$$

$$\begin{aligned} \underline{u}_{LOS} &= \text{UNIT} \left[ \underline{LOS} - (\underline{LOS} \cdot \underline{u}_R) \underline{u}_R \right] \\ \psi &= \cos^{-1} (\underline{u}_{LOS} \cdot \underline{u}_F) \\ \underline{N} &= \underline{u}_{LOS} \times \underline{u}_F \end{aligned} \quad (6.7.7)$$

$$\text{If } \underline{N} \cdot \underline{r}_L < 0; \psi = 2\pi - \psi \quad (6.7.8)$$

#### 5. 6. 8 AGS INITIALIZATION ROUTINE

The Abort Guidance System (AGS) initialization involves a separate routine R-47 in the LGC that prepares a special telemetry downlink list with the required AGS initialization parameters. The AGS-LGC initialization interface is through the telemetry subsystem. At a desired GET, the astronaut may simultaneously make an "ENTER" in both LGC and AEA (Abort Electronic Assembly). The LGC stores the GET at this ENTER and subtracts this time from all state vector reference times used in the AGS initialization. There is also the option of loading an AGS initialization time obtained from an external source.

The AGS Initialization Routine next integrates the CSM and LM state vectors to the current GET. This state vector integration (Sec. 5.2.2) is based on the determination of the earth or lunar orbit condition by examining the CSM primary body indicator  $P_C$  of Section 5.2.2.6. In the earth orbital condition, the CSM and LM state vectors are divided by a factor of four and these modified state vectors are put on the special down-link telemetry list together with the difference between the state vector time and the AGS reference GET. In the lunar orbit condition the CSM and LM state vectors are modified by only the subtraction of the AGS reference GET from the state vector time before being put on the down-link list. This special telemetry down-link list is then sent 10 consecutive times before the normal down-link format is resumed.

Subsequent AGS initializations do not normally require a new AGS time referencing operation involving the simultaneous LGC and AEA ENTER inputs. During AGS initializations following the first, the AEA clock initialization may be omitted. In that case, the special initialization down-link is prepared by modifying the state vector times by subtracting the initial GET reference time determined in the first initialization.

The final operation is to zero the IMU CDU's for the AGS alignment procedure as described in Section 4 provided CDU zeroing is permissible in the LGC at that time.



### 5.6.9 LGC INITIALIZATION

The LGC initialization procedure prior to LM separation is a manual operation which does not involve a numbered LGC program. After the LGC is activated the first requirement is to synchronize the LGC clock with that of the CMC. This is a count-down and mark procedure described in R-33, CMC / LGC Clock Synchronization Routine of Section 4, to obtain an average clock difference which is then used to increment the LGC clock. The CMC and LGC clock synchronization can also be checked by the Mission Ground Control Center using telemetry down-link data, which can provide a more precise difference to increment the LGC clock.

Next, the following parameters are voice-linked from the CSM or uplinked from the earth to the LM to be entered into the LGC:

- 1)  $\underline{r}_C$  : CSM position vector
- 2)  $\underline{v}_C$  : CSM velocity vector
- 3)  $t_C$  : CSM state vector time
- 4)  $\underline{r}_{LS}$  : lunar landing site vector in  
moon-fixed coordinates
- 5)  $t_0$  : time difference between zero GET and  
July 1.0, 1968 universal time.
- 6)  $P_C$  : planet identifier

All of the above parameters are in octal, and all are double precision except item 5,  $t_0$ , which is triple precision and item 6,  $P_C$ , which is one bit.



The estimated CSM and LM state vectors (as defined in Eq. (2.2,28) of Section 5.2.2.6), the estimated landing site state vector, and the reference time  $t_0$  are then initialized as follows:

$$\begin{aligned}
 \underline{r}_{C0} &= \underline{r}_C \\
 \underline{v}_{C0} &= \underline{v}_C \\
 \underline{r}_{Ccon} &= \underline{r}_C \\
 \underline{v}_{Ccon} &= \underline{v}_C \\
 * \underline{\delta}_C &= \underline{0} \\
 * \underline{\nu}_C &= \underline{0} \\
 t_C &= t_C \\
 * \tau_C &= 0 \\
 * x_C &= 0 \\
 P_C &= \begin{cases} 0 & \text{for earth orbit} \\ 1 & \text{for lunar orbit} \end{cases} \\
 \\ 
 \underline{r}_{L0} &= \underline{r}_C \\
 \underline{v}_{L0} &= \underline{v}_C \\
 \underline{r}_{Lcon} &= \underline{r}_C \\
 \underline{v}_{Lcon} &= \underline{v}_C \\
 * \underline{\delta}_L &= \underline{0}
 \end{aligned}$$

$$\begin{aligned}
 * \nu_L &= 0 \\
 t_L &= t_C \\
 * \tau_L &= 0 \\
 * x_L &= 0 \\
 P_L &= \begin{cases} 0 & \text{for earth orbit} \\ 1 & \text{for lunar orbit} \end{cases} \\
 * r_{-LS} &= r_{-LS} \\
 t_0 &= t_0
 \end{aligned}$$

where the items on the left-hand side of the above equations are LGC variables, and the items on the right-hand side are the up-linked or voice-linked parameters. The subscripts C and L refer to CSM and LM, respectively. The variables marked with an asterisk (\*) are initialized during the prelaunch erasable load and do not have to be reset unless the values have been changed during the mission.

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5. 6. 11 LGC IDLING PROGRAM

This program is used to maintain the LGC in a state of readiness for entry into any other program. While the idling program is in operation, the Coasting Integration Routine (Section 5. 2. 2) is used to advance the estimated CSM state vector (and the estimated LM state vector when the LM is not on the surface of the moon) to approximately current time. This procedure has the lowest priority of all programs, and is performed only when no other program is active. This periodic state vector extrapolation is not necessary from a theoretical point of view, but does have two practical purposes. First, it is advisable to maintain current (or at least nearly current) state vector estimates in case an emergency situation arises. Second, a significant amount of computation time is transferred from a period of high computer activity (navigation measurement processing, targeting, etc.) to a period of low activity.

This periodic state vector extrapolation is restricted to near-earth or near-moon orbits since the capability for cislunar-midcourse integration does not exist in the LGC.

In order to use the Coasting Integration Routine in an efficient manner, the maximum value for the integration time step,  $\Delta t_{\max}$ , is computed as described in Section 5. 2. 2. 5. Let  $t_C$  be the time associated with the estimated CSM state vector and  $t_1$  be the current time. The estimated CSM state vector is extrapolated ahead when

$$t_1 > t_C + 4 \Delta t_{\max} \quad (6. 11. 1)$$

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The integration is terminated when  $\Delta t_{\max}$  is more than the integration time-to-go. In this manner no extra and smaller-than-maximum integration time steps are performed, and the periodic integration is accomplished most efficiently.

The estimated LM state vector (if applicable) is then extrapolated to the CSM state vector time in the same manner.

The error transition matrix  $W$  (see Section 5.2.2.4) is extrapolated with the estimated LM or CSM state vector if  $RENDWFLG$  indicates that the  $W$  matrix is valid.  $RENDWFLG$  is defined in Section 5.2.4.2.2. The selection of which state vector to extrapolate the  $W$  matrix with depends upon whether the LM is in flight or on the moon.

The logic for the periodic state vector extrapolation is illustrated in Fig. 6.11-1. The variables  $D$  and  $V$  are indicators which control the Coasting Integration Routine. The quantities  $\underline{x}_L$  and  $\underline{x}_C$  are the estimated LM and CSM state vectors, respectively, and  $\underline{x}$  is a temporary state vector used for integration. Refer to Section 5.2.2.6 for precise definitions of these items. The switch  $SURFFLAG$  indicates whether or not the LM is on the surface of the moon. This flag is set to one (zero) by means of a special  $DSKY$  verb by the astronaut when the LM has landed on (lifted off from) the lunar surface.

As shown in the figure, time synchronization of the two state vectors is achieved and maintained by this program. The purpose of the state vector synchronization is to guarantee correct  $W$  matrix extrapolation during rendezvous navigation.

In order to permit correction of wrong erasable memory parameters which have caused or could cause an invalid and excessively lengthy integration process to begin, there is an emergency special DSKY verb to terminate or inhibit the Coasting Integration Routine. This special verb causes the following to occur:

1. If the Coasting Integration Routine is in operation, it is terminated at the end of the current time step.
2. The current program is terminated.
3. The LGC Idling Program (P-00) is activated.
4. The first P-00 state vector test is bypassed so that no state vector integration test occurs for ten minutes.

Note that this operation does not maintain state vector synchronization and can, therefore, cause incorrect W matrix extrapolation in rendezvous navigation.

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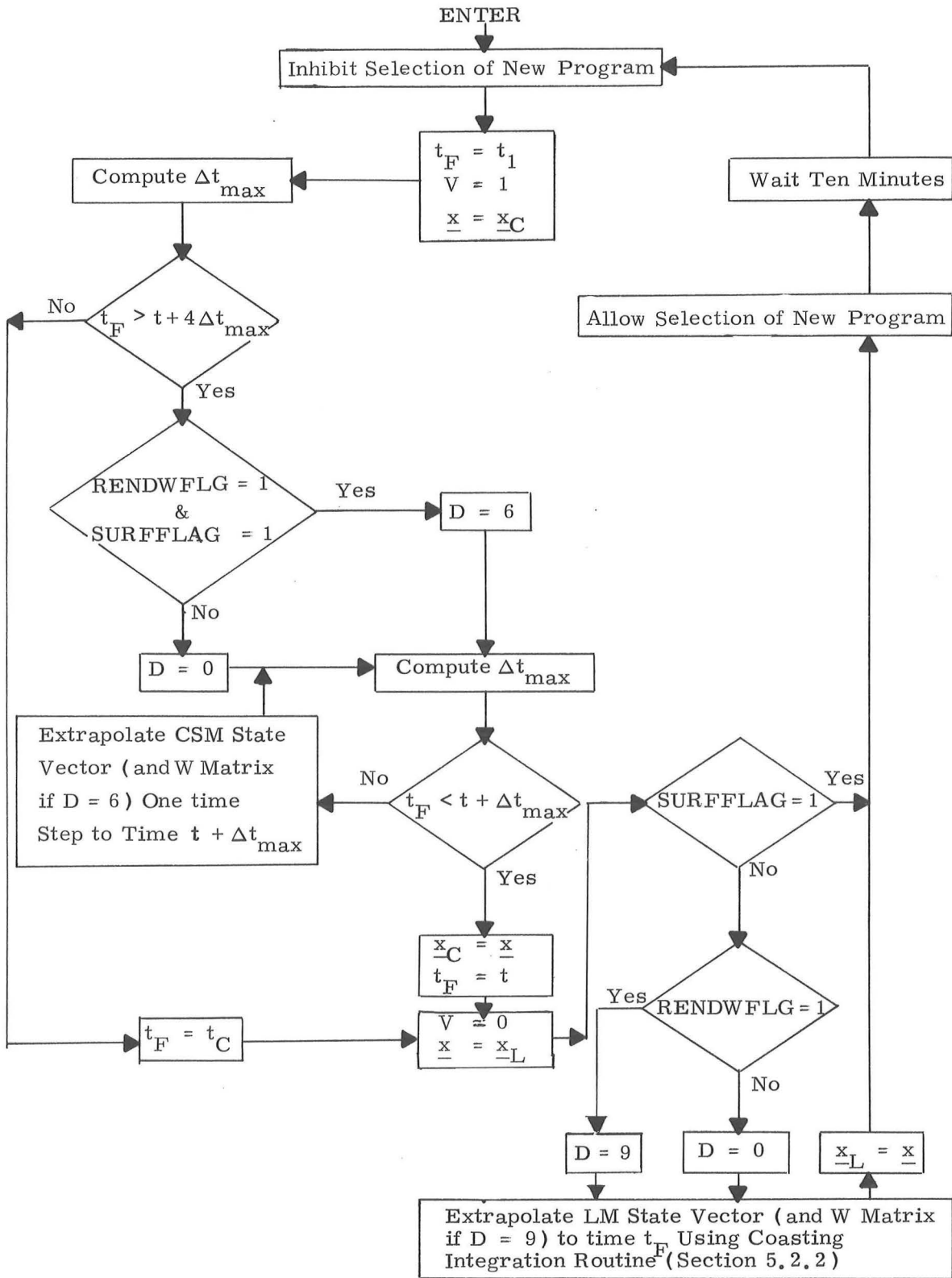


Figure 6.11-1 LGC Idling Program State Vector Extrapolation Logic Diagram

### 5.6.12 FDAI-IMU TRANSFORMATIONS

The following transformations are used to convert IMU gimbal angles to angular readings on the LM FDAI Ball.

#### YAW

$$Y = \sin^{-1} \left[ -\cos (MGA) \sin (OGA) \right]$$

where the display is such that

$$90^{\circ} \geq Y \geq 0^{\circ}$$

or

$$360^{\circ} > Y \geq 270^{\circ}$$

#### PITCH

(If  $|\sin Y| \neq 1.0$ )

$$P = \sin^{-1} \left[ \frac{\sin (IGA) \cos (OGA) + \cos (IGA) \sin (MGA) \sin (OGA)}{\cos Y} \right]$$
$$= \cos^{-1} \left[ \frac{\cos (IGA) \cos (OGA) - \sin (IGA) \sin (MGA) \sin (OGA)}{\cos Y} \right]$$



ROLL

(If  $|\sin Y| \neq 1.0$ )

$$R = \sin^{-1} \left[ \frac{\sin (MGA)}{\cos Y} \right] = \cos^{-1} \left[ \frac{\cos (MGA) \cos (OGA)}{\cos Y} \right]$$

where the displays for both P and R are always positive angles less than  $360^\circ$ .

For  $Y = 90^\circ$  or  $270^\circ$  the preceding equations are indeterminate and the pitch and roll readings become meaningless since they are not uniquely defined in terms of IMU gimbal angles because the FDAI outer and inner gimbal angles are likewise not unique. However, the following equations give the relationships between the IMU gimbal angles and the sum or differences of P and R.

If

1.  $Y = 90^\circ$ , then either

a.  $MGA = 0^\circ$  and  $OGA = -\frac{\pi}{2}$  whence  
 $P + R = IGA$

or

b.  $MGA = \pi$  and  $OGA = +\frac{\pi}{2}$  whence  $P + R = IGA + 180^\circ$ .

2.  $Y = 270^\circ$ , then either

a.  $MGA = 0^\circ$  and  $OGA = +\frac{\pi}{2}$  whence  $P - R = IGA$

or

b.  $MGA = \pi$  and  $OGA = -\frac{\pi}{2}$  whence  $P - R = IGA + 180^\circ$

where

IGA = inner IMU gimbal angle  
MGA = middle IMU gimbal angle  
OGA = outer IMU gimbal angle

The IMU gimbal angles are defined to be zero when the axes of rotation of the gimbals are mutually orthogonal. Each gimbal angle is defined as that angle through which the designated gimbal must be rotated, in the conventional right hand sense, with respect to its outer neighbor to make the X, Y, Z Coordinate Systems of both gimbals coincident.

5.6.13 IMU COMPENSATION

The IMU Compensation is designed to compensate for PIPA bias and scale factor error and at the same time accumulate gyro torquing commands necessary to compensate for the associated bias and acceleration caused gyro drifts. The correction to the PIPA's is

$$\text{PIPA}_C = (1 + \text{SFE}_I) \text{PIPA}_I - \text{BIAS}_I \Delta t$$

where

$\text{PIPA}_C$  is the compensated data for the  $I^{\text{th}}$  PIPA denoted  $\text{PIPAX}_C$ ,  $\text{PIPAY}_C$ ,  $\text{PIPAZ}_C$

$$\text{SFE}_I = \frac{\text{SF} - \text{SF}_{\text{nom}}}{\text{SF}_{\text{nom}}} \quad (\text{erasable load})^*$$

$$\text{SF} = \text{Scale-factor} \frac{\text{CM/Sec}}{\text{Pulse}}$$

$\text{BIAS}_I$  is the bias for the  $I^{\text{th}}$  PIPA (an erasable load)

The compensated data is then used to compute the IRIG torquing necessary to cancel the NBD, ADIA, and ADSRA gyro coefficients. The computations are

$$\text{XIRIG} = -\text{ADIAX} \text{PIPAX}_C + \text{ADSRAX} \text{PIPAY}_C - \text{NBDX} \Delta t$$

$$\text{YIRIG} = -\text{ADIAY} \text{PIPAY}_C + \text{ADSRAY} \text{PIPAZ}_C - \text{NBDY} \Delta t$$

$$\text{ZIRIG} = -\text{ADIAZ} \text{PIPAZ}_C - \text{ADSRAZ} \text{PIPAY}_C + \text{NBDZ} \Delta t$$

\*The term "erasable load" refers to data entered in LGC erasable memory just prior to launch.

where

XIRIG, YIRIG, ZIRIG are gyro drift compensations  
NBDX, NBDY, NBDZ are gyro bias drifts (an erasable load)  
ADSRAX, ADSRAY, ADSRAZ are gyro drifts due to ac-  
celeration along spin reference axis (an erasable load)  
ADIAX, ADIAY, ADIAZ are gyro drifts due to acceleration  
along the input axis (an erasable load)

When the magnitude of any IRIG command exceeds two pulses, the commands are sent to the gyros.

During free-fall only the NBDX, NBDY, NBDZ are the relevant coefficients and the routine is so ordered that only these terms are calculated for the gyro compensation.

5.6.14 RR / LR SELF TEST ROUTINE

The purpose of the RR / LR Self Test Routine (R-04 in Section 4) is to provide suitable DSKY displays and LGC down-link information to support the self tests of the Rendezvous Radar (RR) and the Landing Radar (LR). In doing this the routine also provides a check on the data transmission interface between the LGC and the two radar systems.

The self test of either radar system is accomplished by the astronaut through use of a radar test switch at the LM console. When this switch is in the RR or LR position, artificial target signals are generated within the associated radar system. These signals when acquired by the tracking networks of the radar correspond to certain known values in the measurement parameters (range, etc.).

The self tests of the RR and LR can be conducted with or without the assistance of the LGC. Since these tests can interfere with the normal RR data read and update functions of programs P-20 and P-22, it is essential that the astronaut not have these programs operating when the radar test switch is in one of its test positions (RR or LR).

If the astronaut wishes to have the radar self test parameters displayed on the DSKY and placed on the LGC down-link, he calls the RR / LR Self Test Routine. At the beginning of the routine he indicates which radar is being self tested. If he selects the LR, the routine will start reading continuously the four LR data parameters ( $v_X$ ,  $v_Y$ ,  $v_Z$  and range) once per second. Each parameter is sampled by a single 80 millisecond gate. These parameters are placed on the LGC down-link and are displayed to him in two separate DSKY displays.

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One display contains the range and the present position of the LR antenna. The other display contains the three LR velocity components ( $v_X$ ,  $v_Y$ , and  $v_Z$ ). The four data parameters obtained from the LR are displayed and put on the down-link regardless of the status of the Range and Velocity Data Good discretes. However, whenever a given parameter is obtained from the LR, the associated data good discrete is checked and the Tracker Fail Alarm is turned on if the discrete is missing.

If the RR test mode is selected, a check is made to see if the RR Auto Mode discrete is being received from the RR. If it is present, the RR Track Enable discrete is issued to allow the radar to lock-on the self test signals and subsequently generate the RR Data Good discrete during the data reading process. The purpose of checking the RR Auto Mode discrete is to insure that the mode control switch of the RR is in the LGC position during the data read operation. This insures that the phase of the RR resolver excitation is within tolerance for normal angle measurements by the RR CDU's. If the RR mode control switch is in either of its other two positions, this phase tolerance is not insured. If the RR Auto Mode is absent, a request is issued to the astronaut to place the RR mode control switch in the LGC position. Once this has been accomplished, the routine starts reading the RR CDU angles (shaft and trunnion) every second and the range and range rate on alternate seconds. These quantities are displayed on the DSKY in two separate displays, and thus normally will be down-linked. One display contains the RR CDU angles. It should be noted that the polarity of the displayed RR trunnion angle is opposite that defined in Fig. 6.15-1. The other display contains the range and range rate. The four RR data parameters are displayed and placed on the down-link regardless of the status of the RR Data Good discrete. However, whenever a range or range rate measurement is obtained from the RR, the RR Data Good discrete is checked and the Tracker Fail Alarm is turned on if the discrete is missing.

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It should be noted that the RR or LR data initially displayed by this routine may not be meaningful since the radar tracking loops may still be in the process of locking onto the internally generated target signals of the radar. In addition, the CDU data displayed at the beginning of the RR self test may be that associated with the process of zeroing the RR CDU's. Whenever the RR Auto Mode discrete is first detected by the RR Monitor Routine (R-25) of Section 5.2.4.3, the RR CDU's are zeroed.

It should also be noted that the LR used in this mission is different from that planned for a lunar mission in that the velocity data ( $v_X$ ,  $v_Y$ , and  $v_Z$ ) obtained from the present LR by the LGC is that from the individual velocity beams. In order to display the LR velocity parameters in feet per second, the following computations are made:

$$v_X = k_X (s_X - f_B \tau_{LR})$$

$$v_Y = k_Y (s_Y - f_B \tau_{LR})$$

$$v_Z = k_Z (s_Z - f_B \tau_{LR})$$

where  $s_X$ ,  $s_Y$ , and  $s_Z$  are the binary data words obtained from the LR,  $f_B$  is the bias frequency used in the LR,  $\tau_{LR}$  is the time interval used by the LR when counting the cycles of the doppler and bias frequencies so as to produce the data words ( $s_X$ ,  $s_Y$ , and  $s_Z$ ), and the  $k_X$ ,  $k_Y$ , and  $k_Z$  are the corresponding scale factors used to obtain the above velocities in feet per second. The time interval  $\tau_{LR}$  is 80,001 milliseconds.

The range data is sent to the LGC from the LR as a binary data word  $R_{LR}$ , which represents the count of a certain frequency in the LR during the time interval  $\tau_{LR}$ . Within the LGC, the range  $r_{LR}$  along the range beam is computed as follows:

$$r_{LR} = \begin{cases} k_{LR1} R_{LR} \\ k_{LR2} R_{LR} \end{cases}$$

where  $k_{LR1}$  and  $k_{LR2}$  are the bit weights respectively for the long and short range scales in order to obtain  $r_{LR}$  in feet. When the Range Low Scale discrete is being received from the LR by the LGC,  $k_{LR2}$  is used.

5. 6. 15 RR ANGLE TRANSFORMATIONS

5. 6. 15. 1 Determination of RR Antenna Direction in Navigation Base Coordinates

To obtain a unit vector  $\underline{u}_{RR}$  specifying the present direction of the boresight axis of the RR antenna in navigation base coordinates use is made of the following:

$$\underline{u}_{RR} = \begin{bmatrix} \sin S & \cos T \\ -\sin T \\ \cos S & \cos T \end{bmatrix} \quad (6.15, 1)$$

where S and T are the shaft and trunnion angles of the RR antenna as indicated by the RR CDU's. The positive sense of these angles is shown in Fig. 6.15-1.

5. 6. 15. 2 Equivalent RR Angles for a Desired Pointing Direction

Let  $\underline{u}_D$  be a unit vector defining a desired pointing direction in navigation base coordinates. This direction may be a desired direction for RR designation for which the corresponding RR shaft and trunnion angles must be computed in order to see if they are within the angular coverage modes of the antenna. The corresponding shaft and trunnion angles for this direction are determined as follows:



$$\underline{u}_D = (x, y, z)$$

$$T(1) = -\sin^{-1}(y)$$

$$T(2) = 180^\circ - T(1)$$

$$\underline{u}_P = (x', 0, z') = \text{UNIT}(x, 0, z)$$

$$\cos [S(1)] = z' \quad (6.15.2)$$

$$\sin [S(1)] = x'$$

$$S(1) = \text{ARCTRIG} \left\{ \sin [S(1)], \cos [S(1)] \right\}$$

$$S(2) = 180^\circ + S(1)$$

where  $T(1)$  and  $S(1)$  are the shaft and trunnion angles for Mode 1 of the RR antenna,  $T(2)$  and  $S(2)$  are the corresponding angles for Mode 2, and ARCTRIG implies computing the angle, choosing either  $\sin^{-1}$  or  $\cos^{-1}$  so as to yield maximum accuracy.

5.6.15.3 Determination of RR Gyro Commands during RR Target Designation

When the RR is being designated towards the CSM, the commands issued to the RR CDU's for digital to analog conversion before being sent to the RR gyros are computed approximately every 0.5 seconds as follows: Let  $C_S(1)$  and  $C_T(1)$  be the shaft (S) and trunnion (T) commands to the RR CDU's for the antenna in Mode 1, and  $C_S(2)$  and  $C_T(2)$  be the corresponding commands for the antenna in Mode 2. Initially, the unit vector  $\underline{u}_D$  defining the desired direction of designate in navigation base coordinates is obtained from stable member coordinates as follows:

$$\underline{u}_D = [\text{SMNB}] \text{UNIT}(\underline{r}'_{\text{LOS}}) \quad (6.15.3)$$

where  $[\text{SMNB}]$  is defined in Section 5.6.3.2.1 and  $\underline{r}'_{\text{LOS}}$  is the lag compensated range vector in Section 5.2.4.1. The commands are then computed as follows:

$$C_S(1) = K \left[ \underline{u}_D \cdot \begin{pmatrix} \cos S \\ 0 \\ -\sin S \end{pmatrix} \right] \quad (6.15.4)$$

$$C_S(2) = -C_S(1)$$

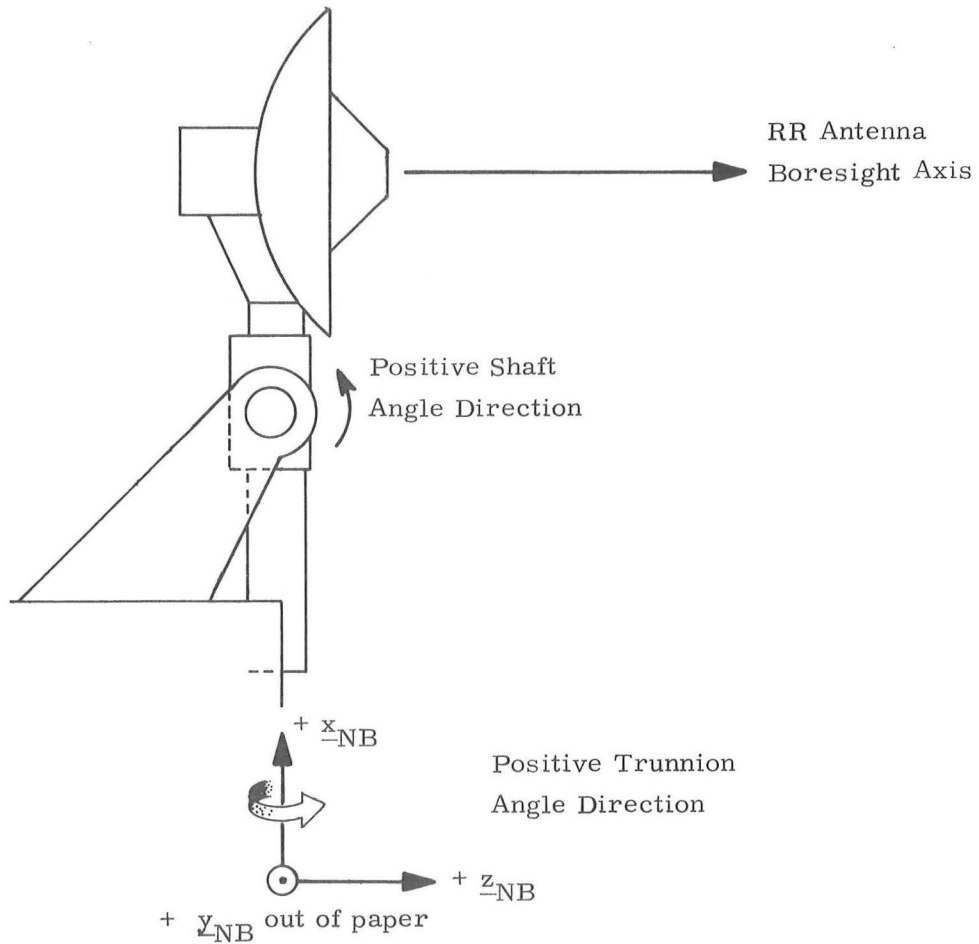
$$C_T(1) = C_T(2) = -K \left[ \underline{u}_D \cdot \begin{pmatrix} \sin T & \sin S \\ & \cos T \\ \sin T & \cos S \end{pmatrix} \right] \quad (6.15.6)$$

where S and T are the present shaft and trunnion angles and K is a scale factor to establish the proper number of bits in the RR CDU's. A limit check is made to insure that no more than 384 bits are sent to an RR CDU.

5.6-46 A

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1. +  $x_{NB}$ , +  $y_{NB}$ , and +  $z_{NB}$  are positive axes of Navigation Base Coordinate System.
2. Zero shaft and trunnion angles place the RR electrical boresight axis parallel to +  $z_{NB}$ .
3. The antenna shaft axis is the outer gimbal axis and is parallel to +  $y_{NB}$ .
4. The antenna trunnion axis is the inner gimbal axis and is orthogonal to the shaft axis and the RR electrical boresight axis.
5. Positive shaft rotation is clockwise motion of antenna as viewed along +  $y_{NB}$ .
6. Positive trunnion rotation is clockwise motion of antenna as viewed along +  $x_{NB}$  when shaft angle is zero.

Figure 6.15-1 Shaft and Trunnion Angles

5.6-47

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5.6.16 TARGET  $\Delta V$  ROUTINE

The purpose of the Target  $\Delta V$  Routine is to update the estimated CSM state vector in accordance with the maneuver  $\Delta V$  which is voice-linked to the LM from the CSM and then entered into the LGC as described in Sections 5.2.1 and 5.2.4.2. The logic for this routine is shown in Fig. 6.16-1. In the figure,  $\Delta V$  is the maneuver velocity change expressed in a CSM local vertical coordinate system, and  $t_{\Delta V}$  is the time of the maneuver.

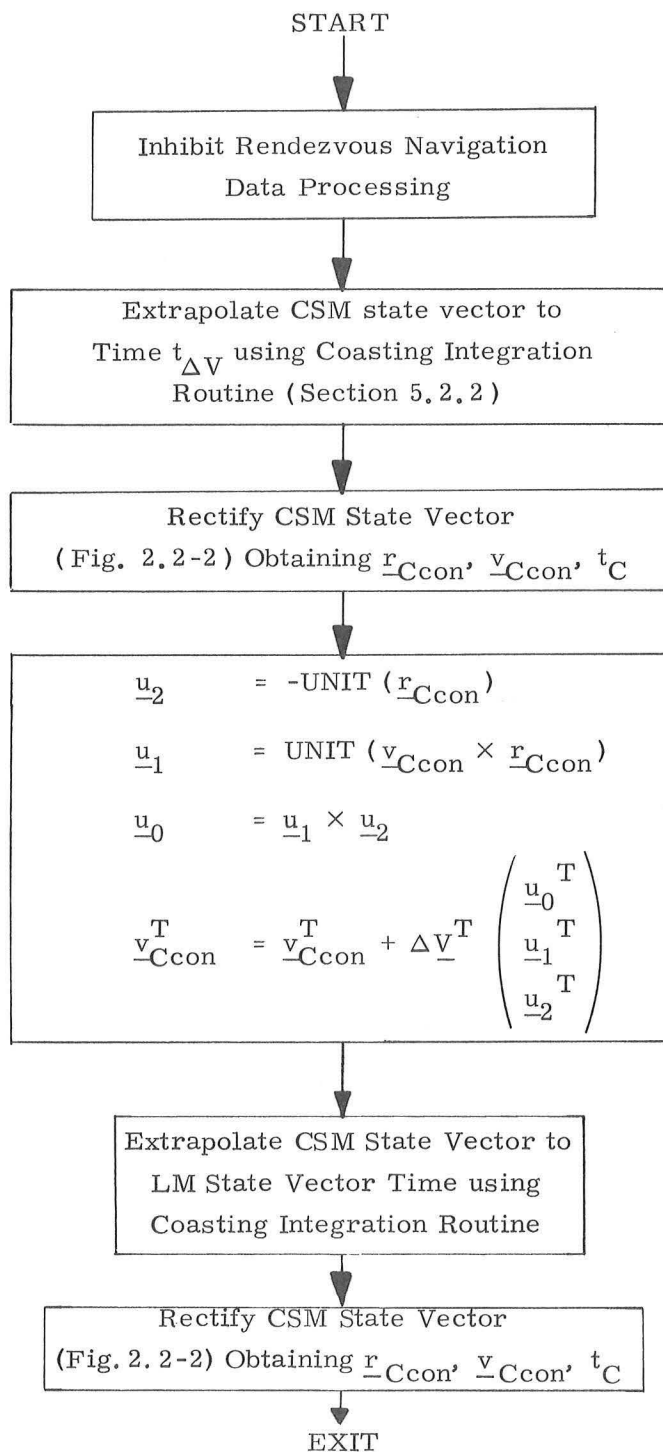


Figure 6.16-1 Target  $\Delta V$  Routine

#### 5.6.17 ORBIT PARAMETER DISPLAY ROUTINE

The Orbit Parameter Display Routine R-30 may be called by the astronaut via an extended verb in order to compute and display certain orbital parameters defined below. This display will be automatically update only when Average G is running. Also the option to select the vehicle for which these orbit parameters are to be displayed will only be available when Average G is not running.

In the normal case the apocenter altitude, pericenter altitude and the time from a reference altitude ( 300,000 ft for Earth orbit, 35,000 for Lunar orbit ) is displayed. If the chosen orbit does not intercept this altitude, the third display is 59B59. Under certain circumstances explained in Section 4 an additional display of time from pericenter may also be requested by the astronaut. The details of each option and acceptable astronaut responses are discussed in Section 4.

The computational logic and equations used in R-30 are contained in Figs. 6.17-1 through 6.17-3. The following is a list of important parameter definitions which apply to these figures.

$\left. \begin{array}{l} \underline{r} \\ \underline{v} \end{array} \right\} :$  State vector of the selected vehicle

$P_C :$  Primary body designator  $\left\{ \begin{array}{l} 0 \text{ Earth} \\ 1 \text{ Moon} \end{array} \right.$

$h_a :$  Apocenter altitude

$h_p$  : Pericenter altitude  
 $r_{LP}$  : Earth launch pad radius  
 $r_{LS}$  : Lunar landing site radius  
 $t_{ff}$  : Time from a reference altitude  
 $t_{PER}$  : Time from pericenter

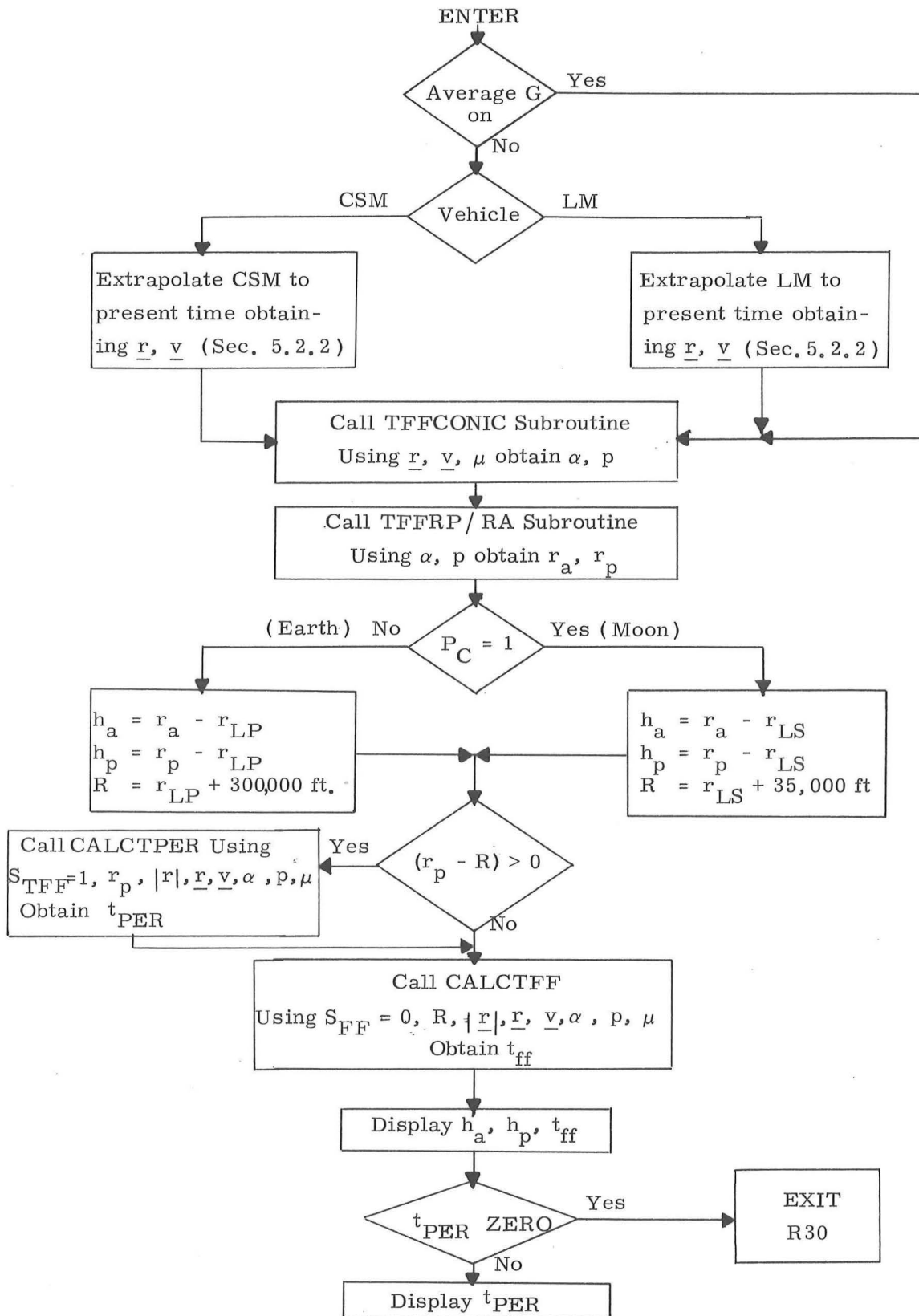


Figure 6.17-1 Orbit Parameter Display Routine

5.6-52



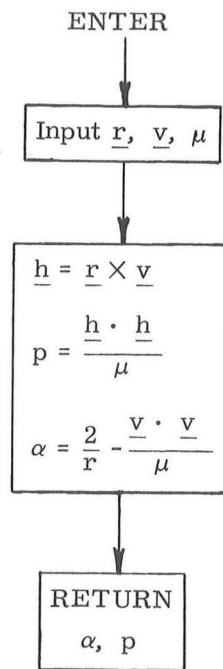


Figure 6.17-2a TFFCONIC Subroutine

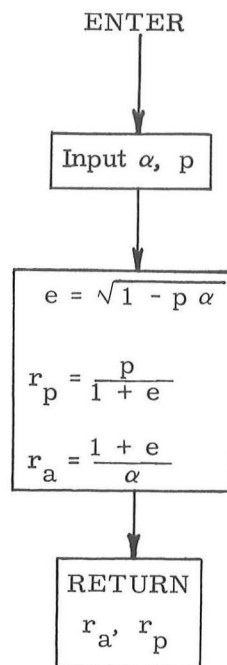
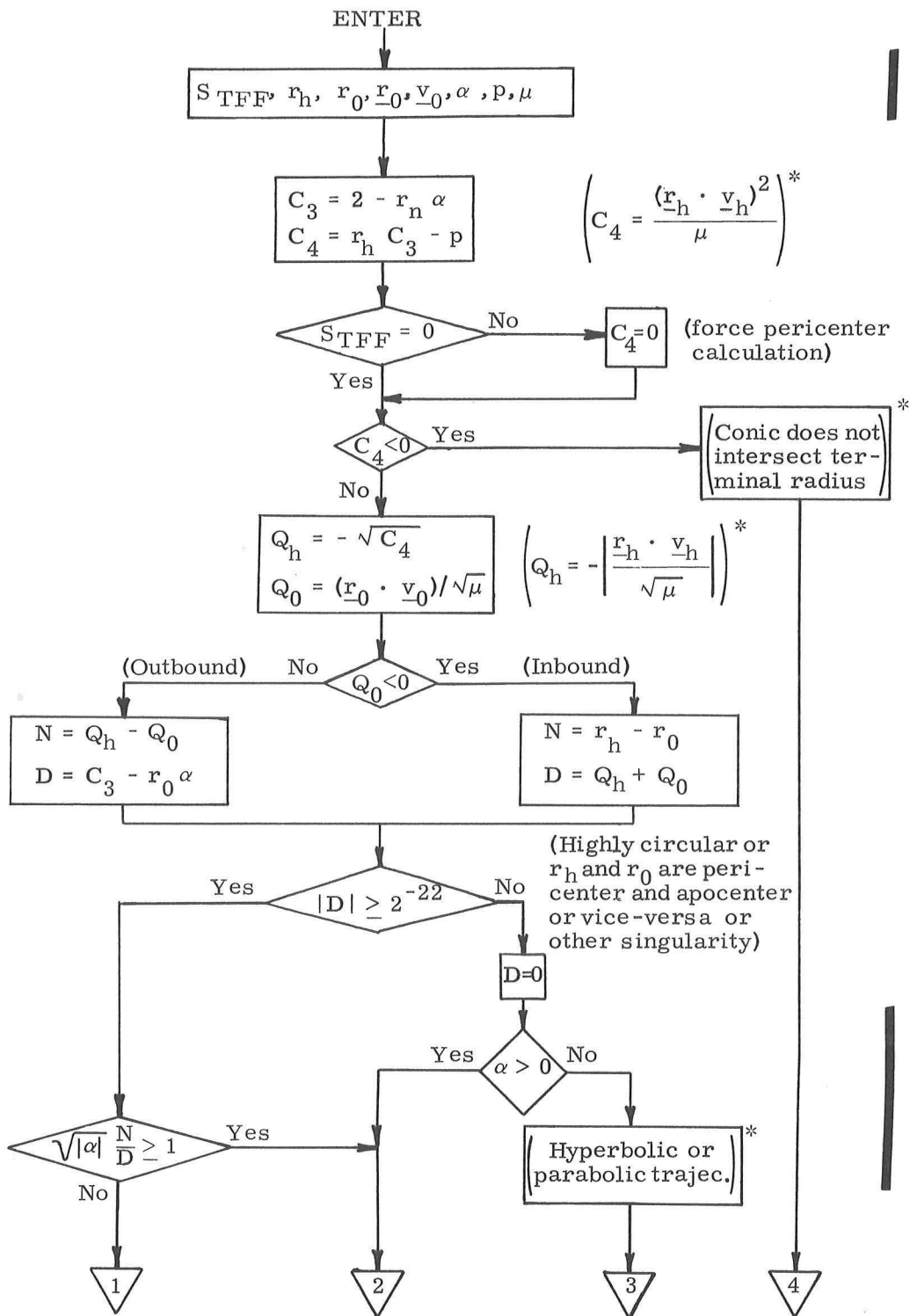


Figure 6.17-2b TFFRP/RA Subroutine



\* Supplementary Information

Fig. 6.17-3 CALCTFF/CALCTPER Subroutine

(page 1 of 2)

5. 6-54

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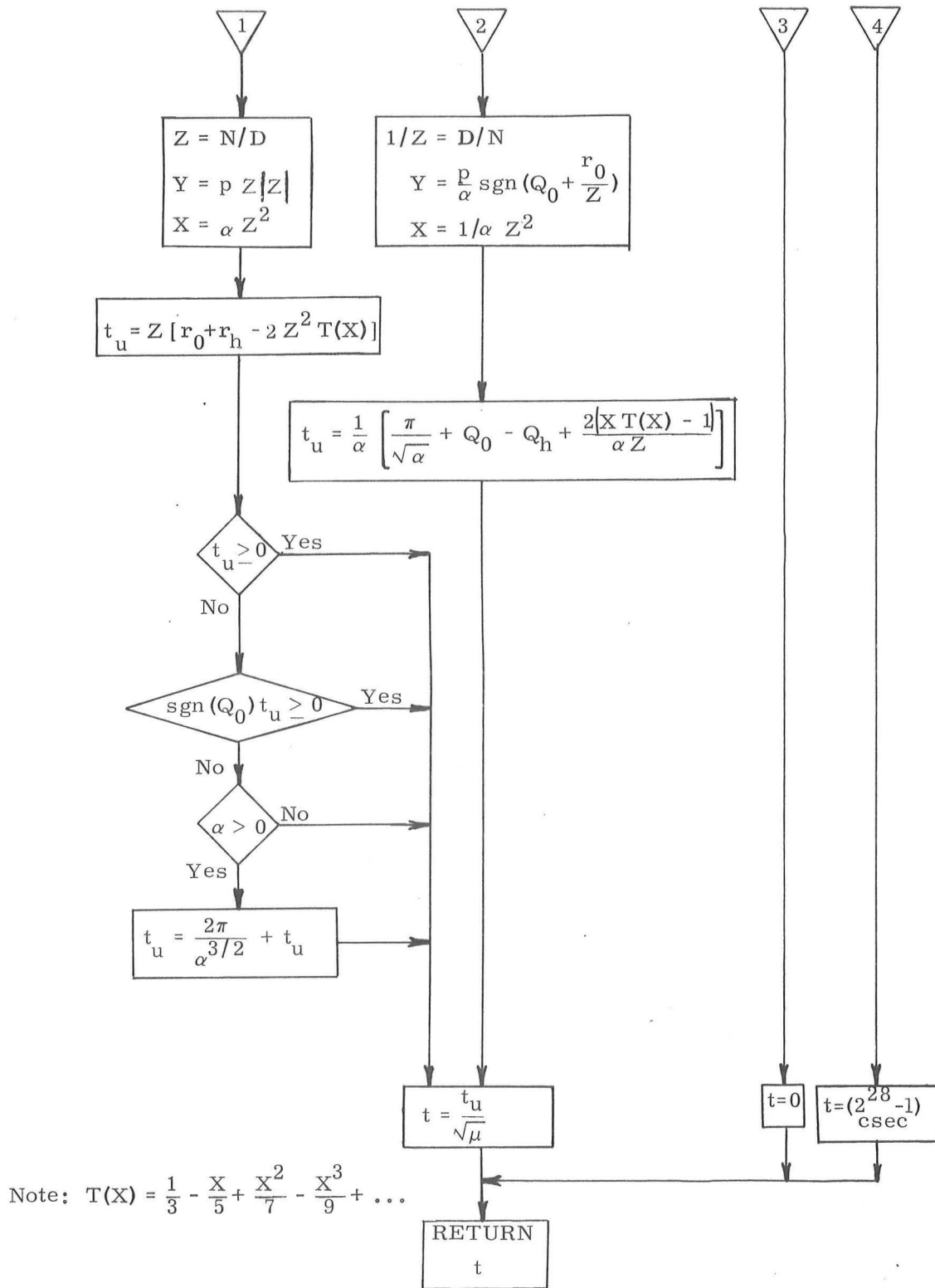


Figure 6.17-3 CALCTFF/CALCTPER Subroutine  
(page 2 of 2)

5.6-55

5. 6. 18 LR SPURIOUS TEST ROUTINE

The purpose of the LR Spurious Test Routine (R-77 of Section 4) is to read out the range and velocity data of the landing radar and place it on the LGC down-link during the landing radar spurious return flight tests. This is a special routine which is not intended to be used on lunar missions. The routine sequentially obtains individual data samples (80 milliseconds duration each) from each of the three velocity beams and the range beam once per second. This data is placed on the down-link regardless of the status of the LR Range Data Good and LR Velocity Data Good discrettes. This routine is instigated and terminated by the astronaut and is capable of performing the above data read-out prior to, during, and after a DPS maneuver.

Before performing the flight test it assumed that the astronaut will place the LR antenna in either of its two positions by use of the LR manual controls provided for this purpose.

It should be noted that the status of all LR discrettes (Range Data Good, Velocity Data Good, Range Low Scale, Antenna Position One, and Antenna Position Two) is already indicated at least once every two seconds on the down-link list.

5. 6. 19     RMS POSITION AND VELOCITY ERROR DISPLAY

In order to provide the capability for astronaut monitoring of the G&N system's estimate of state vector accuracy, there exists a special DSKY verb which causes the RMS position and velocity errors to be computed from the W matrix and to be displayed. Based upon the values in this display and the details of the particular mission, the astronaut will elect to stop the navigation that is in progress, to resume or continue with the current navigation procedure, or to reinitialize the W matrix and continue navigating. The capability of selecting the W matrix initialization parameters is also included in this process.

The logic for the RMS position error ( $\Delta r_{\text{RMS}}$ ) and RMS velocity error ( $\Delta v_{\text{RMS}}$ ) display is illustrated in Fig. 6.19-1. The vectors  $\underline{w}_i$  are partitions of the W matrix as defined in Eq. (2.2.19) of Section 5.2.2.4. The variables  $w_{rr}$  and  $w_{rv}$  are W matrix initialization parameters and RENDWFLG is the W matrix validity flag. See Section 5.2.4.2.2 for further definitions and usage of these terms.

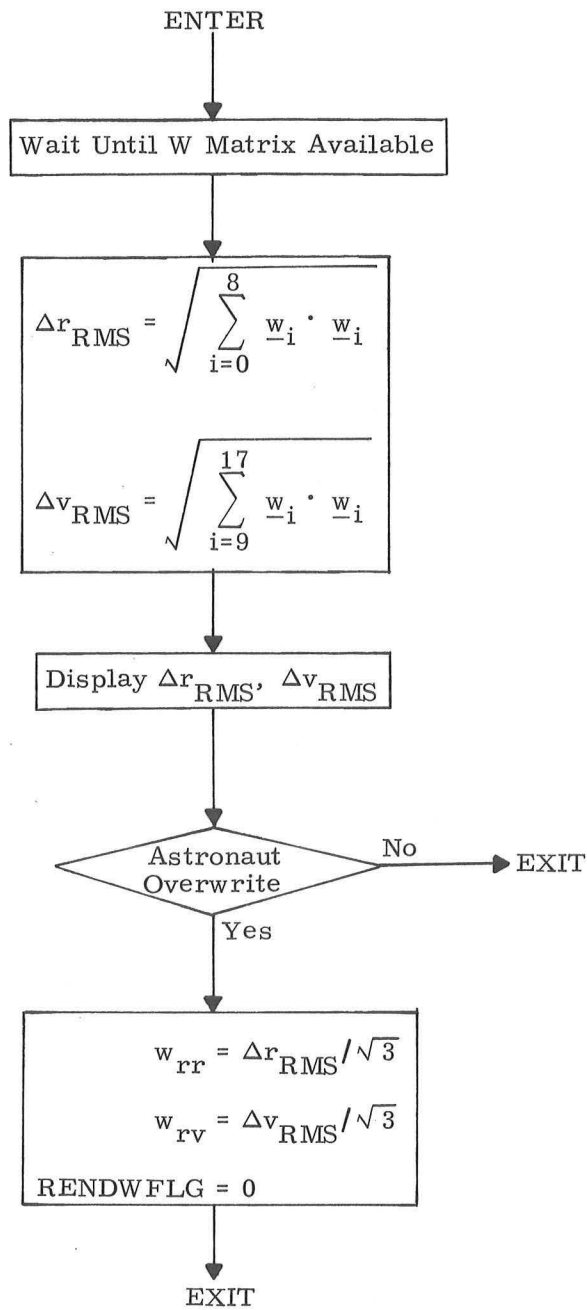


Figure 6.19-1 RMS Position and Velocity Error Display Logic Diagram

ERASABLE MEMORY PARAMETER LIST

This section presents a list of selected parameters required for various program operations. It should be noted that this is a very limited LGC erasable parameter list. The objective of this selected list is to identify those parameters that should be stored in erasable memory and are required primarily to initialize the operation of programs and routines. In most cases these parameters can not be originated within the LGC and must be stored prior to the mission. Some parameters then vary continually throughout the mission (e. g. vehicle state vectors), others are constant for any one mission phase, but may vary between different mission phases, and finally some may be constant for one mission, but be required to change for subsequent missions that use the same LGC program.

<u>Page</u>	<u>Parameter</u>
5.2-21	$\underline{r}_{C0}$ , $\underline{r}_{L0}$ $\underline{v}_{C0}$ , $\underline{v}_{L0}$ $\underline{r}_C$ con, $\underline{r}_L$ con $\underline{v}_C$ con, $\underline{v}_L$ con $\underline{\delta}_C$ , $\underline{\delta}_L$ $\underline{\nu}_C$ , $\underline{\nu}_L$ $t_C$ , $t_L$ $\tau_C$ , $\tau_L$ $x_C$ , $x_L$ $P_C$ , $P_L$
5.2-54	$\text{var}_R$ , $\text{var}_{R\text{min}}$ $\text{var}_V$ , $\text{var}_{V\text{min}}$

5.7-1

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<u>Page</u>	<u>Parameter</u>
5. 2-57	$\text{var}_\beta$
5. 2-58	$\text{var}_\theta$
5. 2-60	$\delta r_{\text{MAX}}$ $\delta v_{\text{MAX}}$
5. 2-62	$w_{rr}$ $w_{rv}$ $w_\beta$ $w_\theta$
5. 3-3	$\Delta v (\Delta t)$
5. 3-19	$\Delta V_{-LV}$ $t_{IG}$
5. 3-22	$S_E$



<u>Page</u>	<u>Parameter</u>
5. 4-5	$t_1$ N E $t_3$
5. 4-12	$t_2$
5. 4-21	$t$ $\omega t$
5. 4-24	$\delta\tau$
5. 4-23	$\Delta V_{TPI}(LV)$

Page                      Parameter

5. 4-24                       $\Delta \underline{V}(LV)$

5. 5-13                       $A_X$   
                                  $A_Y$   
                                  $A_{Z0}$   
                                  $t_0$

5. 5-16                       $\underline{l}_M$

5. 5-22                       $r_{LS}$

5. 6-8                         $AZ_1$  thru  $AZ_3$   
                                  $EL_1$  thru  $EL_3$   
                                  $AZ$   
                                  $EL$

5. 7-4

<u>Page</u>	<u>Parameter</u>
5. 6-31	$\underline{r}_{LS}$ $\underline{t}_{LS}$
5. 6-40	SFE <sub>1</sub> , SFE <sub>2</sub> , SFE <sub>3</sub> BIAS <sub>1</sub> , BIAS <sub>2</sub> , BIAS <sub>3</sub> ADIAX, ADIAY, ADIAZ ADSRAX, ADSRAY, ADSRAZ NBDX, NBDY, NBDZ

5. 7-5

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5. 8      FIXED MEMORY CONSTANTS

Section 5. 8. 1 contains a list and the numerical values of those fixed memory constants in Sections 5. 2 through 5. 6 which have not been specified previously. Those constants which are considered to be control type data are indicated by source references which are listed in Section 5. 8. 2. Explanatory comments are noted in Section 5. 8. 1 where applicable and listed in Section 5. 8. 3. It should be noted that only one page number is given for a constant in the list of Section 5. 8. 1 even though the constant may appear in other parts of Section 5. In these cases the same value is used for the constant as reported in Section 5. 8. 1.

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5. 8. 1      Fixed Constants

<u>Page</u>	<u>Constant</u>	<u>Units</u>	<u>Value</u>	<u>Reference</u> <u>(Sec. 5. 8. 2)</u>	<u>Comments</u> <u>(Sec. 5. 8. 3)</u>
5. 2-9	$\mu_P \left\{ \begin{array}{l} \mu_E \\ \mu_M \end{array} \right.$	$m^3/s^2$	$0.3986032 \times 10^{15}$	2, 3, 4	
		$m^3/s^2$	$0.4902778 \times 10^{13}$	2, 3, 4	
5. 2-12	$J_{2E}$	-	$0.10823 \times 10^{-2}$	2, 3, 4	
	$J_{3E}$	-	$-0.23 \times 10^{-5}$	2, 3, 4	
	$J_{4E}$	-	$-0.18 \times 10^{-5}$	2, 3, 4	
	$r_E$	m	6, 378, 165	2, 3, 4	
5. 2-13	$J_{2M}$	-	$0.207108 \times 10^{-3}$	2, 3, 4	
	$J_{3M}$	-	0	2, 3, 4	
	$J_{4M}$	-	0	2, 3, 4	
	$J_{22M}$	-	$0.207160 \times 10^{-4}$	2, 3, 4	
	$r_M$	m	1, 738, 090	2, 3, 4	
5. 2-22	$\epsilon_t$	csec	3. 0	—	1

<u>Page</u>	<u>Constant</u>	<u>Units</u>	<u>Value</u>	<u>Reference (Sec.5.8.2)</u>	<u>Comments (Sec.5.8.3)</u>	
5.2-50	$f_{BRR}$	cps	212,500	14		
	$f_{BRR} \tau_{RR}$	-	17,000	5		
	$k_{RR}$	fps/bit	-0.6278	5,14,15		
	$k_{R1}$	ft/bit	75.04	5,14,15		
	$k_{R2}$	ft/bit	9.38	5,14,15		
5.2-57	$var_{IMU}$	(mr) <sup>2</sup>	1.0	6		
5.3-22	$F_{DPS}$	pounds	9712.5	5,17		
	$F_{APS}$	pounds	3500	5,17		
	$F_{RCS}$	pounds	400 or 200	5,17		
5.3-26	$\Delta V_K$	LM only	cm/sec	36	19	8
		CSM	cm/sec	12	19	8
		docked	cm/sec	308	19	8
	$\Delta V_K(APS)$	cm/sec	308	19	8	
	$\dot{m}(APS)$	kg/sec	5.14	—		
5.3-29	$F_L$	pounds	200	17		
	$K_1$	kg-m/cs	124.544	18		
	$K_2$	kg-m/cs	31.136	18		
	$K_3$	kg-m/cs <sup>2</sup>	1.5568	18		
	$K_4$	pounds	1050	5,17		

5.8-2

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<u>Page</u>	<u>Constant</u>	<u>Units</u>	<u>Value</u>	<u>Reference</u> (Sec. 5.8.2)	<u>Comments</u> (Sec. 5.8.3)
5.3-32	$\Delta t_{\text{Tail-off}}(\text{DPS})$	csec	-38	20	
	$\Delta t_{\text{Tail-off}}(\text{APS})$	csec	-10	20	
	$V_e$ (DPS)	m/sec	3000	—	
	$V_e$ (APS)	m/sec	3000	5,17	
5.5-8	(Kepler) $\epsilon_t$	-	$2^{-22}$	12	
	(Lambert) $\epsilon_t$	-	$2^{-19}$	12	
	$\epsilon_x$ (moon)	$m^{1/2}$	$2^{-13}$	12	
	$\epsilon_x$ (earth)	$m^{1/2}$	$2^{-12}$	12	
	$\epsilon_c$	-	$2^{-23}$	12	
5.5-11	$k_1$	-	$2^{-17}$	12	
5.5-13	$\omega_E$	rad/sec	$7.29211515 \times 10^{-5}$	8	
5.5-16	$B_0$	rad	0.409164173	7,8	3
	$\Omega_{10}$	rad	-6.03249419	7,8	3
	$F_0$	rad	2.61379488	7,8	3
	$B$	rad/sec	$-7.19756666 \times 10^{-14}$	7,8	3
	$\dot{\Omega}_I$	rad/sec	$-1.07047016 \times 10^{-8}$	7,8	3
	$\dot{F}$	rad/sec	$2.67240019 \times 10^{-6}$	7,8	3
5.5-17	$C_I$	-	0.99964115	7,8	3
	$S_I$	-	0.02678760	7,8	3
5.5-18	a	m	6,378,166	2,3,4	
	b	m	6,356,784	2,3,4	

5.8-3

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Date 12-5-68

<u>Page</u>	<u>Constant</u>	<u>Units</u>	<u>Value</u>	<u>Reference (Sec.5.8.2)</u>	<u>Comments (Sec.5.8.3)</u>
5.5-22	$r_{LP}$	m	6,373,338	11	
5.5-23	$K_1$	-	0.917454125	9,10	4
	$K_2$	-	0.035711642	9,10	4
	$K_3$	-	0.397841585	9,10	4
	$K_4$	-	0.082353868	9,10	4
	$LOM_O$	rad	2.864381	9,10	4
	$LOM_R$	rad/sec	$2.661699 \times 10^{-6}$	9,10	4
	A	deg	6.2503	9,10	4
	B	deg	1.2725	9,10	4
	$OMEGA_A$	rad/sec	$2.6392078 \times 10^{-6}$	9,10	4
	$OMEGA_B$	rad/sec	$2.2725641 \times 10^{-6}$	9,10	4
	$PHASE_A$	rad	1.8002776	9,10	4
	$PHASE_B$	rad	0.7110406	9,10	4
	5.5-24	$LON_O$	rad	0.25069148	9,10
$LON_R$		rad/sec	$-1.07037037 \times 10^{-8}$	9,10	4
$LOS_O$		rad	1.7299995	9,10	4
$LOS_R$		rad/sec	$1.9909838 \times 10^{-7}$	9,10	4
C		deg	1.9190	9,10	4
$OMEGA_C$		rad/sec	$1.996542 \times 10^{-7}$	9,10	4
$PHASE_C$		rad	-0.06544352	9,10	4

<u>Page</u>	<u>Constant</u>	<u>Units</u>	<u>Value</u>	<u>Reference (Sec. 5.8.2)</u>	<u>Comments (Sec. 5.8.3)</u>
5. 5-54	$T_{SS}$	sec	360	—	5
5. 6-44	$f_B$	cps	153, 600	14,16	
	$f_B \tau_{LR}$	-	12, 288. 2	14,16	
	$k_X$	fps/bit	-0. 6440	14,16	
	$k_Y$	fps/bit	1. 212	14,16	
	$k_Z$	fps/bit	0. 8668	14,16	
	$k_{LR1}$	ft/bit	5. 395	14,16	
	$k_{LR2}$	ft/bit	1. 079	14,16	
5. 6-46A	K	bit/rad	2200. 16	—	6
5. 8-6	Star Table		See Fig. 8-1	—	7



Catalogue No.  
(octal)

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43  
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Star Name  
 $\alpha$  Andromedae (Alpheratz)  
 $\beta$  Ceti (Diphda)  
 $\gamma$  Cassiopeiae (Navi)  
 $\alpha$  Eridani (Achernar)  
 $\alpha$  Ursae Minoris (Polaris)  
 $\theta$  Eridani (Acamar)  
 $\alpha$  Ceti (Menkar)  
 $\alpha$  Persei (Mirfak)  
 $\alpha$  Tauri (Aldebaran)  
 $\beta$  Orionis (Rigel)  
 $\alpha$  Aurigae (Capella)  
 $\alpha$  Carinae (Canopus)  
 $\alpha$  Canis Majoris (Sirius)  
 $\alpha_2$  Canis Minoris (Procyon)  
 $\gamma$  Velorum (Regor)  
 $\zeta$  Ursae Majoris (Dnoces)  
 $\alpha$  Hydrae (Alphard)  
 $\alpha$  Leonis (Regulus)  
 $\beta$  Leonis (Denebola)  
 $\gamma$  Corvi (Gienah)  
 $\alpha$  Crucis (Acrux)  
 $\alpha$  Virginis (Spica)  
 $\eta$  Ursae Majoris (Alkaid)  
 $\theta$  Centauri (Menkent)  
 $\alpha$  Bootis (Arcturus)  
 $\alpha$  Coronae Borealis (Alphecca)  
 $\alpha$  Scorpii (Antares)  
 $\alpha$  Trianguli Austr. (Atria)  
 $\alpha$  Ophiuchi (Rasalhague)  
 $\alpha$  Lyrae (Vega)  
 $\sigma$  Sagittarii (Nunki)  
 $\alpha$  Aquilae (Altair)  
 $\beta$  Capricorni (Dabih)  
 $\alpha$  Pavonis (Peacock)  
 $\alpha$  Cygni (Deneb)  
 $\epsilon$  Pegasi (Enif)  
 $\alpha$  Piscis Austr. (Fomalhaut)

Vis: Mag:  
 2.1  
 2.2  
 2.2  
 0.6  
 2.1  
 3.4  
 2.8  
 1.9  
 1.1  
 0.3  
 0.2  
 -0.9  
 -1.6  
 0.5  
 1.9  
 3.1  
 2.2  
 1.3  
 2.2  
 2.8  
 1.0  
 1.2  
 1.9  
 2.3  
 0.2  
 2.3  
 1.2  
 1.9  
 2.1  
 0.1  
 2.1  
 0.9  
 3.2  
 2.1  
 1.3  
 2.5  
 1.3

X  
Coordinate  
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 +.9342726691  
 +.4776746280  
 +.4917355618  
 +.0131955837  
 +.5450662811  
 +.7033883645  
 +.4107492871  
 +.3509587451  
 +.2013426456  
 +.1373948084  
 -.0614360769  
 -.1818957154  
 -.4116502629  
 -.3611937602  
 -.4656165921  
 -.7741360248  
 -.8606970465  
 -.9656240240  
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 -.4524416631  
 -.9171065276  
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 -.6899901699  
 -.7862552143  
 -.5328042377  
 -.3518772846  
 -.1147906312  
 -.1126265542  
 +.1216171923  
 +.2067145272  
 +.4535361097  
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 +.3200014224  
 +.4540570017  
 +.8138753897  
 +.8341953207

Y  
Coordinate  
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 +.7074274193  
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 +.4775804724  
 +.6154234025  
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 -.3500098785  
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 -.9694679589  
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 -.5393383149  
 -.5559063490  
 -.2394362567

Z  
Coordinate  
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 -.8904319187  
 -.1908106439  
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 -.5908647707  
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 -.4436486945  
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 -.2568045150  
 -.8371095679  
 +.7091871552  
 +.1690413589  
 -.4967780649

Figure 8-1 Unit Vectors of the Navigational Stars

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5. 8. 2 References for Fixed Constants

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5.8-8

Revised

SUNDANCE

Rev. 2

Added

GSOP # R-557

PCR # 587.1, 612.1, 657

Date 12-4-68

5. 8. 3      Comments on Fixed Constants

1.      The quantity  $\epsilon_t$  is the limit used for the minimum time step in the Coasting Integration Routine. When the time step drops below  $\epsilon_t$ , the routine is terminated.
  
3.      The values given for the fixed constants used in the Planetary Inertial Orientation Subroutine (Section 5. 5. 2) are obtained by using Chapter 4 of Reference 7 and Sections 4C and 11D of Reference 8.
  
4.      The Lunar and Solar Ephemerides (Section 5. 5. 4) are based upon the technique given in Reference 10. The fixed constants were determined from various data points on the JPL Ephemeris Tape E-9511 (Reference 9 ).
  
5.      The quantity  $T_{SS}$  is a rough estimate of the time between the initiation of the Star Selection Routine in the Fine Alignment Routine (R-51) and the midpoint of the optical sightings on the two celestial bodies.
  
6.      The quantity K is the scale factor used to determine the number of bits to be sent to the RR CDU's during RR target designation. This scaling is applied to the angular difference between the present and desired pointing directions of the RR. No more than 384 bits are sent to an RR CDU because of saturation conditions.

7. The direction of each of the 37 navigation stars in Fig. 8-1 is expressed as the components of a unit vector in the Basic Reference Coordinate System. These star directions are the mean places of the stars at the beginning of the Besselian year 1969. The term "mean place" is defined in Reference 8. The star directions were computed by M. I. T. using essentially the same technique employed by the Nautical Almanac Office.
  
8. The  $\Delta V_K$  (DPS) values and  $\Delta V_K$  (APS) value are given in Reference 19. Two values are given for  $\Delta V_K$  (DPS), depending on spacecraft configuration. The values were selected to be between the maximum possible ullage acceleration and minimum anticipated main engine acceleration.

R-557  
SUNDANCE  
SECTION 5

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