5.2 COASTING FLIGHT NAVIGATION

5.2.1 GENERAL COMMENTS

The CMC Coasting Flight Navigation Routines which are presented in Sections 5.2.2 through 5.2.6 are used during free fall phases of the Apollo mission. The basic objective of these navigation routines is to maintain estimates of the position and velocity vectors of both the CSM and the LM. Let $\underline{r}$ and $\underline{v}$ be the estimates of a vehicle's position and velocity vectors, respectively. Then, the six-dimensional state vector, $\underline{x}$, of the spacecraft is defined by

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \end{pmatrix}$$

Coasting Flight Navigation is accomplished by extrapolating the state vector, $\underline{x}$, by means of the Coasting Integration Routine (Section 5.2.2), and updating or modifying this estimated state using tracking data by the recursive method of navigation (Sections 5.2.3 - 5.2.6).

The Coasting Integration Routine (Section 5.2.2) is used by other navigation and targeting routines to extrapolate the following:

1) Present estimated CSM state vector
2) Present estimated LM state vector
3) An arbitrary specified state vector, such as the predicted result of a maneuver

State vector extrapolation is accomplished by means of Encke's method of differential accelerations. The motion of a spacecraft is dominated by the conic orbit which would result if the spacecraft were in a central force field. In Encke's method the differential equations for the deviations from conic motion are integrated numerically. This technique is in contrast to a numerical integration of the differential equations for the total motion, and it provides a more accurate orbit extrapolation. The numerical integration is accomplished by means of Nystrom's method which gives fourth-order accuracy while requiring only three computations of the derivatives per time step. The usual fourth-order Runge-Kutta integration methods require four derivative computations per time step.

Regardless of the accuracy of the state vector extrapolation, errors in the initial conditions will propagate and soon grow to intolerable size. Thus, it is necessary periodically to obtain additional data in the form of either new state vector estimates or modifications to the current state vector estimates. These state vector modifications are computed from navigation data obtained by means of navigation measurements.

The CSM GNCS uses optical angle data from the scanning telescope (SCT) and the sextant (SXT) and VHF range data to compute state vector changes, while the LM PGNCS uses rendezvous radar (RR) tracking data. Navigation measurement data are used to update state vector estimates during orbit navigation, rendezvous navigation, and cislunar-midcourse navigation procedures. These three navigation procedures will be used normally during the lunar-orbit navigation phase, all LM-CSM lunar-orbit rendezvous phases, and CSM return-to-earth aborts, respectively, in the lunar landing mission. However, in order to provide for alternate mission capability, the orbit and rendezvous navigation procedures can be used near the moon or the earth.

Although the state vector of the CSM is six-dimensional, it is not necessary that the quantities estimated during a particular navigation procedure be the position and velocity vectors of the CSM. A variety of "estimated state vectors", not necessarily of six-dimensions, are used.

In order to achieve desired landing objectives, it is necessary to expand the lunar-orbit navigation procedure to nine dimensions, and to include in the estimation the position vector of the landmark being tracked. The estimated state vector that is used in orbit navigation is given by

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix}$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors and $\underline{r}_\ell$ is the estimated landmark position vector.

During the rendezvous phase, the six-dimensional state vector of either the CSM or the LM can be updated from the measurement data obtained with the CSM-based optics. Normally the LM state vector is updated, but the astronaut can select the CSM update mode. The selection of the update mode is based primarily upon which vehicle's state vector is most accurately known initially, and which vehicle is controlling the rendezvous maneuvers.

The standard six-dimensional CSM state vector is used during cislunar-midcourse navigation.

Navigation data is incorporated into the state vector estimates by means of the Measurement Incorporation Routine (Section 5.2.3) which has both six- and nine-dimensional modes. The Measurement Incorporation Routine is a subroutine of the following CMC navigation routines:

1) Orbit Navigation Routine (Section 5.2.4)

2) Rendezvous Navigation Routine (Section 5.2.5)

3) Cislunar-Midcourse Navigation Routine (Section 5.2.6)

Simplified functional diagrams of the navigation programs which use these routines are given in Figs. 2.1-1, 2.1-2, and 2.1-3, respectively.

In all three navigation programs, estimated position and velocity vectors are obtained at required times by means of the Coasting Integration Routine (Section 5.2.2). The Measurement Incorporation Routine (Section 5.2.3) is used to incorporate the measurement data into the state vector estimates.

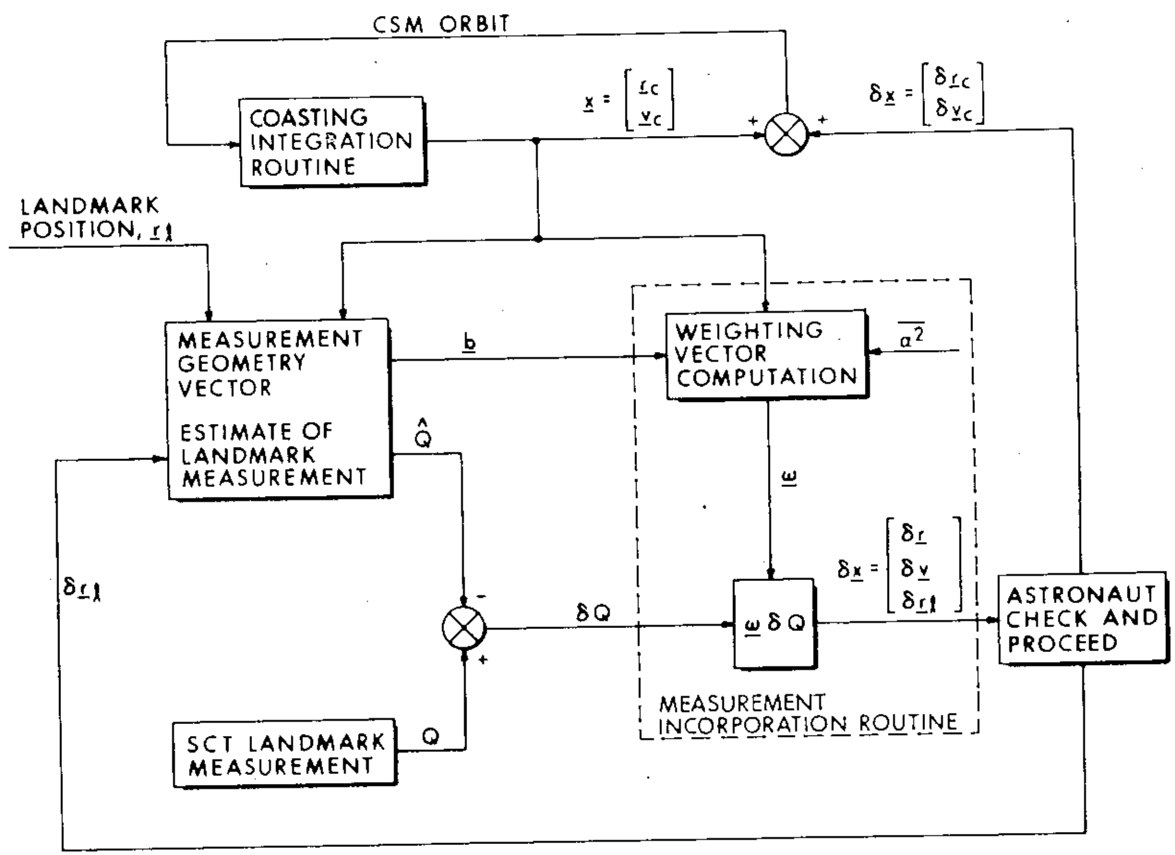

Fig. 2.1-1 Simplified Orbit Navigation Functional Diagram

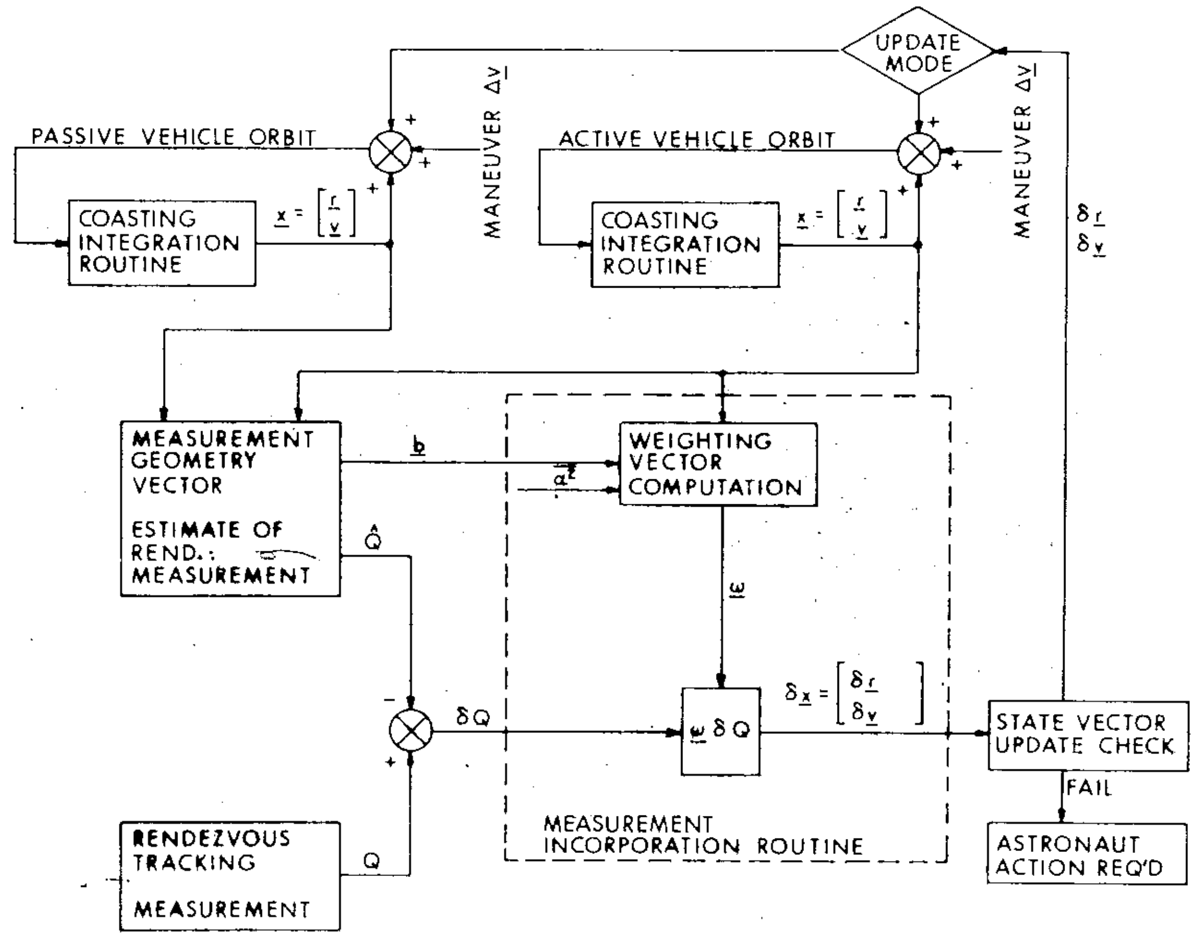

Fig. 2.1-2 Simplified CMC Rendezvous Navigation Functional Diagram

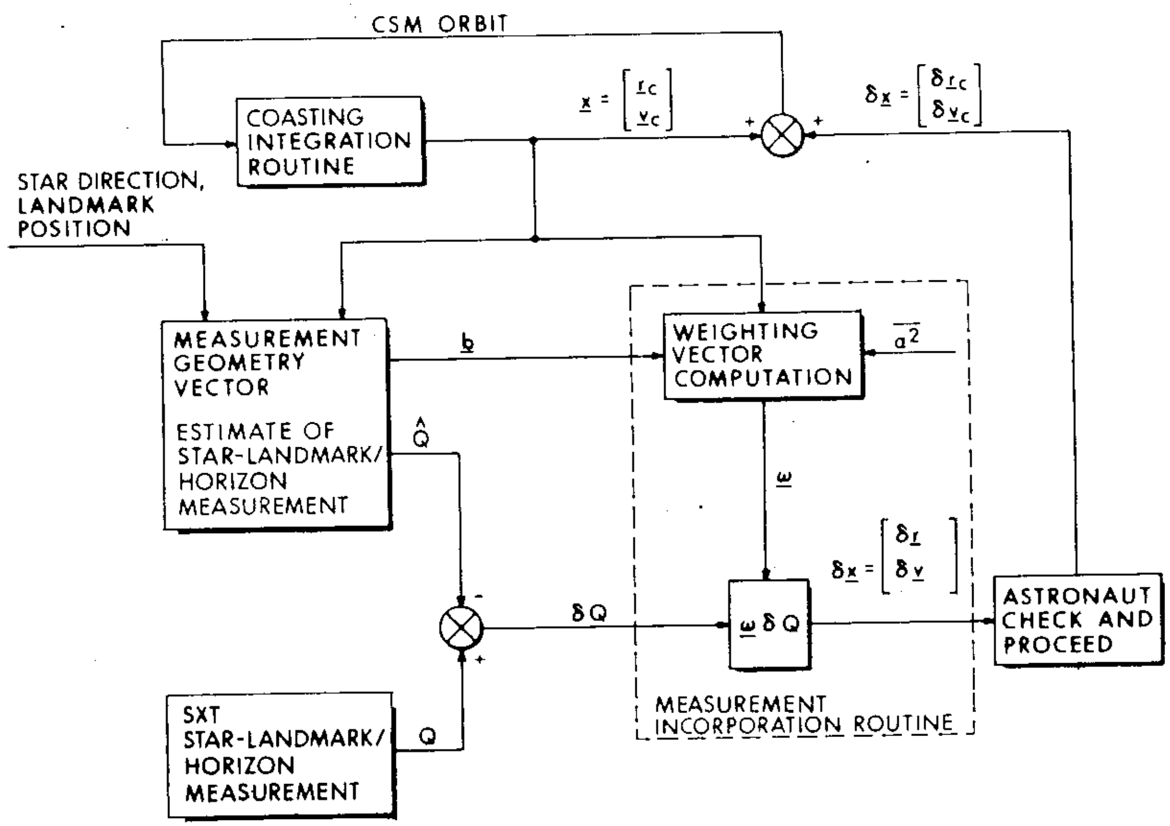

Fig. 2.1-3 Simplified Cislunar-Midcourse Navigation Functional Diagram

The navigation procedure, which is illustrated in simplified form in Figs. 2.1-1 to 2.1-3, involves computing an estimated tracking measurement, $\hat{Q}$, based on the current state vector estimates. This estimated measurement is then compared with the actual tracking measurement Q to form a measured deviation δQ. A statistical weighting vector, $\underline{\omega}$, is computed from statistical knowledge of state vector uncertainties and tracking performance, $\overline{\alpha^2}$, plus a geometry vector, $\underline{b}$, determined by the type of measurement being made. The weighting vector, $\underline{\omega}$, is defined such that a statistically optimum linear estimate of the deviation, $\delta \underline{x}$, from the estimated state vector is obtained when the weighting vector is multiplied by the measured deviation δQ. The vectors $\underline{\omega}$, $\underline{b}$ and $\delta \underline{x}$ are of six or nine dimensions depending upon the dimension of the state vector being estimated.

In an attempt to prevent unacceptably large incorrect state vector changes, certain validity tests have been included in the various CMC navigation routines.

In the Orbit Navigation Routine (Section 5.2.4) the astronaut tracks a landmark and acquires a number of sets of optical angle data before the state vector updating process begins. During the data processing procedure the landmark is out of sight, and it is not possible to repeat the tracking. Before the first set of data is used to update the estimated state vector, the magnitudes of the proposed changes in the estimated CSM position and velocity vectors, δr and δv, respectively, are displayed for astronaut approval. In general, successive accepted values of δr and δv will decrease during the processing of the tracking data associated with one landmark. Thus, if the MARK REJECT button has been used to erase all inaccurate marks, then all state vector updates should be either accepted or rejected. If the first displayed values of δr and δv are judged to be valid, then all data associated with that landmark will be accepted.

The actual values of the first displayed δr and δv will depend upon the statistical parameters stored in the CMC and upon the following types of errors:

Type 1: Errors in the current state vector estimates

Type 2: Errors in alignment of the IMU

Type 3: Reasonable tracking performance errors, including both hardware and astronaut errors

Type 4: A CSM GNCS failure

Type 5: Gross astronaut errors, such as incorrect identification of the landmark

The existence of Type 1 errors is precisely the reason that the landmark tracking is being done. It is the function of the navigation to decrease Type 1 errors in the presence of noise in the form of errors of Types 2 and 3. Since the landmark tracking should not be performed unless the IMU is well aligned and the GNCS is functioning properly, and since bad marks should be rejected, it follows that the purpose of the state vector change validity check is to discover a Type 5 error. This validity check cannot distinguish between a Type 4 error and a Type 5 error.

Based upon the last time that the state vector was updated, when the IMU last was realigned, and an estimate of the tracking performance for the first mark, very crude reasonable values for the first δr and δv can be generated by the astronaut. The CMC will provide no information to assist the astronaut in his estimates of reasonable values for δr and δv.

In the Rendezvous Navigation Routine (Section 5.2.5) measurement data is processed periodically, and it is desirable that the LM be tracked during the entire rendezvous phase up to the manual terminal maneuver. If the magnitudes of the changes in the estimated position and velocity vectors, δ r and δ v, respectively, are both less than preset tracking alarm levels, then the selected vehicle's state vector is automatically updated by the computed deviation, $\delta \underline{x}$, and no special display is presented, except that the tracking measurement counter is incremented by one. If either δ r or δ v exceeds its alarm level, then the state vector is not updated, and the astronaut is alerted to this condition by a special display of δ r and δ v. Included in this display is the source code which indicates whether optical or VHF range-link data caused the display.

If this display should occur because of optics data, then the astronaut should recheck the optical tracking and make sure that he is tracking the LM. Under certain conditions it is possible to mistake a star for LM reflected sunlight, and it may take a period of a few minutes to determine the LM target by watching relative motion of the target and star background. After the tracking has been verified, and navigation data has again been acquired, the astronaut has the option of commanding a state vector update if the tracking alarm is again exceeded, or of repeating further optical checks before incorporating the measurement data. If the astronaut cannot determine the LM target due to no positive acquisition (bright background, etc.) he can terminate the marking procedure and try to achieve tracking conditions at a later time.

The displayed values of δr and δv which have not passed the tracking alarm test will depend upon the statistical parameters stored in the CMC and upon the same five types of errors discussed previously in regard to orbit navigation. The tracking alarm criterion is incorporated in the Rendezvous Navigation Routine to alert the astronaut to the fact that the state vector update is larger than normally expected, and to prevent the estimated state vector from automatically being updated in such cases. The update occurs only by specific command of the astronaut. The tracking alarm level beyond which updating is suspended is primarily chosen to avoid false acquisition and tracking conditions. As previously mentioned, this condition is possible in the CSM if a star is optically tracked by mistake instead of the LM reflected sun light, and it is therefore possible for the alarm level to be exceeded in such cases even though the estimated state vectors are essentially correct. It is also possible for the state vector update alarm level to be exceeded after correct initial acquisition and tracking in the case where a poor estimate of either the CSM or LM state vector exists. In this case the astronaut would have to command the initial state vector update, after which the alarm level would seldom be exceeded during the remainder of the rendezvous phase. It should be noted that this statement is true only if the estimated state vector of the active vehicle performing a powered rendezvous maneuver is updated by the Average-G Routine in the case of the CSM being the active vehicle, or by a DSKY entry (R-32) of the maneuver $\Delta \underline{V}$ if the LM is the active vehicle (Section 5.6.14).

The previously discussed method which the astronaut can use to generate crude estimates of expected δr and δv values in the case of the Orbit Navigation Routine can also be applied to the Rendezvous Navigation Routine. The astronaut must decide whether or not he is tracking the LM. The CMC cannot make this decision.

In the Cislunar-Midcourse Navigation Routine (Section 5.2.6) the astronaut measures the angle between a star and a planetary landmark or horizon. The data from each angle measurement is processed immediately after it is made. The values of δr and δv are displayed for astronaut approval before the state vector is updated by the computed deviation $\delta \underline{x}$. Thus, it is a simple matter to repeat the measurement if the astronaut is uncertain as to the validity of the proposed state vector changes.

The parameters required to initialize the navigation routines (Sections 5.2.4 - 5.2.6) are the initial estimated CSM state vector, plus the initial estimated LM state vector for the Rendezvous Navigation Routine, initial state vector estimation error covariance matrices in the form of prestored diagonal error transition matrices (as defined in Section 5.2.2.4), and a priori measurement error variances. The basic input to the navigation routines is SCT or SXT tracking angle data indicated to the CMC by the astronaut when he presses the MARK button signifying that he has centered the optical reticle on the tracking target (landmark or LM) or superimposed the two objects in the case of a star-landmark/horizon measurement, and automatically-acquired VHF range-link tracking data. The primary output of the navigation routines is the estimated CSM state vector, plus estimated landmark coordinates in the case of the orbit navigation or the estimated LM state vector in the case of the Rendezvous Navigation Routine. The various guidance targeting modes outlined in Section 5.4 are based on the state vector estimates which result from these navigation routines.

5.2.2 COASTING INTEGRATION ROUTINE

5.2.2.1 General Comments

During all coasting phase navigation procedures, an extrapolation of position and velocity by numerical integration of the equations of motion is required. The basic equation may be written in the form

$$\frac{d^2}{dt^2} \underline{r}(t) + \frac{\mu_P^*}{r^3} \underline{r}(t) = \underline{a}_d(t) \qquad (2.2.1)$$

where μ_P is the gravitational constant of the primary body, and $\underline{a}_d(t)$ is the vector acceleration which prevents the motion of the vehicle (CSM or LM) from being precisely a conic with focus at the center of the primary body. The Coasting Integration Routine is a precision integration routine in which all significant perturbation effects are included. The form of the disturbing acceleration $\underline{a}_d(t)$ depends on the phase of the mission.

An approximate extrapolation of a vehicle state vector in which the disturbing acceleration, $\underline{a}_d(t)$ of Eq. (2.2.1), is set to zero may be accomplished by means of the Kepler subroutine (Section 5.5.5).

5.2.2.2 Encke's Method

If $\underline{a}_d$ is small compared with the central force field, direct integration of Eq. (2.2.1) is inefficient. Therefore, the extrapolation will be accomplished using the technique of differential accelerations attributed to Encke.

* In the remainder of Section 5.2 the subscripts P and Q will denote primary and secondary body, respectively. When the body is known, then the subscripts E, M, and S will be used for earth, moon, and sun, respectively. The vehicle will be indicated by the subscripts C for CSM and L for LM.

At time t_0 the position and velocity vectors, $\underline{r}_0$ and $\underline{v}_0$, define an osculating conic orbit. The position and velocity vectors in the conic orbit, $\underline{r}_{con}(t)$ and $\underline{v}_{con}(t)$, respectively, will deviate by a small amount from the actual position and velocity vectors.

The conic position and velocity at time t are computed as shown in Section 5.5.5. Required in this calculation is the variable x which is the root of Kepler's equation. In order to minimize the number of iterations required in solving Kepler's equation, an estimate of the correct solution for x is obtained as follows:

Let

$$\tau = t - t_0 \tag{2.2.2}$$

During the previous computation cycle the values

$$\underline{r}' = \underline{r}_{con}\,(\tau - \frac{\Delta t}{2})$$

$$\underline{v}' = \underline{v}_{con}\,(\tau - \frac{\Delta t}{2}) \tag{2.2.3}$$

$$x' = x\,(\tau - \frac{\Delta t}{2})$$

were computed. A trial value of $x(\tau)$ is obtained from

$$x_t = x' + s\ [1 - \gamma\ s\ (1 - 2\ \gamma\ s) - \frac{1}{6}(\frac{1}{r'} - \alpha)\, s^2] \tag{2.2.4}$$

where

$$s = \frac{\sqrt{\mu_P}}{r'}\left(\frac{\Delta t}{2}\right)$$

$$\gamma = \frac{\underline{r}' \cdot \underline{v}'}{2r'\sqrt{\mu_P}} \tag{2.2.5}$$

$$\alpha = \frac{2}{r_0} - \frac{v_0^2}{\mu_P}$$

After specification of $\underline{r}_0$, $\underline{v}_0$, x_t and τ, the Kepler subroutine (Section 5.5.5) is used to compute $\underline{r}_{con}(\tau)$, $\underline{v}_{con}(\tau)$, and $x(\tau)$.

The true position and velocity vectors will deviate from the conic position and velocity since $\underline{a}_d$ is not zero. Let

$$\begin{aligned} \underline{r}(t) &= \underline{\delta}(t) + \underline{r}_{con}(t) \\ \underline{v}(t) &= \underline{\nu}(t) + \underline{v}_{con}(t) \end{aligned} \qquad (2.2.6)$$

where $\underline{\delta}(t)$ and $\underline{\nu}(t)$ are the position and velocity deviations from the conic. The deviation vector $\underline{\delta}(t)$ satisfies the differential equation

$$\frac{d^2}{dt^2}\,\underline{\delta}(t) = -\frac{\mu_P}{r_{con}^3(t)}\left[f(q)\ \underline{r}(t) + \underline{\delta}(t)\right] + \underline{a}_d(t) \qquad (2.2.7)$$

subject to the initial conditions

$$\underline{\delta}(t_0) = \underline{0}, \quad \underline{\nu}(t_0) = \underline{0} \qquad (2.2.8)$$

where

$$q = \frac{(\underline{\delta} - 2\underline{r}) \cdot \underline{\delta}}{r^2} \qquad (2.2.9)$$

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1 + q)^{3/2}} \qquad (2.2.10)$$

The first term on the right-hand side of Eq. (2.2.7) must remain small, i.e., of the same order as $\underline{a}_d(t)$, if the method is to be efficient. As the deviation vector $\underline{\delta}(t)$ grows in magnitude, this term will eventually increase in size. Therefore, in order to maintain the efficiency of the method, a new osculating conic orbit should be defined by the total position and velocity vectors $\underline{r}(t)$ and $\underline{v}(t)$. The process of selecting a new conic orbit from which to calculate deviations is called rectification. When rectification occurs, the initial conditions for the differential equation for $\underline{\delta}(t)$, as well as the variables τ and x, are again zero.

5.2.2.3 Disturbing Acceleration

The form of the disturbing acceleration $\underline{a}_d(t)$ that is used in Eq. (2.2.1) depends on the phase of the mission. In earth or lunar orbit, only the gravitational perturbations arising from the non-spherical shape of the primary body need be considered. Let $\underline{a}_{dP}$ be the acceleration due to the non-spherical gravitational perturbations of the primary body. Then, for the earth

$$\underline{a}_{dE} = \frac{\mu_E}{r^2} \sum_{i=2}^{4} J_{iE} \left(\frac{r_E}{r}\right)^i \left[P'_{i+1} (\cos \phi)\, \underline{u}_r - P'_i (\cos \phi)\, \underline{u}_z \right] \qquad (2.2.11)$$

where

$$P_2'(\cos\phi) = 3\cos\phi$$

$$P_3'(\cos\phi) = \frac{1}{2}(15\cos^2\phi - 3)$$

$$P_4'(\cos\phi) = \frac{1}{3}(7\cos\phi\, P_3' - 4P_2') \tag{2.2.12}$$

$$P_5'(\cos\phi) = \frac{1}{4}(9\cos\phi\, P_4' - 5P_3')$$

are the derivatives of Legendre polynominals,

$$\cos\phi = \underline{u}_r \cdot \underline{u}_z$$

$$\underline{u}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.2.13}$$

and J_2, J_3, J_4 are the coefficients of the second, third, and fourth harmonics of the earth's potential function. The vectors $\underline{u}_r$ and $\underline{u}_z$ are unit vectors in the direction of $\underline{r}$ and the polar axis of the earth, respectively, and r_E is the equatorial radius of the earth.

In the case of the moon

$$\underline{a}_{dM} = \frac{\mu_M}{r^2}\left\{\sum_{i=2}^{4} J_{iM}\left(\frac{r_M}{r}\right)^i \left[P_{i+1}'(\cos\phi)\,\underline{u}_r - P_i'(\cos\phi)\,\underline{u}_z\right]\right.$$

$$\left. + 3J_{22M}\left(\frac{r_M}{r}\right)^2 \left[4\frac{x_M y_M}{x_M^2 + y_M^2}(\underline{u}_r \times \underline{u}_z) + \frac{x_M^2 - y_M^2}{x_M^2 + y_M^2}((5\cos^2\phi - 3)\,\underline{u}_r - 2\cos\phi\,\underline{u}_z)\right]\right\} \tag{2.2.14}$$

where r_M is the mean lunar radius, J_{22M} is the coefficient of the term in the moon's gravitational potential function which describes the asymmetry of the moon about its polar axis, and x_M and y_M are the X and Y components of $\underline{r}$ expressed in moon-fixed coordinates. The other terms in Eq. (2.2.14) have definitions analogous to those in Eq. (2.2.11). The variables x_M, y_M, and $\underline{u}_z$ are computed by means of the Planetary Inertial Orientation Subroutine (Section 5.5.2).

During cislunar-midcourse flight (translunar and transearth) the gravitational attraction of the sun and the secondary body Q (earth or moon) are relevant forces. The accelerations due to the secondary body and the sun are

$$\underline{a}_{dQ} = - \frac{\mu_Q}{r_{QC}^3} \left[f(q_Q)\, \underline{r}_{PQ} + \underline{r} \right] \qquad (2.2.15)$$

$$\underline{a}_{dS} = - \frac{\mu_S}{r_{SC}^3} \left[f(q_S)\, \underline{r}_{PS} + \underline{r} \right] \qquad (2.2.16)$$

where $\underline{r}_{PQ}$ and $\underline{r}_{PS}$ are the position vectors of the secondary body and the sun with respect to the primary body, r_{QC} and r_{SC} are the distances of the CSM from the secondary body and the sun, and the arguments q_Q and q_S are computed from

$$q_Q = \frac{(\underline{r} - 2\,\underline{r}_{PQ}) \cdot \underline{r}}{r_{PQ}^2} \qquad (2.2.17)$$

$$q_S = \frac{(\underline{r} - 2\,\underline{r}_{PS}) \cdot \underline{r}}{r_{PS}^2} \qquad (2.2.18)$$

The functions $f(q_Q)$ and $f(q_S)$ are calculated from Eq. (2.2.10).

The position vectors of the moon relative to the earth, $\underline{r}_{EM}$, and the sun relative to the earth, $\underline{r}_{ES}$, are computed as described in Section (5.5.4). Then,

$$\underline{r}_{PQ} = \begin{cases} \underline{r}_{EM} & \text{if } P = E \\ -\underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2.2.19}$$

and

$$\underline{r}_{PS} = \begin{cases} \underline{r}_{ES} & \text{if } P = E \\ \underline{r}_{ES} - \underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2.2.20}$$

Finally,

$$\begin{aligned} \underline{r}_{QC} &= \underline{r} - \underline{r}_{PQ} \\ \underline{r}_{SC} &= \underline{r} - \underline{r}_{PS} \end{aligned} \tag{2.2.21}$$

5.2.2.4 Error Transition Matrix

The position and velocity vectors as maintained in the computer are only estimates of the true values. As part of the navigation technique it is necessary also to maintain statistical data in the computer to aid in the processing of navigation measurements.

If $\underline{\epsilon}(t)$ and $\underline{\eta}(t)$ are the errors in the estimates of the position and velocity vectors, respectively, then the six-dimensional correlation matrix E(t) is defined by

$$E_6(t) = \begin{pmatrix} \overline{\underline{\epsilon}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\epsilon}(t)\ \underline{\eta}(t)^T} \\ \\ \overline{\underline{\eta}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\eta}(t)\ \underline{\eta}(t)^T} \end{pmatrix} \qquad (2.2.22)$$

In certain applications it becomes necessary to expand the state vector and the correlation matrix to more than six dimensions so as to include estimation of landmark locations in the CMC during orbit navigation, and rendezvous radar tracking biases in the LGC during the rendezvous navigation procedure. For this purpose a nine-dimensional correlation matrix is defined as follows :

$$E(t) = \begin{pmatrix} & E_6(t) & & \overline{\underline{\epsilon}(t)\ \underline{\beta}^T} \\ & & & \overline{\underline{\eta}(t)\ \underline{\beta}^T} \\ \overline{\underline{\beta}\ \underline{\epsilon}(t)^T} & & \overline{\underline{\beta}\ \underline{\eta}(t)^T} & \overline{\underline{\beta}\ \underline{\beta}^T} \end{pmatrix} \qquad (2.2.23)$$

where the components of the three-dimensional vector $\underline{\beta}$ are the errors in the estimates of three variables which are estimated in addition to the components of the spacecraft state vector.

In order to take full advantage of the operations provided by the interpreter in the computer, the correlation matrix will be restricted to either six or nine dimensions. If, in some navigation procedure, only one or two additional items are to be estimated, then a sufficient number of dummy variables will be added to the desired seven-or eight-dimensional state vector to make it nine-dimensional.

Rather than use the correlation matrix in the navigation procedure, it is more convenient to utilize a matrix W(t), called the error transition matrix, and defined by

$$E(t) = W(t)\, W(t)^T \tag{2.2.24}$$

Extrapolation of the nine-dimensional matrix W(t) is made by direct numerical integration of the differential equation

$$\frac{d}{dt} W(t) = \begin{pmatrix} O & I & O \\ G(t) & O & O \\ O & O & O \end{pmatrix} W(t) \tag{2.2.25}$$

where G(t) is the three-dimensional gravity gradient matrix, and I and O are the three-dimensional identity and zero matrices, respectively. If the W matrix is partitioned as

$$W = \begin{pmatrix} \underline{w}_0 & \underline{w}_1 & \cdots & \underline{w}_8 \\ \underline{w}_9 & \underline{w}_{10} & \cdots & \underline{w}_{17} \\ \underline{w}_{18} & \underline{w}_{19} & \cdots & \underline{w}_{26} \end{pmatrix} \tag{2.2.26}$$

then,

$$\left.\begin{aligned}
\frac{d}{dt}\,\underline{w}_i(t) &= \underline{w}_{i+9}(t) \\
\frac{d}{dt}\,\underline{w}_{i+9}(t) &= G(t)\,\underline{w}_i(t) \\
\frac{d}{dt}\,\underline{w}_{i+18}(t) &= \underline{0}
\end{aligned}\right\} \quad i = 0,\ 1, \ldots, 8 \qquad (2.2.27)$$

The extrapolation may be accomplished by successively integrating the vector differential equations

$$\frac{d^2}{dt^2}\,\underline{w}_i(t) = G(t)\,\underline{w}_i(t) \qquad i = 0, 1, \ldots, 8 \qquad (2.2.28)$$

The gravity gradient matrix G(t) for earth or lunar orbit is given by

$$G(t) = \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] \qquad (2.2.29)$$

During cislunar-midcourse flight

$$\begin{aligned}
G(t) = {} & \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] \\
& + \frac{\mu_Q}{r^5{}_{QC}(t)}\left[3\,\underline{r}_{QC}(t)\,\underline{r}_{QC}(t)^T - r^2{}_{QC}(t)\,I\right]
\end{aligned} \qquad (2.2.30)$$

Thus, if D is the dimension of the matrix W(t) for the given navigation procedure, the differential equations for the $\underline{w}_i(t)$ vectors are

$$\frac{d^2}{dt^2}\underline{w}_i(t) = \frac{\mu_P}{r^3(t)}\left\{3\left[\underline{u}_r(t)\cdot\underline{w}_i(t)\right]\underline{u}_r(t) - \underline{w}_i(t)\right\} + M\frac{\mu_Q}{r^3_{QC}(t)}\left\{3\left[\underline{u}_{QC}(t)\cdot\underline{w}_i(t)\right]\underline{u}_{QC}(t) - \underline{w}_i(t)\right\} \quad i = 0, 1, \ldots, D-1 \tag{2.2.31}$$

where $\underline{u}_r(t)$ and $\underline{u}_{QC}(t)$ are unit vectors in the directions of $\underline{r}(t)$ and $\underline{r}_{QC}(t)$, respectively, and

$$M = \begin{cases} 1 \text{ for cislunar-midcourse flight} \\ 0 \text{ for earth or lunar orbit} \end{cases} \tag{2.2.32}$$

It is possible for a computation overflow to occur during the W matrix integration if any element of the matrix exceeds its maximum value. This event is extremely unlikely because of the large scale factors chosen. The overflow occurs if

1) any element of the position part (upper third) of the W matrix becomes equal to or greater than 2^{19} m,

or

2) any element of the velocity part (middle third) of the W matrix becomes equal to or greater than one m/csec.

In addition, each element of the landmark part (lower third) of the matrix must remain less than 2^{19} m, but this part does not change during integration.

If overflow should occur, an alarm results, and either new state vector estimates must be obtained from RTCC, or a sufficient number of navigation measurements must be made before the state vectors are used in any targeting or maneuver programs.

5.2.2.5 Numerical Integration Method

The extrapolation of navigational data requires the solution of a number of second-order vector differential equations, specifically Eqs. (2.2.7) and (2.2.31). These are all special cases of the form

$$\frac{d^2}{dt^2}\underline{y} = \underline{f}(\underline{y}, t) \tag{2.2.33}$$

Nystrom's method is particularly well suited to this form and gives an integration method of fourth-order accuracy. The second-order system is written

$$\begin{aligned} \frac{d}{dt}\underline{y} &= \underline{z} \\ \frac{d}{dt}\underline{z} &= \underline{f}(\underline{y}, t) \end{aligned} \tag{2.2.34}$$

and the formulas are summarized below.

$$\begin{aligned} \underline{y}_{n+1} &= \underline{y}_n + \underline{\phi}(\underline{y}_n)\,\Delta t \\ \underline{z}_{n+1} &= \underline{z}_n + \underline{\psi}(\underline{y}_n)\,\Delta t \\ \underline{\phi}(\underline{y}_n) &= \underline{z}_n + \frac{1}{6}(\underline{k}_1 + 2\underline{k}_2)\,\Delta t \\ \underline{\psi}(\underline{y}_n) &= \frac{1}{6}(\underline{k}_1 + 4\underline{k}_2 + \underline{k}_3) \\ \underline{k}_1 &= \underline{f}(\underline{y}_n, t_n) \\ \underline{k}_2 &= \underline{f}(\underline{y}_n + \frac{1}{2}\underline{z}_n\,\Delta t + \frac{1}{8}\underline{k}_1\,(\Delta t)^2, \quad t_n + \frac{1}{2}\Delta t) \\ \underline{k}_3 &= \underline{f}(\underline{y}_n + \underline{z}_n\,\Delta t + \frac{1}{2}\underline{k}_2\,(\Delta t)^2, \ t_n + \Delta t) \end{aligned} \tag{2.2.35}$$

For efficient use of computer storage as well as computing time the computations are performed in the following order:

1) Equation (2. 2. 7) is solved using the Nystrom formulas, Eq. (2. 2. 35). It is necessary to preserve the values of the vector $\underline{r}$ at times t_n, $t_n + \Delta t/2$, $t_n + \Delta t$ for use in the solution of Eqs. (2. 2. 31).

2) Equations (2. 2. 31) are solved one-at-a-time using Eqs. (2. 2. 35) together with the values of $\underline{r}$ which resulted from the first step.

The variable Δt is the integration time step and should not be confused with τ, the time since rectification. The maximum value for Δt which can be used for precision integration, Δt_{max}, is computed from

$$\Delta t_{max} = \text{minimum} \left(\Delta t_{\ell im}, \frac{K\, r^{3/2}}{\sqrt{\mu_P}} \right) \tag{2. 2. 36}$$

where

$$\begin{aligned} \Delta t_{\ell im} &= 4000 \text{ sec.} \\ K &= 0.3 \end{aligned} \tag{2. 2. 37}$$

5. 2. 2. 6 Coasting Integration Logic

Estimates of the state vectors of two vehicles (CSM and LM) will be maintained in the computer. In various phases of the mission it will be required to extrapolate a state vector either alone or with an associated W matrix of dimension six or nine.

To accomplish all of these possible procedures, as well as to solve the computer restart problem, three state vectors will be maintained in the computer. Let $\underline{x}_C$ and $\underline{x}_L$ be the estimated CSM and LM state vectors, respectively, and let $\underline{x}$ be a temporary state vector. The state vector $\underline{x}$ is a symbolic representation of the following set of variables:

$$
\left.
\begin{array}{lll}
\underline{r}_0 & = & \text{rectification position vector} \\
\underline{v}_0 & = & \text{rectification velocity vector} \\
\underline{r}_{con} & = & \text{conic position vector} \\
\underline{v}_{con} & = & \text{conic velocity vector} \\
\underline{\delta} & = & \text{position deviation vector} \\
\underline{\nu} & = & \text{velocity deviation vector} \\
t & = & \text{time associated with } \underline{r}_{con},\ \underline{v}_{con},\ \underline{\delta} \text{ and } \underline{\nu} \\
\tau & = & \text{time since rectification} \\
x & = & \text{root of Kepler's equation} \\
P & = & \text{primary body} = \begin{cases} 0 \text{ for earth} \\ 1 \text{ for moon} \end{cases}
\end{array}
\right. \qquad (2.2.38)
$$

The state vectors $\underline{x}_C$ and $\underline{x}_L$ represent an analogous set of variables.

The Coasting Integration Routine is controlled by the calling program by means of the two indicators D and V. The variable D indicates the dimension of the W matrix with

$$D = 0 \qquad (2.2.39)$$

denoting that the state vector only is to be extrapolated. The variable V indicates the appropriate vehicle as follows:

$$V = \begin{cases} 1 \text{ for CSM} \\ 0 \text{ for LM} \\ -1 \text{ for state vector specified by calling program} \end{cases} \qquad (2.2.40)$$

In addition, the calling program must set the desired final time t_F; and, for V equal to -1, the desired state vector $\underline{x}$.

A simplified functional diagram of the Coasting Integration Routine is shown in Fig. 2.2-1. In the figure the indicated state vector is being integrated to time t_F. The value of Δt for each time step is Δt_{max} (Eq. (2.2.36)) or the total time-to-go whichever is smaller. The integration is terminated when the computed value of Δt is less than ϵ_t.

Figures 2.2-2, 2.2-3 and 2.2-4 illustrate in more detail the logic flow of this routine. In these figures certain items which have not been discussed fully in the text are explicitly illustrated. The following is a list of these items together with the number of the figure in which each occurs.

1) Saving of $\underline{r}$ values for W matrix integration: Fig. 2.2-2.
2) Change in origin of coordinates: Fig. 2.2-3.
3) Rectification procedure: Fig. 2.2-3.
4) Selection of disturbing acceleration: Fig. 2.2-4.

The logic flow shown in these figures is controlled by the three flags M, B, and F. Flag M is defined in Eq. (2.2.32), B prevents the recalculation of already available quantities ($\underline{r}_{PQ}$, $\underline{r}_{QC}$), and F is used to distinguish between state vector integration (F = 1) and W matrix integration (F = 0).

If the Coasting Integration Routine is requested to extrapolate the estimated LM state vector and the LM is on the lunar surface, then the routine will use the Planetary Inertial Orientation Subroutine (Section 5.5.2) to compute the desired LM position and the normal integration will not be performed. This procedure is not indicated in the figure.

There is a procedure for the emergency termination of the Coasting Integration Routine in order to permit correction of wrong erasable memory parameters. This emergency function is described in Section 5.6.12.

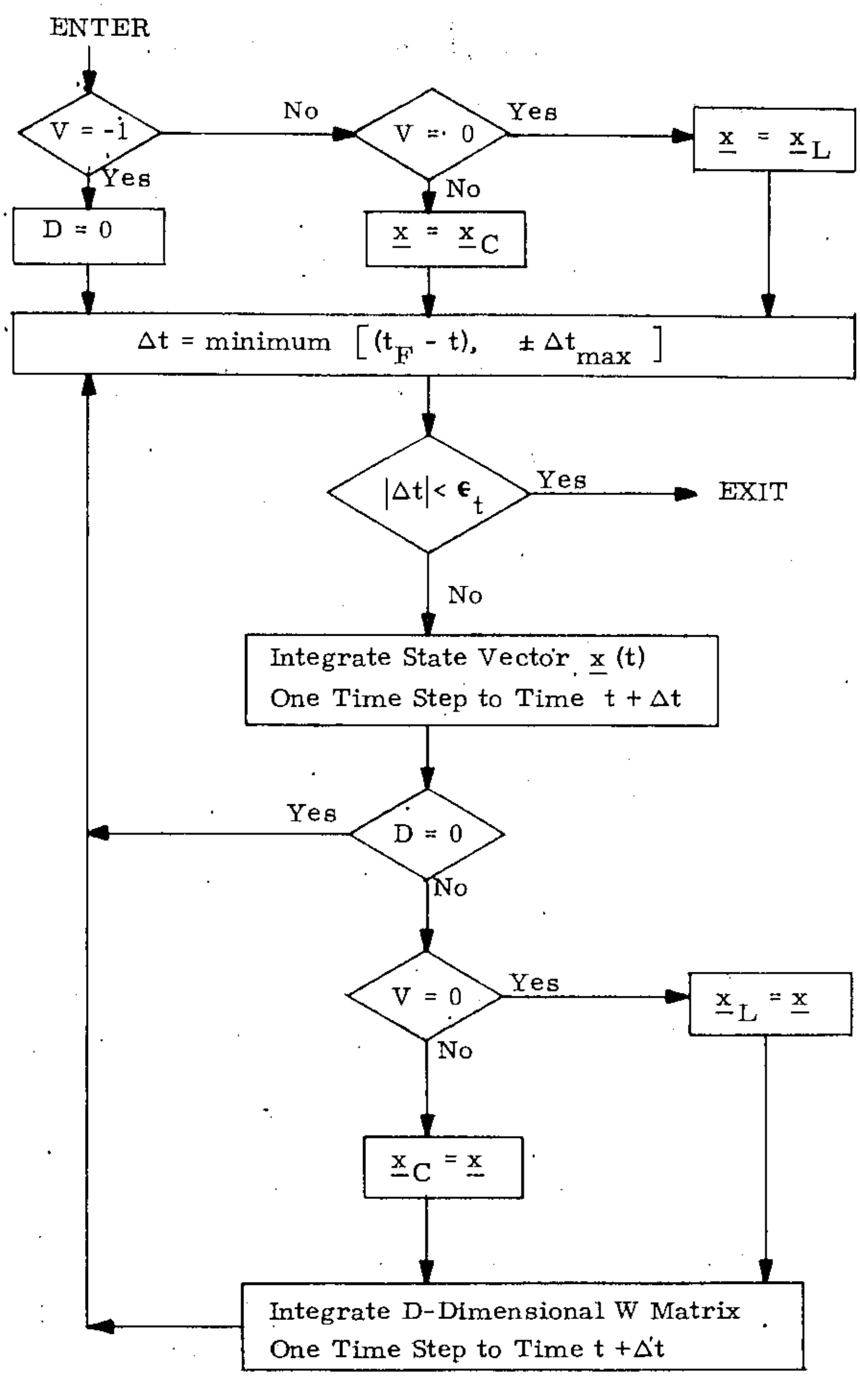

Figure 2.2-1 Simplified Coasting Integration Routine Logic Diagram

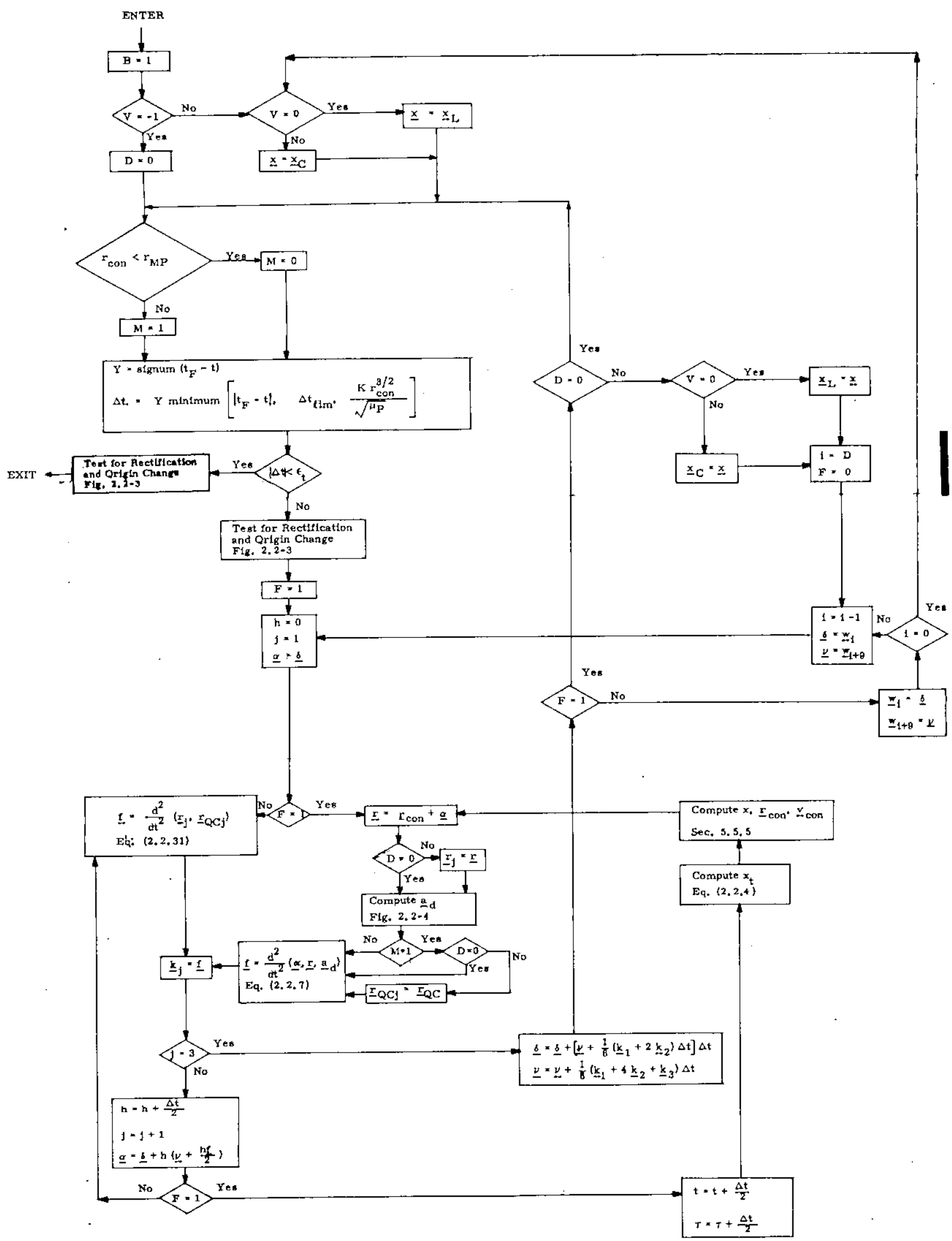

Figure 2.2-2 Coasting Integration Routine

Logic Diagram

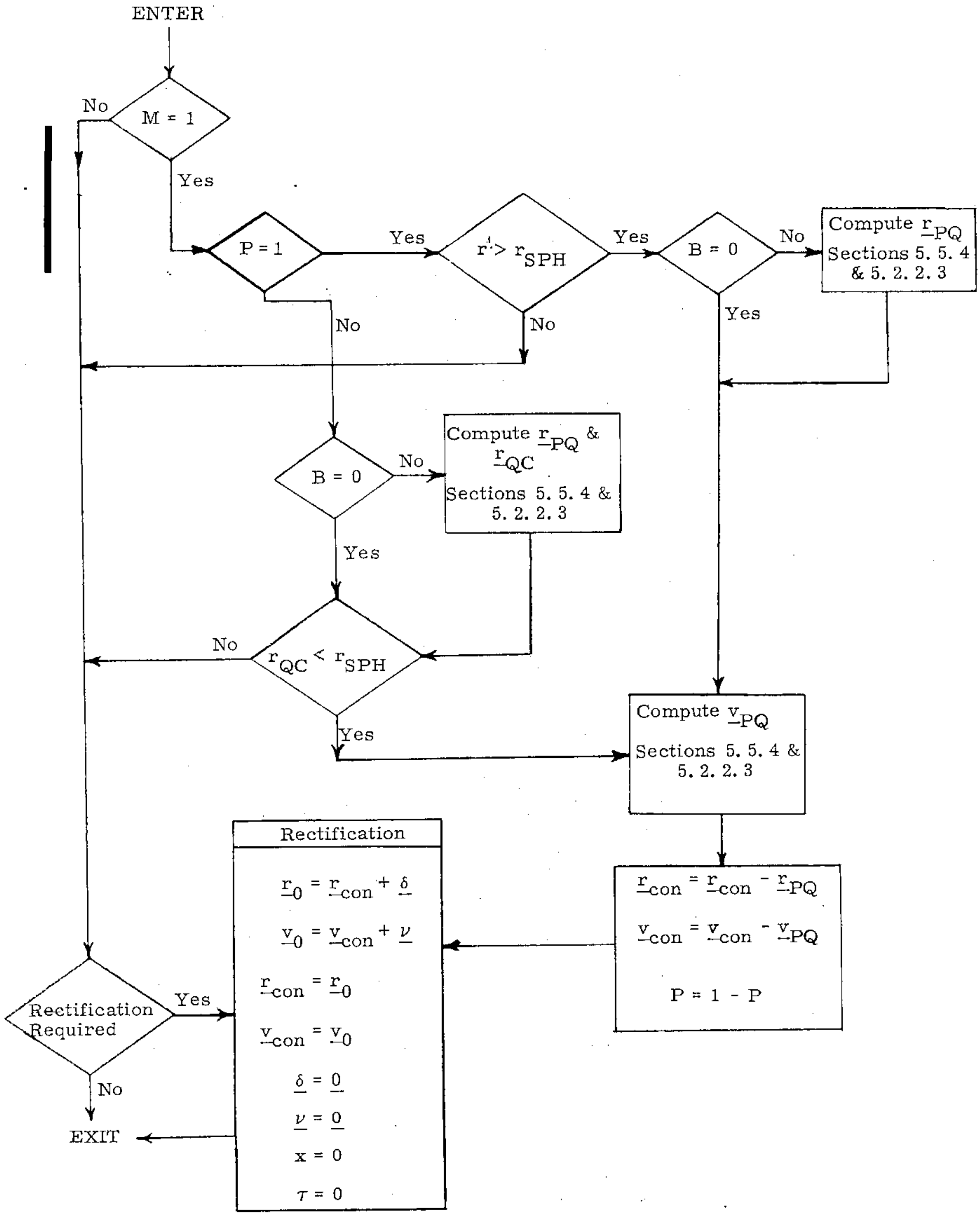

Figure 2.2-3 Rectification and Coordinate System Origin Change Logic Diagram

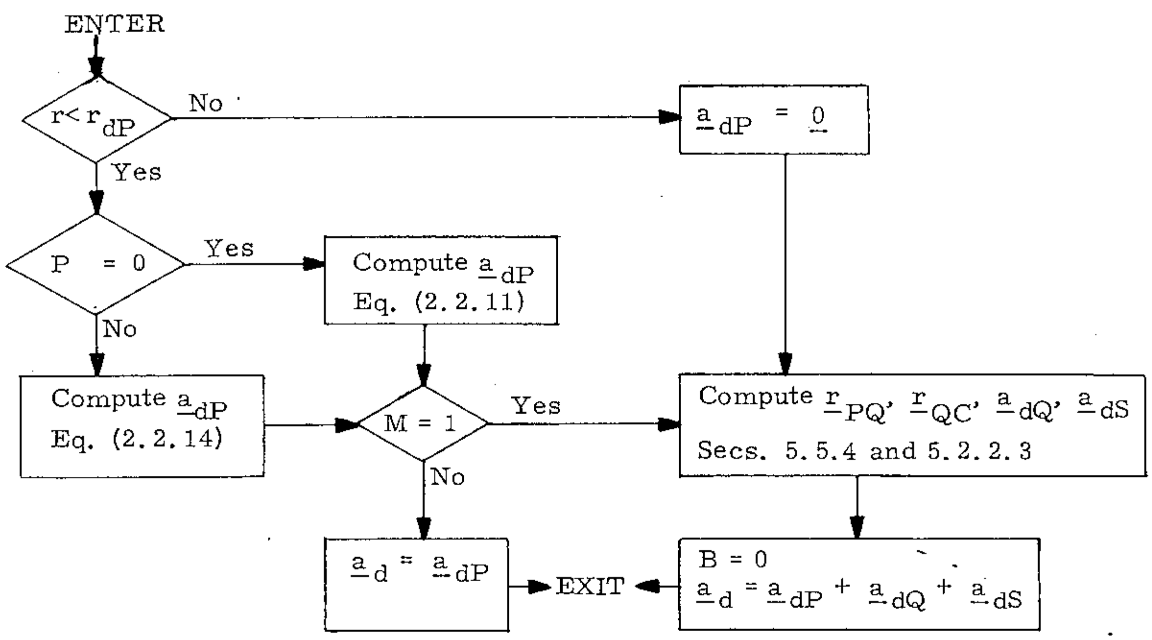

Figure 2.2-4 Disturbing Acceleration Selection Logic Diagram

In addition to the general criterion discussed in Section 5.2.2.2, the requirements for rectification (which are not shown in Fig. 2.2-2) are functions of

1) the computer word length,
2) the fact that the computations are performed in fixed-point arithmetic,
3) the scale factors of the variables, and
4) the accuracy of the Kepler Subroutine (Section 5.5.5).

If

$$\frac{\delta}{r_{con}} > 0.01$$

or if

$$\delta > \begin{cases} 0.75 \times 2^{22} \text{m for } P = 0 \\ 0.75 \times 2^{18} \text{m for } P = 1 \end{cases}$$

or if

$$\nu > \begin{cases} 0.75 \times 2^{3} \text{ m/csec for } P = 0 \\ 0.75 \times 2^{-1} \text{m/csec for } P = 1 \end{cases}$$

then rectification occurs at the point indicated in Fig. 2.2-2. Also, if the calculation of the acceleration (Eq. (2.2.7)) results in overflow (i.e. any component is equal to or greater than 2^{-16} m/csec^2 for P = 0, or 2^{-20} m/csec^2 for P = 1), then the program is recycled to the beginning of the time step and rectification is performed, provided that δ is not identically zero (which may occur if an attempt is made to extrapolate a state vector below the surface). In this exceptional case, an abort occurs with alarm code 00430.

The definitions of the various control constants which appear in Figs. 2.2-1 to 2.2-4 are as follows:

ϵ_t	=	integration time step criterion
r_{SPH}	=	radius of lunar sphere of influence
r_{dE}	=	radius of relevance for earth non-spherical gravitational perturbations
r_{dM}	=	radius of relevance for moon non-spherical gravitational perturbations
r_{ME}	=	distance from the earth beyond which mid-course perturbations are relevant.
r_{MM}	=	distance from the moon beyond which mid-course perturbations are relevant.

5.2.3 MEASUREMENT INCORPORATION ROUTINE

Periodically it is necessary to update the estimated position and velocity vectors of the vehicle (CSM or LM) by means of navigation measurements. At the time a measurement is made, the best estimate of the state vector of the spacecraft is the extrapolated estimate denoted by $\underline{x}'$. The first six components of $\underline{x}'$ are the components of the estimated position and velocity vectors. In certain situations it becomes necessary to estimate more than six quantities. Then, the state vector will be of nine dimensions. From this state vector estimate it is possible to determine an estimate of the quantity measured. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to update the indicated state vector as well as its associated error transition matrix as described in Section 5.2.1. The error transition matrix, W, is defined in Section 5.2.2.4.

This routine is used to compute deviations to be added to the components of the estimated state vector, and to update the estimated state vector by these deviations provided the deviations pass a state vector update validity test as described in Section 5.2.1.

Let D be the dimension (six or nine) of the estimated state vector. Associated with each measurement are the following parameters which are to be specified by the program calling this routine:

$\underline{b}$ = Geometry vector of D dimensions

$\overline{\alpha^2}$ = A priori measurement error variance

δQ = Measured deviation, the difference between the quantity actually measured and the expected value based on the original value of the estimated state vector $\underline{x}'$.

The procedure for incorporating a measurement into the estimated state vector is as follows:

1. Compute a D-dimensional $\underline{z}$ vector from

$$\underline{z} = W'^T \underline{b} \tag{2.3.1}$$

where W' is the error transition matrix associated with $\underline{x}'$.

2. Compute the D-dimensional weighting vector, $\underline{\omega}$, from

$$\underline{\omega}^T = \frac{1}{z^2 + \overline{\alpha^2}} \underline{z}^T W'^T \tag{2.3.2}$$

3. Compute the state vector deviation estimates from

$$\delta \underline{x} = \underline{\omega} \, \delta Q \tag{2.3.3}$$

4. If the data pass the validity test, update the state vector and the W matrix by

$$\underline{x} = \underline{x}' + \delta \underline{x} \tag{2.3.4}$$

$$W = W' - \frac{\underline{\omega} \, \underline{z}^T}{1 + \sqrt{\dfrac{\overline{\alpha^2}}{z^2 + \overline{\alpha^2}}}} \tag{2.3.5}$$

In order to take full advantage of the three-dimensional vector and matrix operations provided by the interpreter in the computer, the nine-dimensional W matrix will be stored sequentially in the computer as follows:

$$\underline{w}_0, \underline{w}_1, \cdots, \underline{w}_{26}$$

Refer to Section 5. 2. 2. 4 for the definition of the W matrix. Define the three-dimensional matrices

$$W_0 = \begin{pmatrix} \underline{w}_0{}^T \\ \underline{w}_1{}^T \\ \underline{w}_2{}^T \end{pmatrix} \qquad W_1 = \begin{pmatrix} \underline{w}_3{}^T \\ \underline{w}_4{}^T \\ \underline{w}_5{}^T \end{pmatrix} \quad \cdots\cdots \quad W_8 = \begin{pmatrix} \underline{w}_{24}{}^T \\ \underline{w}_{25}{}^T \\ \underline{w}_{26}{}^T \end{pmatrix} \tag{2.3.6}$$

so that

$$W = \begin{pmatrix} W_0{}^T & W_1{}^T & W_2{}^T \\ W_3{}^T & W_4{}^T & W_5{}^T \\ W_6{}^T & W_7{}^T & W_8{}^T \end{pmatrix} \tag{2.3.7}$$

Let the nine-dimensional vectors $\delta\underline{x}$, $\underline{b}$, $\underline{\omega}$, and $\underline{z}$ be partitioned as follows:

$$\delta\underline{x} = \begin{pmatrix} \delta\underline{x}_0 \\ \delta\underline{x}_1 \\ \delta\underline{x}_2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} \underline{b}_0 \\ \underline{b}_1 \\ \underline{b}_2 \end{pmatrix} \quad \underline{\omega} = \begin{pmatrix} \underline{\omega}_0 \\ \underline{\omega}_1 \\ \underline{\omega}_2 \end{pmatrix} \quad \underline{z} = \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_8 \end{pmatrix} = \begin{pmatrix} \underline{z}_0 \\ \underline{z}_1 \\ \underline{z}_2 \end{pmatrix} \tag{2.3.8}$$

Then, the computations shown in Eqs. (2. 3. 1) through (2. 3. 3) are performed as follows, using three-dimensional operations:

$$\underline{z}_i = \sum_{j=0}^{\frac{D}{3}-1} W'_{i+3j}\, \underline{b}_j$$

$$a = \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j \cdot \underline{z}_j + \overline{\alpha^2}$$

$$\underline{\omega}_i^T = \frac{1}{a} \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j^T\, W'_{3i+j} \tag{2.3.9}$$

$$\delta \underline{x}_i = \delta Q\, \underline{\omega}_i \qquad \left(i = 0,\ 1,\ \dots,\ \frac{D}{3} - 1\right)$$

Equation (2. 3. 5) is written

$$\gamma = \frac{1}{1 + \sqrt{\overline{\alpha^2}/a}}$$

$$\underline{w}_{i+9j} = \underline{w}'_{i+9j} - \gamma\, z_i\, \underline{\omega}_j \qquad \left(\begin{array}{l} i = 0,\ 1,\ \dots,\ D - 1 \\ j = 0,\ 1,\ \dots,\ \frac{D}{3} - 1 \end{array}\right) \tag{2.3.10}$$

The Measurement Incorporation Routine is divided into two subroutines, INCORP1 and INCORP2. The subroutine INCORP1 consists of Eqs. (2. 3. 9), while INCORP2 is composed of Eqs. (2. 3. 4) and (2. 3. 10). The method of using these subroutines is illustrated in Fig. 2. 3-1.

Since the estimated position and velocity vectors are maintained in two pieces, conic and deviation from the conic, Eq. (2. 3. 4) cannot be applied directly. The estimated position and velocity deviations resulting from the measurement, $\delta \underline{x}_0$ and $\delta \underline{x}_1$, are added to the vectors $\underline{\delta}$ and $\underline{\nu}$, the position and velocity deviations from the conics, respectively. Since $\underline{\delta}$ and $\underline{\nu}$ are maintained to much higher accuracy than the conic position and velocity vectors, a possible computation overflow situation exists whenever Eq. (2. 3. 4) is applied. If overflow does occur, then it is necessary to reinitialize the Coasting Integration Routine (Section 5. 2. 2) by the process of rectification as described in Section 5. 2. 2. 2. The logic flow of the subroutine INCORP2 is illustrated in detail in Fig. 2. 3-2.

Overflow occurs when

$$\delta \geq \begin{cases} 2^{22} \text{ m for } P = 0 \\ 2^{18} \text{ m for } P = 1 \end{cases}$$

or

$$\nu \geq \begin{cases} 2^{3} \text{ m/csec for } P = 0 \\ 2^{-1} \text{ m/csec for } P = 1 \end{cases}$$

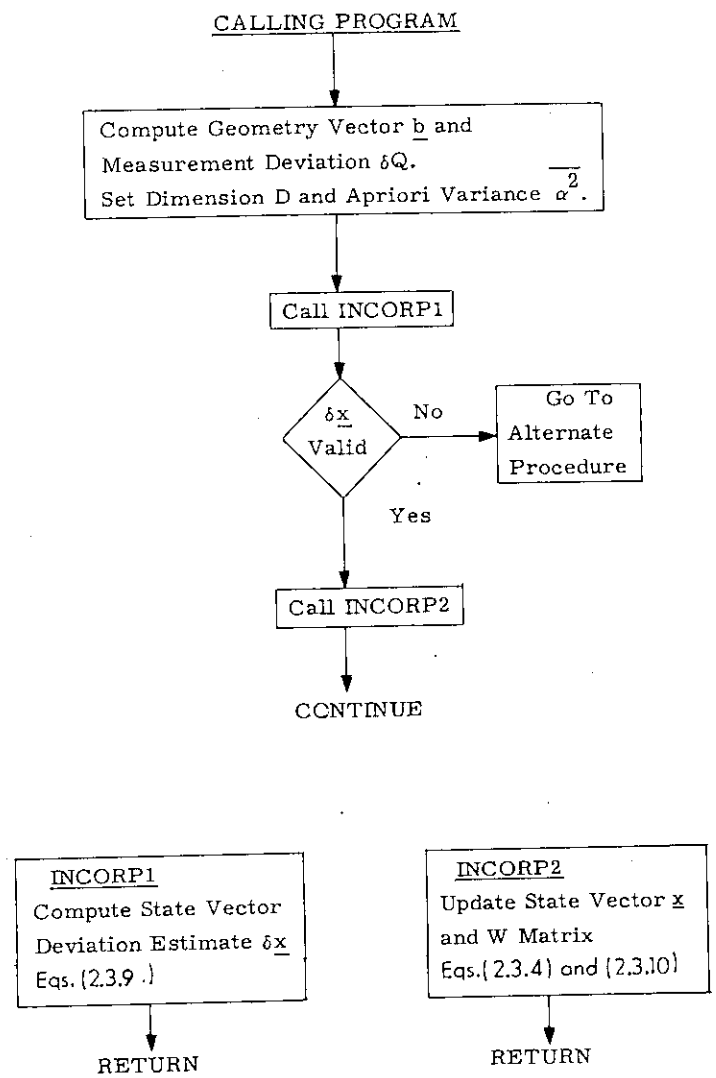

Fig. 2.3-1 Measurement Incorporation Procedure

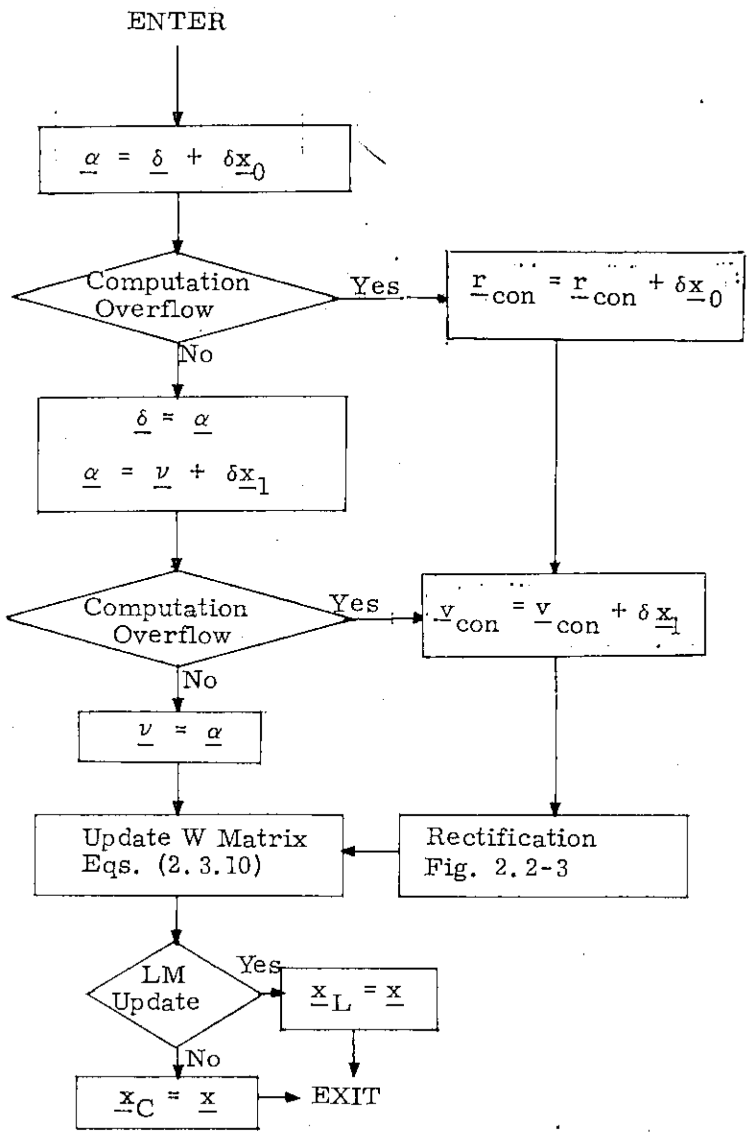

Fig. 2.3-2 INCORP2 Subroutine Logic Diagram

5.2.4 ORBIT NAVIGATION ROUTINE

5.2.4.1 Landmark Tracking Procedure

While the CSM is in lunar or earth orbit, landmark optical tracking data are used to update the estimated CSM state vector and the coordinates of the landmark that is being tracked, as described in Section 5. 2. 1. This routine is used to process the landmark-tracking measurement data, as shown in simplified form in Fig. 2. 1-1, and is used normally in lunar orbit in the lunar landing mission. The routine also can be used in earth orbit during abort situations or alternate missions.

In order to initially acquire and maintain optical tracking, the CSM attitude must be oriented such that the CSM-to-landmark line-of-sight falls within the SCT field of view. In the CSM GNCS there is no automatic vehicle attitude control during the landmark tracking procedure. Any desired attitude control must be accomplished manually by the astronaut.

If the astronaut wishes, he may use the Automatic Optics Designate Routine (Section 5. 6. 8) as an aid in the acquisition of the landmark. This routine has two modes which are relevant to orbit navigation. In the advanced ground track mode (which is useful in lunar orbit for surveillance, selection, and tracking of possible landing sites) the routine drives the CSM optics to the direction of the point on the ground track of the spacecraft at a time slightly more than a specified number of orbital revolutions ahead of current time.

In the landmark mode (which is useful for acquisition of a specified landmark) the routine drives the optics to the estimated direction of the specified landmark. Either the revolution number or the landmark must be specified by the astronaut. The computations and positioning commands in this routine are repeated periodically provided the optics mode switch is set to CMC. Thus, in the advanced ground track mode, the astronaut is shown continuously the ground track of the CSM for a future revolution. The reason for this mode is that it is desirable to select a landing site which is near the CSM orbital plane at the LM lunar landing time.

The Automatic Optics Designate Routine is used in other routines to align the CSM optics to the directions of the following sighting targets:

1) The LM during the rendezvous phase

2) A specified star during IMU alignment procedures

After the astronaut has acquired the desired landmark (not necessarily the one specified to the Automatic Optics Designate Routine), he switches the optics mode to MANUAL and centers the SCT or SXT reticle on the landmark. When accurate tracking is achieved, he presses the optics MARK button, causing the time of the measurement and all optics and IMU gimbal angles to be stored in the CMC. Up to five unrejected navigation sightings of the same landmark may be made during the tracking interval,and all sets of navigation data are acquired before processing of the data begins.

After the astronaut has completed the tracking of a landmark, he is asked by the CMC whether or not he wishes to identify the tracked landmark. If he does, then he enters into the CMC through the keyboard the identification number or the coordinates of the landmark, and the data are processed as described in Section 5. 2. 4. 2 thru 5. 2. 4. 5.

The coordinates of twenty-five preselected lunar landmarks will be stored in the CMC fixed memory. In addition, the coordinates of the landing site will be stored in the CMC erasable memory and can be updated during the mission. In general, code numbers will be used to identify landmarks during a lunar-orbit navigation phase, but it will be necessary for the astronaut to enter the actual coordinates in the case of earth landmarks.

If the astronaut does not identify the landmark, then the Landing Site Designation procedure (Section 5.2.4.3) is used for the navigation data processing. In this process the landmark is considered to be unknown, and the first set of navigation data is used to compute an initial estimate of the landmark location. The remaining sets of data are then processed as described in Section 5. 2. 4. 2 to update the estimated nine-dimensional CSM-landmark state vector.

Whether the landmark is identified or not, one further option is available to the astronaut. He may specify that one of the navigation sightings is to be considered the designator for an offset landing site near the tracked landmark. In this case, the designated navigation data set is saved, the remaining sets of data are processed as described above, and then the estimated offset landing site location is determined from the saved data as described in Section 5. 2. 4. 4. This procedure offers the possibility of designating a landing site in a flat area of the moon near a landmark which is suitable for optical navigation tracking but not for landing.

Each set of navigation data which is used for state vector updating and not for landing site designation or offset produces two updates as described in Section 5. 2. 4. 2. For the first navigation data set the magnitudes of the first proposed changes in the estimated CSM position and velocity vectors, δr and δv, respectively, are displayed for astronaut approval. If the astronaut accepts these proposed changes, then all state vector updates will be performed, and all the information obtained during the tracking of this landmark will be incorporated into the state vector estimates. A detailed discussion of this state vector update validity check is given in Section 5. 2. 1.

After all of the sets of navigation data have been processed, the astronaut has the option of having the updated landmark coordinates (or the coordinates of the unknown landmark) stored in the erasable memory registers allocated to the landing site coordinates. In this manner the original coordinates of the landing site can be revised, or a new landing site can be selected.

The various functions for which the sets of navigation data acquired from the line-of-sight tracking measurements are used are presented in Sections 5. 2. 4. 2 through 5. 2. 4. 4. A detailed description in the form of logic diagrams of the entire Orbit Navigation Routine with all of its options is given in Section 5. 2. 4. 5.

5.2.4.2 State Vector Update from Landmark Sighting

As mentioned in Section 5. 2. 1 the orbit navigation concept involves the nine-dimensional state vector

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix} \qquad (2.\ 4.\ 1)$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors, respectively, and $\underline{r}_\ell$ is the estimated landmark position vector. Both the CSM state vector and the landmark position vector are estimated and updated through the processing of optical tracking data. A simplified functional diagram of the orbit navigation procedure is illustrated in Fig. 2. 1-1. In this section the method of updating the estimated nine-dimensional state vector from a landmark line-of-sight navigation measurement is given.

After the preferred CSM attitude is achieved and optical tracking acquisition is established (Section 5. 2. 4. 1), the astronaut enters tracking data into the CMC by pressing the optics MARK button when he has centered the SCT or SXT reticle on the landmark. As described in Section 5. 2. 4. 1, each set of navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector, $\underline{u}_M$, along the CSM-to-landmark line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = [\text{REFSMMAT}]^T \, [\text{NBSM}] \, \underline{u}_{NB} \qquad (2.\ 4.\ 2)$$

where [REFSMMAT] and [NBSM] are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Eq. (2. 4. 2) are defined in Section 5. 6. 3.

For the purpose of navigation it is convenient to consider the measured unit vector, $\underline{u}_M$, to be the basic navigation data. This navigation measurement of the line-of-sight vector, $\underline{u}_M$, is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the landmark and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-landmark measurements, and using the Measurement Incorporation Routine (Section 5.2.3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_\ell$ be the estimated CSM and landmark position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

1. Calculate the estimated CSM-to-landmark line-of-sight from

$$\begin{aligned} \underline{r}_{CL} &= \underline{r}_\ell - \underline{r}_C \\ \underline{u}_{CL} &= \text{UNIT}\,(\underline{r}_{CL}) \end{aligned} \tag{2.4.3}$$

2. Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_{CL} \times \underline{u}_M) \tag{2.4.4}$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by an angle of less than 2^{-19} rad., then a computation overflow occurs in the execution of Eq. (2.4.4), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\,\underline{x}$, would be negligible for both state vector updates.

(3) Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_s \times \underline{u}_{CL}) \tag{2.4.5}$$

(4) Calculate the nine-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \frac{1}{r_{CL}} \underline{u}_s \tag{2.4.6}$$

$$\underline{b}_1 = \underline{0} \tag{2.4.7}$$

$$\underline{b}_2 = -\underline{b}_0 \tag{2.4.8}$$

(5) Determine the measured deviation, δQ, from

$$\begin{aligned} \delta Q &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \cos^{-1} (\underline{u}_s \cdot \underline{u}_{CL}) \\ &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \end{aligned} \tag{2.4.9}$$

(6) Incorporate the fictitious star-landmark measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step 6 is the state vector update validity check for the first proposed update.

It should be noted that the initialization of the star direction, $\underline{u}_s$, which is given by Eq. (2.4.4), is such that the first artificial star (computed from Eq. (2.4.5)) will yield the maximum value for the measured deviation, δQ, which is obtained from Eq. (2.4.9). The reason for selecting the first $\underline{u}_s$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was accepted by the astronaut, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-landmark line-of-sight vector from Eq. (2.4.3) using the updated values of the estimated CSM and landmark position vectors, $\underline{r}_C$ and $\underline{r}_\ell$, respectively. Then, Steps 3 - 6 are repeated, this time with no state vector update validity check.

If the astronaut rejects the first state vector update, then all of the navigation data is discarded, and no update occurs.

The results of the processing of the measured line-of-sight vector, $\underline{u}_M$, are updated values of the estimated position and velocity vectors of the CSM, $\underline{r}_C$ and $\underline{v}_C$, respectively, and an updated value of the estimated landmark position vector, $\underline{r}_\ell$.

5.2.4.3 Landing Site Designation

As mentioned in Section 5.2.4.1 the nine-dimensional orbit navigation procedure provides the means of mapping on the surface of the planet a point which is designated only by a number of sets of optical tracking data. This process may be used to redesignate the landing site optically, or as an unknown landmark orbit navigation procedure.

Assume that an unmapped landmark has been tracked, and N sets of optical measurement data have been acquired as described in Section 5.2.4.1. Let $\underline{u}_M$ be the measured unit CSM-to-landmark line-of-sight vector obtained from the first set of measurement data by means of Eq. (2.4.2). An estimate of the landmark position at the time of the first navigation sighting, t_M, is given by

$$\underline{r}_\ell = \underline{r}_C + r_C \left[\cos A - \left(\frac{r_0^2}{r_C^2} - \sin^2 A \right)^{1/2} \right] \underline{u}_M \qquad (2.4.10)$$

where

$$\cos A = - \frac{\underline{u}_M \cdot \underline{r}_C}{r_C} \qquad (2.4.11)$$

$\underline{r}_C$ is the estimated CSM position vector at time t_M, and r_0 is the estimated planetary radius. This initial estimated landmark position vector, $\underline{r}_\ell$, and the estimated CSM state vector are then updated by means of the standard Orbit Navigation Routine and the last N - 1 sets of tracking measurement data exactly as if the designated point were a mapped landmark.

The final results of this procedure are a location estimate for the designated point and an improvement in the estimated CSM state vector.

5.2.4.4 Landing Site Offset

During the landing site selection operation any visible landmark may be tracked that is in, or near, the desired landing area. In most cases this visible landmark will not be an acceptable touch down point, and it is desirable to offset the desired landing point away from the visible landmark used for tracking. This is accomplished by tracking the visible landmark and processing this data as previously described for either a mapped or an unknown landmark depending upon the type of landmark tracked. During this tracking operation a designated navigation data set can be taken by positioning the SXT to the desired actual landing point. This designated data set is saved, and, after the tracking data is processed for the visible landmark, the offset landing site location is computed from the saved data by means of Eq. (2.4.10). In this landing site offset calculation, the magnitude of the estimated position vector of the visible landmark is used for the estimated planetary radius r_0 in Eq. (2.4.10).

5.2.4.5 Orbit Navigation Logic

After all optical landmark tracking data have been acquired, the data processing procedure is initialized as illustrated in Fig. 2.4-1. It is assumed that the following items are stored in erasable memory at the start of the procedure shown in this figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5.2.2.6.

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5.2.2.4.

$$\text{ORBWFLAG} = \begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$$

This flag or switch is maintained by programs external to the Orbit

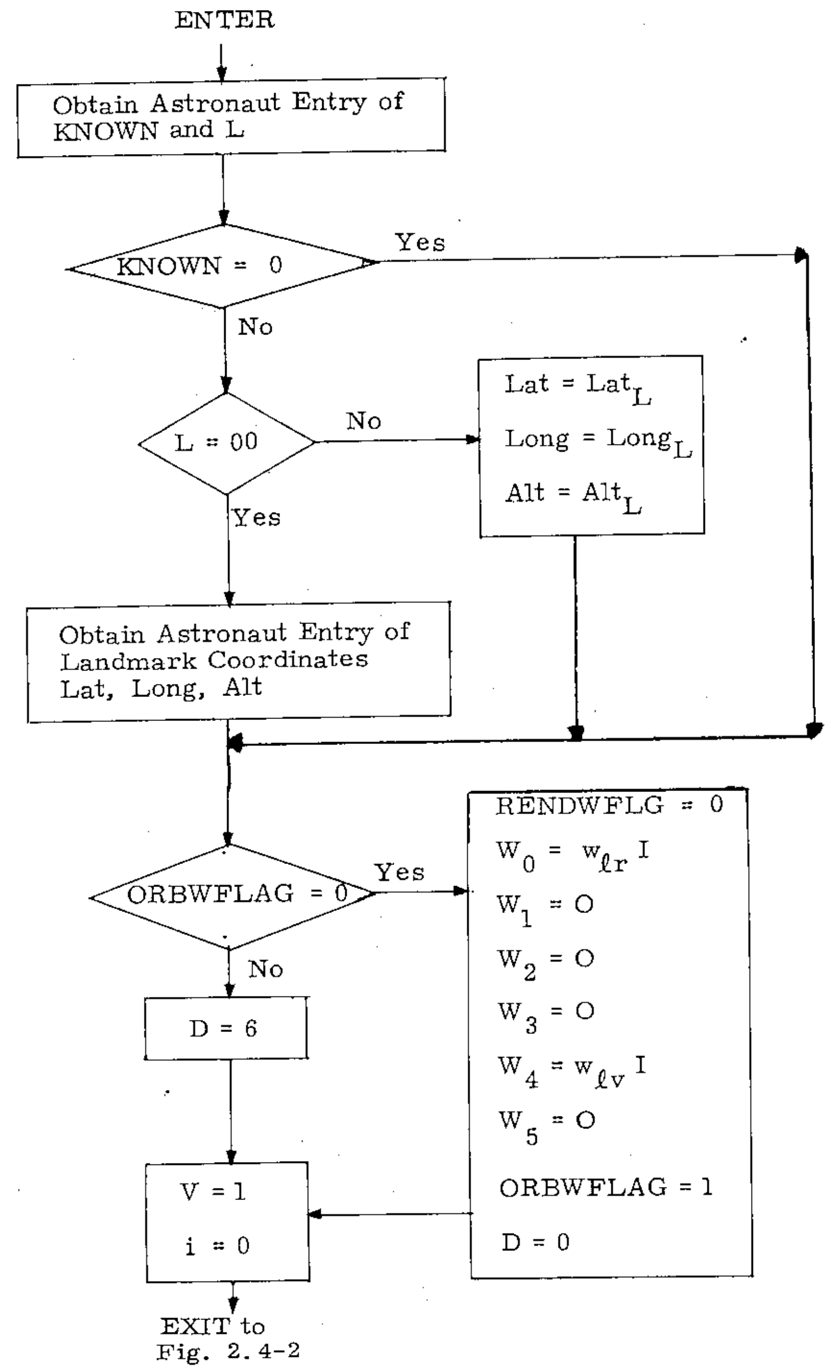

Fig. 2.4-1 Orbit Navigation Routine Initialization

Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing landmark tracking data. The flag is set to zero after each of the following procedures:

1) CSM state vector update from ground
2) Rendezvous navigation
3) Astronaut Command

RENDWFLG = Switch similar to ORBWFLAG but used for rendezvous navigation.

[REFSMMAT] = Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System

N = Number of unrejected sets of navigation data acquired during the tracking of the landmark

t_{M1} to t_{MN} = The N measurement times associated with the N sets of navigation data

N sets of five optics and IMU gimbal angles each

$w_{\ell r}$, $w_{\ell v}$, w_{ℓ} = Preselected W matrix initial diagonal elements

var_{RP} = variance of the primary body radius error

OFF = $\begin{cases} 1 \text{ through 5 for index of landing site offset designator} \\ 0 \text{ for no landing site offset designator} \end{cases}$

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6. I and O are the three-dimensional identity and zero matrices, respectively, and i is an index which is used to count the navigation data sets.

In the initialization routine the astronaut enters into the CMC through the keyboard the following two items:

$$
\text{KNOWN} = \begin{cases} 1 \text{ for mapped or known landmark} \\ 0 \text{ for unmapped landmark or landing site designation} \end{cases}
$$

$$
\text{L} = \begin{cases} 00 \text{ for a landmark whose coordinates are not stored in CMC memory} \\ 01 \text{ for the landing site} \\ 02 \text{ through } 26 \text{ for index number of a landmark whose coordinates are stored in CMC fixed memory} \end{cases}
$$

If

$$
\text{KNOWN} = 1 \quad \text{and} \quad \text{L} = 00
$$

then the astronaut is further requested to enter the coordinates of the landmark; that is, latitude (Lat), longitude (Long) and altitude (Alt). Altitude is defined with respect to the mean lunar radius for lunar landmarks, and the Fischer ellipsoid for earth landmarks.

After completion of the initialization procedure the Orbit Navigation Routine begins processing the data.

For convenience of calculation in the CMC, Eqs. (2.4.5), (2.4.6), (2.4.7), and (2.4.9) are reformulated and regrouped as follows:

$$
\begin{aligned}
\underline{u}_s &= \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \\
\underline{b}_0 &= \underline{u}_s \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL}\left[\cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\right]
\end{aligned}
\qquad (2.4.12)
$$

This set of equations is used both by the Orbit Navigation Routine and the Rendezvous Navigation Routine (Section 5.2.5) in processing optical tracking data. To validate the use of Eqs. (2.4.12) it is necessary only to let

$$
\overline{\alpha^2} = r_{CL}^{\;2}\,(\text{var}_{SCT} + \text{var}_{IMU}) \qquad (2.4.13)
$$

where var_{SCT} and var_{IMU} are the a priori estimates for the SCT and IMU angular error variances per axis, respectively.

The processing of the N sets of landmark-tracking navigation data is illustrated in Fig. 2.4-2. In the figure F is the altitude flag as defined in Section 5.5.3.

As shown in the figure, the CSM state vector is integrated to the time of each measurement, and the measured line-of-sight vector $\underline{u}_M$ is computed. If this data set is an offset designator, then the vector $\underline{u}_M$ and the time are saved, and the program proceeds to the next measurement. If this is the first navigation data set for a known landmark, then the W matrix is initialized and the data are processed to obtain the two state vector updates. If this is the first measurement for an unknown landmark, then the landmark location is computed, and the W matrix is initialized (Fig. 2.4-3) using a procedure in which the geometry of the landmark mapping is explicitly accounted for. For this case no state vector updating occurs. For all other sets of navigation data, two state vector updates are normally obtained.

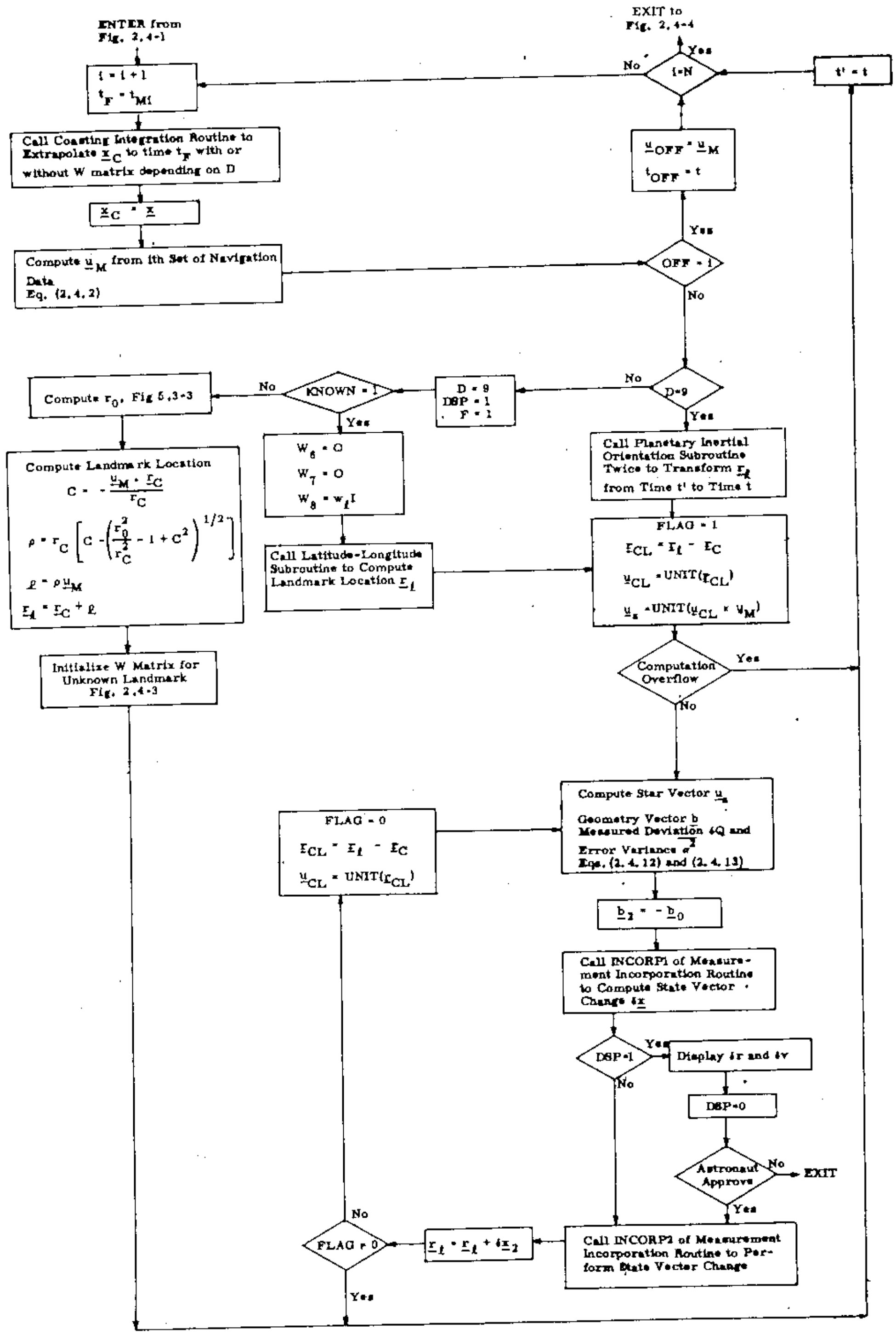

Figure 2.4-2 Orbit Navigation Routine Logic Diagram

ENTER

↓

Compute W_6 and W_7

$$d = \underline{u}_M \cdot \underline{r}_\ell$$

$$U = I - \frac{1}{d}\,\underline{u}_M \underline{r}_\ell^{\,T}$$

$$\underline{w}_{i+18} = U\,\underline{w}_i \quad (i = 0, 1, \ldots, 5)$$

↓

Determine W_8

$$\begin{pmatrix} e_0 & & \\ e_1 & e_2 & \\ e_3 & e_4 & e_5 \end{pmatrix} = \frac{1}{2}\rho^2 \mathrm{var}_{SCT}\,UU^T + \mathrm{var}_{RP}\left(\frac{r_0}{d}\right)^2 \underline{u}_M \underline{u}_M^T$$

$$w_{74} = \sqrt{e_5}$$

$$w_{73} = e_4 / w_{74}$$

$$w_{72} = e_3 / w_{74}$$

$$w_{76} = \sqrt{e_2 - w_{73}^2}$$

$$w_{75} = (e_1 - w_{72} w_{73}) / w_{76}$$

$$w_{78} = \sqrt{e_0 - w_{72}^2 - w_{75}^2}$$

$$W_8 = \begin{pmatrix} w_{72} & w_{73} & w_{74} \\ w_{75} & w_{76} & 0 \\ w_{78} & 0 & 0 \end{pmatrix}$$

↓

EXIT

Fig. 2.4-3 W Matrix Initialization for Unknown Landmark

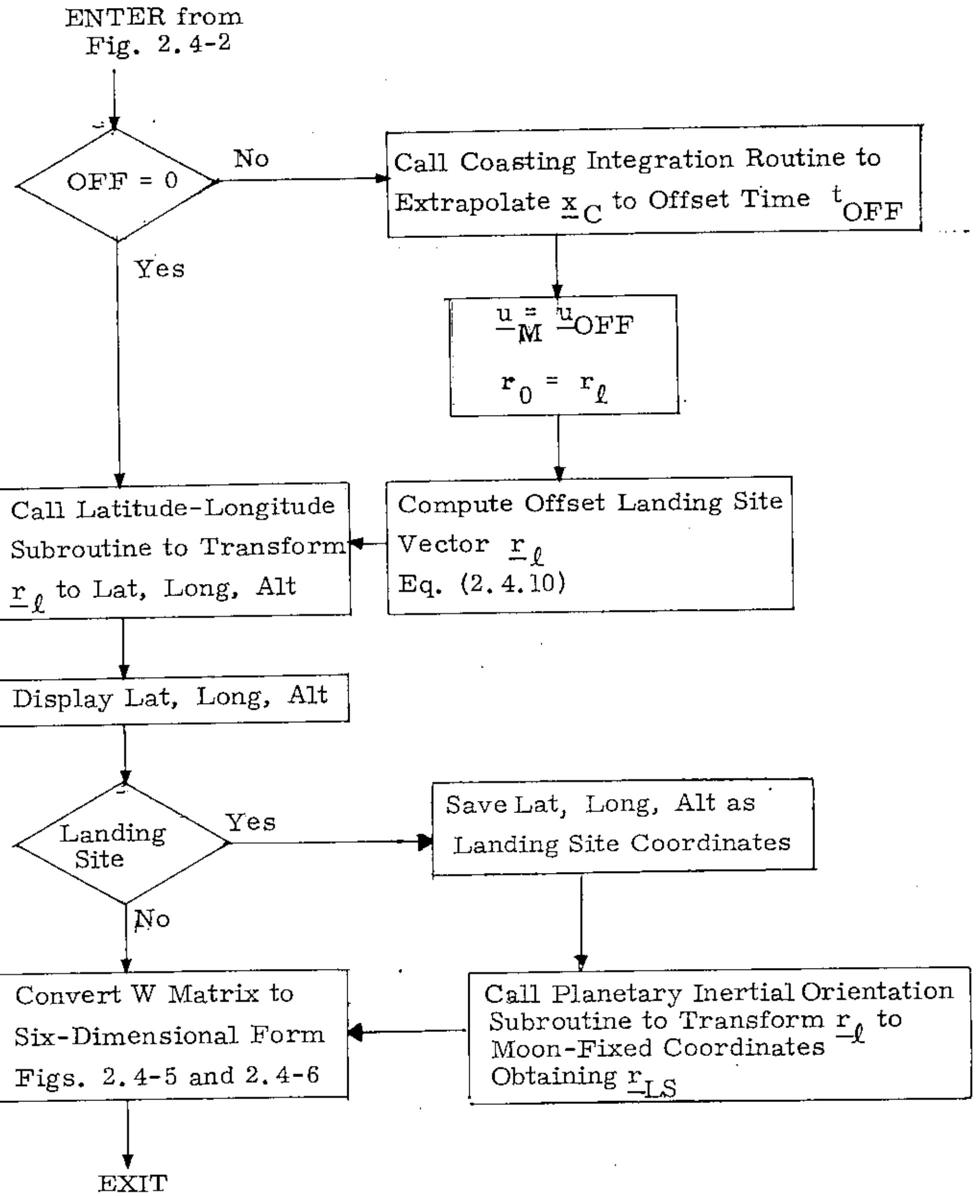

Fig. 2.4-4 Orbit Navigation Routine Termination

After the processing of the data is completed, the location of the offset landing site is computed from the saved data, if a data set was so designated, as shown in Fig. 2.4-4. Then, the final estimated landmark position vector is converted to latitude, longitude, altitude coordinates and these coordinates are displayed. If the tracked landmark is to be the landing site, then the landing site coordinates and the landing site vector ($\underline{r}_{LS}$) are saved in erasable memory.

The final operation of the Orbit Navigation Routine is to convert the nine-dimensional error transition matrix, W, to a six-dimensional matrix with the same CSM position and velocity estimation error variances and covariances. The reason for this procedure is that the W matrix, when it is initialized for processing the data associated with the next landmark, must reflect the fact that the initial landmark location errors are not correlated with the errors in the estimated CSM position and velocity vectors. Of course, after processing measurement data, these cross correlations become nonzero, and it is for this reason that the nine dimensional procedure works, and that it is necessary to convert the final W matrix to six-dimensional form.

The solution to the conversion problem is not unique. A convenient solution is obtained in the following manner.

The error transition matrix, W, has been defined in Section 5.2.3 in terms of the nine three-dimensional submatrices W_0, W_1, ..., W_8 as follows:

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \quad (2.4.14)$$

Let the elements of each of these submatrices be defined by

$$W_i^T = \begin{pmatrix} w_{9i} & w_{9i+3} & w_{9i+6} \\ w_{9i+1} & w_{9i+4} & w_{9i+7} \\ w_{9i+2} & w_{9i+5} & w_{9i+8} \end{pmatrix} \quad (i = 0,\ 1,\ \ldots,\ 8) \tag{2.4.15}$$

The W matrix is then converted to the following six-dimensional form:

$$\begin{pmatrix} W_0^T & W_1^T \\ W_3^T & W_4^T \end{pmatrix} = \begin{pmatrix} w_0 & w_3 & w_6 & w_9 & w_{12} & w_{15} \\ w_1 & w_4 & w_7 & w_{10} & w_{13} & 0 \\ w_2 & w_5 & w_8 & w_{11} & 0 & 0 \\ w_{27} & w_{30} & w_{33} & 0 & 0 & 0 \\ w_{28} & w_{31} & 0 & 0 & 0 & 0 \\ w_{29} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.4.16}$$

$$W_2 = W_5 = O$$

The twenty-one non-zero elements of the converted W matrix are computed by solving the following twenty-one equations:

$$\sum_{k=0}^{5} w_{i+3k} \ w_{j+3k} = \sum_{k=0}^{8} w'_{i+3k} \ w'_{j+3k} = e_p \begin{pmatrix} i,j = 0,1,2,27, \\ 28,29 \\ i \leq j \\ p = 0,1,\ldots,20 \end{pmatrix} \tag{2.4.17}$$

where primes refer to quantities before conversion, and the following table gives p as a function of i and j:

i	29	28	27	2	1	0	28	27	2	1	0	27	2	1	0	2	1	0	1	0	0
j	29	29	29	29	29	29	28	28	28	28	28	27	27	27	27	2	2	2	1	1	0
p	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

The twenty-one elements e_0 to e_{20} are computed as illustrated in Fig. 2.4-5. The converted W matrix is then calculated as shown in Fig. 2.4-6. Included in this procedure are negative radicand and zero divisor checks.

The Orbit Navigation Routine is now ready to process the data acquired in tracking the next landmark.

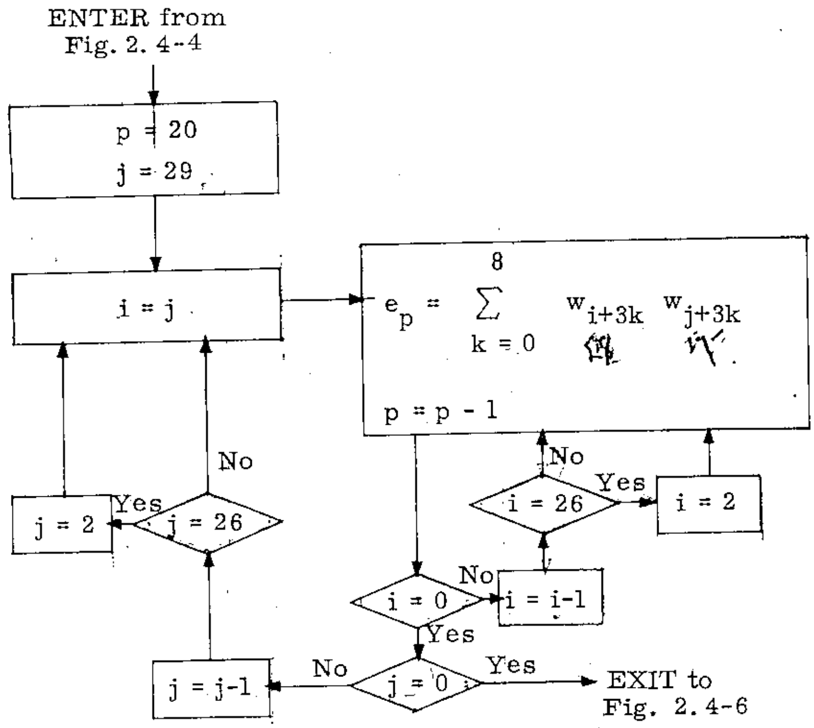

Fig. 2.4-5 Orbit Navigation Routine W Matrix Conversion, Part I

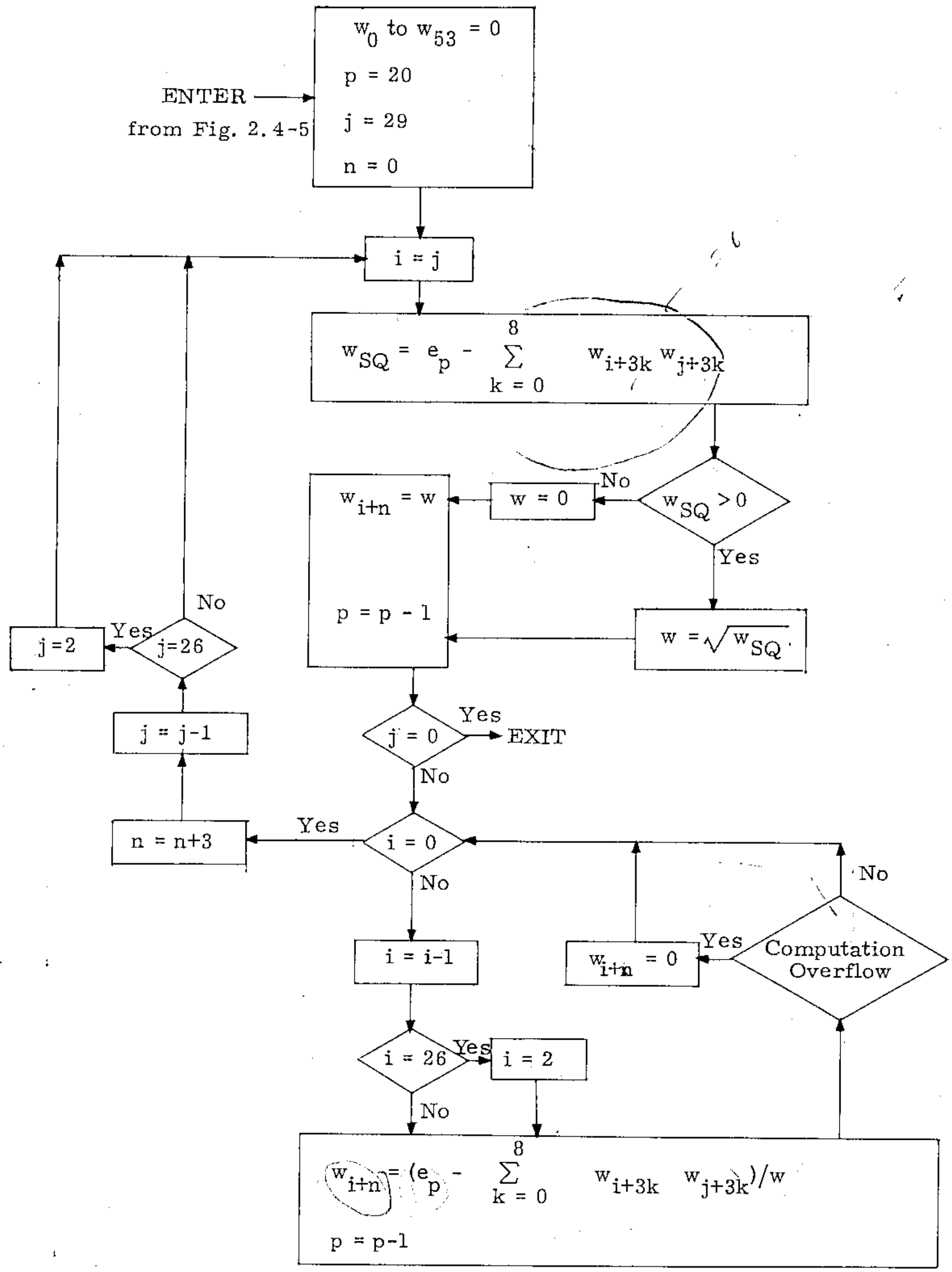

Fig. 2.4-6 Orbit Navigation Routine W Matrix Conversion Part II

5.2.5 RENDEZVOUS NAVIGATION PROGRAM

5.2.5.1 Target Acquisition and Tracking

During mission phases involving rendezvous the Rendezvous Navigation Program (P-20) is used to obtain CSM optical and/or VHF range-link tracking data of the LM to update the estimated state vector of either the CSM or the LM, as discussed in Section 5.2.1 and outlined in Fig. 2.1-2. A general block diagram of program P-20 is shown in Fig. 2.5-1. At the beginning of program P-20 certain preliminary steps are performed such as setting the Rendezvous, Track, Update, and Preferred Attitude flags. The Rendezvous flag is set to denote that program P-20 is being used. When this flag is reset, program P-20 is permanently terminated. The purpose of the other flags is given in the following sections. In addition to initializing the above flags, the state vector update option is automatically set to the LM, the Optics and VHF Range Mark counters are set to zero, and the LM and CSM state vectors are extrapolated to the current time using the Coasting Integration Routine (Section 5.2.2).

Afterwards, the Tracking Attitude Routine (R-61) is used to orient the vehicle to a preferred attitude for optical tracking of the LM. This attitude is achieved by maneuvering the vehicle so as to establish coincidence between a body-fixed reference vector and the estimated line-of-sight (LOS) to the LM. In addition routine R-61 establishes a vehicle attitude rate in order to maintain this coincidence. The LOS in this case is determined by advancing the estimated state vectors of the CSM and LM to the current time using the Kepler Subroutine (Section 5.5.5). The vehicle orientation about the body-fixed reference vector is not controlled by the Tracking Attitude Routine and must be corrected by the astronaut if it is unsatisfactory. There are actually two different body-fixed reference vectors

Functions Under Separate Control of Astronaut:

1. Rendezvous Tracking Sighting Mark Routine (R-21)
2. Backup Rendezvous Tracking Sighting Mark Routine (R-23)
3. Processing of VHF range data in routine R-22

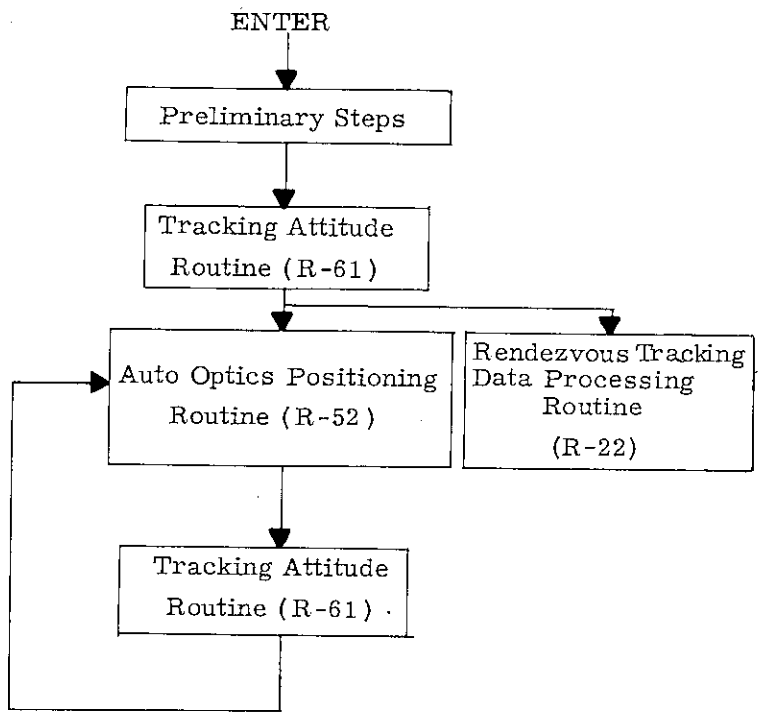

Fig. 2.5-1 Rendezvous Navigation Program

which may be aligned to the LOS by the Tracking Attitude Routine during rendezvous navigation. If the Preferred Attitude flag is set, as is the case at the beginning of program P-20, the body-fixed reference vector used by Tracking Attitude Routine is that shown in Fig. 2.5-2. This vector is the preferred vector if the sextant (SXT) is going to be used to optically track the LM. The vector is so chosen to be approximately in the center of the common coverage sector of the SXT and the RR transponder. If the astronaut is going to use the back-up optical device (Crew Optical Alignment Sight), he should reset the Preferred Attitude flag so that the Tracking Attitude Routine will use the other body-fixed reference vector which is the +X-axis of the CSM.

The quantities issued to the RCS DAP by the Tracking Attitude Routine (R-61) in order to establish the preferred tracking attitude and attitude rate are:

1. $\underline{\omega}_{CA}$ - desired LOS rate in control axis coordinates
2. IGA_D, MGA_D, OGA_D - desired IMU gimbal angles which define the desired vehicle attitude to the RCS DAP.
3. $\Delta \underline{GA}$ - a vector defining the desired incremental changes in the IMU gimbal angles every 0.1 second.

The desired LOS rate $\underline{\omega}_{CA}$ in control axis coordinates is obtained as follows:

$$\underline{\omega}_{LOS} = \left[\text{REFSMMAT} \right] \left\{ \frac{\text{UNIT}\,(\underline{r}_L - \underline{r}_C) \times (\underline{v}_L - \underline{v}_C)}{\left| \underline{r}_L - \underline{r}_C \right|} \right\}$$

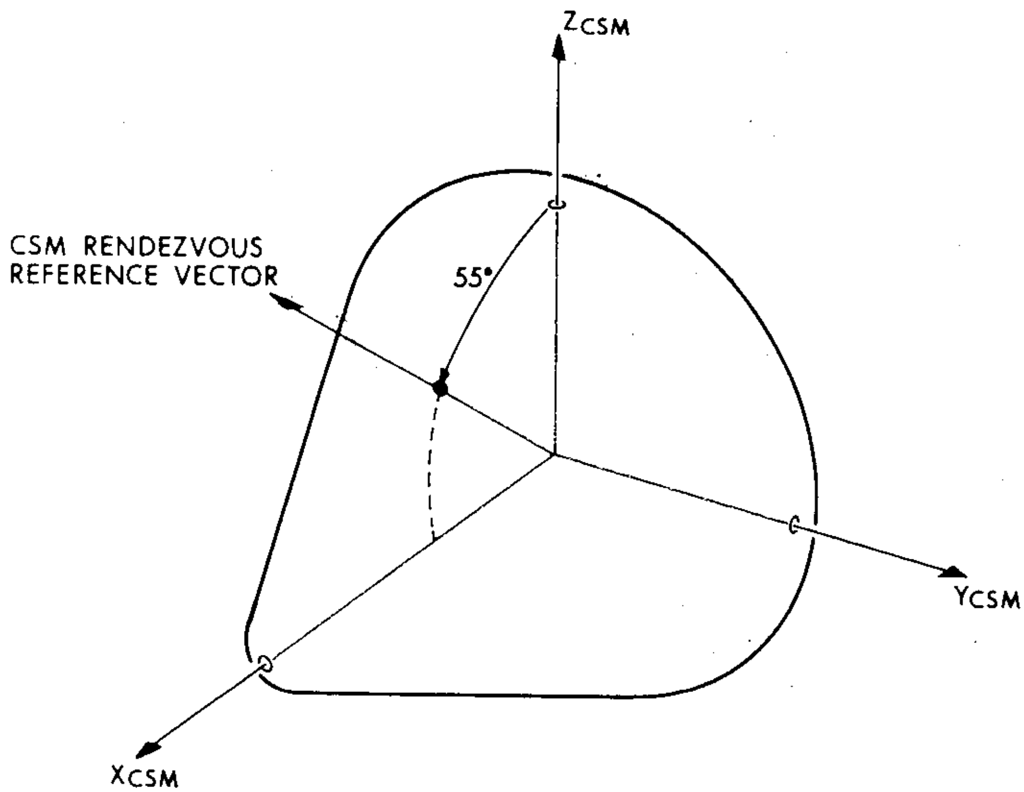

Figure 2.5-2 CSM Rendezvous Reference Vector for the SXT

$$\underline{\omega}_{CA} = \left[NBCA\right] \left[SMNB\right] \underline{\omega}_{LOS}$$

where $\underline{\omega}_{LOS}$ is the LOS rate in stable member coordinates, $\left[SMNB\right]$ is the transformation matrix defined in Section 5.6.3.2.1, $\left[NBCA\right]$ is the matrix for transforming a vector from navigation base to control axis coordinates as defined in Section 5.3.3.3.3, and $\underline{r}_L$, $\underline{v}_L$, $\underline{r}_C$, and $\underline{v}_C$ are the position and velocity vectors of the LM and CSM in basic reference coordinates obtained by using the Kepler Subroutine of Section 5.5.5.

The vector $\Delta \underline{GA}$ is computed as follows:

$$\Delta \underline{GA} = \begin{bmatrix} \cos IGA \sec MGA & 0 & -\sin IGA \sec MGA \\ -\cos IGA \tan MGA & 1 & \sin IGA \tan MGA \\ \sin IGA & 0 & \cos IGA \end{bmatrix} (0.1)\, \underline{\omega}_{LOS}$$

where $\underline{\omega}_{LOS}$ is the LOS rate in stable member coordinates computed previously.

The desired IMU gimbal angles IGA_D, MGA_D, and OGA_D are obtained by using the routine VECPOINT of Section 3 where the input to this routine is the unit vector $\underline{u}_{LOS}$ defining the line-of-sight from the CSM to the LM in stable member coordinates and the unit vector $\underline{u}_{BF}$ defining one of the previously mentioned body-fixed reference vectors in vehicle or navigation base coordinates. The vector $\underline{u}_{LOS}$ is computed as follows:

$$\underline{u}_{LOS} = \left[REFSMMAT\right] \text{UNIT} (\underline{r}_L - \underline{r}_C)$$

Once the desired attitude and attitude rate are achieved by routine R-61, it is seen in Fig. 2.5-1 that the Auto Optics Positioning Routine (R-52) and the Rendezvous Tracking Data Processing Routine (R-22) are called by program P-20. The purpose of the Auto Optics Positioning Routine (Section 5.6.8) is to drive the SXT to the line-of-sight of the LM, a star, or a landmark. To insure that the above line-of-sight is that of the LM, program P-20 sets the LM Target flag just before calling routine R-52. If the Preferred Attitude and Update flags are set and the optics mode switch is in the CMC position, routine R-52 will periodically compute the line-of-sight to the LM and drive the optics to it. In addition, routine R-52 will call the Tracking Attitude Routine (R-61) each time to re-establish the desired tracking attitude and attitude rate. However, it should be noted that routine R-61 will only perform its task every fourth time it is called by routine R-52 due to control exercised over a counter in routine R-61.

The purpose of the Rendezvous Tracking Data Processing Routine (R-22) is to periodically process the optical and/or the VHF range-link tracking data to update the state vector of either the CSM or LM as defined by the state vector update option. This option is automatically set to the LM at the beginning of program P-20 but can later be set to the CSM by the astronaut if it is considered necessary. The optical tracking data processed by this routine is obtained by using concurrently either the Rendezvous Tracking Sighting Mark Routine (R-21) or the Backup Rendezvous Tracking Sighting Mark Routine (R-23). Either routine can be called and terminated by the astronaut during operation of program P-20. If the astronaut wishes to have the VHF range-link data processed, he must set the VHF Range flag after routine R-22 has been put into operation by program P-20. It is assumed that the astronaut has activated the VHF range-link and has assured himself of target acquisition by the range-link before he sets the VHF Range flag. The VHF range-link operates in conjunction with the voice-link and measures the range between the CSM and the LM. Target acquisition can be ascertained by the astronaut by observing the range display provided with the VHF range-link. The range data is sent to the CMC upon request by the Rendezvous Tracking Data Processing Routine (R-22). A detailed discussion of this routine is given in Section 5. 2. 5. 2. The astronaut should not use the VHF range-link for navigation updates beyond 200 nm since the VHF range measurement accuracy beyond this range is unspecified.

As previously mentioned, the optical tracking data processed by routine R-22 is obtained by using either the Rendezvous Tracking Sighting Mark Routine (R-21) or the Backup Rendezvous Tracking Sighting Mark Routine (R-23), both of which are described in detail in Section 4. Under normal circumstances the astronaut will call routine R-21 during the operation of program P-20 and use the sextant (SXT) to track the LM because of its greater accuracy and target detection capability. After the preferred tracking attitude has been established and the SXT has been driven to the LOS of the LM, the astronaut acquires the target through the SXT and switches the optics mode switch to the MANUAL position so that he may manually center the SXT reticle on the target. When accurate tracking is achieved, he presses the MARK button, which causes the measurement time and the SXT and IMU gimbal angles to be recorded. This data is stored in a certain erasable memory location denoted as Position 1. When the next mark is made, the data from the previous mark is transferred to Position 2 where it becomes available to the Rendezvous Tracking Data Processing Routine (R-22) and the new mark data is placed in Position 1.

In order to achieve the desired accuracy levels of the vehicle state vector with optical tracking data, it is important that this data be taken at essentially uniform intervals over the tracking period. Uniform tracking intervals are defined to be about once every minute throughout the phase, as opposed to all of the tracking data being taken over a short period. The reason for this requirement is given in Section 5. 2. 5. 2. 2.

If it becomes necessary to use the Crew Optical Alignment Sight (COAS) instead of the SXT to obtain optical tracking data, use is made of the Backup Rendezvous Tracking Sighting Mark Routine (R-23). At the beginning of this routine the astronaut loads the coordinates of the COAS. These coordinates are the equivalent SXT shaft and trunnion angles of the COAS, which would be the same as those indicated for the SXT if it were possible to point the SXT in the same direction as the COAS. Afterwards, the astronaut acquires the LM with the COAS and performs an optical mark by keying a PROCEED into the DSKY instead of depressing the MARK button since the button cannot be reached by him when using the COAS. Whenever he keys in a PROCEED, the measurement time, the IMU gimbal angles, and the previously loaded coordinates of the COAS are stored in the same Position 1 used by routine R-21 and are transferred to Position 2 when the next set of mark data is placed in Position 1.

Optical tracking of the LM from the CSM and request for data from the VHF range-link are suspended during rendezvous maneuvers by either vehicle. If the CSM is the passive vehicle and is tracking the LM for monitoring and possible abort retrieval, LM rendezvous maneuvers are voice-linked to the CSM as an ignition time and three velocity components in a LM local vertical coordinate system, and then entered as updates to the estimated LM state vector in the CMC. Upon receipt of this data, SXT tracking and data processing should be suspended until after the maneuver. The update is accomplished by means of the Target ΔV Program, P-76. If the CSM is the active vehicle then the estimated CSM state vector is updated by means of the Average-G Routine (Section 5. 3. 2) during the maneuver.

5.2.5.2 Rendezvous Tracking Data Processing Routine

5.2.5.2.1 General Operation

As indicated in Section 5.2.5.1 the purpose of the Rendezvous Tracking Data Processing Routine (R-22) is to periodically process the optical and/ or VHF range-link tracking data to update the state vector of either the CSM or LM.

The logic associated with this routine is given in Fig. 2.5-3 where it is seen at the top of the figure that routine R-22 alternates between checking the status of the VHF Range flag and checking to see if optics mark data is in Position 2. Whenever optics mark data is found in Position 2 it is transferred to Position 3 and used to calculate the correction (or update) to the state vector. Whenever the VHF Range flag is found to be set, the routine reads the range from the VHF range-link if at least 60 seconds have expired since the last time range was read. Immediately after reading the range, a check is made to see if the Data Good discrete is being received from the VHF range-link, signifying that the range tracking network is tracking the target satisfactorily. If the Data Good discrete is present, the range data is used to calculate the correction to the state vector. The time t_{VHF} is used as the time of range measurement. If the Data Good discrete is not present, the Tracker Fail Light is turned on.

The manner in which the state vector correction is calculated for either optics mark or range-link data is given in Section 5.2.5.2.2. The connections labeled E, F, and G in Fig. 2.5-3 correspond to those given for the logic flow of the rendezvous navigation computations given in Fig. 2.5-4. To distinguish between the two types of data in Fig. 2.5-3 use is made of a Source Code (SC) which is equal to one or two depending on whether it is optics mark or range-link data, respectively.

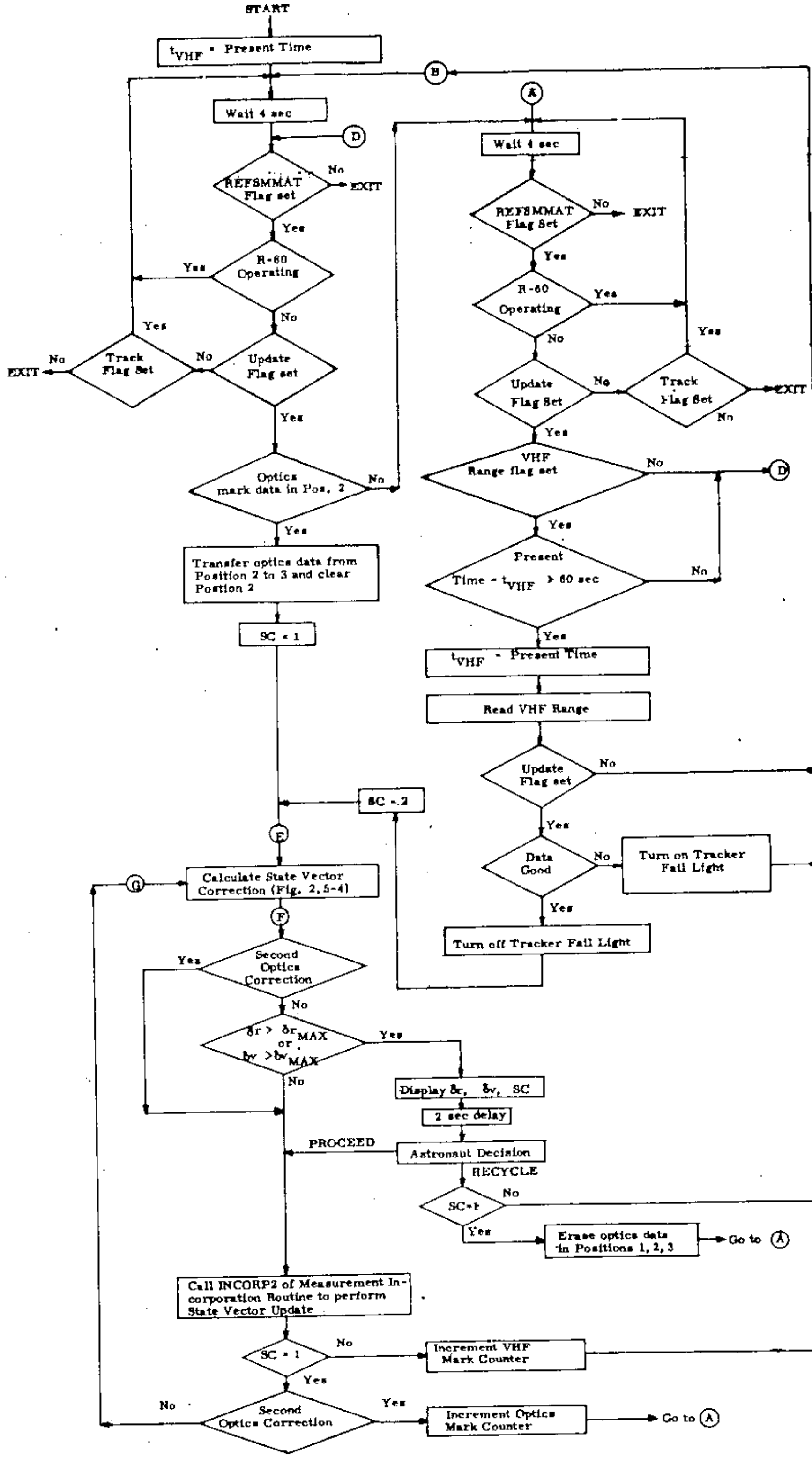

Figure 2.5-3 Rendezvous Tracking Data Processing

After the state vector correction has been calculated it is seen in Fig. 2.5-3 that a check is made to see if the magnitudes of the proposed correction in position and velocity (δ r and δ v) exceed certain threshold limits (δr_{MAX} and δv_{MAX}) stored in erasable memory. The same threshold limits are used in this check whether the state vector correction is based on optics mark or range-link data. The purpose of this threshold check is to insure the validity of the proposed state vector correction (update). If the proposed correction exceeds either threshold limit, the magnitudes of the correction in position and velocity (δ r and δ v) and the Source Code (SC) are displayed to the astronaut. If the correction is the result of optical tracking data and the astronaut is sure that he is tracking the LM, he should command the update. Otherwise he should reject the data and recheck the optical tracking. Additional details on the threshold check are given in Section 5.2.1. It should be noted that if optical data is being processed, there will actually be two separate corrections to the state vector instead of one due to the manner in which the optics data is used to calculate the state vector correction as explained in Section 5.2.5.2.2. Since the second optics correction is calculated using the state vector updated with the first optics correction, it is unlikely that the magnitude of second correction will exceed that of the first. Consequently, a threshold check is made in Fig. 2.5-3 only on the first optics correction.

At various points in Fig. 2.5-3 it is seen that a check is made to see if the Update flag is present (set). This flag is removed when there is no desire to process the optics and range-link data. It is removed by the CMC during CSM ΔV maneuvers and during certain time consuming computations, and by the Target Delta V Program. If the Update flag is not present, a check is made on the Track flag. The Track flag is removed when it is desired to temporarily terminate the rendezvous navigation process. If the Track flag is present, it is seen in Fig. 2.5-3 that the routine will continue to monitor the Track and Update flags in a standby status until one of the flags changes state.

The range obtained from the VHF range-link by the Rendezvous Tracking Data Processing Routine is that measured by the range-link between the CSM and the LM. This data is sent to the CMC from the range-link as a 15 bit binary data word R_{RL}. In the CMC the range R_M in nautical miles is obtained as follows:

$$R_M = k_{RL} R_{RL}$$

where k_{RL} is the bit weight in nautical miles.

5.2.5.2.2 Rendezvous Navigation Computations

Each set of optical navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector, $\underline{u}_M$, along the CSM-to-LM line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = \left[\mathrm{REFSMMAT}\right]^T \left[\mathrm{NBSM}\right] \underline{u}_{NB} \qquad (2.5.1)$$

where $\left[\mathrm{REFSMMAT}\right]$ and $\left[\mathrm{NBSM}\right]$ are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Eq. (2.5.1) are defined in Section 5.6.3.

For the purpose of navigation it is convenient to consider the measured unit vector, $\underline{u}_M$, to be the basic navigation data. This navigation measurement of the line-of-sight vector, $\underline{u}_M$, is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the LM and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-LM measurements, and using the Measurement Incorporation Routine (Section 5. 2. 3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_L$ be the estimated CSM and LM position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

1. Calculate the estimated CSM-to-LM line-of-sight from

$$\begin{aligned} \underline{r}_{CL} &= \underline{r}_L - \underline{r}_C \\ \underline{u}_{CL} &= \text{UNIT}\ (\underline{r}_{CL}) \end{aligned} \tag{2.5.2}$$

2. Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_{CL} \times \underline{u}_M) \tag{2.5.3}$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by an angle of less than 2^{-19} rad., then a computation overflow occurs in the execution of Eq. (2.5.3), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\,\underline{x}$, would be negligible for both state vector updates.

3 Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_s \times \underline{u}_{CL}) \tag{2.5.4}$$

4 Calculate the six-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \pm \frac{1}{r_{CL}} \underline{u}_s \tag{2.5.5}$$

$$\underline{b}_1 = \underline{0} \tag{2.5.6}$$

where the + (−) sign is selected if the CSM (LM) state vector is being updated.

5 Determine the measured deviation, δQ, from

$$\begin{aligned} \delta Q &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \cos^{-1} (\underline{u}_s \cdot \underline{u}_{CL}) \\ &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \end{aligned} \tag{2.5.7}$$

6 Incorporate the fictitious star-LM measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step 6 is the state vector update validity check for the first proposed update, as described in Section 5.2.1.

It should be noted that the initialization of the star direction, $\underline{u}_s$, which is given by Eq. (2.5.3), is such that the first artificial star (computed from Eq. (2.5.4)) will yield the maximum value for the measured deviation, δQ, which is obtained from Eq. (2.5.7). The reason for selecting the first $\underline{u}_s$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was valid, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-LM line-of-sight vector from Eq. (2.5.2) using the updated values of the estimated CSM and LM position vectors, $\underline{r}_C$ and $\underline{r}_L$, respectively. Then, Steps 3 - 6 are repeated, this time with no state vector update validity check.

If the first proposed state vector update does not pass the validity check, then the magnitudes of the proposed changes in the estimated position and velocity vectors, δr and δv, respectively, are displayed. If the astronaut is sure that he is tracking the LM, then he should command the update. Otherwise he should reject the data and recheck the optical tracking. A detailed discussion of this state vector update validity check is given in Section 5.2.1.

The results of the processing of the measured line-of-sight vector, $\underline{u}_M$, are updated values of the estimated position and velocity vectors of the CSM or the LM. These two estimated state vectors are used to compute required rendezvous targeting parameters as described in Section 5.4.4.

For convenience of calculation in the CMC, Eqs. (2.5.4)-(2.5.7) are reformulated and regrouped as follows:

$$
\begin{aligned}
\underline{u}_s &= \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \\
\underline{b}_0 &= \pm\, \underline{u}_s \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL}\left[\cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\right]
\end{aligned}
\qquad (2.5.8)
$$

This set of equations is used both by the Rendezvous Navigation Routine and the Orbit Navigation Routine (Section 5.2.4) in processing optical tracking data. To validate the use of Eqs.(2.5.8) it is necessary only to let

$$
\overline{\alpha^2} = r_{CL}^{\;2}\,(var_{SXT} + var_{IMU}) + var_{INT} \qquad (2.5.9)
$$

where var_{SXT} and var_{IMU} are the a priori estimates for the SXT and IMU angular error variances per axis, respectively. The variable var_{INT} is included in Eq. (2.5.9) for the purpose of smoothing the effects of coasting integration inaccuracies.

After VHF range-link acquisition is established (Section 5.2.5.1) the measured CSM-to-LM range, R_M, is automatically acquired at approximately one minute intervals. The geometry vector and measured deviation for a VHF range measurement are given by

$$
\begin{aligned}
\underline{b}_0 &= \pm\, \underline{u}_{CL} \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= R_M - r_{CL}
\end{aligned}
\qquad (2.5.10)
$$

where the + (-) sign is selected if the LM (CSM) state vector is being updated. The measurement error variance is computed from

$$\overline{\alpha^2} = \text{maximum}\ (r_{CL}^2\ \text{var}_R,\ \text{var}_{R\,min}) \qquad (2.5.11)$$

where var_R is the range error variance corresponding to a percentage error and $\text{var}_{R\,min}$ is the minimum range error variance.

The rendezvous navigation computations are illustrated in Fig. 2.5-4. As shown in the figure, this set of computations is entered from two points (E and G) of Fig. 2.5-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5.2.2.6.

$\underline{x}_L$ = Estimated LM state vector.

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ or $\underline{x}_L$ as defined in Section 5.2.2.4.

$$\text{RENDWFLG} = \begin{cases} 1 & \text{for valid W matrix} \\ 0 & \text{for invalid W matrix} \end{cases}$$

This flag or switch is maintained by programs external to the Rendezvous Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing LM tracking data. The flag is set to zero after each of the following procedures:

1) State vector update from ground

2) Orbit or Cislunar-Midcourse Navigation

3) Astronaut Command

ORBWFLAG = Switch similar to RENDWFLG but used for orbit or cislunar-midcourse navigation.

[REFSMMAT] = Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System.

t_F = Measurement time.

Five optics and IMU gimbal angles.

(or)

R_M = Measured Range.

w_{rr} and w_{rv} = Preselected W matrix initial diagonal elements. There is one value for each of these two initial diagonal W matrix elements stored in the CMC erasable memory. These parameters nominally represent rendezvous injection conditions. They can be changed during the mission by the astronaut or by RTCC.

SC = Source Code $\begin{cases} 1 & \text{Optics Measurement} \\ 2 & \text{Range Measurement} \end{cases}$

The variables D and V are indicators which control the Coasting Integration Routine (Section 5. 2. 2) as described in Section 5. 2. 2. 6, and I and O are the three-dimensional identity and zero matrices, respectively.

The optical measurement incorporation procedure outlined above should be repeated at about one minute intervals throughout the rendezvous phase except during powered maneuvers, as described in Section 5. 2. 5. 1. As indicated in Section 5.2.5.1, it is important that the SXT tracking data be taken over as large an angular sector in inertial space swept out by the line-of-sight as possible to achieve desired state vector accuracy levels. This essentially uniform tracking operation is required since the SXT tracking provides information only in directions normal to the line-of-sight, as indicated by Eq. (2. 5. 5), and the line-of-sight must be allowed to rotate in inertial space to achieve more complete update data. As mentioned previously, the VHF range data is obtained automatically, approximately every minute.

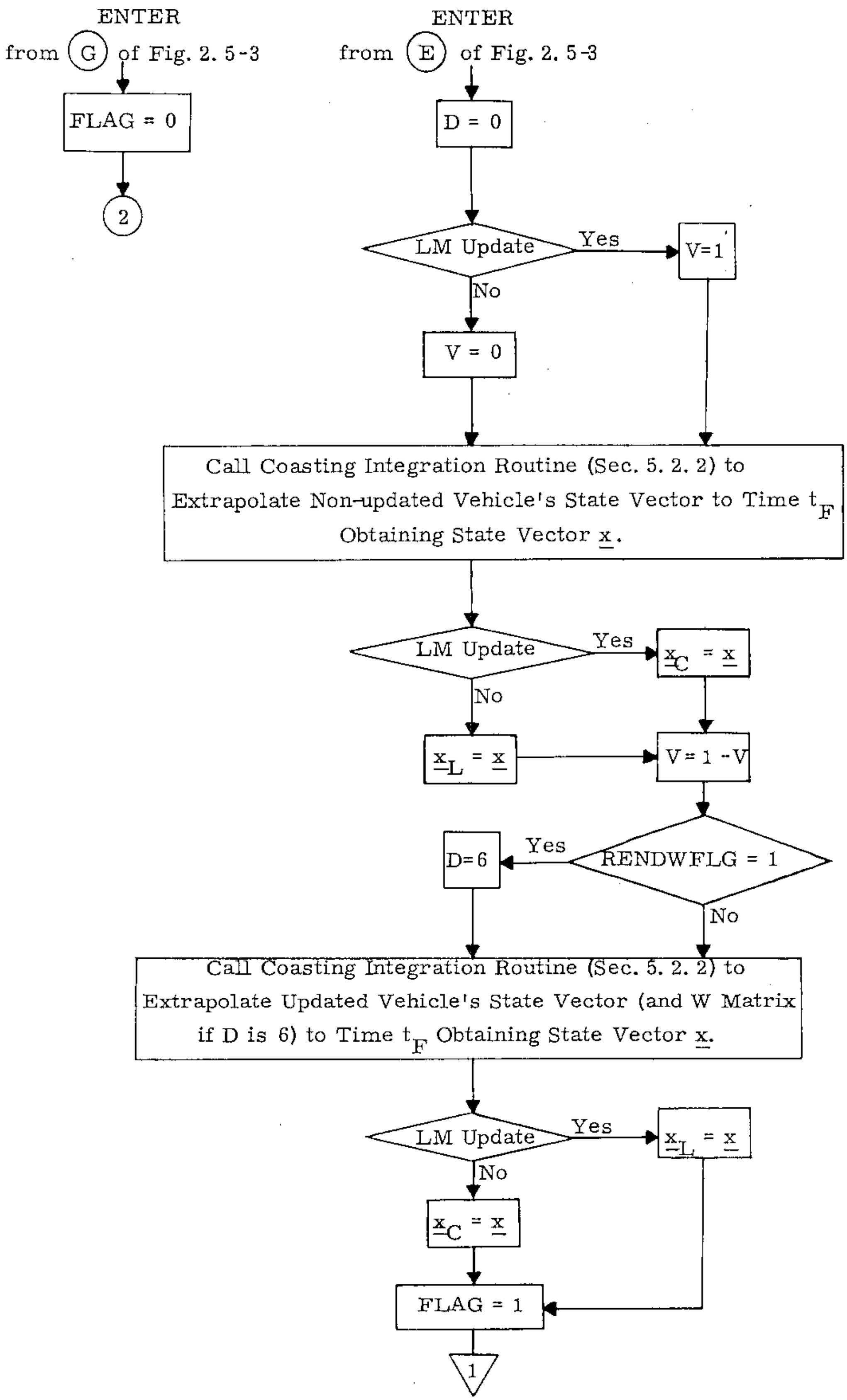

Figure 2. 5-4 Rendezvous Navigation Computations (page 1 of 3)

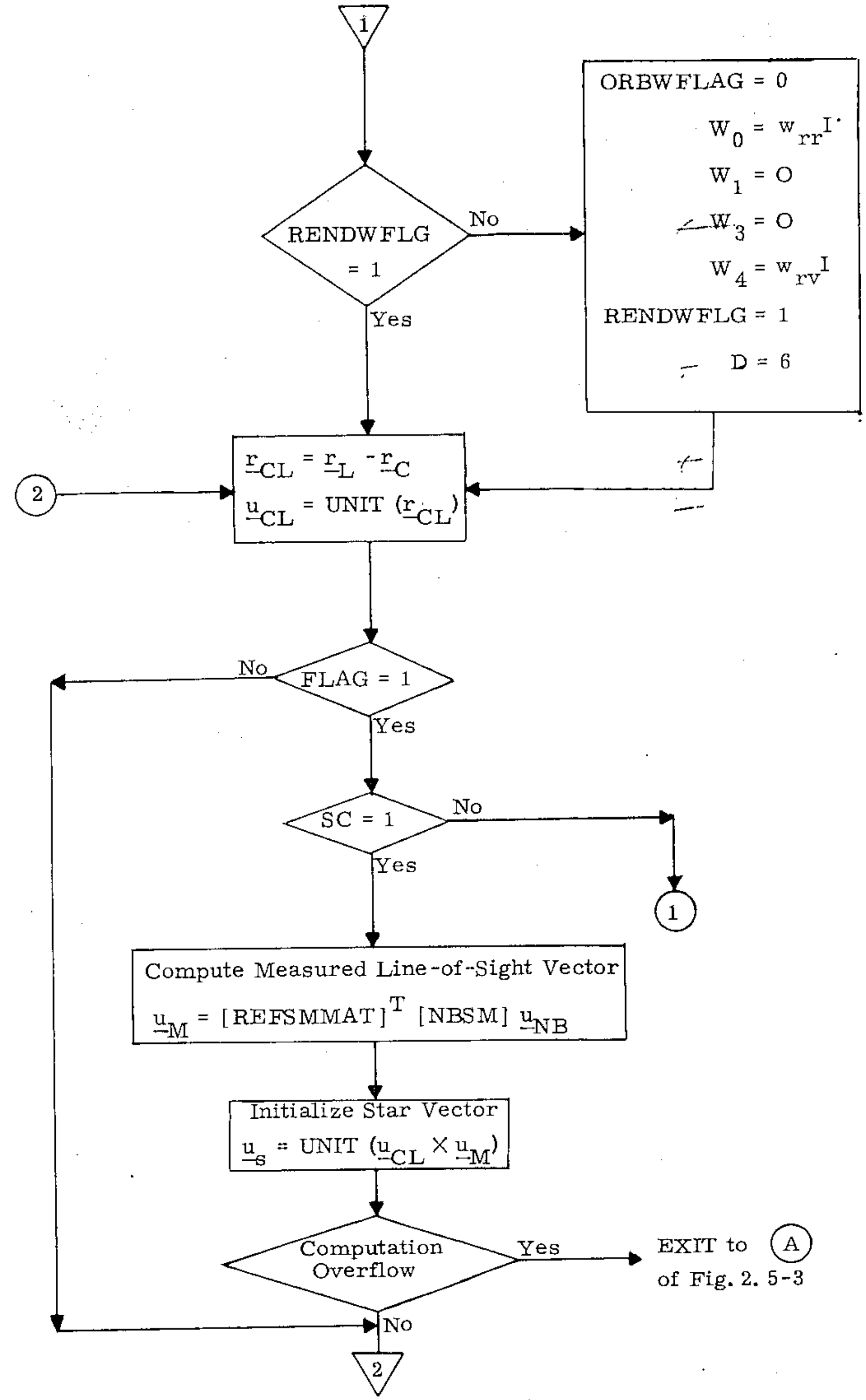

Figure 2. 5-4 Rendezvous Navigation Computations (page 2 of 3)

Compute Star Vector, Geometry Vector, Measured Deviation, and Error Variance for Optics Measurement

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_s \times \underline{u}_{CL})$$

$$\underline{b}_0 = \pm\ \underline{u}_s$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = r_{CL} \left[\cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \right]$$

$$\overline{\alpha^2} = r_{CL}{}^2\ (\text{var}_{SXT} + \text{var}_{IMU})$$

Compute Geometry Vector, Measured Deviation, and Error Variance for Radar Measurement

$$\underline{b}_0 = \pm\ \underline{u}_{CL}$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = R_M - r_{CL}$$

$$\overline{\alpha^2} = \text{maximum}\ (r_{CL}{}^2\ \text{var}_R,\ \text{var}_{Rmin})$$

Call INCORP1 of Measurement Incorporation Routine to Compute State Vector Change $\delta \underline{x}$

EXIT to F of Fig. 2. 5-3

Figure 2. 5-4 Rendezvous Navigation Computations (page 3 of 3)

5.2.5.2.3 Rendezvous Navigation Computations (Alternate Line-of-sight)

During rendezvous optical tracking data are normally obtained by means of SXT sightings of the LM from the CSM and processed as described in Section 5.2.5.2.2. Navigation data can also be obtained by means of a backup optical device. However, since data obtained in this manner is much less accurate than SXT sighting data, the backup device should be used for the sightings only if the SXT has failed, or if the astronaut cannot return to the lower equipment bay to make SXT sightings.

The processing of the data from a backup sighting is identical to the procedure described in Section 5.2.5.2 with the following two exceptions:

1. The values of the shaft and trunnion angles associated with the particular device used are astronaut input items.

2. The measurement error variance, Eq. (2.5.9), is replaced by

$$\overline{\alpha^2} = r_{CL}^2 \left(var_{ALT} + var_{IMU} \right) \qquad (2.5.12)$$

where var_{ALT} is the a priori estimate for the angular error variance of an alternate line-of-sight measurement per axis.

5.2.6 CISLUNAR-MIDCOURSE NAVIGATION ROUTINE

5.2.6.1 General Comments

During the midcourse phase of the lunar mission, navigation data can be obtained by the measurement of the angle between the directions to a star and a planetary horizon or landmark, as described in Section 5.2.1. This routine is used to process the star-landmark/horizon measurement data, as illustrated in simplified form in Fig. 2.1-3, and is normally used only in an abort situation in conjunction with a return-to-earth targeting and maneuver procedure after the loss of ground communication. The Return-to-Earth Routine (Section 5.4.5) and this routine provide the CMC with the capability for guiding the CSM back to the earth and to safe entry conditions.

The acquisition of the star and landmark/horizon may be accomplished either automatically or manually. In the manual mode it is not necessary to have the IMU aligned or even on for this measurement since only the optics trunnion angle is used as measurement data. In the automatic acquisition mode, however, the IMU must be on and aligned prior to the initiation of this routine.

In the processing of the navigation data, it is necessary to distinguish between earth and lunar measurements, and between primary and secondary body measurements. This is accomplished by means of the variable Z, which denotes measurement planet, and which is part of the data loaded by the astronaut after the measurement. Also included in the data load are star and landmark or horizon identification.

5.2.6.2 Star-Landmark Measurement

Let $\underline{r}_C$ and $\underline{v}_C$ be the estimated CSM position and velocity vectors and W the error transition matrix. The SXT star-landmark angle measurement processing procedure is as follows:

1. Use the Coasting Integration Routine (Section 5.2.2) to extrapolate the estimated CSM state vector and the W matrix to the time of the measurement obtaining $\underline{r}'_C$, $\underline{v}'_C$, and W'.

2. Let $\underline{r}_{ZC}$ be the estimated CSM position vector relative to the measurement planet Z. Then

$$\underline{r}_{ZC} = \begin{cases} \underline{r}'_C & \text{if } Z = P \\ \underline{r}_{QC} & \text{if } Z = Q \end{cases} \tag{2.6.1}$$

where $\underline{r}_{QC}$ is the estimated position of the CSM relative to the secondary body Q, and is computed as described in Sec. 5.2.2.3.

3. Compute $\underline{r}_\ell$, the location of the landmark at the measurement time, by means of the Latitude-Longitude Subroutine (Section 5.5.3).

4. Compute the estimated pointing vector from

$$\begin{aligned} \underline{r}_{CL} &= \underline{r}_\ell - \underline{r}_{ZC} \\ \underline{u}_{CL} &= \text{UNIT}(\underline{r}_{CL}) \end{aligned} \tag{2.6.2}$$

5 Let $\underline{u}_s$ be a unit vector in basic reference coordinates which defines the direction to the star whose coordinates are in the CMC fixed memory or were loaded by the astronaut. It is necessary to correct the star vector for aberration, i.e., the change in the observed star direction caused by velocity perpendicular to the direction. The observed star direction, $\underline{u}_s^*$, is given by

$$\underline{u}_s^* = \text{UNIT}\left(\underline{u}_s + \frac{\underline{v}'_C - \underline{v}_{ES}}{c}\right) \qquad (2.6.3)$$

where $\underline{v}_{ES}$ is the velocity of the sun relative to the earth, and c is the speed of light. The velocity vector of the sun relative to the earth, $\underline{v}_{ES}$, is assumed constant for the duration of the mission, as described in Section 5.5.4. The velocity of the moon relative to the earth, for the case in which the moon is the primary body, is negligible. The coordinates of a planet should not be loaded by the astronaut for cislunar navigation unless they correctly indicate the direction of the planet with respect to the CSM and allowance has been made for the difference in aberration between that indicated in Eq. 2.6.3 and that which truly exists for the planet.

6 Correct $\underline{u}_{CL}$ for aberration as follows:

$$\underline{u}_{CL}^* = \text{UNIT}\left(\underline{u}_{CL} + \frac{\underline{v}'_C}{c}\right) \qquad (2.6.4)$$

7 Then, if A is the measured angle, the six dimensional geometry vector, $\underline{b}$, and the measured deviation, δQ, are given by

$$\text{COSQ} = \underline{u}_s^* \cdot \underline{u}_{CL}^*$$

$$\underline{b}_0 = \frac{1}{r_{CL}} \text{ UNIT } (\underline{u}_s^* - \text{COSQ } \underline{u}_{CL}^*)$$

$$\underline{b}_1 = \underline{0} \qquad (2.6.5)$$

$$\delta Q = A - \cos^{-1} (\text{COSQ})$$

8 Incorporate the measurement into the CSM state vector estimate by means of the Measurement Incorporation Routine (Section 5.2.3) after astronaut approval.

5.2.6.3 Star-Horizon Measurement

The processing of a star-horizon measurement is the same as that of a star-landmark measurement except for Step 3 above. The estimated location of the landmark (horizon) $\underline{r}_\ell$ must be obtained from geometrical considerations.

Referring to Fig. 2.6-1, it is seen that the star unit vector, $\underline{u}_s$, and the estimated CSM position vector, $\underline{r}_{ZC}$, determine a plane. Assuming that the measurement planet is the earth and that the horizon of the earth is at a constant altitude, the intersection of this plane and the horizon of the earth is approximately an ellipse, called the horizon ellipse.

To determine the orientation of the horizon ellipse, define the following three mutually orthogonal unit vectors:

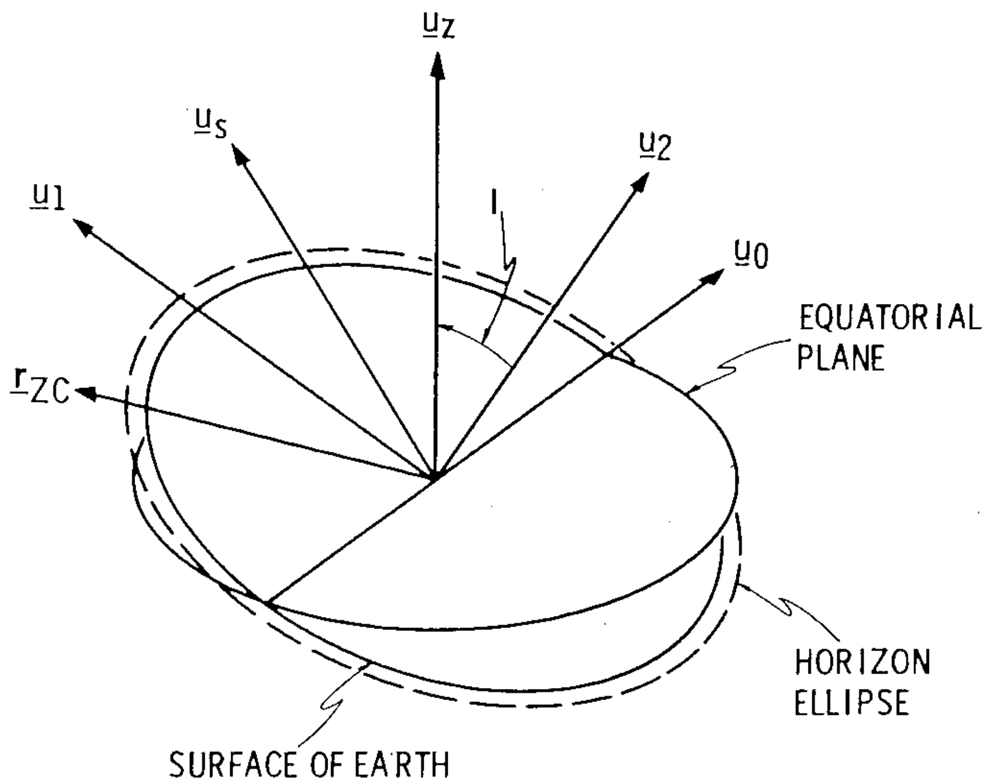

Figure 2. 6-1 Definition of Horizon Coordinate System

$$\underline{u}_2 = \text{UNIT}\ (\underline{u}_s \times \underline{r}_{ZC})$$

$$\underline{u}_0 = \text{UNIT}\ (\underline{u}_Z \times \underline{u}_2) \qquad (2.6.6)$$

$$\underline{u}_1 = \underline{u}_2 \times \underline{u}_0$$

where $\underline{u}_Z$ is a unit vector along the earth's polar axis and is given by

$$\underline{u}_Z = \begin{pmatrix} A_Y \\ -A_X \\ 1 \end{pmatrix} \qquad (2.6.7)$$

The angles A_X and A_Y are defined in Section 5.5.2. Then, as seen in Fig. 2.6-1, the vectors $\underline{u}_0$ and $\underline{u}_1$ are along the semi-major and semi-minor axes of the horizon ellipse, respectively; and $\underline{u}_2$ is perpendicular to the ellipse. The inclination angle I of the horizon ellipse with respect to the equatorial plane of the earth is obtained from

$$\sin I = \underline{u}_1 \cdot \underline{u}_Z \qquad (2.6.8)$$

The shape of the ellipse is defined by its major and minor axes. The semi-major axis a_H is given by

$$a_H = a + h \qquad (2.6.9)$$

where a is the semi-major axis of the Fischer ellipsoid and h is the horizon altitude. Let r_F be the radius of the Fischer ellipsoid at the latitude equal to the inclination angle of the horizon ellipse computed from Eq. (5.3.1) of Section 5.5.3. Then, the semi-minor axis of the ellipse, b_H, is obtained from

$$b_H = r_F + h \tag{2.6.10}$$

The problem of determining the vector $\underline{r}_\ell$ can now be reduced to a two dimensional one. Define the Horizon Coordinate System to have its X- and Y-axes along $\underline{u}_0$ and $\underline{u}_1$, respectively, as illustrated in Fig. 2.6-2. Let

$$M = \begin{pmatrix} \underline{u}_0^T \\ \underline{u}_1^T \\ \underline{u}_2^T \end{pmatrix} \tag{2.6.11}$$

The matrix M is the transformation matrix from the Basic Reference Coordinate System to the Horizon Coordinate System. The vectors $\underline{r}_{ZC}$ and $\underline{u}_s$ are transformed to the Horizon Coordinate System as follows:

$$\begin{aligned} \underline{r}_H &= M\,\underline{r}_{ZC} \\ \underline{u}_{sH} &= M\,\underline{u}_s \end{aligned} \tag{2.6.12}$$

Let x_H and y_H be the two non-zero components of $\underline{r}_H$, and let the two points of tangency from $\underline{r}_H$ to the horizon be $\underline{t}_0$ and $\underline{t}_1$.

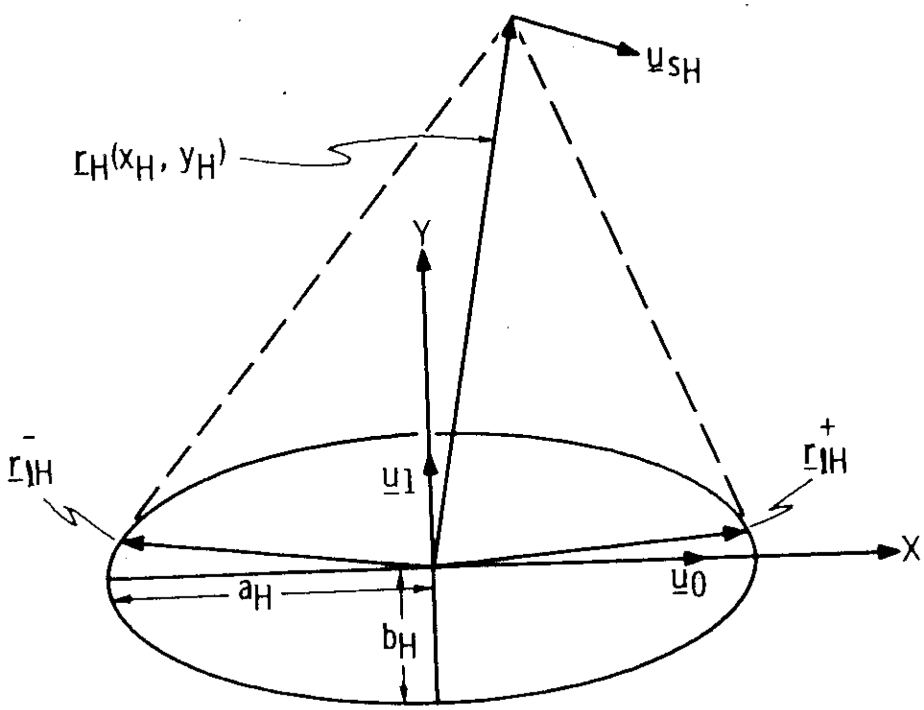

Figure 2.6-2 Geometry of Star-Horizon Measurement

The vectors $\underline{t}_0$ and $\underline{t}_1$ are obtained by solving simultaneously the equation of the horizon ellipse

$$\frac{x^2}{a_H^2} + \frac{y^2}{b_H^2} = 1 \qquad (2.6.13)$$

and the equation of the line which is tangent to the ellipse and which passes through the point (x_H, y_H)

$$\frac{xx_H}{a_H^2} + \frac{yy_H}{b_H^2} = 1 \qquad (2.6.14)$$

The following equations result:

$$\underline{t}_0 = \frac{1}{A} \begin{pmatrix} x_H + \dfrac{a_H}{b_H} y_H \sqrt{A - 1} \\ \\ y_H - \dfrac{b_H}{a_H} x_H \sqrt{A - 1} \\ \\ 0 \end{pmatrix} \qquad (2.6.15)$$

$$\underline{t}_1 = \frac{1}{A} \begin{pmatrix} x_H - \dfrac{a_H}{b_H} y_H \sqrt{A - 1} \\ \\ y_H + \dfrac{b_H}{a_H} x_H \sqrt{A - 1} \\ \\ 0 \end{pmatrix} \qquad (2.6.16)$$

where

$$A = \frac{x_H^2}{a_H^2} + \frac{y_H^2}{b_H^2} \qquad (2.6.17)$$

The two points of tangency, $\underline{t}_0$ and $\underline{t}_1$, correspond to the two horizon points, $\underline{r}_{\ell H}^{+}$ (near horizon) and $\underline{r}_{\ell H}^{-}$ (far horizon). To determine which $\underline{t}_i$ corresponds to which horizon, compute the two quantities

$$A_i = \underline{u}_{sH} \cdot \mathrm{UNIT}\,(\underline{t}_i - \underline{r}_H) \quad (i = 0, 1) \qquad (2.6.18)$$

Then, the $\underline{t}_i$ which yielded the larger A_i is the near horizon and the other is the far horizon. Let $\underline{t}_k$ be the horizon which was used in the measurement. The horizon vector is then

$$\underline{r}_\ell = M^T\,\underline{t}_k \qquad (2.6.19)$$

The measurement processing is completed by following steps 4 - 8 of the star-landmark measurement procedure.

The star-horizon measurement processing procedure has been based upon the assumption that the planet involved in the measurement is the earth. If the moon is the measurement planet, and it is assumed that the moon is a sphere, then the entire procedure presented above is valid except for the computation of a_H and b_H. For a lunar-horizon measurement Eqs. (2. 6. 8) - (2. 6. 10) are replaced by

$$
\begin{aligned}
a_H &= r_M \\
b_H &= r_M
\end{aligned}
\qquad (2.6.20)
$$

where r_M is the mean radius of the moon.

5. 2. 6. 4 Angle Measurement Processing Logic

The computational logic for the Cislunar-Midcourse Navigation Routine is illustrated in Fig. 2. 6-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5. 2. 2. 6

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5. 2. 2. 4

ORBWFLAG = $\begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$

This flag or switch is maintained by programs external to the Cislunar-Midcourse Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing star-landmark/horizon angle measurement data. The flag is set to zero after each of the following procedures:

1) CSM state vector update from ground
2) Rendezvous navigation
3) Astronaut Command

RENDWFLG = Switch similar to ORBWFLG but used for rendezvous navigation.

t_F = Measurement time

A = Measured angle

$\underline{u}_s$ = Measurement star

Z = Measurement planet = $\begin{cases} 0 \text{ for earth} \\ 1 \text{ for moon} \end{cases}$

L = Landmark switch or flag = $\begin{cases} 1 \text{ for landmark measurement} \\ 0 \text{ for horizon measurement} \end{cases}$

w_{mr} and w_{mv} = Preselected W matrix initial diagonal elements

For convenience of calculation in the CMC, Eqs. (2. 6. 5) are reformulated as follows:

$$
\begin{aligned}
COSQ &= \underline{u}_s^* \cdot \underline{u}_{CL}^* \\
\underline{b}_0 &= \text{UNIT}\left(\underline{u}_s^* - COSQ\ \underline{u}_{CL}^*\right) \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL}\left[A - \cos^{-1}(COSQ)\right]
\end{aligned}
\qquad (2.\ 6.\ 21)
$$

To make Eqs. (2. 6. 21) valid it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^2\ var_{TRUN} + var_L \qquad (2.\ 6.\ 22)$$

where var_{TRUN} and var_L are the a priori estimates of the SXT trunnion-angle and landmark or horizon error variances, respectively.

The variables D and V are indicators which control the Coasting Integration Routine (Section 5. 2. 2) as described in

Section 5.2.2.6, I and O are the three-dimensional identity and zero matrices, respectively, and F is the altitude flag as defined in Section 5.5.3.

The calculation of the estimated pointing vector, $\underline{u}_{CL}$, is illustrated in Fig. 2.6-4. This subroutine is used both in the processing of the navigation data and by Routine R-60 in the automatic acquisition mode to point the SXT landmark line-of-sight at the specified target.

Finally, there is available either the landmark coordinates or the far horizon flag H defined by

$$H = \begin{cases} 1 \text{ for far horizon} \\ 0 \text{ for near horizon} \end{cases}$$

In the case of a horizon measurement the computational logic for determining the horizon vector $\underline{r}_\ell$ is shown in Fig. 2.6-5.

The landmark coordinates (latitude, longitude, altitude) are either in fixed memory or entered into the computer by the astronaut. The altitude is referenced to the Fischer ellipsoid for earth landmarks, and the mean lunar radius for lunar landmarks.

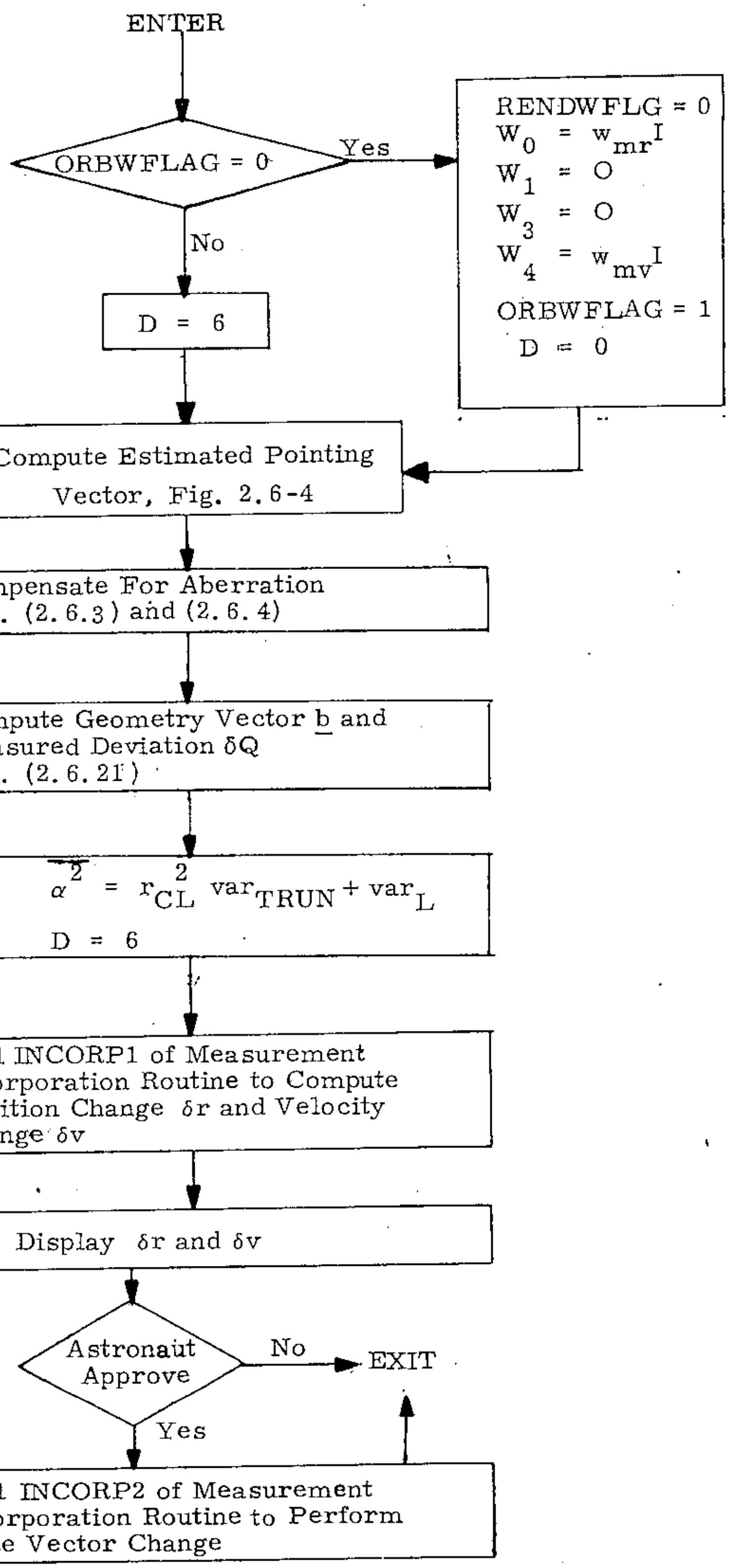

Figure 2.6-3 Cislunar-Midcourse Navigation Routine Logic Diagram

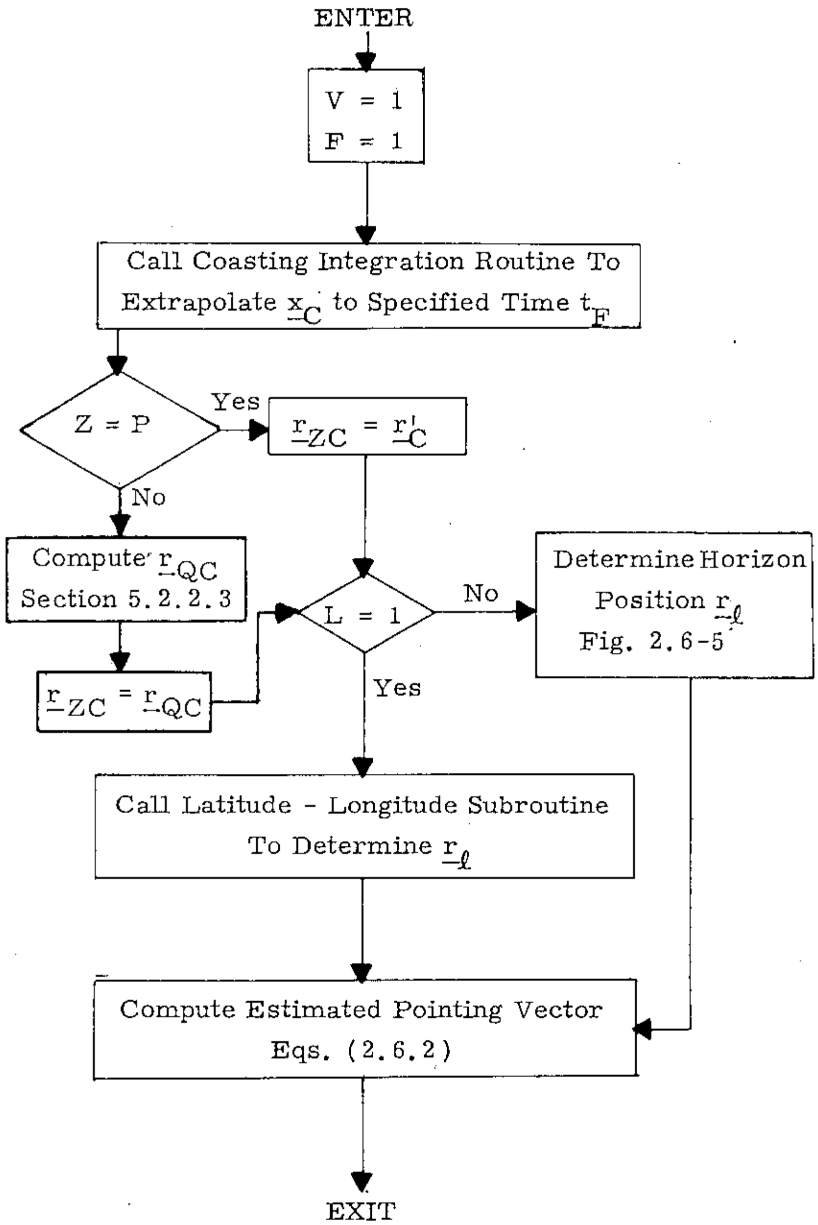

Fig. 2.6-4 Estimated Pointing Vector Computation Subroutine Logic Diagram

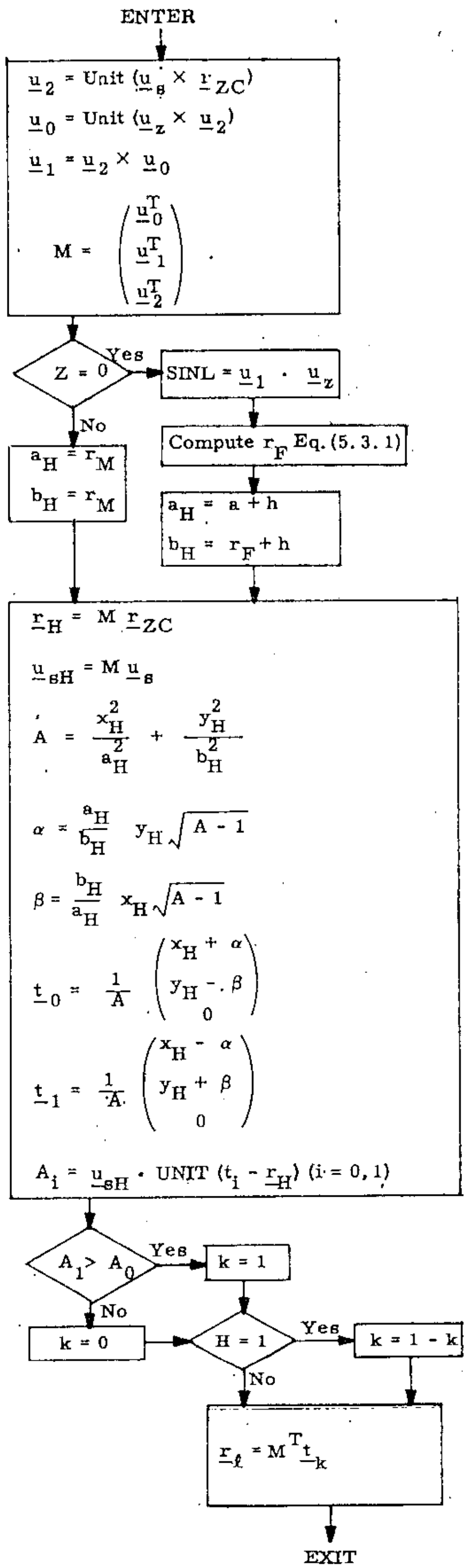

Figure 2.6-5 Horizon Vector Determination Logic Diagram