# Star Tracker Error Analysis

## Roll-to-Pitch Nonorthogonality



Mission Planning and Analysis Division

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National Aeronautics and Space Administration

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SHUTTLE PROGRAM

# STAR TRACKER ERROR ANALYSIS

## ROLL-TO-PITCH NONORTHOGONALITY

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## 1.0 INTRODUCTION

Incorporation of the four gimbal IMU model into the IMUCAL bench program has isolated an anomaly in the star tracker software line of sight (LOS) rate test. During verification testing of the revised IMU model, it was noted that the LOS rate cosine was greater than one in certain cases. This implies that either one or both of the star tracker measured end-point unit vectors that are used to compute the LOS rate cosine have lengths greater than unity. The search for the software rogue that is stretching these vectors came to an end at the roll/pitch nonorthogonality matrix in the TNB_CL module of the IMU software.

## 2.0 DISCUSSION AND RESULTS

The star tracker software (Ref. 1) generates three measurement vectors for each star that is tracked. Each vector is rotated through the navigation base to inertial platform transformation matrix that is computed in the TNB_CL module. One of the three vectors is computed from averaged measurement data and is used for IMU alignments. The remaining two vectors are computed from the first and last measurement samples of the twenty-one sample set and are used internally in the LOS rate test. Any corruption of the vector to be used for alignment is of primary importance.

The source of the anomaly is the roll/pitch gimbal nonorthogonality (DP) matrix in the TNB_CL module (Ref. 2). The ideal matrix

$$(\bar{\Delta}) = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is approximated in the current software design by

$$(\Delta) = \begin{bmatrix} 1 & -\delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\delta$ is the roll/pitch gimbal nonorthogonality, DP. In other words, the nonorthogonality matrix is nonorthogonal.

### 2.1 EFFECT OF THE DP MATRIX APPROXIMATION ON THE IMU ALIGNMENT ACCURACY

Star measurement vectors that are used for IMU alignments are unitized in the aberration correction equations and, therefore, are not corrupted in length.

However, there is some concern that the DP matrix may introduce error into the measurement vector direction. If $\varepsilon$ is defined to be the angular error in the measurement vector direction, then

$$\cos\varepsilon = \text{UNIT}\{(\Delta)\vec{V}\} \cdot \{(\bar{\Delta})\vec{V}\}$$

$$\cos\varepsilon = \frac{(V_1^2 + V_2^2)(\cos\delta + \delta\sin\delta) + V_3^2}{((1+\delta^2)(V_1^2 + V_2^2) + V_3^2)^{1/2}}$$

The error, $\varepsilon$, will be greatest when $V_3 = 0$, therefore,

$$\varepsilon_{max} = \cos^{-1}\left[\frac{\cos\delta + \delta \sin\delta}{(1 + \delta^2)^{1/2}}\right] \qquad (1)$$

which is plotted as a function of $\delta$ in Figure 1. Since the range of $\delta$ for the Shuttle IMUs is $|\delta| < 0.002$ radians, Figure 1 shows that measurement vector directional errors induced by rotation through the DP matrix are insignificant.

## 2.2 EFFECT OF THE DP MATRIX APPROXIMATION ON THE LOS RATE TEST

The end-point measurement vectors are not corrected for aberration and, therefore, are not unitized after rotation through the DP matrix. The LOS rate test assumes that these vectors are of unit length and computes the LOS rate cosine by

$$\cos\rho = \vec{U} \cdot \vec{V}$$

where $\rho$ is the star LOS "rate", $\vec{U}$ the first measurement vector, and $\vec{V}$ the twenty-first measurement vector. Since the end-point vectors are greater than unity in length, the computed LOS rate is greater than the ideal value by the factor $1/|\vec{U}||\vec{V}|$. Measurement data are accepted whenever

$$\vec{U} \cdot \vec{V} > \text{TOL12}$$

Therefore, the probability of accepting debris data as star data is increased due to this anomaly. The end result of this error is to make the LOS rate test less sensitive.

The angular error, $\varepsilon$, corresponding to the cosine error can be computed by comparing the flight software formulation with the ideal solution. If $P$ is defined as the corrupted inner product using the approximation of the DP matrix and $\bar{P}$ the ideal inner product, then

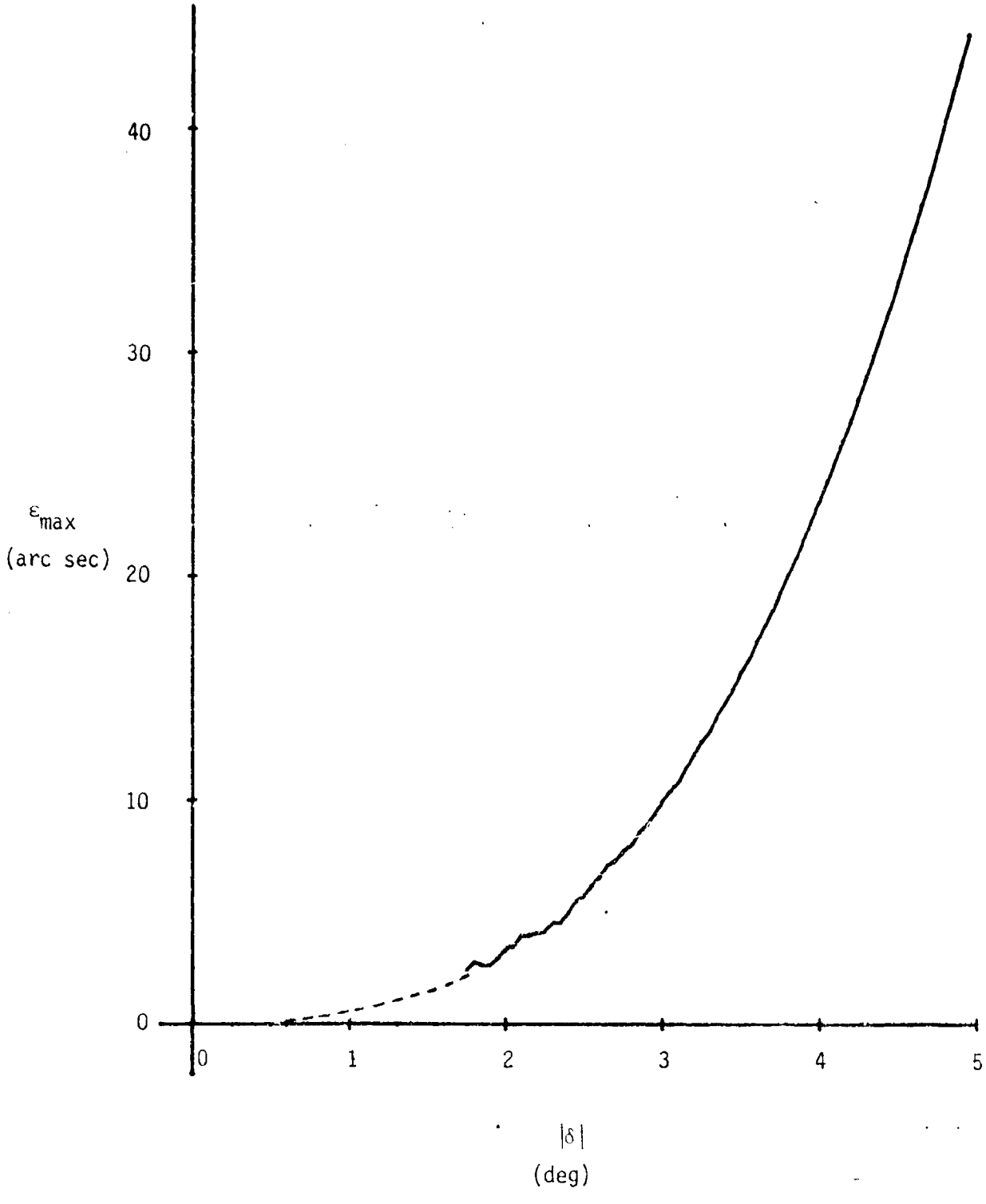


FIGURE 1. STAR MEASUREMENT VECTOR DIRECTIONAL ERROR VERSUS THE OUTER ROLL TO PITCH GIMBAL NONORTHOGONALITY ANGLE

$$P = (\Delta)\vec{U} \cdot (\Delta)\vec{V}$$

$$\overline{P} = (\overline{\Delta})\vec{U} \cdot (\overline{\Delta})\vec{V}$$

which in expanded form are given by

$$P = U_1V_1 + U_2V_2 + U_3V_3 + \delta^2(U_1V_1 + U_2V_2)$$

$$\overline{P} = U_1V_1 + U_2V_2 + U_3V_3$$

The error in P will be greatest when $U_3 = V_3 = 0$. Assuming these worst case conditions ($U_3 = V_3 = 0$), $\overline{P}$ and P take the form

$$P = \vec{U} \cdot \vec{V}(1 + \delta^2) = \cos \rho$$

$$\overline{P} = \vec{U} \cdot \vec{V} = \cos \overline{\rho}$$

Therefore

$$\cos \rho = \vec{U} \cdot \vec{V}(1 + \delta^2) = (1 + \delta^2) \cos \overline{\rho} \qquad (2)$$

The error in the equivalent angle ($\varepsilon = |\rho - \overline{\rho}|$) will be greatest when $\overline{\rho} \simeq 0$; however, if we set $\overline{\rho} = 0$ we cannot solve for $\rho$ because

$$\rho = \cos^{-1}(1 + \delta^2)$$

is undefined for $\delta \neq 0$.

Alternatively, we can set the computed angle, $\rho$, equal to zero and solve for the actual angle, $\overline{\rho}$, that would yield a computed angle of zero.

$$\overline{\rho} = \cos^{-1}\left[\frac{1}{1 + \delta^2}\right] = \varepsilon_{max} \qquad (3)$$

The maximum equivalent angular error in the computed star LOS rate is plotted as a function of the roll/pitch gimbal nonorthogonality angle in Figure 2. Also plotted are average errors, $\varepsilon_{mean}$, that were generated by a simulation program. Figure 2 illustrates that the DP matrix approximation introduces large errors into the computed star LOS rate.

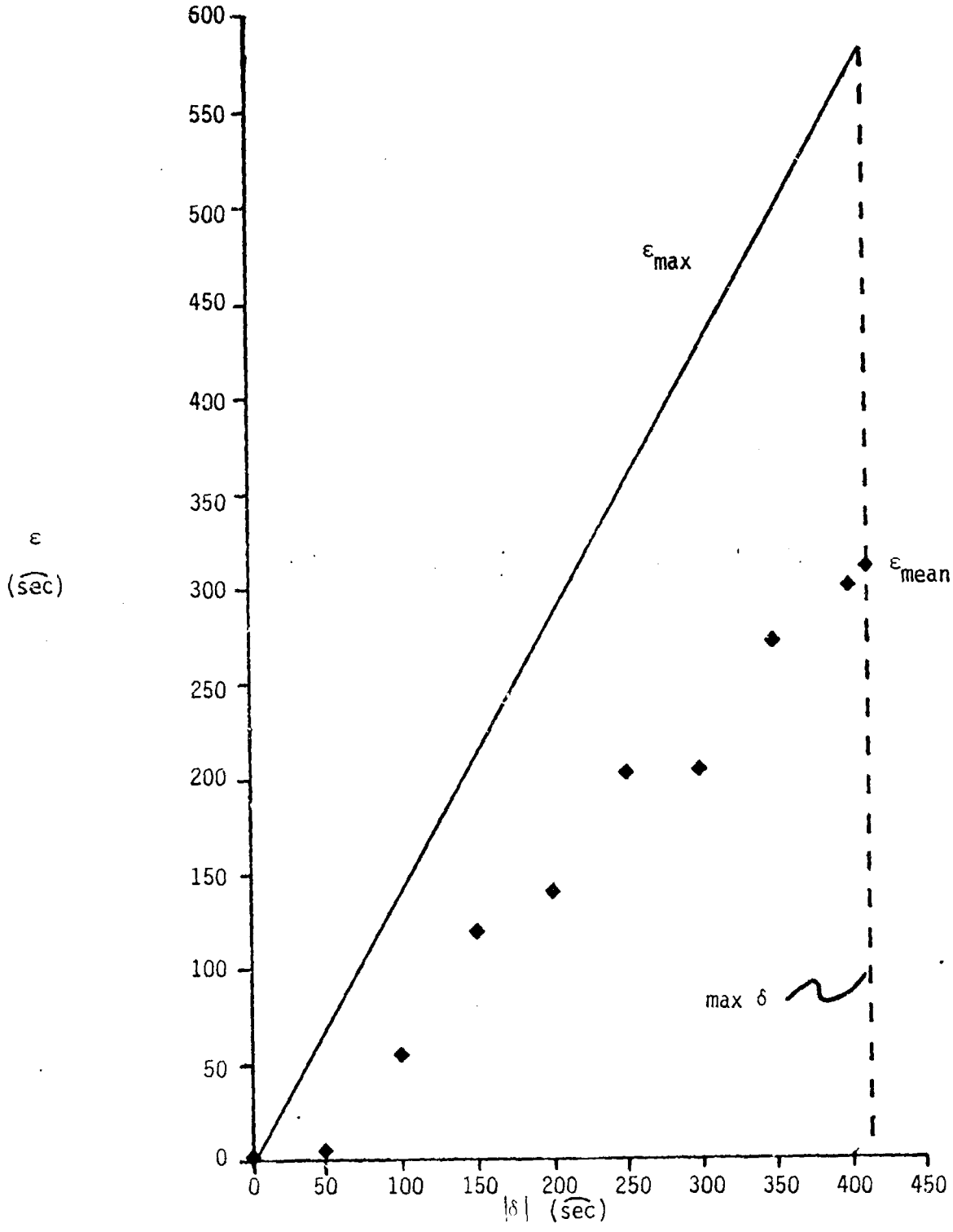


FIGURE 2. STAR LOS RATE TEST ERROR VERSUS THE OUTER ROLL TO PITCH GIMBAL NONORTHOGONALITY ANGLE

The significance of the LOS rate error is more easily understood by expressing the error in terms of a LOS rate sensitivity, $\lambda$. If the DP matrix approximation were replaced with the exact matrix, the LOS rate sensitivity would be defined as

$$\lambda_{ideal} = \left[\frac{\cos^{-1}(TOL12)}{3.2 \text{ sec}}\right] \tag{4}$$

where TOL12 is the LOS rate test tolerance limit and 3.2 sec is the time between the first and last star sightings. By substituting TOL12 for cos $\rho$ in equation (2), the largest angular change between the first and last measurements, $\bar{\rho}$, is given by

$$\bar{\rho} = \cos^{-1}\left[\frac{TOL12}{1 + \delta^2}\right]$$

The LOS rate sensitivity, therefore, is

$$\lambda = \frac{1}{3.2 \text{ sec}} \cos^{-1}\left[\frac{TOL12}{1 + \delta^2}\right] \tag{5}$$

The LOS rate sensitivity is plotted as a function of $\delta$ for three cases in Figure 3 (solid lines). The uppermost curve corresponds to the current STS-1 software configuration (the STS-1 value for TOL12 is padded to circumvent the unforeseen). The dashed line with the same $\lambda$-axis intercept illustrates the LOS rate sensitivity that could be obtained by using the exact DP matrix or by unitizing the end-point measurement vectors before forming the inner product. The middle curve was generated using the minimum recommended value for TOL12 (Ref. 3). This case corresponds to the limit of the current formulation of the LOS rate test. As with the previous case, the dashed line indicates the capabilities that could be obtained by using the exact DP matrix or unitizing the end-point measurement vectors. Since the present inner product algorithm is inherently inaccurate, the bottom curve is plotted to illustrate capabilities that could be realized by using the vector cross product of the end-point measurement vectors to compute the sine of the angle change. The cross product algorithm has proved to be very accurate; and, in addition, its accuracy is not degraded by the approximation of the DP matrix. The bottom curve, therefore, defines the LOS rate sensitivity limit imposed by the hardware inaccuracies.

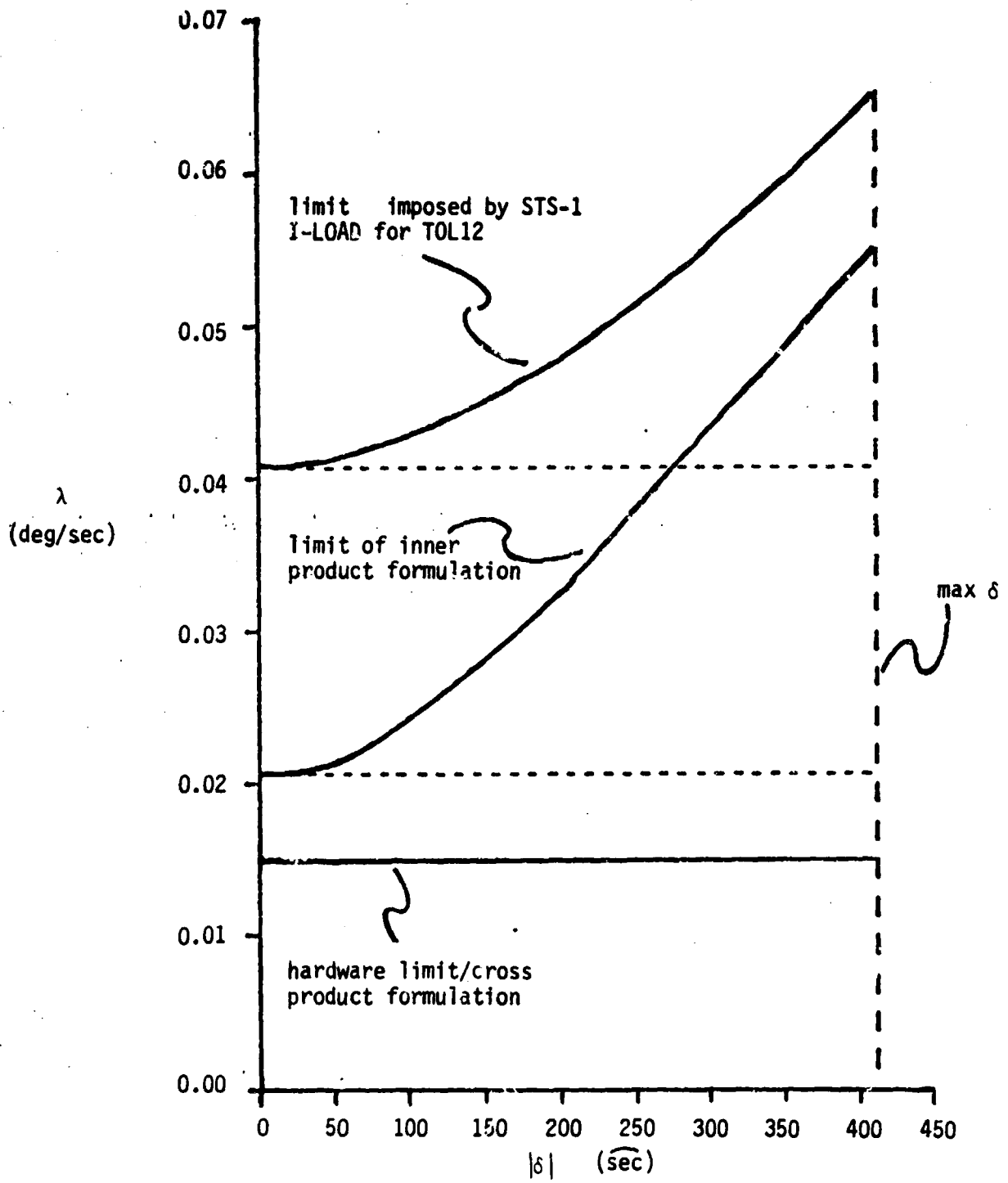


FIGURE 3. LOS RATE TEST SENSITIVITY VERSUS THE OUTER ROLL TO PITCH GIMBAL NONORTHOGONALITY ANGLE

Returning to the STS-1 case, the ideal LOS rate sensitivity (eqn. 4) is

$$\lambda_{ideal} = 0.041 \text{ deg/sec}$$

The maximum error resulting from the DP matrix approximation ($\delta = 0.002$) limits the LOS rate sensitivity to

$$\lambda = 0.065 \text{ deg/sec}$$

This limit is 59% greater than the ideal limit and, therefore, the LOS rate test will accept significantly more debris as alignment data as a result of the errors in the DP matrix approximation.

## 3.0 CONCLUSIONS AND RECOMMENDATIONS

The approximation of the DP matrix does not corrupt the directional information in star measurement vectors and, therefore, will not degrade IMU alignment accuracy. The DP matrix approximation does, however, increase the length of vectors that are rotated through it. As a result, significant errors are introduced into the star LOS rate test. Therefore, the possibility of accepting debris position vectors as measurement data for IMU alignments is increased. It is recommended that the approximation of the DP matrix in the TNB_CL module be replaced with the exact matrix

$$\begin{bmatrix} CDP & -SDP & 0 \\ SDP & CDP & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where SDP and CDP are the sine and cosine, respectively of the outer roll/pitch gimbal nonorthogonality angle. SDP and CDP are currently supplied by the IMU ground calibration software and, therefore, additional flight software code is not required to compute these constants. The nine equations for the elements of the Nav base to cluster transformation matrix, however, would have to be rederived to include the gimbal nonorthogonality sines and cosines, and then these equations implemented into the flight software. In addition to this recommendation, two additional solutions exist. Unitizing the end-point measurement vectors would restore their lengths to unity thereby eliminating that source of error. The second alternative is to reformulate the star LOS rate test using the cross product algorithm. Choice of one of these alternatives will be made via discussions with RI, JSC and IBM in the near future.

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