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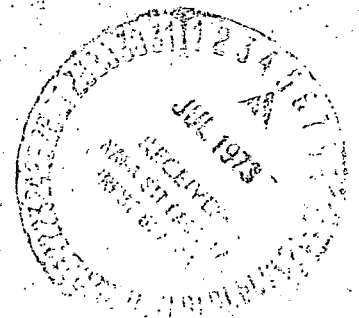
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SPACE SHUTTLE GN&C EQUATION DOCUMENT  
 No. 20  
 FINE ALIGNMENT GYRO TORQUING ANGLES  
 By  
 Joseph St. Amand

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**SPACE SHUTTLE GN&C EQUATION DOCUMENT**

**No. 20**

**Fine Alignment Gyro Torquing Angles**

**by**

**Joseph St. Amand**

**M. I. T. Charles Stark Draper Laboratory**

**September 1972**

**NAS9-10268**

**for**

**National Aeronautics and Space Administration**

**Systems Analysis Branch**

**Guidance and Control Division**

**Manned Spacecraft Center, Houston, Texas**

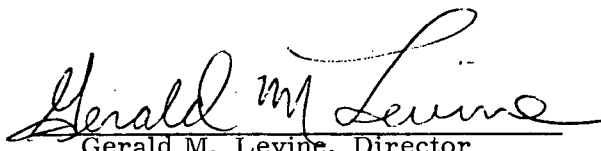
## ACKNOWLEDGEMENT

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

## FOREWORD

This document is one of a series of candidates for inclusion in a future revision of MSC-04217, "Space Shuttle Guidance, Navigation and Control Design Equations". The enclosed has been prepared under NAS9-10268, Task No. 15-A, "GN&C Flight Equation Specification Support", and applies to functions 1 and 2 of the Inertial Reference Module (OS4) as defined in MSC-03690, Rev. B, "Space Shuttle Orbiter Guidance, Navigation and Control Software Functional Requirements - Vertical Flight Operations", dated 15 December 1971.



Gerald M. Levine, Director  
APOLLO Space Guidance Analysis Division

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## NOMENCLATURE

$\underline{i}_{1R}$	Unit vector to star number one in reference coordinates.
$\underline{i}_{2R}$	Unit vector to star number two in reference coordinates.
$\underline{i}_{1S}$	Unit vector to star number one in present stable member coordinates.
$\underline{i}_{2S}$	Unit vector to star number two in present stable member coordinates.
$M_{R-DS}$	Transformation matrix from reference coordinates to desired stable member coordinates.
$\underline{i}_{1DS}$	Unit vector to star number one in desired stable member coordinates.
$\underline{i}_{2DS}$	Unit vector to star number two in desired stable member coordinates.
$\underline{i}_{XR}, \underline{i}_{YR}, \underline{i}_{ZR}$	Unit vectors of the reference coordinate system.
$\underline{i}_{XS}, \underline{i}_{YS}, \underline{i}_{ZS}$	Unit vectors of the present stable member.
$\underline{i}_{XD}, \underline{i}_{YD}, \underline{i}_{ZD}$	Unit vectors of the desired stable member.
$\underline{i}_{XU}, \underline{i}_{YU}, \underline{i}_{ZU}$	An orthogonal triad of unit vectors defined in terms of $\underline{i}_{1S}$ and $\underline{i}_{2S}$ .
$\underline{i}_{XUD}, \underline{i}_{YUD}, \underline{i}_{ZUD}$	An orthogonal triad of unit vectors in terms of $\underline{i}_{1DS}$ and $\underline{i}_{2DS}$ .
$\theta_x, \theta_y, \theta_z$	Gyro torquing angles
$x_{Dj}, y_{Dj}, z_{Dj}$ (j = 1, 2, 3)	Projections of the desired stable member onto the present stable member.
$\underline{i}_Z = (z_1, 0, z_3)$	A unit vector -- in the plane defined by $\underline{i}_{XS}$ and $\underline{i}_{ZS}$ -- used in the final determination of the gyro torquing angles.

## 1. INTRODUCTION

At various points in a Shuttle mission, the need arises to align the Inertial Reference Unit (IRU). By sighting on two stars and making use of the stored transformation from the optics frame to the navigation base frame, and the gimbal angle transformation from the navigation base to the stable member, unit vectors to the target stars in the present stable member coordinates are generated. Furthermore, as discussed in Ref. 1, the coordinates of these two stars, in the reference coordinate frame, are stored in the computer. Also stored in the computer is the transformation matrix from the reference frame to the desired stable member coordinate system. From the above, unit vectors to the target stars in the desired stable member frame are constructed. This information, when combined with the preceding, is sufficient to express the present stable member in terms of the desired stable member.

This document details how the gyro torquing angles required to go from the present stable member to the desired stable member are determined. The inputs are the transformation matrix from the reference to the desired stable member coordinates and the unit vectors to the two target stars in both the reference and present stable member coordinates. The method of determining the gyro torquing angles is exactly that used in project APOLLO (Ref. 2).

While the equations as derived are correct for arbitrarily large torquing angles, the alignments under consideration are Fine Alignments (i. e., torquing angles  $\lesssim 1^\circ$ ).

## 2. FUNCTIONAL FLOW DIAGRAM

The functional flow diagram for the calculation of Fine Alignment Gyro Torquing Angles is shown in Figure 1. The routine is entered after optical sightings have been made on two stars and the sighting data is considered acceptable on the basis of the star angle difference check. Reference 1 details the steps preceding entry into this routine.

Upon derivation of the gyro torquing angles, the angles are checked on the basis of previous gyro drift rates. Reference 1 details the logic which checks the reasonableness of the computed angles.



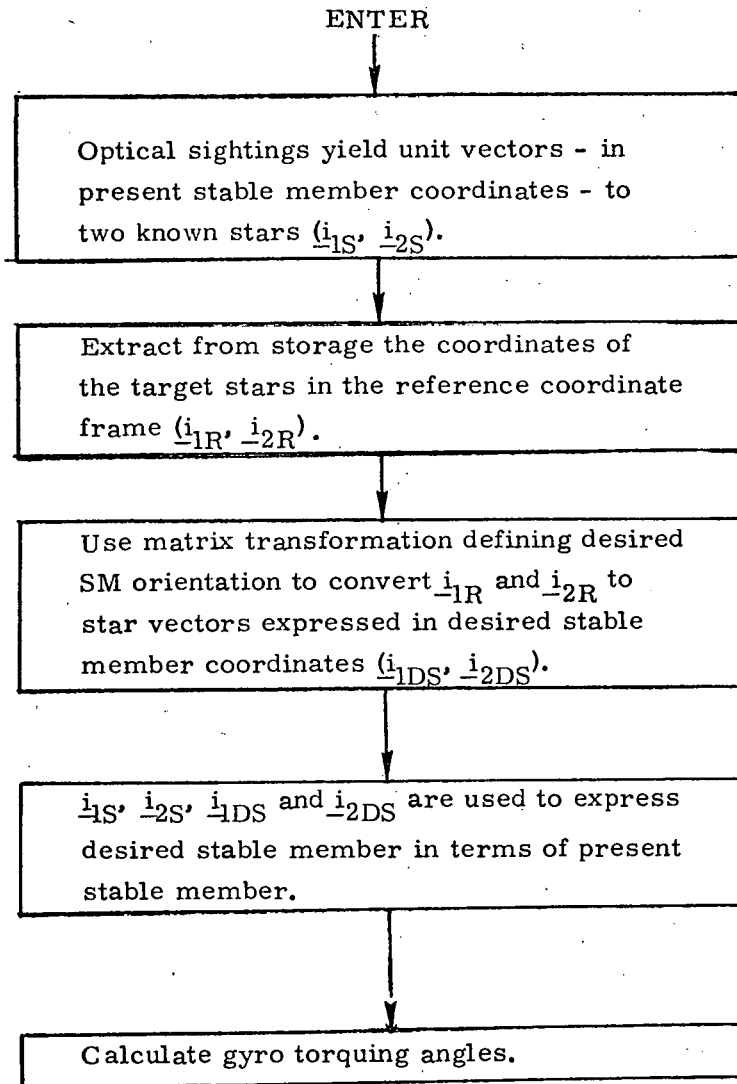


Figure 1. Fine Alignment Gyro Torquing Angles Functional Flow Diagram.

### 3. INPUT AND OUTPUT VARIABLES

#### Input Variables

$\underline{i}_{1S}$	Unit vector of star number one in present stable member coordinates.
$\underline{i}_{2S}$	Unit vector of star number two in present stable member coordinates.
$\underline{i}_{1R}$	Unit vector of star number one in the reference frame.
$\underline{i}_{2R}$	Unit vector of star number two in the reference frame.
$M_{R-DS}$	Transformation matrix from reference to desired SM coordinates.

#### Output Variables

$\theta_x, \theta_y, \theta_z$	Gyro torquing angles required for fine alignment of the IRU.
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#### 4. DESCRIPTION OF EQUATIONS

##### 4.1 Introduction

In this section the gyro torquing angles which allow for the fine alignment of the stable member from its present orientation to a desired orientation are described.

The present orientation of the stable member is determined (implicitly) by sighting on two known\* stars. Upon being processed by the computer, the mark data allows the positions of the stars to be expressed in terms of the present stable member coordinates. The unit vectors to the stars, in present stable member coordinates ( $\underline{i}_S, \underline{i}_{2S}$ ) are given explicitly as:

$$\underline{i}_S = s_{1X}\underline{i}_{XS} + s_{1Y}\underline{i}_{YS} + s_{1Z}\underline{i}_{ZS} \quad (1a)$$

and

$$\underline{i}_{2S} = s_{2X}\underline{i}_{XS} + s_{2Y}\underline{i}_{YS} + s_{2Z}\underline{i}_{ZS} \quad (1b)$$

Unit vectors of the same two stars in reference coordinates are given by  $\underline{i}_{1R}$  and  $\underline{i}_{2R}$ .

##### 4.2 Star vectors in terms of desired stable member coordinates

The desired orientation of the stable member is stored in the computer. The unit vectors of the reference coordinate system are given by  $\underline{i}_{XR}$ ,  $\underline{i}_{YR}$ , and  $\underline{i}_{ZR}$ ; corresponding unit vectors in the desired stable member are  $\underline{i}_{XDS}$ ,  $\underline{i}_{YDS}$ , and  $\underline{i}_{ZDS}$ . The two sets of unit vectors are related by the relation

$$\begin{pmatrix} \underline{i}_{XD} \\ \underline{i}_{YD} \\ \underline{i}_{ZD} \end{pmatrix} = M_{R-DS} \begin{pmatrix} \underline{i}_{XR} \\ \underline{i}_{YR} \\ \underline{i}_{ZR} \end{pmatrix} \quad (2)$$

---

\*Here "known" means that the unit vector of the star in the reference coordinate system is stored in the computer.

where  $M_{R-DS}$  is the transformation matrix between the two coordinate systems.

The unit vectors to the two stars are stored in the computer as

$$\underline{i}_R = r_{1X} \underline{i}_X + r_{1Y} \underline{i}_Y + r_{1Z} \underline{i}_Z \quad (3a)$$

$$\underline{i}_R = r_{2X} \underline{i}_X + r_{2Y} \underline{i}_Y + r_{2Z} \underline{i}_Z \quad (3b)$$

The same stellar unit vectors, expressed in terms of the desired stable member coordinates, are given by

$$\underline{i}_{DS} = d_{1X} \underline{i}_{XD} + d_{1Y} \underline{i}_{YD} + d_{1Z} \underline{i}_{ZD} \quad (4a)$$

$$\underline{i}_{DS} = d_{2X} \underline{i}_{XD} + d_{2Y} \underline{i}_{YD} + d_{2Z} \underline{i}_{ZD} \quad (4b)$$

where

$$\begin{pmatrix} d_{1X} \\ d_{1Y} \\ d_{1Z} \end{pmatrix} = M_{R-DS} \begin{pmatrix} r_{1X} \\ r_{1Y} \\ r_{1Z} \end{pmatrix} \quad (5a)$$

and

$$\begin{pmatrix} d_{2X} \\ d_{2Y} \\ d_{2Z} \end{pmatrix} = M_{R-DS} \begin{pmatrix} r_{2X} \\ r_{2Y} \\ r_{2Z} \end{pmatrix} \quad (5b)$$

#### 4.3 Desired stable member in terms of present stable member

At this point we have the position vectors to two stars expressed in terms of both the present stable member and the desired stable member.

It is now desired to express the desired stable member in terms of the present stable member. To this end we define two ortho-normal

coordinate systems; one in the present stable member coordinates and the other in the desired stable member coordinates. The system defined in terms of the present stable member is given by

$$\underline{i}_{XU} = \underline{i}_{1S} \quad (6a)$$

$$\underline{i}_{YU} = \text{unit}(\underline{i}_{1S} \times \underline{i}_{2S}) \quad (6b)$$

$$\underline{i}_{ZU} = \underline{i}_{XU} \times \underline{i}_{YU} \quad (6c)$$

while

$$\underline{i}_{XUD} = \underline{i}_{1DS} \quad (7a)$$

$$\underline{i}_{YUD} = \text{unit}(\underline{i}_{1DS} \times \underline{i}_{2DS}) \quad (7b)$$

$$\underline{i}_{ZUD} = \underline{i}_{XUD} \times \underline{i}_{YUD} \quad (7c)$$

defines the system in terms of the desired stable member frame.

Explicitly,

$$\begin{pmatrix} \underline{i}_{XUD} \\ \underline{i}_{YUD} \\ \underline{i}_{ZUD} \end{pmatrix} = \begin{pmatrix} u_{X1} & u_{X2} & u_{X3} \\ u_{Y1} & u_{Y2} & u_{Y3} \\ u_{Z1} & u_{Z2} & u_{Z3} \end{pmatrix} \begin{pmatrix} \underline{i}_{XD} \\ \underline{i}_{YD} \\ \underline{i}_{ZD} \end{pmatrix} \quad (8)$$

Because the transformation matrix is orthogonal, ( $M^T = M^{-1}$ ), we have upon inverting Eq. (8)

$$\begin{pmatrix} \underline{i}_{XD} \\ \underline{i}_{YD} \\ \underline{i}_{ZD} \end{pmatrix} = \begin{pmatrix} u_{X1} & u_{Y1} & u_{Z1} \\ u_{X2} & u_{Y2} & u_{Z2} \\ u_{X3} & u_{Y3} & u_{Z3} \end{pmatrix} \begin{pmatrix} \underline{i}_{XUD} \\ \underline{i}_{YUD} \\ \underline{i}_{ZUD} \end{pmatrix} \quad (9)$$

At this point it should be recalled that there exists only two physical vectors, albeit expressed in two frames of reference. Accordingly, if in Eq. (9) the set of unit vectors

$$\begin{pmatrix} \underline{i}_{XUD} \\ \underline{i}_{YUD} \\ \underline{i}_{ZUD} \end{pmatrix}$$

is replaced by

$$\begin{pmatrix} \underline{i}_{XU} \\ \underline{i}_{YU} \\ \underline{i}_{ZU} \end{pmatrix}$$

we end up with

$$\begin{pmatrix} \underline{i}_{XD} \\ \underline{i}_{YD} \\ \underline{i}_{ZD} \end{pmatrix} = \begin{pmatrix} u_{X1} & u_{Y1} & u_{Z1} \\ u_{X2} & u_{Y2} & u_{Z2} \\ u_{X3} & u_{Y3} & u_{Z3} \end{pmatrix} \begin{pmatrix} \underline{i}_{XU} \\ \underline{i}_{YU} \\ \underline{i}_{ZU} \end{pmatrix} \quad (10)$$

Thus we have succeeded in expressing the desired stable member in terms of the present. This is so because  $\underline{i}_{XU}$ ,  $\underline{i}_{YU}$ , and  $\underline{i}_{ZU}$  are expressed in terms of  $\underline{i}_{1S}$  and  $\underline{i}_{2S}$ , each of which in turn is expressed in terms of  $\underline{i}_{XS}$ ,  $\underline{i}_{YS}$ , and  $\underline{i}_{ZS}$ .

Explicitly, the unit vectors of the desired stable member are expressed in terms of those of the present stable member as follows;

$$\underline{i}_{XD} = x_{D1} \underline{i}_{XS} + x_{D2} \underline{i}_{YS} + x_{D3} \underline{i}_{ZS} \quad (11a)$$

$$\underline{i}_{YD} = y_{D1} \underline{i}_{XS} + y_{D2} \underline{i}_{YS} + y_{D3} \underline{i}_{ZS} \quad (11b)$$

$$\underline{i}_{ZD} = z_{D1} \underline{i}_{XS} + z_{D2} \underline{i}_{YS} + z_{D3} \underline{i}_{ZS} \quad (11c)$$

The exact expressions for  $x_{D1}$ ,  $x_{D2}$ , etc. are obtained by expressing  $\underline{i}_{XU}$ ,  $\underline{i}_{YU}$ , and  $\underline{i}_{ZU}$  in terms of  $\underline{i}_{XS}$ ,  $\underline{i}_{YS}$ , and  $\underline{i}_{ZS}$  in Eq. 10.

In the computations of the torquing angles, the coefficients  $x_{D1}$ ,  $x_{D2}$ , etc. are used as inputs.

#### 4.4 Calculation of Gyro Torquing Angles

Having expressed the desired stable member in terms of the present stable member, three rotations that bring the two into coincidence are readily defined. The definition of these angles is the same as that given in Ref. 2.

A rotation of  $\theta_Y$  is first performed about the present y axis ( $\underline{i}_{YS}$ ) yielding  $\underline{i}_X$ ,  $\underline{i}_{YS}$ , and  $\underline{i}_Z$ . Then a rotation of  $\theta_Z$  is performed about  $\underline{i}_Z$  yielding  $\underline{i}_{XD}$ ,  $\underline{i}_Y$ , and  $\underline{i}_Z$ . Thirdly, a rotation of  $\theta_X$  about  $\underline{i}_{XD}$  results in  $\underline{i}_{XD}$ ,  $\underline{i}_{YD}$ , and  $\underline{i}_{ZD}$ .

The unit vector  $\underline{i}_Z$ , obtained after the first rotation, is given by

$$\underline{i}_Z \equiv (z_1, 0, z_3) \quad (12a)$$

$$= \left( \frac{-x_{D3}}{\sqrt{x_{D1}^2 + x_{D3}^2}}, 0, \frac{x_{D1}}{\sqrt{x_{D1}^2 + x_{D3}^2}} \right) \quad (12b)$$

Expressed in terms of  $\underline{i}_Z$  and the projection of the present stable member onto the desired stable member, the rotation angles are given by:

$$\begin{cases} \sin \theta_Y = z_1 & (13a) \\ \cos \theta_Y = z_3 & (13b) \end{cases}$$

$$\begin{cases} \sin \theta_Z = x_{D2} & (14a) \\ \cos \theta_Z = z_3 x_{D1} - z_1 x_{D3} & (14b) \end{cases}$$

$$\begin{cases} \sin \theta_X = \underline{i}_Z \cdot \underline{i}_{YD} = z_1 y_{D1} + z_3 y_{D3} & (15a) \\ \cos \theta_X = \underline{i}_Z \cdot \underline{i}_{ZD} = z_1 z_{D1} + z_3 z_{D3} & (15b) \end{cases}$$

## 5. DETAILED FLOW DIAGRAM

This section contains detailed flow diagrams of the sequential equations used in the calculation of gyro torquing angles by the fine alignment routine.

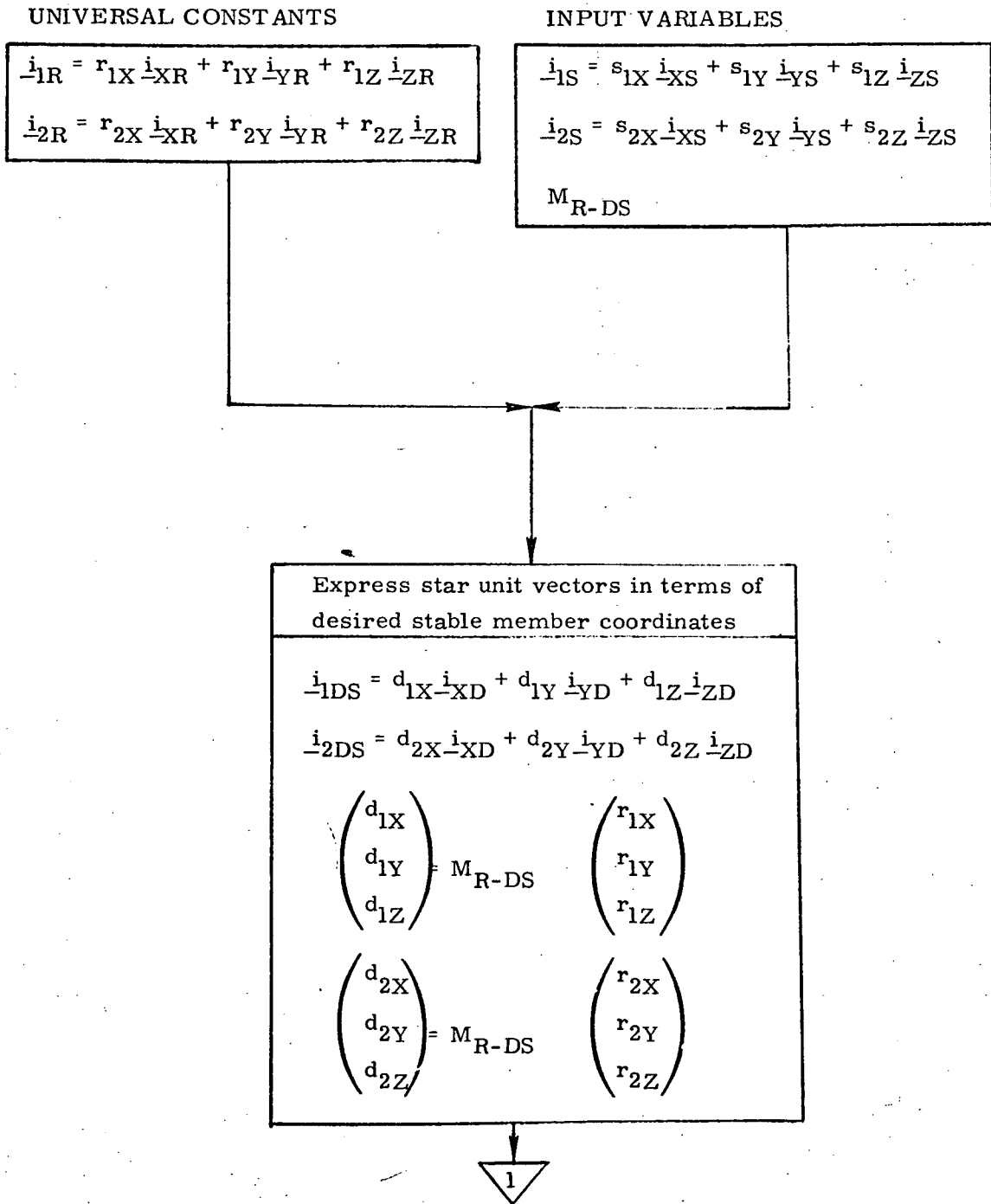


Figure 2a. Fine Alignment Gyro Torquing Angles





Express desired stable member coordinates in terms of present stable member coordinates.

$$\begin{cases} i_{XU} = i_{1S} \\ i_{YU} = \text{unit } (i_{1S} \times i_{2S}) \\ i_{ZU} = i_{XU} \times i_{YU} \end{cases}$$

$$\begin{cases} i_{XUD} = i_{1DS} \\ i_{YUD} = \text{unit } (i_{1DS} \times i_{2DS}) \\ i_{ZUD} = i_{XUD} \times i_{YUD} \end{cases}$$

$$\begin{aligned} i_{XD} &= i_{XUD, 1} i_{XU} + i_{YUD, 1} i_{YU} + i_{ZUD, 1} i_{ZU} \\ &= x_{D1} i_{XS} + x_{D2} i_{YS} + x_{D3} i_{ZS} \end{aligned}$$

$$\begin{aligned} i_{YD} &= i_{XUD, 2} i_{XU} + i_{YUD, 2} i_{YU} + i_{ZUD, 2} i_{ZU} \\ &= y_{D1} i_{XS} + y_{D2} i_{YS} + y_{D3} i_{ZS} \end{aligned}$$

$$\begin{aligned} i_{ZD} &= i_{XUD, 3} i_{XU} + i_{YUD, 3} i_{YU} + i_{ZUD, 3} i_{ZU} \\ &= z_{D1} i_{XS} + z_{D2} i_{YS} + z_{D3} i_{ZS} \end{aligned}$$

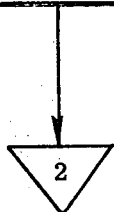


Figure 2b. Fine Alignment Gyro Torquing Angles  
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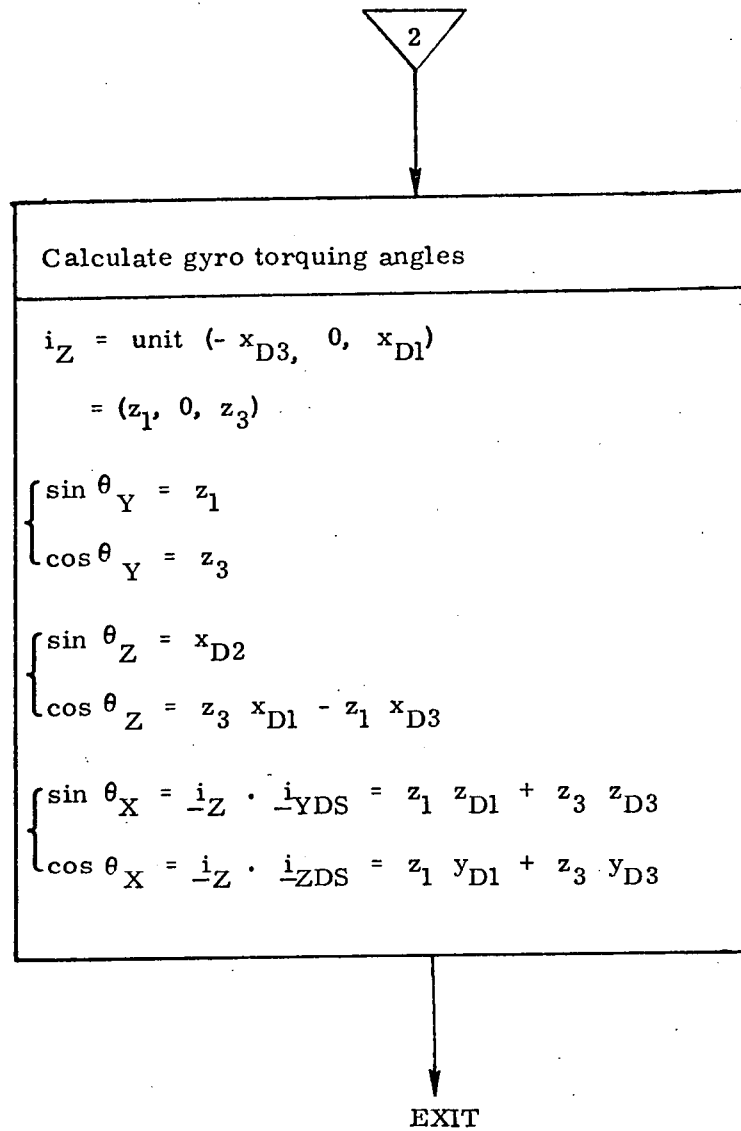


Figure 2c. Fine Alignment Gyro Torquing Angles

## 6. SUPPLEMENTARY INFORMATION

The equations derived in this document define three rotations which are sufficient to bring the present stable member into coincidence with the desired stable member. Because of the arbitrariness in defining the order of successive rotations and the axes of rotation (i. e., with respect to the present stable member or the desired stable member), care must be exercised in calling the derived angles "gyro torquing angles". It is conceivable that different manufacturers might employ different definitions for "gyro torquing angles".

In that a specific IRU has not been selected for Shuttle use, no final statement can presently be made as regards "gyro torquing angles" for fine alignment unless there are to be no other routines that might possibly operate under a conflicting definition.

In this document (as in Ref. 2) the rotations were executed in the sequence Y, Z, X. Should for any reason a different sequence be desired (e. g., X, Y, Z), the derived equations are correct provided a proper transformation of variables takes place (i. e.,  $Y \longrightarrow X$ ,  $Z \longrightarrow Y$ ,  $X \longrightarrow Z$ ).

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