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SHUTTLE PROGRAM

SPACE SHUTTLE ASTRODYNAMICAL CONSTANTS

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1.0 SUMMARY

This document provides basic space Shuttle astrodynamical constants for use in mission planning and construction of ground and onboard software input loads. The data included here are provided to facilitate the use of consistent numerical values throughout the project. The document supersedes reference 1.

2.0 INTRODUCTION

The astrodynamic constants presented in this document are taken primarily from reference 2, "Natural Environment and Physical Standards for Apollo and AAP". Those data have been expanded, and other data have been included when there was an indication that additional information would be useful. The values of the constants in reference 3 have been informally adopted by the Interplanetary Trajectory Committee.

3.0 DISCUSSION OF DATA

The data included in this document are described briefly in this section. Detailed information may be found at the beginning of sections 4 and 5.

3.1 ASTRODYNAMIC CONSTANTS

The trajectory prediction and astrodynamic constants are presented in section 4. The values included are based upon (ref. 2) and the Apollo Navigation Working Group document (ref. 4), but additional data have been included.

3.2 CONVERSION FACTORS

A list of conversion factors that were used to generate the data in section 4 is given in section 5.0. These are the minimum conversions required for systems of units commonly in use at JSC.

4.0 ASTRODYNAMIC CONSTANTS AND PARAMETERS

4.1 INTRODUCTION

This section presents the values and associated uncertainties of the constants and models used in trajectory prediction. A list of conversion factors and a description of the gravitational potential equation are also included.

Uncertainties (1^o) are presented if available. For consistent conversion between units, more decimal digits are given for some quantities than are justified by the uncertainties. The values presented are in agreement with those adopted by NASA Headquarters (ref. 2).

4.2 CONSTANTS AND PARAMETERS

4.2.1 Angular Velocity of the Earth's Rotation With Respect to the Vernal Equinox

The Earth's angular rotational velocity with respect to a precessing equinox (ω_p) and inertial equinox (ω_I) for OFT (calendar year 1979) based on references 6 and 16 is

$$\omega_p = .7292115854918357 - 004 \text{ radians per second}$$

$$\omega_I = .7292115146459210 - 004 \text{ radians per second}$$

4.2.2 Other Constants

This section defines values for other constants used in the ground and onboard software. Many of these parameters are not basic constants but are derived using some defining assumption.

- a. Stellar aberration constant - the maximum aberration of a star observed from the Earth occurs when direction of motion of observer is at right angles to line of sight to star.

$$c_A = .993674 \times 10^{-4} \text{ rad}$$

- b. Mean molecular weight of air - the mean molecular weight of the 10 air constituents (N₂, O₂, Ar, CO₂, Ne, He, Kr, Xe, CH₄, H₂) found by:

$$M_O = \frac{\sum (n_i \cdot M_i)}{\sum n_i} = 28.9644 \text{ Kg/Kmol}$$

where n_i is number density and M_i is molecular weight

- c. The gas constant - the constant factor in the equation of state for perfect gases.

$$R^* = 8.31432 \times 10^{-3} \text{ Newton meter (Kmol}\cdot\text{K)}$$

- d. $g M_o / R^*$ - derived constant used in onboard calculations of density involving sea level gravity acceleration, mean molecular weight and the gas constant

$$g_OVER_R = .01041294 \text{ K}^0/\text{ft}$$

- e. Speed of sound constant - derived constant used in the calculation of the speed of sound. The speed of sound is defined by:

$$c_s = \left(\frac{\gamma R^* T_m}{M_o} \right)^{1/2}$$

where $\gamma = 1.4$ and is the ratio of the specific heat of air at constant pressure to that at a constant volume (dimensimlers).

Because γ , R^* , and M_o are constants, they are combined for computational ease and the equation is written:

$$c_s = K \sqrt{T_m}$$

where K is the speed of sound constant

$$K = 65.77035 \text{ ft/sec/} \sqrt{K^0}$$

4.2.3 Gravitational Potential Function

The classical expression for the gravitational potential V exerted at a point in space located at a distance r from the center of the attracting body of radius R_E and gravitational parameter μ_E is given in equation as

$$V(r, \phi, \lambda) = \sum_{n=0}^{\infty} \frac{\mu_E}{r} \left(\frac{R_E}{r} \right)^n \sum_{m=0}^n P_{nm} (\sin \phi) C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)$$

ϕ is latitude (geodetic)

λ is longitude

where C_{nm} and S_{nm} are the harmonic coefficients of the potential function and $P_{nm}(\sin \phi)$ represents the associated Legendre functions of the first kind, of degree n and order m . Because $\sin \phi = z/r = u$, where u is a direction cosine, the associated Legendre functions may be expressed as:

$$P_{nm}(u) = \frac{\cos^m \phi}{2^n n!} \frac{d^{n+m}}{du^{n+m}} (u^2 - 1)^n$$

The constants are for real time operations are:

$$C_{2,0} = -1082.7 \times 10^{-6}$$

$$C_{3,0} = 2.56 \times 10^{-6}$$

$$C_{4,0} = 1.58 \times 10^{-6}$$

$$C_{22} = 1.57 \times 10^{-6}$$

$$S_{22} = -.897 \times 10^{-6}$$

all others are zero.

(Higher order models are sometimes used for analysis and postmission trajectory reconstruction.)

4.2.3.1 Equatorial Earth Radius (Gravitational)

1 E.r. = .6378160000000000+007 \pm .50000+001 m (ref. 2).
 1 E.r. = .2092572178477690+008 \pm .16404+002 int. ft
 1 E.r. = .3443930885519158+004 \pm .26998-002 n. mi.
 1 E.r. = .9999992160754699+000 \pm .78392-006 E.r. (MCC)

4.2.3.2 Gravitational Parameter ($GM_e = \mu_e = \mu_{\text{Earth}}$)

μ_e = .3986012000000000+015 \pm .40000+009 m^3/sec^2 (ref. 2).
 μ_e = .1407646853278542+017 \pm .14126+001 $(\text{int. ft})^3/\text{sec}^2$
 μ_e = .6275027808522208+005 \pm .62970-001 $(\text{n. mi.})^3/\text{sec}^2$
 μ_e = .1990931661816326+002 \pm .19979-004 $(\text{E.r.})^3/\text{hr}^2$ (MCC)

4.2.3.3 Mass of the Earth

M_e = .5973343323842350+025 KG
 M_e = .1316896781981220+026 LBM

4.2.4 Lunar Constants

4.2.4.1 Earth-Moon Mass Ratio (ref. 2)

M_e/M_m = 81.3010 (± 0.0010)

4.2.4.2 Mean Lunar Radius

R_m = .1738090000000000 + 007 \pm .70000 + 002 m (ref. 2)
 R_m = .5702395013123360 + 007 \pm .22966 + 003 int. ft
 R_m = .9384935205183585 + 003 \pm .37797 - 001 n. mi.
 R_m = .2725062772756741 + 000 \pm .10975 - 004 E.r. (MCC)

4.2.4.3 Principal Axes

a = .1738570000000000 + 007 \pm .70000 + 002 m (ref. 2)
 a = .5703969816272966 + 007 \pm .22966 + 003 int. ft
 a = .9387526997840173 + 003 \pm .37797 - 001 n. mi.
 a = .2725815340305558 + 000 \pm .10975 - 004 E.r. (MCC)

b = .1738210000000000 + 007 \pm .70000 + 002 m (ref. 2)
 b = .5702788713910761 + 007 \pm .22966 + 003 int. ft
 b = .9385583153347732 + 003 \pm .37797 - 001 n. mi.
 b = .2725250914643945 + 000 \pm .10975 - 004 E.r. (MCC)

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$$\begin{aligned}
 c &= .1737490000000000 + 007 \pm .70000 + 002 \text{ m (ref. 2)} \\
 c &= .5700426509186352 + 007 \pm .22966 + 003 \text{ int. ft} \\
 c &= .9381695464362851 + 003 \pm .37797 - 001 \text{ n. mi.} \\
 c &= .2724122063320720 + 000 \pm .10975 - 004 \text{ E.r. (MCC)}
 \end{aligned}$$

where a is directed toward the center of the Earth, c is coincident with the Moon's rotational axis, and b is perpendicular to a and c .

4.2.4.4 Gravitational parameter for the Moon ($GM_m = \mu_m = \mu_{\text{Moon}}$)

$$\begin{aligned}
 \mu_{\text{Moon}} &= .4902780000000000 + 013 \pm .60000 + 008 \text{ m}^3/\text{sec}^2 \text{ (ref. 2)} \\
 \mu_{\text{Moon}} &= .1731400417087798 + 015 \pm .21189 + 010 \text{ (int. ft)}^3/\text{sec}^2 \\
 \mu_{\text{Moon}} &= .7718260968373028 + 003 \pm .94456 - 002 \text{ (n. mi.)}^3/\text{sec}^2 \\
 \mu_{\text{Moon}} &= .2448838571715250 + 000 \pm .29969 - 005 \text{ E.r.}^3/\text{hr}^2 \text{ (MCC)}
 \end{aligned}$$

4.2.5 General Constants

4.2.5.1 Astronomical Unit

$$\begin{aligned}
 AU &= .1495978930000000 + 012 \pm .50000 + 004 \text{ m (ref. 2)} \\
 AU &= .4908067355643045 + 012 \pm .16404 + 005 \text{ int. ft} \\
 AU &= .8077640010799136 + 008 \pm .26998 + 001 \text{ n. mi.} \\
 AU &= .2345469159233102 + 005 \pm .78393 - 003 \text{ E.r. (MCC)}
 \end{aligned}$$

4.2.5.2 Velocity of Light in a Vacuum

$$\begin{aligned}
 c &= .2997925000000000 + 009 \pm .30000 + 003 \text{ m/sec (ref. 2)} \\
 c &= .9835711942257218 + 009 \pm .98425 + 003 \text{ int. ft/sec} \\
 c &= .1618750000000000 + 006 \pm .16199 + 000 \text{ n. mi./sec} \\
 c &= .1692105801590269 + 006 \pm .16933 + 000 \text{ E.r./hr (MCC)}
 \end{aligned}$$

4.2.5.3 Gravitational Parameters for the Sun

$$\begin{aligned}
 \mu_{\text{Sun}} &= .1327124990000000 + 021 \pm .15000 + 014 \text{ m}^3/\text{sec}^2 \text{ (ref. 2)} \\
 \mu_{\text{Sun}} &= .4686697671960888 + 022 \pm .52972 + 015 \text{ (int. ft)}^3/\text{sec}^2 \\
 \mu_{\text{Sun}} &= .2089242635906454 + 011 \pm .23614 + 004 \text{ (n. mi.)}^3/\text{sec}^2 \\
 \mu_{\text{Sun}} &= .6628718533157138 + 007 \pm .74922 + 000 \text{ E.r.}^3/\text{hr}^2 \text{ (MCC)}
 \end{aligned}$$

4.3 EPHemeris TAPE SYSTEMS

4.3.1 DE19 Tape

The ephemeris tape system to be used for all missions is provided by JPL and is called the JPL Development Ephemeris Number 19 (DE19). For additional information, see references 7 and 8.

4.3.3 Tape System Conversion Factors

The following values are to be used to convert DE19 units to kilometers (refs. 7, 8, and 11).

$$AU = 149\ 597\ 893 \text{ km} \text{ (scale factor for planetary ephemerides)}$$

$$R_{em} = 6378.1492 \text{ km} \text{ (scale factor for lunar ephemerides)}$$

$$\mu^{-1} = 81.301 \text{ (ratio of Earth mass to Moon mass)}$$

$$GE = 398\ 601.2 \text{ km}^3/\text{sec}^2 \text{ (gravitational parameter of the Earth)}$$

4.4 DRAG MODEL

4.4.1 Reference Atmospheres

Seven reference atmospheres to be used are listed below.

- a. Jacchia (ref. 12)
- b. U. S. Standard Atmosphere, 1962 (ref. 13)
- c. U. S. Standard Atmosphere Supplements, 1966 (ref. 14)
- d. Cape Kennedy Reference Atmosphere (ref. 15)
- e. GLOBAL Reference Atmosphere (ref. 17)
- f. Edwards Atmosphere (ref. 18)
- g. Vandenberg Atmosphere (ref. 19)

The MSFC Modified Jacchia Model is a computerized version of Jacchia's Static Diffusion Model (ref. 12). This model and other mechanizations may be used for density computations between 120 and 1000 km. This model is dynamic because the values of the atmospheric parameters vary widely with geomagnetic activity, season, solar activity, and latitude. If predicted values of solar

activity or geomagnetic activity are required, they may be obtained from the Mathematical Physics Branch, Johnson Space Center, 483-4751.

The U. S. Standard Atmosphere (1962) is a static model in that the values of density are fixed with altitude. This model is primarily used as a standard for defining atmospheric properties and is accepted as the worldwide reference for air data devices. Note, however, that for altitudes in the interval of 300 to 500 km, the 1962 standard atmosphere is as much as 75-percent more dense than the Jacchia model. The standard 1962 atmosphere model is used in the MCC for mission operations in the Encke free-flight predictor whenever the spacecraft altitude is between 125 km and entry interface.

The U. S. Standard Atmosphere Supplements (1966) is a dynamic model very similar to the MSFC modified Jacchia. It is currently in use for Apollo entry studies and will be used for other entry evaluations.

The Cape Kennedy Reference Atmosphere, which is sometimes referred to as the Patrick atmosphere, should be used for vehicle launch analysis and for launches from the Eastern Test Range. For launches from the Western Test Range, the Vandenberg reference should be used.

The four D global and Edwards AFB models are used for entry analysis, the generation of operational profiles and dispersion analysis.

4.4.2 Drag Equations (for Orbit Operations)

$$R_D = -\frac{1}{2} C_D \rho (A/M) (R - \Omega \times R) | R - \Omega \times R |$$

where

$$C_D = (CDF + CDN|\sin \alpha|^N)(1 - |\sin \beta|) + CDS|\sin \beta| + CDA|\sin^2 \beta \sin \alpha|$$

and

α = angle of attack

β = angle of side slip

CDF, CDN, CDS, CDA, N = fit coefficients associated with vehicle configuration

$C_D = 2.0$ (above 400 000 feet for constant area drag)

A = effective cross sectional area of vehicle

M = mass of vehicle

ρ = density of atmosphere at given altitude (z_1)

\dot{R} = inertial velocity vector of vehicle

R = position vector to vehicle from the center of Earth

\ddot{R}_D = acceleration vector caused by drag

Ω = Earth's rotational vector = $(0, 0, \omega)$

$$z_1 = r - \frac{r}{d}$$

$$d = \left(\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} \right)^{1/2}$$

$$r = (x^2 + y^2 + z^2)^{1/2}$$

for the ellipsoid used in the 1962 U. S. Standard Atmosphere,

$$a = 6.378\ 178 \times 10^6 \text{ m}$$

$$b = 6.356\ 797 \times 10^6 \text{ m}$$

4.5 FISCHER EARTH MODEL

The following constants describe the Fischer Earth model (1960), which is used for the location of radar stations and other Earth surface features (ref. 2)

4.5.1 Equatorial Earth Radius

$$a = .6378166000000000 + 007 \text{ m}$$

$$a = .2092574146981627 + 008 \text{ int. ft}$$

$$a = .3443934125269978 + 004 \text{ n. mi.}$$

$$a = .1000000156784905 + 001 \text{ E.r. (MCC)}$$

4.5.2 Flattening

$$f = \text{flattening} = 1 - b/a$$

$$f = 1/298.30 = 0.3352329869259135 \times 10^{-2}$$

4.5.3 Polar Earth Radius

$$\begin{aligned}b &= .6356784283607107 + 007 \quad \text{m} \\b &= .2085559148165061 + 008 \quad \text{int. ft} \\b &= .3432388922034075 + 004 \quad \text{n. mi.} \\b &= .9966478263900521 + 000 \quad \text{E.r. (MCC)}\end{aligned}$$

4.5.4 Eccentricity of Ellipsoid

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$e = .8181333401693114 - 001$$

$$e^2 = 2f - f^2$$

$$e^2 = .6693421622965943 - 002$$

5.0 EQUIVALENTS AND CONVERSION FACTORS

1 int. ft	= .3048000000000000 + 000	m (exact) (ref. 2)
1 n. mi.	= .1852000000000000 + 001	km (exact) (ref. 2)
1 E.r.	= .6378165000000000 + 004	km (exact for scaling) for MCC internal use
1 lbm	= .4535923700000000 + 000	kg (exact) (ref. 2)
1 hr	= 3600.000000000000 + 000	sec (exact)
1 rad	= $180/\pi$ deg	
1 deg	= 3600 arc sec	
1 km	= .5399568034557235 + 000	n. mi.

1 m = .3280839695013123 + 001 int. ft
1 n. mi. = .6076115485564304 + 004 int. ft
1 rad = .5729577951308233 + 002 deg
1 deg = .1745329251994329 - 001 rad
1 kg = .2204622621848776 + 001 lbm
1 int. stat. mi. = 5280 ft (exact)
1 lbf = 32.174048556 (int. ft/sec²)lbm
 π = .3141592653589793 + 001 (ref.2)

REFERENCES

1. Lanier, R.: Trajectory Prediction Parameters for Skylab, Space Station/Space Shuttle and Interplanetary Missions. Rev 1, JSC IN 70-FM-121, Aug. 31, 1970.
2. Natural Environment and Physical Standards for the Apollo Program and the Apollo Applications Program. NASA M-DE-8020.00C, SE 015-001-1B, July 10, 1969.
3. Melbroune, William G.; et al.: Constants and Related Information for Astrodynamic Calculations, 1968. TR-32-1306, JPL, July 15, 1968.
4. Apollo Navigation Working Group; Apollo Missions and Navigation Systems Characteristics. Technical Report AN-1.3, Dec. 15, 1967, as amended.
5. Mechtly, E. A.: The International System of Units, Physical Constants and Conversion Factors. Revised, NASA SP-7012, 1969.
6. Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac. Her Majesty's Stationery Office (London), 1961.
7. Devine, Charles J.: JPL Development Ephemeris Number 19. JPL Technical Report 32-1181, Nov. 15, 1967.
8. Mulholland, J. Derral: Revised Constants for Use with JPL Lunar Ephemeris Number 4. JPL Reference no. 311.13-180, May 14, 1968.
9. York, W.: Estimated DE19 Ephemeris Errors for Apollo 12 Mission. NASA/MSC memo, 69-FM49-308, Sept. 30, 1969.
10. The Deep Space Network: Space Programs Summary 37-57, Volume II. Jet Propulsion Laboratory.
11. O'Handley, Douglas A.; Holdridge, D. B.; and Mulholland, J. D.: JPL Development Ephemeris Number 69, Preliminary Documentation. JPL External Technical Report (undated).
12. Jacchia, L. G.: New Static Models of the Thermosphere and Exosphere With Empirical Temperature Profiles. Smithsonian Astrophysical Observatory, 1970.
13. U. S. Standard Atmosphere, 1962. Prepared under sponsorship of NASA, USAF, and U. S. Weather Bureau, Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1962.
14. U. S. Standard Atmosphere Supplements, 1966. Prepared under sponsorship of ESSA, NASA, and USAF, Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1966.

15. Smith, O. E.; and Weidner, D. K.: A Reference Atmosphere for Patrick AFB, Florida, Annual (1963 Revision). NASA TM X-53139, September 23, 1964.
16. Cockrell, B. F.: New numbers for the Earth's rotational angular velocity. NASA/JSC memo FM83 (78-148), May 19, 1976.
17. Justus, C.; Woodrum, A.; Roper, R.; and Smith, D. E.: Four-D Global Reference Atmosphere. NASA TM X-64872, September 1974.
18. Johnson, D. L.: Hot, Cold, and Annual Reference Atmospheres for Edwards Air Force Base, California (1975 Version). NASA TM X-64970, November 1975.
19. Carter, E. A.; and Brown, S. C.: A Reference Atmosphere for Vandenberg AFB, California Annual (1971 Version). NASA TM X-64590, May 10, 1971.