## SPACE SHUTTLE GN\&C EQUATION DOCUMENT

No. 17
(Revision 3)
Entry and Transition Guidance
by
Fred J. Marcus


## CHARLES STARK DRAPER <br> LABORATORY

## MASSACHUSETTS IF\& TTUTE OF TECHNOLOGY

CAMBRIDGE, MASSACHUSETTS, O2139


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## Charles Stark Draper Laboratory <br> June 1973 <br> NAS9-10268

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

## REVISION 3 CHANGES

This document is a revised version of the previous one dated December 1972 and supersedes that document. Major changes have been made in the guidance equations to increase the flexibility of the system and improve its performance characteristics.

The guidance system now includes a transition mode following the normal entry mode. Significant changes have also been made in the constant heating rate mode and in the roll angle command logic.

## FOREWORD

This document is one of a series of candidates for inclusion in a future revision of JSC-04217, 'Space Shuttle Guidance, , Navigation and Control Design Equations ". The enclosed has been prepared under NAS9-10268, Task No. 15-A, "GN \& C Flight Equation Specification Support", and applies to functions 6 through 9 of the Entry Guidance Module (OG5) as defined in JSC-03690, Rev. D, "Space Shuttle Orbiter Guidance, Navigation and Control Software Functional Requirements," dated January 1973.


Gerald M. Levine
Division Leader, Guidance Analysis
NASA Programs Department

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## NOMENCLATURE

## Notational Conventions

Upper-case letters represent matrices
Lower-case and Greek letters are reserved for scalars and vectors
Vector quantities are underlined, e.g. $\underline{x}$
Vectors are assumed to be column vectors unless explicitly noted
Symbols
a Effective aerodynamic area for vehicle
$a_{D} \quad$ Desired value of vertical acceleration
$c_{D} \quad$ Aerodynamic drag coefficient for vehicle
$c_{\rho} \quad$ Coefficient in desired-density relation for constant heatingrate mode
$c_{\dot{q}} \quad$ Coefficient used in relation for desired $c_{D}$
${ }^{c} D_{\dot{q}} \quad$ Desired value of $c_{D}$ for constant heating-rate mode
${ }^{c_{R_{1}}}, c_{R_{2}} \quad$ Coefficients used in relation for $r_{G D}$ (transition guidance mode)
d Drag force per unit mass
ee Square of eccentricity of Fischer ellipsoid
$\underline{h}_{\mathrm{N}} \quad$ Stored array of reference trajectory altitudes
h Vehicle altitude above Fischer ellipsoid
$h_{\rho} \quad$ Density altitude
${ }^{h_{D}}, \dot{h}_{D}, \ddot{h_{D}} \quad$ Desired density altitude and derivatives w. r.t. time


IMU-derived vertical velocity and derivatives w. r.t. time
$\dot{h}_{f} \quad \quad \quad$ Filtered vertical velocity
$h_{\rho} \dot{h}_{\rho} \quad$ Altitude at which $\rho$ would occur and its derivative w.r.t. time
$\underset{\sim}{i} \quad$ Unit vector along $\underline{r}_{\mathrm{E}}$ (earth-fixed coordinates)
ípole Unit vector along North pole (earth-fixed coordinates)
i $\mathrm{EE} \quad$ Unit vector directed towards local East (earth-fixed coordinates)
i SE Unit vector directed towards local South (earth-fixed coordinates)
$\underline{\underline{i}}$ GCE Unit vector along desired great-circle heading direction (earth-

- $\mathrm{RDE} \quad$ Unit vector along desired terminal position (earth-fixed coordinates)
$\underline{i}^{\mathbf{i}}$ VRHE $\quad$ Unit vector along $\underline{v}_{\text {RHE }}$ (earth-fixed coordinates)
$\mathrm{k} \quad$ Index variable used for reference-trajectory lookup

$k_{f_{3}}, k_{f_{4}} \quad$ Coefficients used in desired vertical acceleration filter (dependent upon $\omega_{\mathrm{f}_{2}}$ and $\Delta t$ )
$\mathrm{k}_{\ell} \quad$ Coefficient used in relation for latitude rate
$\mathrm{k}_{\alpha} \quad$ Sensitivity factor in angle-of-attack relation
$\mathrm{k}_{\phi 0}, \mathrm{k}_{\phi 1}, \quad$ Coefficients used in roll-command relation
$\mathrm{k}_{\phi 2}$

| $k_{\phi 3}$, | Value used for $\mathrm{k}_{\phi 2}$ during transition mode |
| :---: | :---: |
| $\mathrm{k}_{\phi 4}$ | Initial value of $\mathrm{k}_{\phi 2}$ |
| ${ }^{\mathrm{k}} \eta_{1}$ | Fraction of $\eta_{\mathrm{D}}$ at which roll angle should be reversed |
| ${ }^{\mathbf{k}} \eta_{2}$ | Fraction of $\eta_{D}$ at which roll angle used to correct heading error (transition mode) |
| $\ell_{\text {A }}$ | Latitude of vehicle (geocentric) |
| $\ell_{\text {AD }}$ | Desired latitude at the end of entry (geocentric) |
| ${ }^{\ell} \mathrm{OD}$ | Desired longitude at the end of entry |
| $\mathrm{M}_{\mathrm{SM}-\mathrm{E}}$ | Transformation matrix from stable member to earth-fixed coordinates |
| m | Mass of vehicle |
| n | Index for computation-cycle time |
| $n_{\text {mach }}$ | Mach number |
| $\dot{q}_{\text {D }}$ | Desired heating rate during constant heat-rate mode |
| $\underline{r}$ | Vehicle position (stable-member coordinates) |
| $\mathrm{r}_{\mathrm{e}}$ | Earth radius (at target point) |
| $\underline{r}_{\underline{E}}$ | Vehicle position (earth-fixed coordinates) |
| $\dot{\mathbf{r}}_{\mathrm{f}}$ | $R$ ate of change of earth radius along $\underline{r}_{E}$ (Fischer ellipsoid) |
| $\mathrm{r}_{\mathrm{G}}$ | Range to go to target point |
| $r_{\text {GD }}$ | Desired value of $r_{G}$ at current $v_{R E}$ (from stored trajectory) |
| $\mathrm{r}_{\mathrm{G}_{\dot{\mathrm{q}}}}$ | Desired range to be covered in the constant heating-rate mode |
| $r_{\text {GEF }}$ | Nominal rad ge to be covered in the reference trajectory mode (at relative velocity ${ }^{v_{F} 1_{\dot{q}}}$ ) |


| $\mathrm{r}_{\mathrm{G}_{\text {APP }}}$ | Nominal range to be covered in approach phase |
| :---: | :---: |
| ${ }^{r_{G}}{ }_{\text {MIN }}$ | Lower limit for ${ }^{r_{G}}$ in angle-of-attack computation |
| $\mathrm{r}_{\mathrm{G}}^{*}$ | Range-to-go used in reference-trajectory mode guidance computations |
| $\underline{-r N}_{\text {GN }}$ | Stored array of reference-trajectory range-to-go |
| ${ }^{\text {p pole }}$ | Polar earth radius |
| $s_{\text {HED }}$ | Switch used for roll angle command (transition mode) |
| $s_{\phi}$ | Dummy variable used in roll-reversal logic |
| $s_{\phi 1}$ | Switch used to indicate reference table index (k) has changed |
| $\mathrm{s}_{\mathrm{q}}$ | Switch used to start constant heating rate mode |
| ${ }^{\text {S REF }}$ | Switch used to start reference-trajectory mode |
| $s_{t}$ | Total specific force on vehicle |
| ${ }^{s} \mathrm{t}_{\text {PROG }}$ | Value of specific force at which constant heating rate guidance is begun |
| $\mathrm{s}_{\mathrm{v}}$ | Vertical component of specific force on vehicle |
| t | Current time |
| $\stackrel{v}{F}_{\dot{\mathrm{q}}}$ | Relative velocity at which constant heating-rate mode is terminated |
| ${ }^{v_{\mathrm{F} 1}}{ }_{\dot{\mathrm{q}}}$ | Relative velocity used in relation for ${ }^{c_{D}}{ }_{\dot{\mathbf{q}}}$ |
| ${ }^{\mathrm{v}} \mathrm{N}$ | Reference-trajectory array of vehicle speed w.r.t. air mass |
| $\mathrm{v}_{\mathrm{R}}$ | Velicle velocity w.r.t. air mass (stable-member coordinates) |


| $\underline{v}_{\text {RE }}$ | Vehicle velocity w.r.t. air mass (earth-fixed coordinates) |
| :---: | :---: |
| $\mathrm{v}_{\text {RE }}$ | Magnitude of $\underline{\mathrm{v}}_{\mathrm{RE}}$ |
| $\underline{v}_{\text {- }}$ RHE | Horizontal component of $\mathrm{v}_{\mathrm{RE}}$ |
| $\mathrm{v}_{\mathrm{TR}}^{1}$ | Value of relative velocity at which gains are changed preceding transition mode |
| ${ }^{\mathrm{v}_{\mathrm{TR}}^{2}}$ | Value of relative velocity at which transition guidance is begun |
| ${ }^{v_{T R}}$ | Value of relative velocity at which transition guidance is terminated |
| ${ }^{\alpha} \mathrm{C}$ | Angle-of-attack command |
| ${ }^{\alpha}$ CONST | Angle-of-attack to be maintained before constant heating-rate mode |
| ${ }^{\alpha}$ CMAX | Maximum permissible value of ${ }^{\alpha} \mathrm{C}$ |
| ${ }^{\alpha}$ CMIIN | Minimum permissible value of ${ }^{\alpha} \mathrm{C}$ |
| ${ }^{\alpha} \mathrm{D}$ | Desired value of $\alpha$ at current $\mathrm{v}_{\mathrm{RE}}$ (from stored trajectory) |
| ${ }^{\alpha_{\operatorname{MAX}}^{\dot{q}}}$ | Maximum permissible angle-of-attack in constant heating-rate mode |
| ${ }^{\alpha_{\mathrm{MIN}_{\mathrm{q}}}}$ | Smallest permissible angle-of-attack in constant heating-rate mode |
| $\underline{\underline{\alpha}}^{\mathrm{N}}$ | Stored array of reference-trajectory angle-of-attack |
| $\bar{\alpha}^{P}$ | Average angle-of-attack during previous time step |
| ${ }^{\varphi} \mathrm{C}$ | Roll angle command |
| ${ }^{\text {CONSST }}$ | Roll-angle to be maintained before constant heating-rate mode |
| ${ }^{\bar{\phi}}{ }_{P}$ | Average roll angle during previous time step |
| ${ }^{\Delta a_{v_{C}}}$ | Commanded change in vertical acceleration |
| $\Delta \psi$ | Difference between current and desired heading of vehicle w.r.t. air mass |


| $\Delta \psi_{0}$ | Value of $\Delta \psi$ on first pass |
| :---: | :---: |
| $\Delta \underline{v}$ | Accelerometer-measured velocity change from previous to present computation-cycle time |
| $\Delta \mathrm{v}_{\mathrm{TR}}^{1} 1$ | Difference between $\mathrm{v}_{\mathrm{TR}}{ }_{1}$ and ${ }^{\text {V }} \mathrm{TR}_{2}$ |
| $\Delta \mathrm{v}_{\mathrm{TR}_{2}}$ | Difference between $\mathrm{v}^{\mathrm{TR}} \mathrm{T}_{2}$ and ${ }^{\mathrm{v}} \mathrm{TR}_{3}$ |
| $\Delta t$ | Time interval from previous to present computation-cycle time |
| $\Delta t^{\text {OLD }}$ | Previous value of $\Delta t$ |
| $\omega_{\mathrm{f}_{1}}$ | Parameter in vertical velocity filter |
| $\omega_{\mathrm{f}_{2}}$ | Parameter in desired vertical acceleration filter |
| $\begin{aligned} & \phi_{\text {MAX }}, \\ & \phi_{\mathrm{MIN}} \end{aligned}$ | Levels used in ${ }^{\varphi} \mathrm{C}$ computations |
| $\phi *$ | Dummy variable used in roll-angle computations |
| $\phi_{\mathrm{OLD}}$ | Previous value of ${ }^{\phi} \mathrm{C}$ |
| $\theta$ | Great-circle angle from the current position to be desired target point |
| $\eta$ | Absolute value of heading error |
| $\eta_{\mathrm{D}}$ | Desired value of $\eta$ at current $\mathrm{v}_{\mathrm{RE}}$ (from stored trajectory) |
| $\underline{\eta}_{\mathrm{N}}$ | Stored array of reference-trajectory $\eta$ |
| $\hat{\rho}$ | Estimated density from specific force measurements |
| $\rho_{\text {q }}$ | Desired density for constant heating-rate mode |
| $\zeta$ | Dummy variable used in reference trajectory lookup |
| $\zeta_{1}$ | Dummy variable used in gain computation preceding transition mode |

Special Notation
( ) A-priori estimated value prior to measurement incorporation
( ) Average over previous computation cycle of ( )
|( )| Magnitude of ( )
()$^{\mathrm{T}} \quad$ Transpose of ( )
unit (__) Unit vector for ( _ )
sign ( ) Algebraic sign associated with ( ). Value is +1 or -1 , with $\operatorname{sign}(0) \triangleq+1$

## 1. INTRODUCTION

The Entry-Guidance Routine presented here is designed to take the orbiter vehicle from entry interface ( $h \approx 400,000 \mathrm{ft}$ ) through the critical heating phase of entry down to the start of the approach phase ( $h \approx 70,000 \mathrm{ft}$ ). The basic ideas are outlined in Refs. (1) - (5), (7). Simulation results demonstrating the feasibility of the concept are given in Refs. 2, 5, 6.

There are four basic guidance modes:
(1.) An initial programmed-maneuver mode in which the vehicle is oriented with a constant roll angle and angle-of-attack. This mode is used until the specific force is greater than a pre-selected level.
(2.) A constant heating-rate mode during which the stag-nation-point heating rate is held constant at a preselected value, chosen essentially to minimize heat loads on the vehicle without violating maximum temperature constraints.
(3.) A reference trajectory mode during which the vehicle follows a pre-stored trajectory designed to get the vehicle to the terminal point with a minimum required TPS weight, and without violating operational constraints on the vehicle.
(4.) A transition mode during which the vehicle's angle-of-attack is linearly ramped as a function of velocity from the back to the front side of the L/D curve. This mode also provides terminal conditions compatible with the start of an approach guidance system.

Thermal control is provided by varying the magnitude of the roll angle so as to follow a density-vs. -speed profile. Density information is derived from IMU measurements of the aerodynamic specific force acting on the vehicle. A-priori knowledge of the vehicle's mass, effective aerodynamic area, and drag coefficient $\left(c_{D}\right)$ are required in the process.

During the constant heat rate and reference trajectory modes, range control is provided by changing the angle-of-attack of the vehicle. Upper and lower limits on angle-of-attack are required in order not to violate operational constraints on the vehicle. Lateral trajectory control is obtained by reversing the direction of the roll angle.

In the transition-guidance mode, roll-angle magnitude is used to provide both altitude and range control. Limits on roll angle magnitude are used in order to alleviate potential problems due to sonic boom focal effects.

## 2. FUNCTIONAL FLOW DIAGRAM

The basic information flow in the Entry Guidance Routine is shown in Figure 1. This is based on the guidance concept of Refs. 2, 5, 7.

After the routine is entered, a series of targeting computations are made. This involves the computation of quantities such as the vehicle's current heading error $(\Delta \psi)$, and range to the target point $(\theta)$.

The particular guidance mode to be entered is next determined. There are four possible guidance modes:
(1.) Initial programmed maneuver
(2.) Constant stagnation-point heating rate guidance
(3.) Stored reference-trajectory guidance
(4.) Transition guidance

The constant heating-rate mode is entered when the measured specific force on the vehicle is greater than a preselected value. The reference-trajectory mode is entered when the magnitude of the vehicle's relative velocity is less than a preselected value. The transition mode is also started on a relative-velocity criterion.

In the programmed maneuver mode the vehicle is oriented with a constant roll angle and an angle-of-attack. This orientation is maintained until the heatingrate mode is entered.

In the constant heating-rate mode, the density altitude $\left(h_{D}\right)$ required to attain the desired stagnation point heating rate ( $\dot{q}_{\mathrm{D}}$ ) is first computed. An angle-of-attack command ( $\alpha_{C}$ ) is next computed, based on the desired range-to-go for the heating-rate mode. Finally, roll-angle magnitude commands ( $\phi_{C}$ ) are computed to control the vertical-plane motion of the vehicle so as to follow a densityaltitude vs. speed profile. No roll reversals take place in this mode.

In the reference-trajectory mode, the required reference-trajectory quantities are first obtained from the stored table at the current speed. These include angle-of-attack ( $\alpha_{D}$ ), altitude ( $h_{D}$ ), range-to-go ( $r_{G D}$ ), and the nominal heading error ( $\eta_{\mathrm{D}}$ ). The angle-of-attack command ( $\alpha_{\mathrm{C}}$ ) is then computed as a perturbation from the reference value ( $\alpha_{D}$ ) based on the difference between the stored and measured values of range-to-go. Roll-angle magnitude commands are computed in the same manner as for the constant heating-rate mode, excpet that the desired density altitude ( $h_{D}$ ) is from the stored table. Roll-angle direction is based on a comparison between the current heading error and the reference-trajectory value.


During the transition mode, desired values of angle-of-attack, density altitude, and allowable heading error are obtained from a stored trajectory as in the reference-trajectory mode. Range-to-go ( $r_{G D}$ ) is computed from a parabolic fit, relating nominal range to speed. The commanded angle-of-attack is preprogrammed as a function of speed, and during this mode $\alpha_{\mathrm{C}}$ is set equal to $\alpha_{\mathrm{D}}$.

Roll-angle magnitude commands are again computed in the same manner as for the constant heat rate mode except that the desired density altitude ( $h_{D}$ ) from the table lookup is modified by a term proportional to the vehicle's range error ( $r_{G D}-r_{G}$ ). Roll angle magnitude and direction are also based on the vehicle's current heading error. If $\eta$ is not too large, roll reversals are performed as in the reference trajectory mode. If, however, $\eta$ becomes excessive, both the rollangle magnitude and roll direction are commanded to reduce this error.
3. INPUT AND OUTPUT VARIABLES

Input Variables
$\mathrm{M}_{\text {SM-E }}$
$\underline{r}$
$\mathrm{v}_{\mathrm{R}} \quad$ Vehicle velocity w.r.t. air mass (stable-member coordinates)
$\Delta \underline{\mathbf{v}} \quad \mathrm{IMU}$-measurement velocity change
$\Delta t \quad$ Time interval over which $\Delta \underline{v}$ is taken
$\bar{\alpha}_{\mathbf{P}} \quad$ Average value of angle-of-attack over previous $\Delta t$
$\bar{\phi}_{\mathrm{P}} \quad$ Average value of roll-angle over previous $\Delta t$

## Output Variables

${ }^{\alpha} \mathrm{C}$
Angle-of-attack command
${ }^{\phi} \mathrm{C}$
Roll angle command

## 4. DESCRIPTION OF EQUATIONS

### 4.1 Targeting Calculations

The range from the present location of the vehicle to the desired target point is computed from

$$
\begin{equation*}
\theta=\cos ^{-1}\left(\underline{i_{R E}} \cdot \underline{i} R \mathrm{RDE}\right) \tag{1}
\end{equation*}
$$

where $\underline{-}_{R E}$ and ${\underset{\sim}{i}}^{-}$DE are unit vectors at the present and target-point locations on the earth.

The vehicle's current heading error $(\Delta \psi)$, defined as the difference between the current vehicle heading and the great-circle heading to the target point,is computed as:

$$
\begin{equation*}
\Delta \psi=\cos ^{-1}\left[\underline{i}_{\mathrm{GCE}} \cdot \underline{i}_{\mathrm{VRHE}}\right] \operatorname{sign}\left[\left(\underline{\mathrm{i}}_{\mathrm{GCE}} \times \underline{i}_{\mathrm{VRHE}}\right) \cdot \underline{i}_{\mathrm{RE}}\right] \tag{2}
\end{equation*}
$$

where $\underline{v}_{\text {RHE }}$ is the horizontal component of vehicle's relative velocity, and $\underline{i}$ VRHE is a unit vector along $\underline{V}_{\mathrm{V}}^{\mathrm{RHE}}$. The term $\underline{\mathrm{i}}_{\mathrm{GCE}}$ is a unit vector which is horizontal at the current vehicle location, and lies in the desired great-circle trajectory plane.

### 4.2 Initial Programmed-Maneuver Mode

The roll-angle command ( $\varphi_{C}$ ) w. r.t. the relative velocity vector is for a constant roll attitude, i.e.

$$
\begin{equation*}
\phi_{\mathrm{C}}=\phi_{\mathrm{CONST}} \tag{3}
\end{equation*}
$$

The angle-of-attack command ( $\alpha_{C}$ ) is also maintained constant during this mode, i.e.

$$
\begin{equation*}
{ }^{\alpha} \mathrm{C}={ }^{\alpha} \text { CONST } \tag{4}
\end{equation*}
$$

4.3 Constant Heating-Rate Mode

### 4.3.1 Thermal Control

The required density ( $\rho_{D}$ ) corresponding to a constant stagnation-point heating rate ( $\dot{q}_{D}$ ) is computed from the expression:

$$
\begin{equation*}
\rho_{\mathrm{D}}=c_{1} \dot{q}_{\mathrm{D}}^{2} / v_{R E}{ }^{6.3} \tag{5}
\end{equation*}
$$

where $v_{R E}$ is the velocity of the vehicle w. r.t. the air mass, and $c_{1}$ is a constant. This relation can be obtained from the stagnation-point heating-rate equation:

$$
\begin{equation*}
\dot{q}=c_{0} \rho^{0.5} v_{R E} 3.15 \tag{6}
\end{equation*}
$$

where $q$ is the heating rate, $\rho$ is the density of the atmosphere, and $c_{0}$ is a constant.

Thermal control is obtained by causing the vehicle to follow the desired density ( $\rho_{D}$ ) vs. speed ( $v_{R E}$ ) profile. To accomplish this objective, an estimate of density at the current vehicle location is obtained from IMU-derived drag measurements. The basic relation for the density estimate ( $\hat{\rho}$ ) is:

$$
\begin{equation*}
\hat{\rho}=2 \mathrm{~d} \frac{\mathrm{~m}}{\mathrm{c}_{\mathrm{D}}{ }^{a}} / \mathrm{v}_{\mathrm{RE}} \tag{7}
\end{equation*}
$$

where $m$ is the vehicle's mass, a is the effective aerodynamic area, and $c_{D}$ is the computed drag coefficient. The quantity $v_{R E}$ is the magnitude of the vehicle's velocity w. r.t. the air mass. The measured drag ( d ) is computed from IMU ve-locity-change date ( $\Delta \underline{v}$ ) using the relation:

$$
\begin{equation*}
\mathrm{d}=\left|\Delta \underline{\mathrm{v}} \cdot \underline{\mathrm{i}}_{\mathrm{VRE}} / \Delta \mathrm{t}\right| \tag{8}
\end{equation*}
$$

where $\underline{i}_{V R E}$ is a unit vector along $\underline{v}_{R E}$, and $\Delta t$ is the time interval over which $\Delta \underline{v}$ is measured.

Rather than working with $\rho_{D}$ and $\rho$ directly, it has been found to be more convenient to transform these quantities to equivalent density altitudes above the earth, i.e. $h_{D}$ and $h_{\rho}$. In early simulations a simple exponential model was used to implement the transformation, based on the relation

$$
\begin{equation*}
\rho=\rho_{0} \epsilon^{-h / h_{S}} \tag{9}
\end{equation*}
$$

where $\rho_{0}$ is the sea-level density of the atmosphere, and $h_{s}$ is its scale height. Currently a stored density-vs. altitude table for the 1962 standard atmosphere is used here.

Using $h_{D}$ and $h_{\rho}$ as the primary guidance variables, a simple control law has been formulated which permits accurate control of density during the entire entry phase. The basic command is the desired change in vertical acceleration $\left(\Delta a_{v_{D}}\right)$, which is computed from the relation:

$$
\begin{equation*}
\left.\Delta \mathrm{a}_{\mathrm{v}}=\mathrm{k}_{\phi 1}\left(\mathrm{~h}_{\mathrm{D}}-\mathrm{h}_{\rho}\right)+\mathrm{k}_{\phi 2}\left(\dot{\mathrm{~h}}_{\mathrm{D}}-\dot{\mathrm{h}}_{\rho}\right)+{\left.\ddot{\left(\mathrm{h}_{\mathrm{D}}\right.}-\ddot{\mathrm{h}}_{\mathrm{I}}\right)}\right) \tag{10}
\end{equation*}
$$

where $\dot{h}_{\rho}, \dot{h}_{D}, \ddot{h}_{D}$ are the first and second derivatives of $h_{\rho}$ and $h_{D}$. The quantities $\mathrm{k}_{\phi_{1}}$ and $\mathrm{k}_{\phi_{2}}$ are maintained as constant sensitivities during the constant-heat-rate and reference-trajectory modes. Prior to the start of the transition phase they are gradually increased to higher values which are then held constant during the transition mode. To obtain best performance in the presence of measurement noise, smoothing of the altitude-rate term ( $\dot{h}_{\rho}$ ) and the nominal vertical acceleration ( $\mathrm{h}_{\mathrm{D}}$ ) is required.

The quantity $\ddot{h}_{\mathrm{I}}$ is computed by back differencing the vehicle's altitude rate ( $\dot{\mathrm{h}}_{\mathrm{I}}$ ). This altitude rate is computed using navigated information and a correction term to account for the change in earth radius with changing geocentric latitude (Fischer-ellipsoid earth model). The quantity $\dot{\mathrm{h}}_{\mathrm{I}}$ is evaluated according to the following equations:

$$
\begin{equation*}
\dot{h}_{\mathrm{I}}=\underline{\mathrm{v}}_{\mathrm{RE}} \cdot \underline{i}_{\mathrm{RE}}-\dot{\mathrm{r}}_{\mathrm{f}} \tag{11}
\end{equation*}
$$

where $\dot{r}_{f}$ is the time rate of change of earth radius due to the vehicle's latitude rate. $r_{f}$ is computed as:

$$
\begin{equation*}
\dot{r}_{f}=-\frac{r_{\text {pole }} \text { ee } \cos \left(\ell_{A}\right) \sin \left(\ell_{A}\right) \dot{\ell}_{A}}{\left[1-e e \cos ^{2}\left(\ell_{A}\right)\right]^{1.5}} \tag{12}
\end{equation*}
$$

where $r_{\text {pole }}$ is the value of earth radius at the pole, ee is the square of the eccentricity of the Fischer-ellipsoid; and the terms $\ell_{A}, \dot{\ell}_{A}$ are the vehicle's latitude and its time rate of change, respectively.

The roll-angle command ( $\varphi_{\mathrm{C}}$ ) required to obtain $\Delta \mathrm{a}_{\mathrm{v}_{\mathrm{D}}}$ is obtained by commanding a change from the current roll angle ( $\phi_{\mathrm{P}}$ ), according to the relation

$$
\begin{equation*}
\phi_{\mathrm{C}}=\phi_{\mathrm{P}^{-}} \frac{\left(\Delta \mathrm{a}_{\mathrm{v}_{\mathrm{D}}}\right) /\left|\tan \phi_{\mathrm{P}}\right|}{\mathrm{s}_{\mathrm{v}}} \tag{13}
\end{equation*}
$$

where the current vertical specific force ( $\mathrm{s}_{\mathrm{v}}$ ) is obtained from the inertial measurement unit. Special logic is provided to circumvent problems with Eq. (13) when $\phi$ is very small.

### 4.3.2 R ange Control

Range control during the constant heating-rate mode is obtained by varying the angle-of-attack of the vehicle. The desired great-circle range to be covered in the constant heating-rate mode $\left(r_{G_{\dot{q}}}\right)$ is the difference between the present greatcircle range to the terminal point $\dot{q}_{\left(r_{G}\right)}$ and the nominal range to be travelled in the final reference-trajectory mode ( $r_{G_{R E F}}$ ), and in the approach-guidance routine $\left(r_{G_{A P P}}\right)$, i.e.

$$
\begin{equation*}
r_{G_{\dot{q}}}=r_{G}-r_{G_{R E F}}-r_{G_{A P P}} \tag{14}
\end{equation*}
$$

It should be noted that the constant heating-rate mode is terminated at a selected final velocity $\left({ }_{\mathrm{v}_{\mathrm{q}}}\right)$.

If various reasonable assumptions are made for the constant heating-rate mode, such as small flight-path angle, small changes in heading, small changes in altitude compared to the radius of the earth, and constant $c_{D}$, then a simple closedform solution is obtained for $r_{G_{\dot{q}}}$. The mathematical relation is:

$$
\begin{equation*}
r_{G_{\dot{q}}}=\frac{c_{2}\left(v_{R E}{ }^{6.3}-v_{F} \dot{q}\right.}{\dot{q}^{2} c_{D}(a / m)} \tag{15}
\end{equation*}
$$

where $c_{2}$ is a constant. By rearranging Eq. (15), the desired value of $\quad\left(c_{D_{\dot{q}}}\right)$
for the present $\dot{q}$ and required $r_{\dot{q}}$ is obtained as: for the present $\dot{q}$ and required $r_{\dot{q}}$ is obtained as:

$$
\begin{equation*}
\mathrm{c}_{\mathrm{D}_{\dot{q}}}=\frac{\mathrm{c}_{2}\left(\mathrm{v}_{\mathrm{RE}}{ }^{6.3}-\mathrm{v}_{\mathrm{F}_{\dot{q}}}^{6.3}\right)}{\dot{\mathrm{q}}^{2} \mathrm{r}_{\mathrm{G}_{\dot{\mathrm{q}}}}(\mathrm{a} / \mathrm{m})} \tag{16}
\end{equation*}
$$

The angle of attack command $\left(\alpha_{C}\right)$ required to achieve ${ }^{c_{D_{q}}}$; is then computed as:

$$
\begin{equation*}
\alpha_{\mathrm{C}}=\alpha_{\mathrm{P}}+\left(\mathrm{c}_{\mathrm{D}}{ }_{\dot{\mathrm{q}}}-\mathrm{c}_{\mathrm{D}}\right) \mathrm{k}_{\alpha_{\mathrm{q}}} \tag{17}
\end{equation*}
$$

where $\alpha_{P}$ is the average value of angle-of-attack over the previous time step and $\mathrm{k}_{\alpha_{\dot{q}}} \quad$ is a constant gain approximately equal to $\partial \mathrm{c}_{\mathbf{D}} / \partial \alpha$.

### 4.3.3 Lateral Control

Roll reversals are not commanded during this guidance mode. The direction of the roll angle is chosen to reduce the vehicle's initial heading error.
4.4 Reference Trajectory Mode

### 4.4.1 Stored-Table Lookup

The guidance concept makes use of a stored reference trajectory designed to minimize TPS weight for the maximum cross-range case of interest. The stored variables are angle-of-attack ( $\underline{\alpha}_{N}$ ), altitude ( $\underline{h}_{N}$ ), great-circle range-to-go to the target point ( $\underline{r}_{G N}$ ), and allowable heading error $\left(\eta_{N}\right)$. The independent or index variable is vehicle speed w.r.t. the air mass ( $\underline{v}_{\mathrm{N}}$ ). In current simulation studies a 30-element stored array is used.

A linear interpolation is used between the tabulated variables. For a typical case where the vehicle's speed ( $\mathrm{v}_{\mathrm{RE}}$ ) lies between the stored values $\mathrm{v}_{\mathrm{N}_{\mathrm{K}}}$ and $v_{N_{k+1}}$, a dummy variable $\zeta$ is computed as:

$$
\begin{equation*}
\zeta=\left(\mathrm{v}_{\mathrm{RE}}-\mathrm{v}_{\mathrm{N}_{\mathrm{k}}}\right) /\left(\mathrm{v}_{\mathrm{N}_{\mathrm{k}+1}}-\mathrm{v}_{\mathrm{N}_{\mathrm{k}}}\right) \tag{18}
\end{equation*}
$$

The desired or reference-trajectory angle-of-attack ( $\alpha_{D}$ ) is then computed from:

$$
\begin{equation*}
\alpha_{\mathrm{D}}=(1-\zeta) \alpha_{\mathrm{N}_{\mathrm{k}}}+\zeta \alpha_{\mathrm{N}_{\mathrm{k}+1}} \tag{19}
\end{equation*}
$$

Similar relations are used to find $h_{D}, r_{G D}$, and $\eta_{D}$.

### 4.4.2 Thermal Control

The basic idea is to roll the vehicle about its velocity vector so as to follow a stored density vs. speed profile. The roll-angle magnitude computations are essentially the same as for the constant heating-rate mode, except that the desired density altitude ( $h_{D}$ ) is obtained from the stored reference table (rather than from Eq. 9).

### 4.4.3 Range Control

Angle-of-attack commands ( $\alpha_{C}$ ) are computed as a combination of the desired and reference-trajectory range-to-go ( $r_{G}$ and $r_{G D}$ ). The basic relation is

$$
\begin{equation*}
\alpha_{\mathrm{C}}=\alpha_{\mathrm{D}}+\mathrm{k}_{\alpha}\left(\mathrm{r}_{\mathrm{G}}-\mathrm{r}_{\mathrm{GD}}\right) / \mathrm{v}_{\mathrm{RE}} \tag{20}
\end{equation*}
$$

The multiplying factor associated with the range-difference term is varied inversely with vehicle velocity ( $\mathrm{k}_{\alpha} / \mathrm{v}_{\mathrm{RE}}$ ). The maximum value of angle-of-attack is limited to a preselected level $\alpha^{\alpha}$ CMAX .

### 4.4.4 Lateral Control

The lateral control of the vehicle's trajectory is accomplished by roll modulation, i. e. reversing the direction of roll. For this purpose it is necessary first to see if the present roll angle ( $\phi_{\mathrm{OLD}}$ ) is directed so as to increase the heading error $(\Delta \psi)$. This is accomplished by looking at the test quantity $s_{\phi}$ defined by:

$$
\begin{equation*}
s_{\phi}=\Delta \psi \phi \text { OLD } \tag{21}
\end{equation*}
$$

where $\phi_{\mathrm{OLD}}$ is the preceding roll-angle command. If it turns out that $\phi_{\mathrm{OLD}}$ is tending to increase $\Delta \psi$, then a check is made to see if the magnitude of the current heading error $(\eta)$ is greater than a specified fraction $k_{\eta_{1}}$ of the reference-trajectory value $\left(\eta_{D}\right)$. If it is then seen that $\eta$ is greater ${ }^{1}$ than $k_{\eta_{1}} \eta_{D}$, then the roll-command direction is reversed from the previous command ${ }^{1}$ ( $\phi_{\mathrm{OLD}}$ ).

### 4.5 Transition Mode

### 4.5.1 Nominal Trajectory Description

The transition mode uses a stored trajectory as does the reference-trajectory mode. The nominal trajectory for this phase is not based on temperature or TPS considerations, but rather on providing maximum footprint capability (to null post-blackout navigation errors) while avoiding violation of vehicle constraints (elevon deflections, hinge moments, g loads,etc.)

Angle-of-attack, altitude, and allowable heading error are stored as a function of speed as in the reference-trajectory mode. Nominal range to the target $\left(\mathrm{r}_{\mathrm{GN}}\right)$, however, is computed from a parabolic function of range vs. speed, i. e.

$$
\begin{equation*}
r_{G N}=c_{R_{1}}\left(v_{R E}-v_{F}\right)+c_{R_{2}}\left(v_{R E}-v_{F}\right)^{2} \tag{22}
\end{equation*}
$$

where $v_{F}$ is the value of velocity at which the guidance mode is terminated. This form is used for the nominal range computation rather than a table lookup because it provides smoother control histories.

Angle-of-attack is pre-programmed as a function of velocity during transition. This technique accomplishes the three following objectives; (1) keeps the vehicle in a controllable orientation ( $\alpha$ vs. Mach no.), (2) helps avoid violation of elevon deflection and hinge-moment constraints, and (3) keeps the vehicle flying at near maximum L/D (thus providing good maneuver capability).

### 4.5.3 Range Control

$R$ ange is controlled by altering the altitude-vs.-velocity profile flown by the vehicle. An increase in altitude provides a lower density (or drag) thereby increasing range. The opposite is true for a decrease in altitude. The altitude-vs.velocity level at which the vehicle is commanded to fly is computed from the nominal reference altitude and the vehicle's current range error as follows:

$$
\begin{equation*}
h_{D}=h_{N}+k(v)\left(r_{G}-r_{G N}\right) \tag{23}
\end{equation*}
$$

where $k(v)$ is a gain that is linearly increased with decreasing velocity. This value of desired altitude is then used to compute roll-angle magnitude commands as in the constant heat rate and reference-trajectory modes. During this mode the limits on maximum roll-angle are modified to keep the vehicle from violating sonic boom over-pressure constraints.

### 4.5.4 Lateral Control

Lateral control during transition is accomplished by use of a double-deadband logic. These deadbands are defined by the quantities $\mathrm{k}_{\eta_{1}}$ and $\mathrm{k}_{\eta_{2}}$ (Note: ${ }_{\mathrm{k}_{\eta_{2}}}>\mathrm{k}_{\eta_{1}}$ ). If the vehicle's current heading error $\eta$ is less than ${ }_{\text {reversal logic of the reference-trajectory mode is used for lateral }}{ }^{2}{ }^{2} \eta_{\mathrm{D}}$, control.

If the vehicle's heading error is greater than $\mathrm{k}_{\eta_{2}} \eta_{\mathrm{D}}$, and if the vehicle is currently too far from the target point (i.e., $r_{G}>r_{G D}$ ), then both roll-angle magnitude and roll-angle direction are commanded to decrease $\eta$. This is accomplished by commanding a roll-angle magnitude of $45^{\circ}$ (if the normally commanded roll-angle is less than $45^{\circ}$ ) with the proper sign to reduce the heading error. The final output roll command is first limited by the maximum roll-angle constraints.

A range-error criterion is used in this lateral logic because in those cases where the vehicle is too far from the target, the normal roll angle commands are small (in order to increase in-plane $L / D$ to increase range). These small roll angles tend to decrease the vehicle's lateral capability, and the vehicle would fly past the target if some explicit lateral control were not employed.

## 5. DETAILED FLOW DIAGRAMS

This section contains detailed flow diagrams of the Entry Guidance Routine.
PROGRAM CONSTANTS


Figure 2a. Entry Guidance Routine, Detailed Flow Diagram


Figure 2b. Entry Guidance Routine, Detailed Flow Diagram


Figure 2c. Entry Guidance Routine, Detailed Flow Diagram


Figure 2d. Entry Guidance Routine, Detailed Flow Diagram


Figure 2e. Entry Guidance Routine, Detailed Flow Diagram


Figure 2f. Entry Guidance Routine, Detailed Flow Diagram


Figure 2g. Entry Guidance Routine, Detailed Flow Diagram


Figure 2h. Entry Guidance Routine, Detailed Flow Diagram


Figure 2i. Entry Guidance Routine, Detailed Flow Diagram

(Fig. 21)

Figure 2j. Entry Guidance Routine, Detailed Flow Diagram


Figure 2k. Entry Guidance Routine, Detailed Flow Diagram


Figure 21. Entry Guidance Routine, Detailed Flow Diagram

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