

# **PERTURBATIONS**

**BOOST MONITOR**

**TLI**

**LUNAR ORBIT**

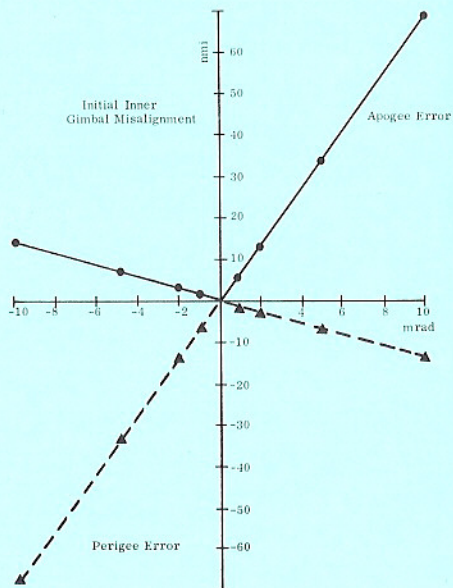
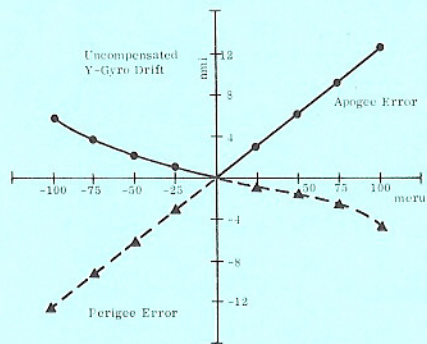
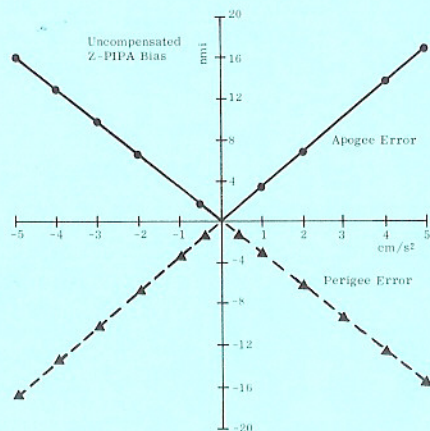
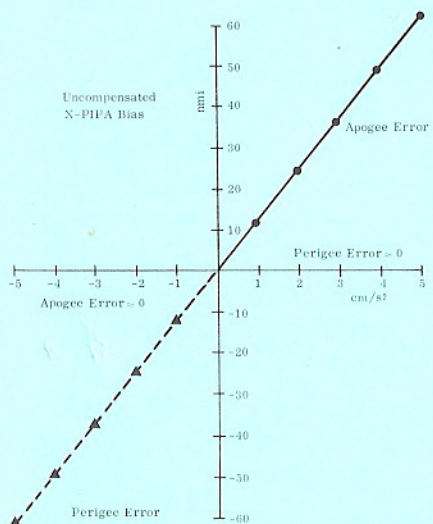
**LM DESCENT**

**LM ASCENT**

**TEI**

**ENTRY**

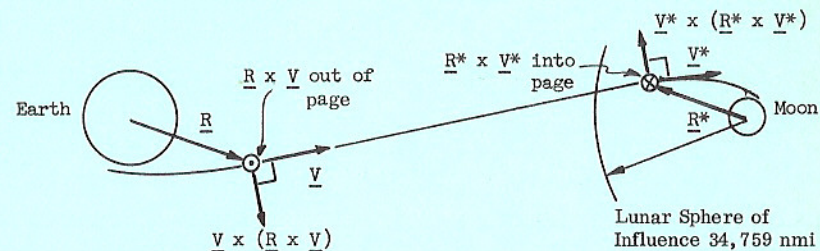
BOOST MONITOR ERRORS  
(Significant IMU Errors Only)



ERRORS IN PERICYNTHION  
DUE TO VELOCITY ERRORS DURING TRANSLUNAR COAST

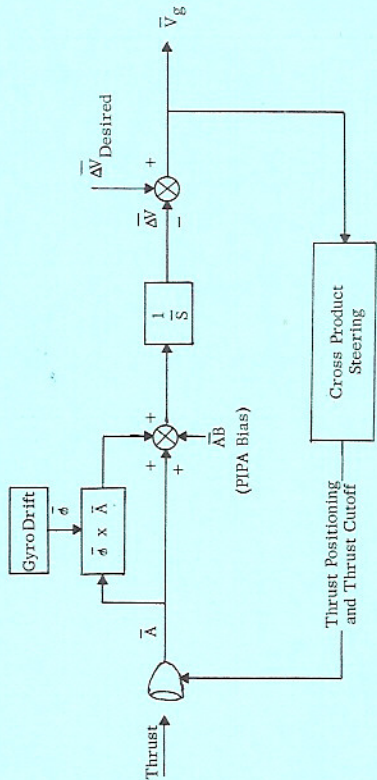
DIRECTION OF VELOCITY ERROR	ERROR IN PERICYNTHION (nmi/ft/s)
ERROR EXISTING AT TLI CUTOFF (2:38:50.0 GET)	
Along $\underline{V}$	129.35
Along $\underline{\bar{R}} \times \underline{V}$	0.38
Along $\underline{\bar{V}} \times (\underline{\bar{R}} \times \underline{V})$	-4.10
ERROR EXISTING AT MCC-1 (TLI + 9 hours) (11:38:50 GET)	
Along $\underline{V}$	33.63
Along $\underline{\bar{R}} \times \underline{V}$	5.48
Along $\underline{\bar{V}} \times (\underline{\bar{R}} \times \underline{V})$	-9.38
ERROR EXISTING AT MCC-2 (TLI + 28 hours) (30:38:50 GET)	
Along $\underline{V}$	19.03
Along $\underline{\bar{R}} \times \underline{V}$	4.74
Along $\underline{\bar{V}} \times (\underline{\bar{R}} \times \underline{V})$	-6.99
ERROR EXISTING AT MCC-3 (LOI - 22 hours) (52:28:38 GET)	
Along $\underline{V}$	9.70
Along $\underline{\bar{R}} \times \underline{V}$	2.73
Along $\underline{\bar{V}} \times (\underline{\bar{R}} \times \underline{V})$	-3.33
ERROR EXISTING AT MCC-4 (LOI - 5 hours) (69:28:38 GET)	
Along $\underline{V}^*$	0.36
Along $\underline{\bar{R}}^* \times \underline{V}^*$	0
Along $\underline{\bar{V}}^* \times (\underline{\bar{R}}^* \times \underline{V}^*)$	2.86

\*  $\underline{R}$  and  $\underline{V}$  are with respect to moon centered inertial system.



EFFECT OF INSTRUMENT ERRORS ON THRUST GUIDANCE

The thrust guidance used on Apollo, in simplified form, is as follows



The above steering loop attempts to drive  $V_g$  (velocity) to zero. The inability of the steering loop to drive  $V_g$  to zero will result in velocity residuals at the end of the burn as displayed in P-40.

Since the thrust guidance loop performs its calculations on the difference between  $\Delta V$  measured and  $\Delta V$  desired, the velocity residuals displayed by P-40 at the end of each burn are not caused by instrument errors but by the inability of the steering loop to drive  $V_g$  to zero.

Instrument errors result in a difference between the measured incremental velocity change and the true incremental velocity change. Assuming perfect cross product steering ( $V$  Residuals = 0 in P-40) the resultant orbit will differ from the planned orbit. A measure of the difference between the actual and planned orbits is the error between actual and planned apolune and perilune.

The following tables relate incremental spacecraft velocity errors, during lunar orbit maneuvers, to errors in apolune and perilune. Knowing apolune and perilune errors, the table is used to obtain the required error in velocity, expressed in body axes coordinates. This velocity error is then converted to possible instrument errors by using the table relating PIPA bias and gyro drift to spacecraft velocity.

For Example: Assume a perilune error of +5.0 m/s and zero spacecraft residuals, as shown in P-40, after the DOI maneuver.

1. From the "Lunar Orbit Maneuver Perturbation" table, the most sensitive spacecraft velocity error is  $\Delta V_x$ .

2. The required velocity error is

$$\Delta V_x = \frac{5.0 \text{ m/s}}{-1.70} = -2.94 \text{ m/s}$$

3. The weighting factors relating instrument errors to velocity errors yield

$$-0.69 \times 10^{-2} AB_x + 53.2 \times 10^{-2} AB_y + 53.2 \times 10^{-2} AB_z = -2.94 \text{ m/s}$$

Therefore:

$$-0.37 AB_x - 1.71 AB_z = -7.14 \text{ ft/sec}$$

or

$$\text{Uncompensated } AB_z = +19 \text{ cm/sec}^2$$

$$\text{Uncompensated } AB_x = +4.17 \text{ cm/sec}^2$$

EFFECT OF INSTRUMENT ERRORS ON THRUST GUIDANCE (CONTINUED)

The "Lunar Orbit Maneuver Perturbation" table also gives the transformation from stable member space to spacecraft body control axes space as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S/C \text{ CSM}} = \begin{bmatrix} R_x (-7.25^\circ) \\ \text{SM-NB} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{SM}$$

where:

$\begin{bmatrix} \text{SM-NB} \end{bmatrix}$  = Transformation from stable member to the nav base using gimbal angles

$\begin{bmatrix} R_x (-7.25^\circ) \end{bmatrix}$  = Rotation matrix about  $X_{NB}$  of  $-7.25$  degrees to transform from nav base space to spacecraft control axes space for the CSM.

For the LM

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S/C \text{ LM}} = \begin{bmatrix} \text{SM-NB} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{SM}$$

The purpose of this transformation is to indicate the location of the stable member with respect to thrust. The assumed gimbal angles are as follows:

	OUTER GIMBAL (degrees)	INNER GIMBAL (degrees)	MIDDLE GIMBAL (degrees)
LOI	0	0	0
DOI	0	282	359
CIRCULARIZATION	0	141	358
LOPC 1	0	0	0
LM TPI	0	103	0
LOPC 2	0	0	0
SHAPING	3	58	3
TEI	180	0	0

For all maneuvers thrust is assumed along  $+X_{S/C}$  axis.

MANEUVER	LOI	DOI	Circularization	LOPC-1	TPI	LOPC-2	SHAPING	TEI
BODY AXES VELOCITY ERRORS	$\Delta V_x$	-0.15	0.11	-	-	-	-	-
	$\Delta V_y$	-0.11	0.16	-	-	-	-	-
	$\Delta V_z$	0.11	0.41	-	-	-	-	-
ERROR IN PERIUNE (amt./lt/s)	$\Delta V_x$	-0.13	-	-	-	-	-	-
	$\Delta V_y$	-0.11	-	-	-	-	-	-
	$\Delta V_z$	0.11	-	-	-	-	-	-
ERROR IN APOLUNE (amt./lt/s)	$\Delta V_x$	-0.13	-	-	-	-	-	-
	$\Delta V_y$	-0.11	-	-	-	-	-	-
	$\Delta V_z$	0.11	-	-	-	-	-	-
ERROR IN CONTROL AXES TRANSFORMATION [SC] = [M] [SM]	$\Delta V_x$	-0.11	-	-	-	-	-	-
	$\Delta V_y$	-0.11	-	-	-	-	-	-
	$\Delta V_z$	0.11	-	-	-	-	-	-
STABLE MEMBER TO SPACECRAFT	$\Delta V_x$	0	-0.078	-0.108	0.619	0	-0.050	0
	$\Delta V_y$	0	-0.075	-0.091	0.126	0	-0.050	0
	$\Delta V_z$	0	-0.628	0.075	-0.775	-0.973	0	0
RHS = BASIC REFERENCE SYSTEM	$\Delta V_x$	-0.993	0.437	-0.670	-0.791	-0.452	-0.217	-0.063
	$\Delta V_y$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
RHS = REFMAT [BHS]	$\Delta V_x$	-0.066	0.529	-0.840	-0.462	-0.452	-0.217	-0.063
	$\Delta V_y$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
Lunar Landing Site	$\Delta V_x$	0.846	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
	$\Delta V_y$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
Lunar	$\Delta V_x$	-0.002	-0.533	-0.840	-0.462	-0.452	-0.217	-0.063
	$\Delta V_y$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
Lunar Lifting	$\Delta V_x$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_y$	-0.002	-0.533	-0.840	-0.462	-0.452	-0.217	-0.063
	$\Delta V_z$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
Lunar Lifting Preferred	$\Delta V_x$	-0.002	-0.533	-0.840	-0.462	-0.452	-0.217	-0.063
	$\Delta V_y$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063
Lunar Lifting Preferred	$\Delta V_x$	-0.002	-0.533	-0.840	-0.462	-0.452	-0.217	-0.063
	$\Delta V_y$	0.846	0.665	-0.486	-0.797	-0.452	-0.217	-0.063
	$\Delta V_z$	-0.101	-0.437	-0.600	-0.797	-0.452	-0.217	-0.063

-Indicates coefficient is less than 0.05 mnt/lt/s.

$\Delta V_{SC}$  ERRORS CAUSED BY GYRO DRIFTS [V X  $\hat{A}$ ] AND PIPA BIASES

GYRO DRIFTS - MERU  
 PIPA BIASES - CM/SEC<sup>2</sup>  
 $\Delta V_{SC}$  - FT/SEC  
 $\Delta V_{SC}$  - SEC (LENGTH OF BURN + 90 SECONDS FOR STARTUP OF AVERAGE ROUTINE)  
 $\Delta T_p$  - SEC (TIME FROM LAST ALIGNMENT TO BURN)  
 $\Delta T_g$  - SEC (TIME FROM LAST ALIGNMENT TO BURN)

WEIGHT FACTORS RELATE PIPA BIAS AND GYRO DRIFTS TO VELOCITY ERROR.

LOI  
 $\Delta V_{SC}$  =  $-3.28 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $+2.58 \times 10^{-5} NBDY \Delta T_g^2 - 2.03 \times 10^{-4} NBDZ \Delta T_g^2 - 3.25 \times 10^{-2} A_x \Delta T_p + 0.41 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $-2.03 \times 10^{-4} NBDY \Delta T_g^2 - 2.58 \times 10^{-5} NBDZ \Delta T_g^2 - 3.25 \times 10^{-2} A_x \Delta T_p - 3.25 \times 10^{-2} A_z \Delta T_p$

DOI  
 $\Delta V_{SC}$  =  $-0.69 \times 10^{-2} A_x \Delta T_p - 3.21 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $-1.46 \times 10^{-5} NBDY \Delta T_g^2 + 0.19 \times 10^{-5} NBDY \Delta T_g - 0.38 \times 10^{-5} NBDZ \Delta T_g - 0.41 \times 10^{-2} A_x \Delta T_p - 3.25 \times 10^{-2} A_z \Delta T_p + 0.05 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $-0.19 \times 10^{-5} NBDY \Delta T_g^2 - 1.49 \times 10^{-5} NBDY \Delta T_g + 3.18 \times 10^{-2} A_x \Delta T_p - 0.41 \times 10^{-2} A_z \Delta T_p$

CIRCULARIZATION

$\Delta V_{SC}$  =  $2.55 \times 10^{-2} A_x \Delta T_p + 0.12 \times 10^{-2} A_y \Delta T_p + 2.06 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $+4.50 \times 10^{-6} NBDX \Delta T_g^2 - 0.92 \times 10^{-6} NBDY \Delta T_g^2 + 5.63 \times 10^{-6} NBDZ \Delta T_g^2 + 0.35 \times 10^{-2} A_x \Delta T_p - 3.25 \times 10^{-2} A_y \Delta T_p - 0.25 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $+0.78 \times 10^{-6} NBDX \Delta T_g^2 - 7.20 \times 10^{-6} NBDY \Delta T_g^2 + 0.55 \times 10^{-6} NBDZ \Delta T_g^2 - 2.03 \times 10^{-2} A_x \Delta T_p - 0.41 \times 10^{-2} A_y \Delta T_p + 2.54 \times 10^{-2} A_z \Delta T_p$

PLANE CHANGE #1

$\Delta V_{SC}$  =  $-3.28 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $0.15 \times 10^{-5} NBDY \Delta T_g - 1.15 \times 10^{-5} NBDZ \Delta T_g - 3.25 \times 10^{-2} A_x \Delta T_p + 0.41 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $-1.15 \times 10^{-5} NBDY \Delta T_g - 0.15 \times 10^{-5} NBDZ \Delta T_g - 0.41 \times 10^{-2} A_x \Delta T_p - 3.25 \times 10^{-2} A_z \Delta T_p$

LM TPI

$\Delta V_{SC}$  =  $0.76 \times 10^{-2} A_x \Delta T_p - 3.19 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $5.12 \times 10^{-6} NBDX \Delta T_g - 1.22 \times 10^{-6} NBDZ \Delta T_g - 3.28 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $-5.26 \times 10^{-6} NBDY \Delta T_g^2 - 3.19 \times 10^{-2} A_x \Delta T_p + 0.76 \times 10^{-2} A_z \Delta T_p$

PLANE CHANGE #2

$\Delta V_{SC}$  =  $-3.28 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $0.26 \times 10^{-5} NBDY \Delta T_g^2 - 2.04 \times 10^{-5} NBDZ \Delta T_g^2$   
 $\Delta V_{SC}$  =  $-3.25 \times 10^{-2} A_x \Delta T_p + 0.41 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $-2.04 \times 10^{-5} NBDY \Delta T_g^2 - 0.26 \times 10^{-5} NBDZ \Delta T_g^2$   
 $\Delta V_{SC}$  =  $-0.41 \times 10^{-2} A_x \Delta T_p - 3.25 \times 10^{-2} A_z \Delta T_p$

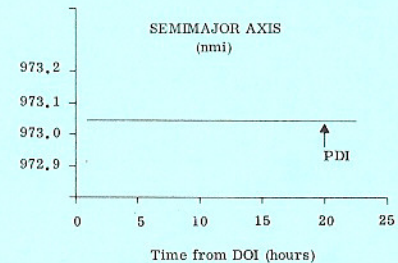
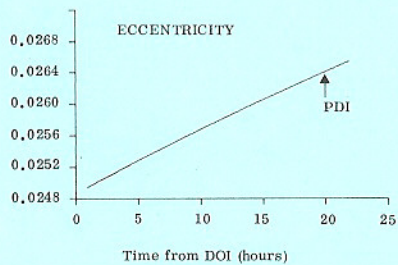
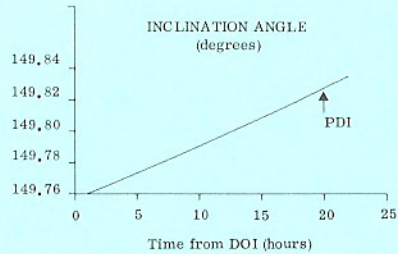
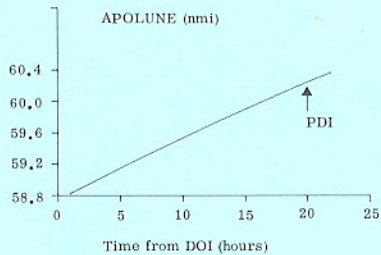
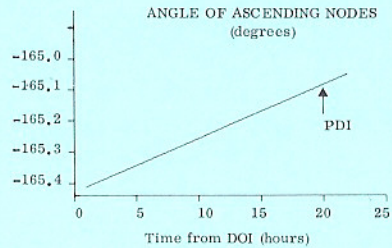
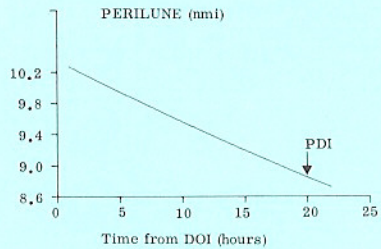
SHAPING

$\Delta V_{SC}$  =  $-1.73 \times 10^{-2} A_x \Delta T_p - 0.16 \times 10^{-2} A_y \Delta T_p + 2.78 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $2.34 \times 10^{-6} NBDX \Delta T_g^2 + 0.21 \times 10^{-6} NBDY \Delta T_g^2 - 1.47 \times 10^{-6} NBDZ \Delta T_g^2$   
 $\Delta V_{SC}$  =  $+0.29 \times 10^{-2} A_x \Delta T_p - 3.27 \times 10^{-2} A_y \Delta T_p$   
 $\Delta V_{SC}$  =  $0.25 \times 10^{-6} NBDX \Delta T_g^2 - 2.76 \times 10^{-6} NBDY \Delta T_g^2 - 2.77 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $-0.25 \times 10^{-2} A_y \Delta T_p - 1.74 \times 10^{-2} A_z \Delta T_p$

TEI

$\Delta V_{SC}$  =  $-3.28 \times 10^{-2} A_x \Delta T_p$   
 $\Delta V_{SC}$  =  $-0.30 \times 10^{-4} NBDY \Delta T_g^2 + 2.32 \times 10^{-4} NBDZ \Delta T_g^2 + 3.25 \times 10^{-2} A_x \Delta T_p - 0.41 \times 10^{-2} A_z \Delta T_p$   
 $\Delta V_{SC}$  =  $+2.32 \times 10^{-4} NBDY \Delta T_g^2 + 0.30 \times 10^{-4} NBDZ \Delta T_g^2 + 0.41 \times 10^{-2} A_x \Delta T_p + 0.25 \times 10^{-2} A_z \Delta T_p$

## APOLLO 16 LUNAR ORBIT PARAMETERS (TYPICAL)

EFFECT OF INSTRUMENT ERRORS ON  
LUNAR LANDING POSITION  
APOLLO 16

INSTRUMENT ERROR	CROSS-RANGE ERROR (ft)	DOWN RANGE ERROR (ft)
PIPA BIAS (cm/sec <sup>2</sup> )		
X	0	-178
Y	-2985	0
Z	0	-2984
IMU MISALIGNMENTS (mrad.)		
About X <sub>SM</sub>	777	0
About Y <sub>SM</sub>	0	-86
About Z <sub>SM</sub>	42	0
GYRO DRIFT (meru)*		
NBDX	10.4	0
NBDY	0	-3.2
NBDZ	2.8	0

\* Gyro drift was assumed to start at ignition.

LM INSERTION ORBIT APOLONE AND PERILUNE ERRORS  
 APOLLO 16

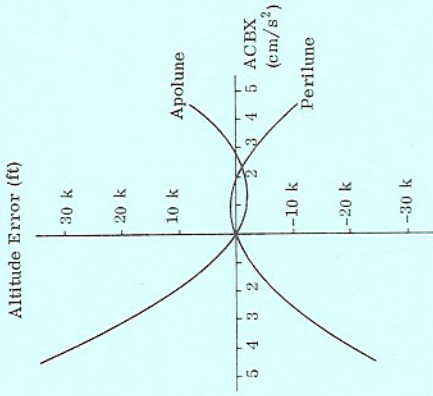
Effect of Instrument Errors on LM Insertion Orbit  
 Apollo 16

Instrument Error	Radial Velocity Error (ft/s)	*Crossrange Velocity Error (ft/s)	Downrange Velocity Error (ft/s)	Apolune Error (ft)	Perilune Error (ft)	Crossrange Error (ft)	Inclination Error (deg)
PIPA Bias (cm/s <sup>2</sup> )							
ACBX	-14.0	-	2.4	Graph A	Graph A	-	-
ACBY	-	-13.5	-	-	-	-2,947	-0.097
ACBZ	-5.3	-	-13.4	Graph B	Graph B	-	-
IMU Misalignment at Liftoff (arc-seconds)							
$\phi_X$	-	-0.026	-	-	-	-4.8	$-1.8 \times 10^{-4}$
$\phi_Y$	0.024	-	-0.011	Graph C	Graph C	-	-
$\phi_Z$	-	0.006	-	-	-	2.2	$5.0 \times 10^{-4}$
Gyro Drift Starting at Liftoff (meru)							
NBDX	-	-0.099	-	-	-	-12.4	$-6.3 \times 10^{-4}$
NBDY	0.094	-	-0.025	Graph D	Graph D	-	-
NBDZ (+@ Z <sub>SM</sub> )	-	-0.008	-	-	-	-3.2	$-6.7 \times 10^{-5}$

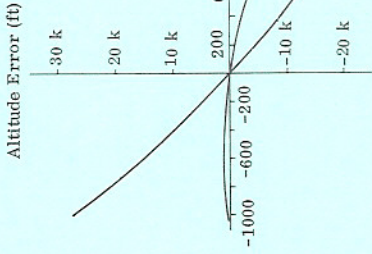
\*CR = DR x R

NOMINAL VALUES:

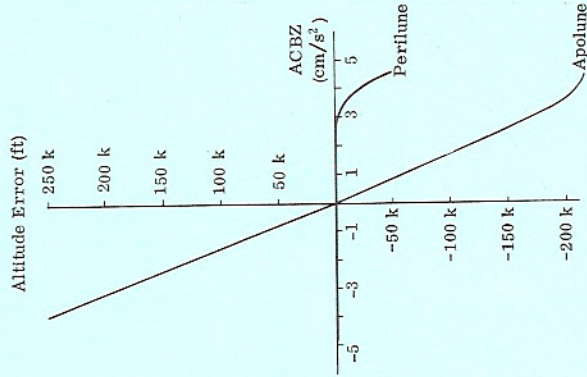
Radial Velocity	32 ft/s	Apolune	45.6 nmi
Crossrange Velocity	0 ft/s	Perilune	9.0 nmi
Downrange Velocity	5,531.3 ft/s	Crossrange Inclination	0 nmi
			147 deg



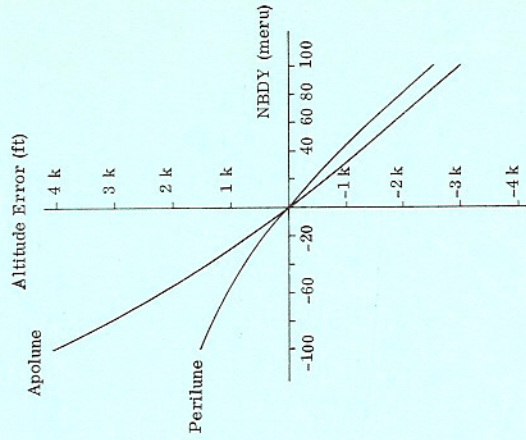
Graph A



Graph C



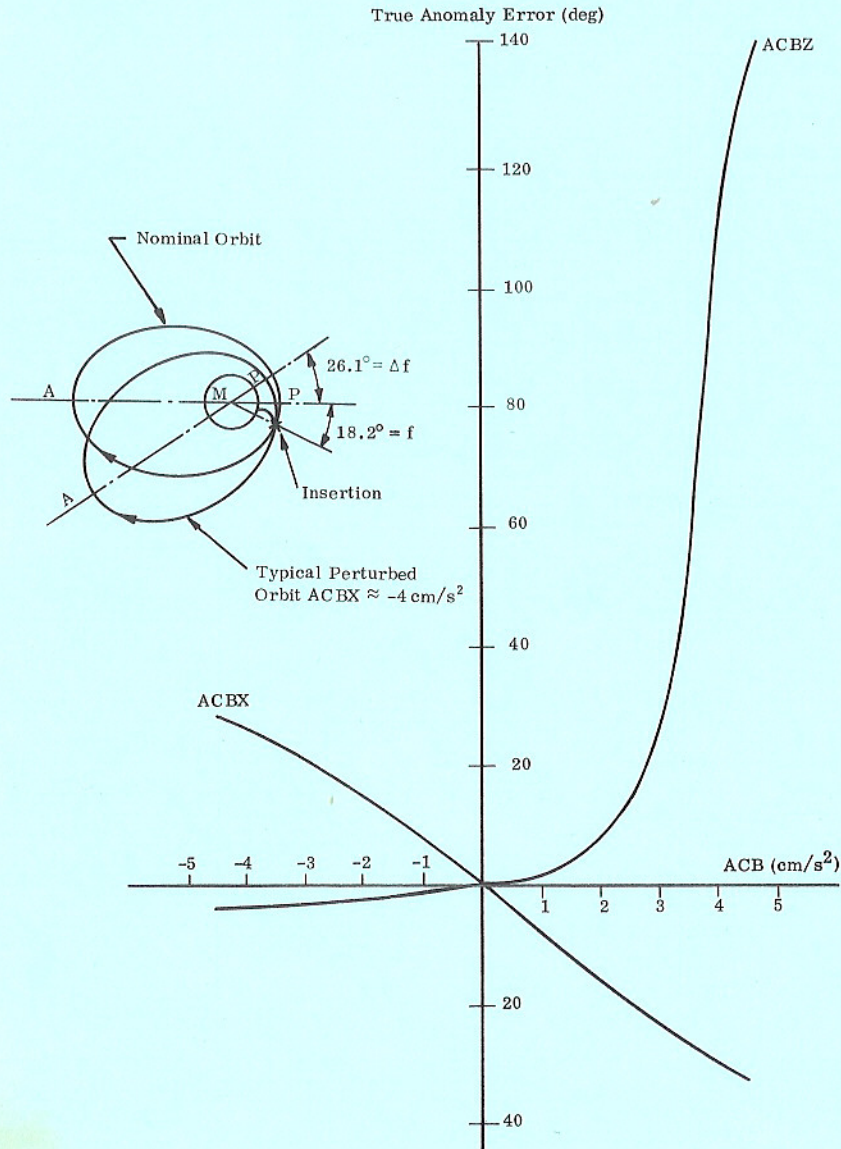
Graph B



Graph D

LM INSERTION ORBIT APOLONE AND PERILUNE ERRORS  
 APOLLO 16

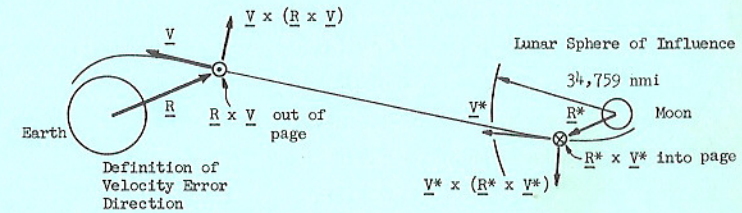
LM INSERTION ORBIT TRUE ANOMALY ERRORS DUE TO  
UNCOMPENSATED PIPA BIAS  
APOLLO 16



ERRORS AT THE ENTRY INTERFACE (400,000 FT. ALT.)  
DUE TO VELOCITY ERRORS DURING TRANSEARTH COAST

DIRECTION OF VELOCITY ERROR	ERROR IN VACUUM PERIGEE (nmi/ft/s)	ERROR IN TIME OF ARRIVAL (s/ft/s)	ERROR IN VELOCITY AT EI (ft/s/ft/s)	ERROR IN FLIGHT PATH ANGLE AT EI (degrees/ft/s)
ERROR EXISTING AT TEI CUTOFF (222:23:03.3 GET)				
Along $\underline{V}^*$	-22.89	-96.95	2.84	-1.95
Along $\underline{R}^* \times \underline{V}^*$	2.04	10.00	-0.05	0.14
Along $\underline{V}^* \times (\underline{R}^* \times \underline{V}^*)$	-7.02	0.85	0.05	-0.49
ERROR EXISTING AT MCC-5 (TEI + 17 hours) (239:23:03.3 GET)				
Along $\underline{V}$	1.67	-21.60	0.05	0.12
Along $\underline{R} \times \underline{V}$	0.68	1.85	0.02	0.05
Along $\underline{V} \times (\underline{R} \times \underline{V})$	9.97	30.90	-0.04	0.72
ERROR EXISTING AT MCC-6 (EI - 22 hours) (268:22:45.4 GET)				
Along $\underline{V}$	1.24	-5.25	0.23	0.09
Along $\underline{R} \times \underline{V}$	0.02	0.05	0.02	0
Along $\underline{V} \times (\underline{R} \times \underline{V})$	5.93	14.85	0.01	0.41
ERROR EXISTING AT MCC-7 (EI - 3 hours) (287:22:45.4 GET)				
Along $\underline{V}$	0.50	0.30	0.51	0.04
Along $\underline{R} \times \underline{V}$	0	0	0	0
Along $\underline{V} \times (\underline{R} \times \underline{V})$	1.48	2.55	0.01	0.10

\*  $\underline{R}$  and  $\underline{V}$  are with respect to moon centered inertial system



TYPICAL LUNAR RETURN ENTRY POSITION ERROR SENSITIVITIES  
AT GUIDANCE TERMINATION

ERROR SOURCE ( $\epsilon$ )		NOMINAL 1,250 NMI ENTRY	
		Downrange Error (nmi/ $\epsilon$ )	Crossrange Error (nmi/ $\epsilon$ )
GYRO BIAS DRIFT (meru)	NBDX	0.0	-0.17
	NBDY	-0.12	0.0
	NBDZ	0.0	-0.43
ACCELEROMETER BIAS (cm/sec <sup>2</sup> )	ACBX	-4.49	0.0
	ACBY	0.0	4.65
	ACBZ	-1.45	0.0

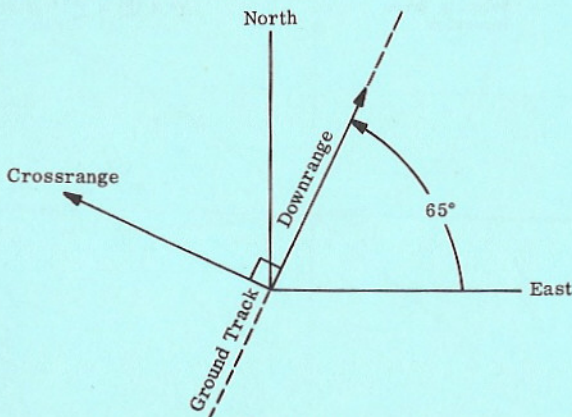
- NOTES: 1. Assumed accelerometer operation 15 minutes prior to EI (altitude of 400,000 feet).  
2. Assumed IMU drift from 1.5 hours prior to EI.  
3. Assumed IMU orientation such that at EI:

$$\bar{Z}_{SM} = -\text{Unit}(\bar{R})$$

$$\bar{Y}_{SM} = -\text{Unit}(\bar{R} \times \bar{V})$$

$$\bar{X}_{SM} = \bar{Y}_{SM} \times \bar{Z}_{SM}$$

4. Crossrange-downrange definition:



Nominal 1,250 Nautical Miles Entry