

SCIENCE

NAVIGATION
CSM DIGITAL AUTOPILOT
LM DIGITAL AUTOPILOT
LUNAR LANDING
ORBITS

COASTING FLIGHT NAVIGATION

The purpose of Coasting Flight Navigation is to estimate spacecraft position and velocity. The estimates are computed using orbital mechanics and navigation sightings to improve the accuracy of the orbital mechanics. The navigation sightings are incorporated into the position and velocity estimates using a modified Kalman filter which is a recursive optimal estimator that is characterized by the following computational procedure.

1. Extrapolate the state vector ahead to time t_n , using the best estimate of the state at time t_{n-1} .

$$\hat{\underline{x}}'_n = [c_{n-1}] \hat{\underline{x}}'_{n-1}$$

2. Extrapolate the error covariance matrix in a similar manner.

$$[E'_n] = [c_{n-1}] [E_{n-1}] [c_{n-1}]^T + [U_{n-1}] \quad (U_{n-1} = \text{process noise})$$

3. Compute the optimal gain matrix.

$$[K_n^*] = [E'_n] [H_n]^T [H_n E'_n H_n^T + V_n]^{-1} \quad (V_n = \text{measurement noise})$$

4. Calculate a measurement vector for time t_n .

$$\hat{\underline{y}}'_n = [H_n] \hat{\underline{x}}'_n$$

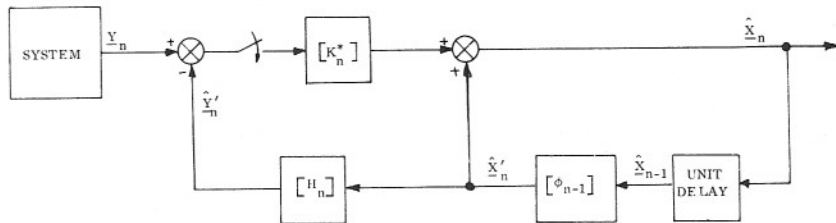
5. Update the estimate of the state vector, using the extrapolated state $\hat{\underline{x}}'_n$, the optimal gain $[K_n^*]$, the extrapolated measurement $\hat{\underline{y}}'_n$, and the actual measurement at time t_n , \underline{y}_n .

$$\hat{\underline{x}}_n = \hat{\underline{x}}'_n + [K_n^*] (\underline{y}_n - \hat{\underline{y}}'_n)$$

6. Update the error covariance matrix in a similar manner.

$$[E_n] = [E'_n] - [K_n^*] [H_n] [E'_n]$$

This procedure is illustrated by the following block diagram.



Basic Recursive Procedure of Kalman Filtering

MODIFICATIONS TO THE BASIC KALMAN FILTER

The basic Kalman filter outlined on the previous page was modified for use in Coasting Flight Navigation in the following aspects.

1. Instead of extrapolating the error covariance matrix, $[E]$, the square root of the error covariance matrix is extrapolated. This insures that the covariance matrix will always be positive semi-definite and avoids the usual difficulties that occur when computation techniques yield a negative error covariance matrix. The square root of the error covariance matrix is called the error transition matrix $[W]$.
2. The state vector and error transition matrix are extrapolated by integrating their respective second order differential equations via the coasting integration routine. This is done in lieu of the state transition approach and eliminates the necessity of computing a new time varying transition matrix for each measurement interval.
3. The update of the state vector after each measurement must first be displayed and approved by the astronaut before it is incorporated into the state vector. This eliminates the possibility of an erroneous update due to an improper mark.
4. Only one measurement is incorporated at a time. This reduces the dimension of the filter equations and changes the matrix inversion in the optimal gain equation to a scalar division.

STATE VECTOR DEFINITION

The state vector for coasting flight navigation is defined as the deviation of the spacecraft position and velocity from a reference conic.

$$\text{State Vector} = \underline{x}(t) = \begin{bmatrix} \delta r_x \\ \delta r_y \\ \delta r_z \\ \delta v_x \\ \delta v_y \\ \delta v_z \end{bmatrix} = \begin{bmatrix} \text{Deviations from} \\ \text{conic position} \\ \\ \text{Deviations from} \\ \text{conic velocity} \end{bmatrix}$$

Deviations from the reference conic are assumed to be Gaussian distributed with a known mean and variance. The mean is estimated via the precision integration routines or obtained from MSFN. The variance is given by the error covariance matrix, which is precomputed and entered via erasable data load.

CORRELATION BETWEEN BASIC KALMAN FILTER TERMINOLOGY AND COASTING FLIGHT TERMINOLOGY

The following table lists the correlation between basic Kalman filter terminology as used on the previous page and Coasting Flight Navigation terminology to be used on the following pages.

CELESTIAL NAVIGATION TERMINOLOGY	KALMAN FILTER TERMINOLOGY	CORRELATION
\underline{b} = geometry vector of dimension "D"	$[H]$ = measurement matrix	$\underline{b}^T \Rightarrow [H]$
$\underline{\omega}$ = weighting vector of dimension "D"	$[K_n^*]$ = optimal gain matrix	$\underline{\omega} \Rightarrow [K_n^*]$
$[W]$ = error transition matrix of dimension "D x D"	$[E]$ = error covariance matrix	$[E] \Rightarrow [W W^T]$
\underline{x} = state vector	\underline{x} = state vector	$\underline{x} \Rightarrow \underline{x}$
$\bar{\sigma}^2$ = a priori measurement error variance (scalar)	$[V]$ = covariance of the measurement noise	$\bar{\sigma}^2 \Rightarrow [V]$
δQ = measurement deviation (scalar)	$(\underline{y}_n - \hat{\underline{y}}'_n)$ = measurement residual	$\delta Q \Rightarrow (\underline{y}_n - \hat{\underline{y}}'_n)$

COASTING INTEGRATION ROUTINE

The Coasting Integration routine is a standardized subroutine used to integrate spacecraft state vectors to specific times. It is used during each of the three navigation programs (P20-Orbital Navigation, P22-Rendezvous Navigation, and P23 Cislunar Midcourse Navigation) to extrapolate the state vector and error transition matrix ahead to the measurement time by direct numerical integration of their differential equations.

STATE VECTOR EQUATIONS

The basic equation describing spacecraft motion is

$$\begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ -\frac{\mu}{|\mathbf{r}|^3} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{a}_d \end{bmatrix} \quad (1)$$

where

- \mathbf{r} = spacecraft position vector
- \mathbf{v} = spacecraft velocity vector
- \mathbf{a}_d = disturbance acceleration vector
- μ = primary planet gravitation constant

When the disturbance, \mathbf{a}_d , is small, then Encke's method of differential accelerations can be used to solve Equation 1. Encke's method divides spacecraft motion into two parts: (1) conic or osculating orbital motion which would result if $\mathbf{a}_d = \mathbf{0}$ and (2) deviation from conic motion as a result of $\mathbf{a}_d \neq \mathbf{0}$.

$$\mathbf{r} = \mathbf{r}_{\text{conic}} + \hat{\delta}\mathbf{r} \quad (2)$$

$$\mathbf{v} = \mathbf{v}_{\text{conic}} + \hat{\delta}\mathbf{v} \quad (3)$$

where

- \mathbf{r}, \mathbf{v} = spacecraft position and velocity
- $\mathbf{r}_{\text{conic}}, \mathbf{v}_{\text{conic}}$ = conic position and velocity
- $\hat{\delta}\mathbf{r}, \hat{\delta}\mathbf{v}$ = deviations from conic position and velocity

Substitution of Equations 2 and 3 into the basic equation of motion, Equation 1, yields differential equations for the conic position and velocity and the deviation from conic position and velocity.

Conic Motion Equation

$$\begin{bmatrix} \dot{\mathbf{r}}_{\text{conic}} \\ \dot{\mathbf{v}}_{\text{conic}} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ -\frac{\mu}{|\mathbf{r}_{\text{conic}}|^3} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{\text{conic}} \\ \mathbf{v}_{\text{conic}} \end{bmatrix} \quad (4)$$

Deviation Equation

$$\begin{bmatrix} \hat{\delta}\dot{\mathbf{r}} \\ \hat{\delta}\dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{G} & 0 \end{bmatrix} \begin{bmatrix} \hat{\delta}\mathbf{r} \\ \hat{\delta}\mathbf{v} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{a}_d \end{bmatrix} \quad \text{Subject to initial conditions } \begin{cases} \hat{\delta}\mathbf{r}(t_0) = \mathbf{0} \\ \hat{\delta}\mathbf{v}(t_0) = \mathbf{0} \end{cases} \quad (5)$$

where

$$\mathbf{G} = \frac{\mu}{r_{\text{conic}}^3} \begin{bmatrix} 3\mathbf{r}_{\text{conic}}\mathbf{r}_{\text{conic}}^T - r_{\text{conic}}^2\mathbf{I} \end{bmatrix}$$

The conic motion equation is solved explicitly using Kepler's subroutine. The deviation equation is solved by direct numerical integration and is called the "state equation"; it can be written in the form

$$\dot{\hat{\mathbf{x}}} = [\mathbf{F}] \hat{\mathbf{x}} + \mathbf{C} \quad (6)$$

ERROR TRANSITION MATRIX EQUATIONS

The accuracy of the state vector estimation process is characterized by the error covariance matrix which expresses the mean squared error of each state vector element in matrix form

state vector estimation error

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x} \quad (7)$$

where

- \mathbf{e} = estimation error
- $\hat{\mathbf{x}}$ = estimated state vector
- \mathbf{x} = actual state vector

error covariance matrix

$$[\mathbf{E}] = \langle \mathbf{e} \mathbf{e}^T \rangle \quad (8)$$

where

- $[\mathbf{E}]$ = error covariance matrix
- $\langle \rangle$ = denotes "expected value"

If the state vector is described by Equation 6, then the error covariance matrix is described by

$$\dot{\mathbf{E}} = \mathbf{F}\mathbf{E} + \mathbf{E}\mathbf{F}^T \quad (\text{neglecting process noise}) \quad (9)$$

and the error transition matrix is described by

$$\dot{\mathbf{W}} = \mathbf{F}\mathbf{W} \quad (10)$$

where

$$[\mathbf{W}\mathbf{W}^T] = [\mathbf{E}]$$

Equation 10 is the differential equation describing the error transition matrix and is solved by direct numerical integration.

SUMMARY

TIME DOMAIN EQUATIONS	STATE TRANSITION EXTRAPOLATION	NUMERICAL INTEGRATION EXTRAPOLATION
$\dot{\hat{\mathbf{x}}} = \mathbf{F}\hat{\mathbf{x}} + \mathbf{C}$	$\hat{\mathbf{x}}' = [\phi] \hat{\mathbf{x}}$	$\hat{\mathbf{x}}' = \hat{\mathbf{x}}(t_n) + \int_{t_n}^{t_{n+1}} \mathbf{F} \hat{\mathbf{x}} dt$
$\dot{\mathbf{E}} = \mathbf{F}\mathbf{E} + \mathbf{E}\mathbf{F}^T$	$\mathbf{E}' = \phi \mathbf{E} \phi^T$	
$\dot{\mathbf{W}} = \mathbf{F}\mathbf{W}$		$\mathbf{W}' = \mathbf{W}(t_n) + \int_{t_n}^{t_{n+1}} \mathbf{F}\mathbf{W} dt$

MEASUREMENT INCORPORATION ROUTINE

The Measurement Incorporation routine, like the Coasting Integration routine, is used by all three navigation programs. Its purpose is to compute a state vector update based on the information obtained from the navigation sighting. The procedure for updating the state vector can be divided into three parts.

1. Compute a geometry vector, \underline{b} , based on the state vector and type of navigation sighting that is being made.
2. Compute a state vector update.
3. Update the state vector and error transition matrix if the astronaut approves of the incorporation.

GEOMETRY VECTOR

The geometry vector relates the measurement, \hat{Y}'_n , to the state vector, $\hat{\Sigma}'_n$, according to Equation 11

$$\hat{Y}'_n = \underline{b}^T \hat{\Sigma}'_n \quad (11)$$

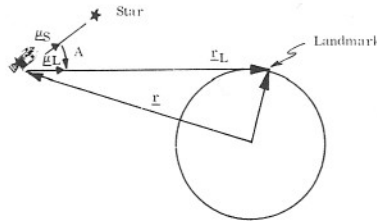
where

\hat{Y}'_n = the expected measurement (a scalar)

\underline{b} = geometry vector ($n \times 1$ column vector)

$\hat{\Sigma}'_n$ = state vector extrapolated to the measurement time via the Coasting Integration routine.

The state vector for coasting flight navigation is a deviation from a reference (osculating) orbit and the measurement variable can also be thought of as a deviation from a reference or nominal. Thus the measurement vector can be determined by taking the derivative of the measurement with respect to the state vector. This can be illustrated for the case of cislunar midcourse navigation, where the navigation sighting is the angle between a known near planetary landmark and a star as shown below



$$r_L \cos A \cong \underline{u}_S^T \underline{r}_L$$

Taking partials

$$\delta r_L \cos A - r_L \sin A \delta A = \underline{u}_S^T \delta \underline{r}_L$$

$$\delta A = (\delta r_L \cos A - \underline{u}_S^T \delta \underline{r}_L) / r_L \sin A$$

By definition

$$\delta r_L r_L^{-1} \text{ a scalar} = \underline{r}_L^{-T} \delta \underline{r}_L$$

therefore

$$\delta r_L = \frac{\underline{r}_L^T}{r_L} \delta \underline{r}_L = \underline{r}_L^{-T} \delta \underline{r}_L$$

and

$$\delta A = \frac{\underline{r}_L^{-T} \delta \underline{r}_L \cos A - \underline{u}_S^T \delta \underline{r}_L}{r_L \sin A}$$

$$\delta A = \frac{(\underline{r}_L^{-T} \cos A - \underline{u}_S^T) \delta \underline{r}_L}{r_L \sin A} = \frac{(\underline{r}_L^{-T} \cos A - \underline{u}_S^T)}{r_L \sin A} (-\delta \underline{r}_L')$$

$$\underline{b} = \frac{\underline{u}_S - \underline{r}_L \cos A}{r_L \sin A} = \frac{1}{r_L} \text{Unit}(\underline{u}_S - \underline{r}_L(\underline{u}_S^T \underline{r}_L))$$

The geometry vector can also be determined by geometrical relationship between the state vector and the type of measurement. Since \underline{b} is different for each navigation program, its derivations are presented on the following pages which describe the individual navigation programs.

COMPUTE THE STATE VECTOR UPDATE

The computation of the state vector update encompasses Steps 3, 4, and 5 of the basic Kalman filtering procedure.

1. Compute the optimal gain matrix

$$[K_n^*] = [E' H^T] [HE' H^T + V]^{-1} \quad (12)$$

or, in the terminology of coasting flight navigation,

$$\begin{aligned} \underline{\omega} &= [W' W'^T \underline{b}] [b^T W' W'^T \underline{b} + \bar{\sigma}^2]^{-1} \\ &= \frac{W' \underline{z}}{(z^2 + \bar{\sigma}^2)} \end{aligned} \quad (13)$$

where

$\underline{\omega}$ = optimal gain vector

W' = error transition matrix extrapolated forward by the Coasting Integration routine

\underline{z} = $W'^T \underline{b}$ (a vector used to simplify computations)

z^2 = $\underline{z}^T \underline{z} = \underline{b}^T W' W'^T \underline{b}$ (analogous to $HE' H^T$)

$\bar{\sigma}^2$ = measurement noise covariance (an erasable data load or fixed memory constant which depends on which navigation program is in use)

$(z^2 + \bar{\sigma}^2)$ is a scalar and eliminates the necessity of matrix inversion

2. Compute the expected measurement based on the extrapolated state vector.

$$\hat{Y}'_n = \underline{b}^T \hat{\Sigma}'_n \quad (14)$$

where

\hat{Y}'_n = expected measurement from current navigation sighting

\underline{b} = geometry vector

$\hat{\Sigma}'_n$ = extrapolated state vector

3. Compute the state vector update

$$\delta \underline{x} = \underline{\omega} \delta Q = \underline{\omega} (Y_n - \hat{Y}'_n) \quad (15)$$

where

$\delta \underline{x}$ = state vector update

$\underline{\omega}$ = optimal weighting vector

δQ = measurement residual

Y_n = information from current navigation sighting

\hat{Y}'_n = expected measurement

UPDATE THE STATE VECTOR AND ERROR TRANSITION MATRIX

Before the state vector update, $\delta \underline{x}$, is incorporated into the state vector, it is displayed to the astronaut for his approval. This is to prevent erroneous tracking data, such as improperly identified stars or landmarks, from being used to update the state. When astronaut approval has been issued, the state vector and error transition matrix are updated as follows:

UPDATE THE STATE VECTOR

$$\hat{\Sigma}_n = \hat{\Sigma}'_n + \delta \underline{x} \quad (16)$$

where

$\hat{\Sigma}_n$ = best estimate of state at time t_n

$\hat{\Sigma}'_n$ = state vector extrapolated ahead from time t_{n-1} to t_n

$\delta \underline{x}$ = state vector update

UPDATE THE ERROR TRANSITION MATRIX

The equation governing updating of the error covariance matrix is given in Step 6 of the basic Kalman filtering procedure.

$$[E_n] = [E'_n] - [K_n^*] [H_n] [E'_n] \quad (17)$$

This can be rewritten in terms of coasting flight navigation terminology

$$[WW^T] = [W'W'^T] - \omega \underline{z}^T [W'W'^T] \quad (18)$$

Equation 18 can be separated to obtain an equation for updating only the error transition matrix [W] instead of the product [WW^T].

$$[W] = [W'] - \frac{\omega \underline{z}^T}{1 + \frac{\omega^2 \underline{z}^T \underline{z}}{z^2 + \alpha^2}} \quad (19)$$

where

- [W] = updated error transition matrix
- [W'] = extrapolated error transition matrix
- ω = optimal gain vector
- \underline{z} = $W'^T \underline{b}$
- z^2 = $\underline{z}^T \underline{z}$
- α^2 = measurement noise covariance

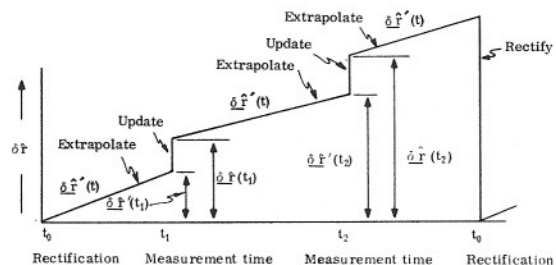
SUMMARY

The total spacecraft position and velocity is kept current through extrapolation, updating, and rectification. Rectification is a process used to redefine the reference trajectory by adding the deviations, $\hat{\delta r}$ and $\hat{\delta v}$, to the osculating elements, \underline{r}_{conic} and \underline{v}_{conic} , thereby redefining the osculating orbit and reducing the deviation state vector to zero.

$$\begin{bmatrix} \underline{r}_{conic} \\ \underline{v}_{conic} \end{bmatrix} = \begin{bmatrix} \underline{r}_{conic} \\ \underline{v}_{conic} \end{bmatrix} + \begin{bmatrix} \hat{\delta r} \\ \hat{\delta v} \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} \hat{\delta r} \\ \hat{\delta v} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (21)$$

This process is used to preserve the efficiency of Encke's method and is illustrated below.



INITIALIZING THE COASTING FLIGHT NAVIGATION PROGRAMS

Each of the navigation programs is initialized prior to use by specifying the measurement variance, $\bar{\sigma}^2$, and the initial error transition matrix, [W]. The measurement variance gives a confidence level for the navigation instruments by specifying the variance of all the error sources associated with the instrument. The W matrix gives a confidence level for the initial estimate of the state vector by specifying the mean squared error in the position and velocity estimates. [W] is initialized as a diagonal matrix which says that initially the position and velocity errors are independent.

MEASUREMENT VARIANCE	INITIAL "W" MATRIX
CISLUNAR MIDCOURSE NAVIGATION ROUTINE P-23	
$\bar{\sigma}^2 = \text{Var}_{\text{Trun}} + \text{Var}_1 / r_{CL}^2$ $= (0.05 \text{ mr})^2 + \frac{1 \text{ nmi}^2}{r_{CL}^2}$	$[W] = \begin{bmatrix} w_{rr}^* & & 0 \\ & w_{vv}^* & \\ 0 & & w_{mv}^* \end{bmatrix}$
ORBITAL NAVIGATION ROUTINE P-22	
$\bar{\sigma}^2 = \text{Var}_{\text{SCT}} + \text{Var}_{\text{IMU}}$ $= (1 \text{ mr})^2 + (1.0 \text{ mr})^2$	$[W] = \begin{bmatrix} w_{rr}^* & & 0 & & 0 \\ & w_{vv}^* & & & \\ 0 & & w_{\beta}^* & & 0 \\ & & & w_{\theta}^* & \\ 0 & & 0 & & w_{\phi}^* \end{bmatrix}$
RENDEZVOUS NAVIGATION PROGRAM (LEM) P-20	
<p>RANGE MEASUREMENT</p> $\bar{\sigma}^2 = \text{Max} \left\{ \text{Var}_{R_r}^*, \frac{\text{Var}_{R_{min}}^*}{r_{CL}^2} \right\}$ <p>RANGE RATE MEASUREMENT</p> $\bar{\sigma}^2 = \text{Max} \left\{ r^2 \text{Var}_{V_r}^*, \text{Var}_{V_{min}}^* \right\}$ <p>SHAFT ANGLE MEASUREMENT</p> $\bar{\sigma}^2 = \text{Var}_{\beta}^* + \text{Var}_{\text{IMU}} = \text{Var}_{\beta}^* + (1.0 \text{ mr})^2$ <p>TRUNNION ANGLE MEASUREMENT</p> $\bar{\sigma}^2 = \text{Var}_{\theta}^* + \text{Var}_{\text{IMU}} = \text{Var}_{\theta}^* + (1.0 \text{ mr})^2$	<p>FOR RENDEZVOUS</p> $[W] = \begin{bmatrix} w_{rr}^* & & 0 & & 0 \\ & w_{vv}^* & & & \\ & & w_{\beta}^* & & 0 \\ & & & w_{\theta}^* & \\ & & & & w_{\phi}^* \end{bmatrix}$ <p>FOR LUNAR SURFACE NAVIGATION</p> $[W] = \begin{bmatrix} w_{rr}^* & & 0 \\ & w_{vv}^* & \\ 0 & & w_{mv}^* \end{bmatrix}$
RENDEZVOUS NAVIGATION PROGRAM (CSM) P-20	
<p>OPTICAL TRACKING</p> $\bar{\sigma}^2 = \text{Var}_{\text{SXT}} + \text{Var}_{\text{IMU}} + \frac{\text{Var}_{\text{INT}}^*}{r_{CL}^2}$ $= (0.2 \text{ mr})^2 + (1.0 \text{ mr})^2 + \frac{\text{Var}_{\text{INT}}^*}{r_{CL}^2}$ <p>VHF RANGING</p> $\bar{\sigma}^2 = \text{Max} \left\{ \text{Var}_{R_r}^*, \frac{\text{Var}_{R_{min}}^*}{r_{CL}^2} \right\}$ <p>ALTERNATE LOS</p> $\bar{\sigma}^2 = \text{Var}_{\text{ALT}}^* + \text{Var}_{\text{IMU}}$ $= \text{Var}_{\text{ALT}}^* + (1.0 \text{ mr})^2$	$[W] = \begin{bmatrix} w_{rr}^* & & 0 \\ & w_{vv}^* & \\ 0 & & w_{mv}^* \end{bmatrix}$

* These values are stored in erasable memory.

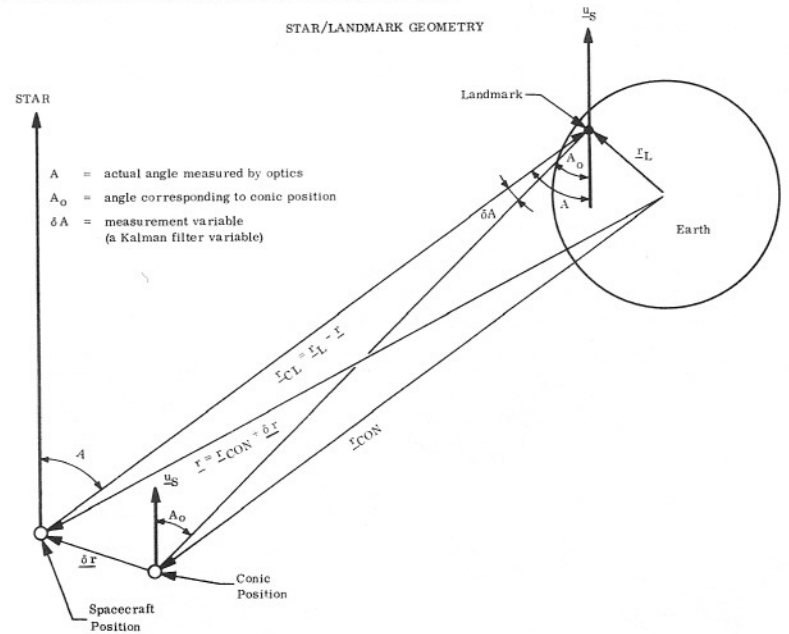
ERASABLE DATA LOAD PARAMETERS

PARAMETER (\bar{r}^2)	DATA LOAD MNEMONIC	VALUE	PARAMETER [W]	DATA LOAD MNEMONIC	VALUE
Var _R (LM)	RANGEVAR	1.1111×10^{-5}	W _{mR} (CSM)	WMIDPOS	30,000 ft
Var _{R min} (LM)	RVARMIN	66 m ²	W _{mV} (CSM)	WMIDVEL	30 ft/s
Var _V (LM)	RATEVAR	1.8777×10^{-5}	W _{rR} (CSM)	WORBPOS	0.0
Var _{V min} (LM)	VVARMIN	$0.017445 \text{ m}^2/\text{s}^2$	W _{rV} (CSM)	WORBVEL	0.0
Var _β (LM)	SHAFTVAR	(1 mrad) ²	W _r (CSM)	S22WSUBL	10,000 m
Var _θ (LM)	TRUNVAR	(1 mrad) ²	W _{rR} (LM)	WRENDPOS	10,000 ft
Var _{INT} (CSM)	INTVAR	(14 m) ²	W _{rV} (LM)	WRENDVEL	10 ft/s
Var _R (CSM)	RVAR	0.0	W _β (LM)	WSHAFT	15 mrad
Var _{R min} (CSM)	RVARMIN	(200 ft) ²	W _θ (LM)	WTRUN	15 mrad
Var _{ALT} (CSM)	ALTVAR	(3.9 mrad) ²	W _{rR} (LM)	WSURFPOS	0.0
			W _{rV} (LM)	WSURFVEL	0.0
			W _{rR} (CSM)	WRENDPOS	10,000 ft
			W _{rV} (CSM)	WRENDVEL	10 ft/s

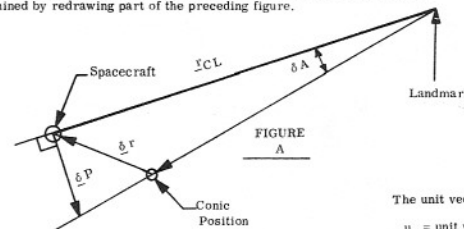
CISLUNAR MIDCOURSE NAVIGATION PROGRAM

(P23)

During cislunar midcourse navigation the angle between the lines of sight to a known star and a planetary landmark is measured. This angle measurement is then used to update the state vector via the measurement incorporation routine. The geometry relating the angular measurement to the deviation state vector is given below.



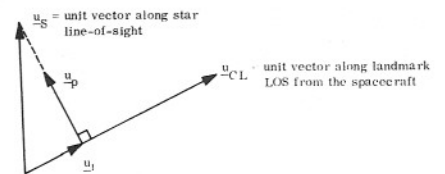
The relationship between the state vector and measurement variable can be determined by redrawing part of the preceding figure.



$$\delta A = \frac{|\delta P|}{|r_{CL}|} = \frac{u_p^T \delta r}{|r_{CL}|} = \frac{u_p^T}{|r_{CL}|} \delta r = b^T \delta x$$

where u_p is a unit vector perpendicular to r_{CL}

The unit vector, u_p , can be determined from Figure A



$$\therefore b = \frac{1}{|r_{CL}|} \left[\text{Unit}(u_S - (u_S \cdot u_{CL}) u_{CL}) \right]^T$$

During orbital navigation, the inertial line of sight from the spacecraft to a planetary landmark is measured by recording the optics shaft and trunnion angles, the IMU gimbal angles, and the time of the mark. Up to five marks are made on each landmark before the Measurement Incorporation routine is called to update the state vector.

Classical celestial mechanics says that if the angles between a planetary landmark and two different stars are measured then the line of sight from the spacecraft to the landmark can be determined, or that two star/landmark measurements are equivalent to one line-of-sight measurement. Orbital navigation uses this equivalency to incorporate the line-of-sight measurement by treating it as two star/landmark measurements. The two fictitious star/landmark measurements are then incorporated in the same manner as real star/landmark measurements in P23.

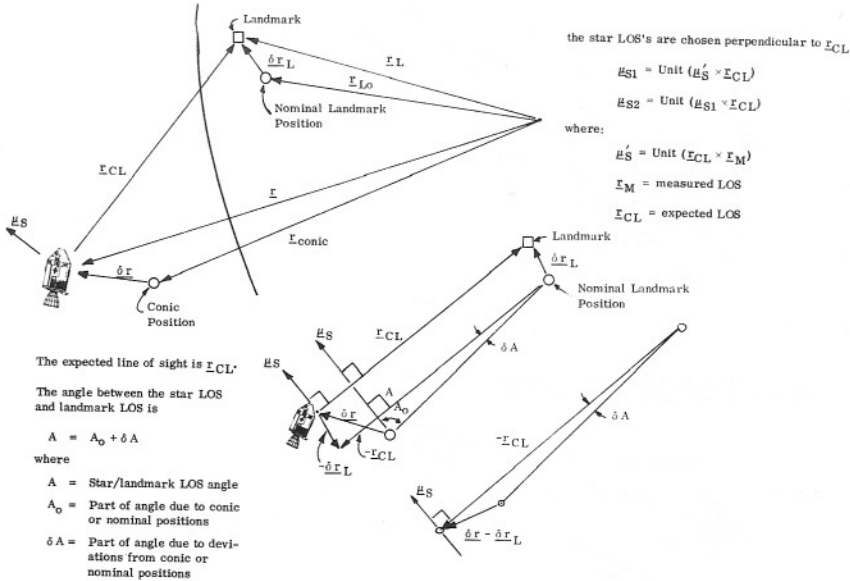
The dimension of the state vector for orbital navigation is expanded from six to nine to include the landmark position vector. If the concept of "deviation state vector" is carried over to include the landmark, then

$$\underline{x} = \begin{bmatrix} \delta r \\ \delta v \\ \delta r_L \end{bmatrix}$$

where

- \underline{x} = orbital navigation state vector
- δr = deviation of spacecraft position from the reference conic
- δv = deviation of spacecraft velocity from the reference conic
- δr_L = deviation of landmark position from the nominal

The geometry vector, \underline{b} , can be determined using a procedure similar to that used for P23.



From the above figure

$$\delta A = \frac{\underline{e}_S^T (\delta \underline{r} - \delta \underline{r}_L)}{|\underline{r}_{CL}|} = \frac{\underline{e}_S^T}{|\underline{r}_{CL}|} (\delta \underline{r} - \delta \underline{r}_L)$$

$$\delta A = \underline{b}^T \underline{x} = \frac{1}{|\underline{r}_{CL}|} [\underline{e}_S^T \quad 0 \quad -\underline{e}_S^T] \begin{bmatrix} \delta r \\ \delta v \\ \delta r_L \end{bmatrix}$$

$$\therefore \underline{b} = \frac{1}{|\underline{r}_{CL}|} \begin{bmatrix} \underline{e}_S \\ 0 \\ -\underline{e}_S \end{bmatrix}$$

The Rendezvous Navigation program, P20, is used during the rendezvous phases of flight. Both the CSM and LM computers have Program P20 so that the CSM can do rendezvous navigation by tracking the LM, or the LM can navigate by tracking the CSM. The CSM can navigate by measuring the line of sight to the LM using the SXT or the COAS, and/or the VHF ranging link. The LM uses the Rendezvous Radar to measure range, range rate, RR shaft angle bias, and RR trunnion angle bias.

The state vectors for rendezvous navigation are

$$\underline{x}_{CSM} = \begin{bmatrix} \delta r \\ \delta v \\ \delta \beta \end{bmatrix} \quad \underline{x}_{LM} = \begin{bmatrix} \delta r \\ \delta v \\ \delta \beta \\ \delta \theta \\ 0 \end{bmatrix}$$

where

- δr = deviation from conic position
- δv = deviation from conic velocity
- $\delta \beta$ = RR shaft angle bias
- $\delta \theta$ = RR trunnion angle bias

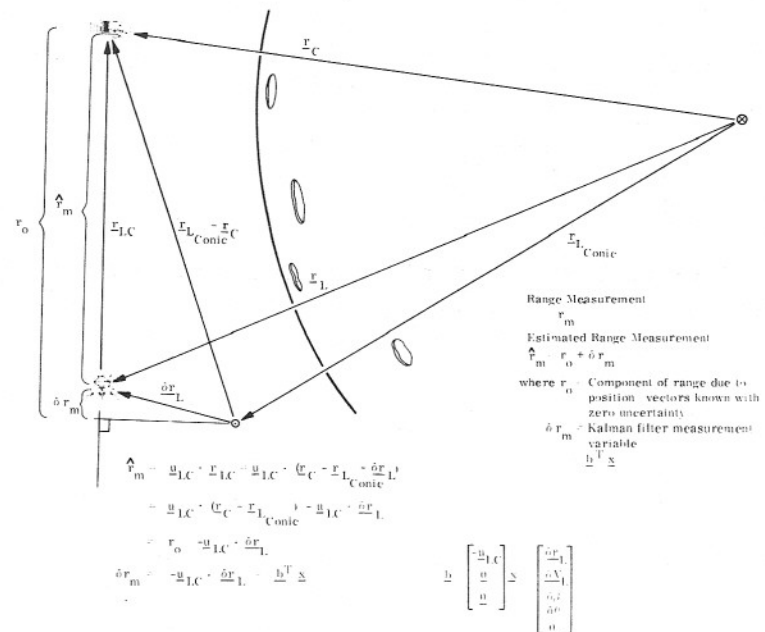
The Measurement Incorporation routine is used to incorporate the rendezvous measurements, and the geometry vectors are determined as follows.

- Line-of-Sight Measurements Made with the SXT or COAS. The line-of-sight measurements are incorporated into the state vector by adopting two fictitious star/landmark sightings just as with the LOS measurements made during orbital navigation.
- VHF Range Measurements Made by the CSM. The geometry vector for VHF ranging is derived in the same manner as the RR range measurement vector. The only difference is the dimension of the CSM which is only 6 versus 9 for the LM.

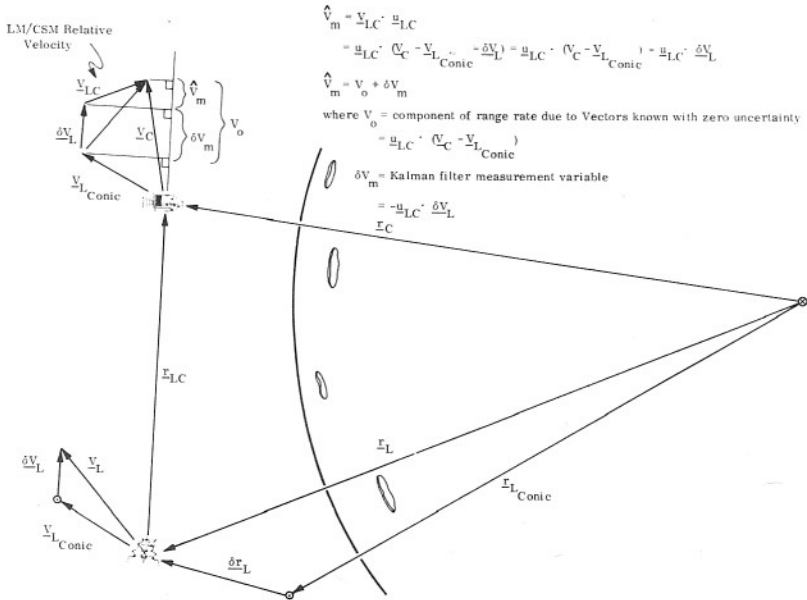
$$\underline{b} = \begin{bmatrix} \pm \underline{u}_{CL} \\ 0 \end{bmatrix} \quad (\pm \text{ is used depending on whether the LM or CSM state vector is being updated.})$$

The geometry relating the RR measurements to the state vector is described on the following pages.

RENDEZVOUS RANGE (r_m) MEASUREMENT



Rendezvous range rate measurement has a term due to deviation from conic velocity ($\frac{\partial V_L}{\partial t}$) and a term due to deviation from conic position ($\frac{\partial r_L}{\partial t}$). The term due to velocity deviation ($\frac{\partial V_L}{\partial t}$) is:



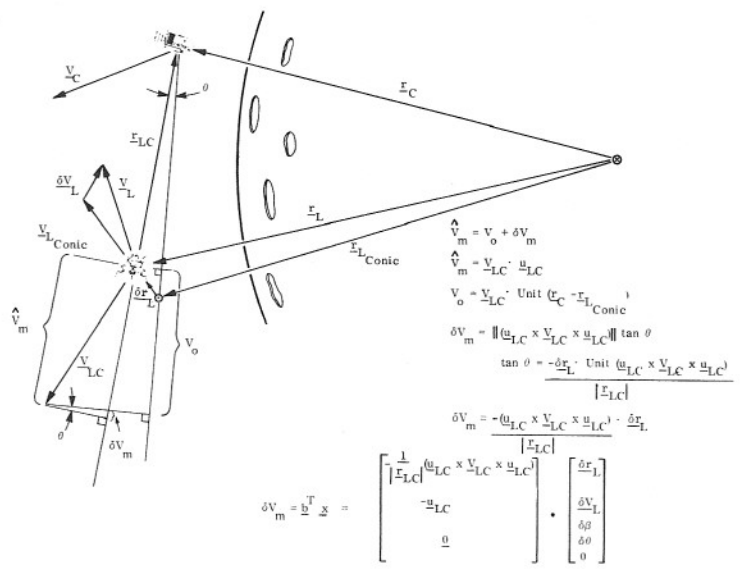
$$\hat{V}_m = V_o + \delta V_m$$

where V_o = component of range rate due to Vectors known with zero uncertainty

$$\delta V_m = \text{Kalman filter measurement variable}$$

$$= -\frac{\partial V_L}{\partial t} \cdot \frac{\partial r_L}{\partial t}$$

The rendezvous range rate term due to position deviation is:



$$\hat{V}_m = V_o + \delta V_m$$

$$\hat{V}_m = V_{LC} \cdot \frac{\partial r_L}{\partial t}$$

$$V_o = V_{LC} \cdot \text{Unit}(\epsilon_C - \epsilon_{LCConic})$$

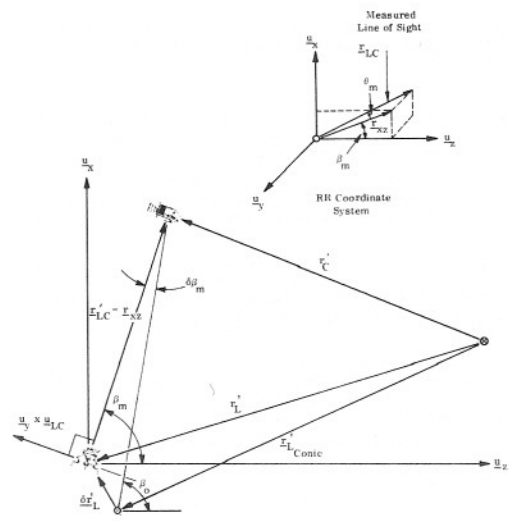
$$\delta V_m = \frac{\| \frac{\partial r_L}{\partial t} \times V_{LC} \times \frac{\partial r_L}{\partial t} \| \tan \theta}{|\epsilon_{LC}|}$$

$$\tan \theta = \frac{-\frac{\partial r_L}{\partial t} \cdot \text{Unit}(\epsilon_C - \epsilon_{LCConic})}{|\epsilon_{LC}|}$$

$$\delta V_m = \frac{-\frac{\partial r_L}{\partial t} \times V_{LC} \times \frac{\partial r_L}{\partial t}}{|\epsilon_{LC}|} \cdot \frac{\partial r_L}{\partial t}$$

$$\delta V_m = \frac{1}{|\epsilon_{LC}|} \begin{bmatrix} \frac{\partial r_L}{\partial t} \times V_{LC} \times \frac{\partial r_L}{\partial t} \\ -\frac{\partial r_L}{\partial t} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial r_L}{\partial t} \\ \frac{\partial V_L}{\partial t} \\ 0 \end{bmatrix}$$

$$\delta V_m = \frac{1}{|\epsilon_{LC}|} \begin{bmatrix} \frac{\partial r_L}{\partial t} \times V_{LC} \times \frac{\partial r_L}{\partial t} \\ -\frac{\partial r_L}{\partial t} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial r_L}{\partial t} \\ \frac{\partial V_L}{\partial t} \\ 0 \end{bmatrix}$$



The rendezvous radar shaft angle is defined in the radar X-Z coordinate plane. The effect of LM and CSM position vectors on the shaft angle can be determined by looking at the projection of these vectors on the x-z plane:

$$\hat{\theta}_m = \beta_m + \delta \theta_m$$

where β_m = estimate of RR shaft angle due to ϵ_{LC}

$$\beta_m = \text{portion of RR shaft angle due to } \epsilon_C \text{ and } \epsilon_{LCConic}$$

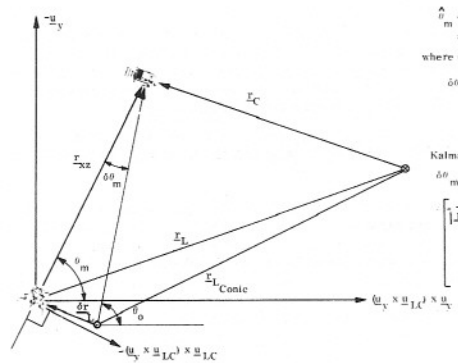
$$\delta \theta_m = \frac{1}{|\epsilon_{xz}|} \left| \text{Unit}(\epsilon_C \times \frac{\partial r_L}{\partial t} \times \frac{\partial r_L}{\partial t}) \cdot \frac{\partial r_L}{\partial t} \right|$$

The actual measured shaft angle has two components a nominal one due to spacecraft position and an additional deviation term ($\delta \theta$) which is an element of the state vector.

$$\hat{\theta}_m = \beta_m + \delta \theta = \beta_m + \delta \theta + \delta \theta$$

The Kalman filter measurement variable $\delta \theta_m + \delta \theta = \frac{1}{|\epsilon_{xz}|} \begin{bmatrix} \frac{\partial r_L}{\partial t} \times \frac{\partial r_L}{\partial t} \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial r_L}{\partial t} \\ \frac{\partial V_L}{\partial t} \\ \delta \theta \\ \delta \theta \\ 0 \end{bmatrix}$

The rendezvous radar trunnion angle is defined in the $\frac{\partial r_L}{\partial t} - \frac{\partial r_L}{\partial t}$ plane and can be illustrated by projecting the spacecraft position vectors onto this plane. Like the shaft angle it is divided into two components, a nominal one due to spacecraft positions and a deviation term ($\delta \theta$) which is an element of the state vector.



$$\hat{\theta}_m = \theta_m + \delta \theta$$

$$= \theta_m + \delta \theta + \delta \theta$$

where θ_m = component of θ_m due to $\epsilon_C - \epsilon_{LCConic}$

$$\delta \theta_m = \text{component of } \theta_m \text{ due to } \frac{\partial r_L}{\partial t}$$

$$= \frac{\frac{\partial r_L}{\partial t} \cdot \frac{1}{|\epsilon_{xz}|} \left| \text{Unit}(\epsilon_C \times \frac{\partial r_L}{\partial t} \times \frac{\partial r_L}{\partial t}) \right| \times \frac{\partial r_L}{\partial t}}{|\epsilon_{xz}|}$$

Kalman filter measurement variable $\delta \theta_m + \delta \theta = \frac{1}{|\epsilon_{xz}|} \begin{bmatrix} \frac{\partial r_L}{\partial t} \times \frac{\partial r_L}{\partial t} \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial r_L}{\partial t} \\ \frac{\partial V_L}{\partial t} \\ \delta \theta \\ \delta \theta \\ 0 \end{bmatrix}$

CSM DIGITAL AUTOPILOT

The CSM digital autopilot (DAP) provides a primary stabilization and control function and attitude error display for CSM or CSM/LM coasting flight, CSM or CSM/LM powered flight and CM entry, as well as a backup SATURN takeover function.

COASTING FLIGHT

The CSM coasting flight autopilot or reaction control system autopilot (RCS DAP) provides attitude and translation control in three CSM axes during nonthrusting phases of flight. The RCS DAP has three major modes of operation, Auto, Hold and Free, as commanded by the S/C CONTROL switch.

The Hold mode maintains or holds the spacecraft (S/C) at a desired attitude within the limits of an attitude deadband specified by the crew. Rotational hand controller (RHC) commands will be processed as the discrete commanded rate specified in the DAP variable load; a new desired attitude will be established and maintained upon release of the RHC.

The Auto mode enables rate and attitude commands from the steering routines to be processed by the DAP for maneuvering the S/C to a desired attitude at a specified rate. With the absence of maneuver commands, the Auto mode functions as an attitude hold mode. RHC commands are processed in the Auto mode as a discrete commanded rate and automatic maneuvering is terminated until resumed by crew action via the DSKY.

The Free mode of operation releases the S/C from all maneuvers and attitude hold commands, other than minimum impulse commands, and allows the S/C to drift freely. Minimum impulse commands are single 14-millisecond control jet firings which are commanded by the RHC or, if there are no RHC commands, by the minimum-impulse controller (MIC).

Translation hand controller (THC) commands are processed in any mode and are combined with rotation commands for the desired maneuver. When a combination rotation and translation is not possible, due to a quad failure, the rotation command has priority.

ERROR DISPLAY

The DAP also provides attitude error display to the crew via the FDAI attitude error meters. There are three types of attitude error displays available.

- To provide a monitor of autopilot performance, the autopilot following errors or phase plane errors in control axis coordinates can be displayed by keying a V61E.
- To aid the crew in executing a manual maneuver, the total attitude error with respect to the desired maneuver angles in N22 can be displayed by keying V62E.
- Total Astronaut attitude error with respect to preloaded N17 angles can be displayed by keying V63E providing another manual maneuver aid. N17 can be loaded with a snapshot of the present CDU angles by keying V60E.

POWERED FLIGHT

The powered flight autopilot stabilizes and controls the attitude of the spacecraft and maintains thrust vector control (TVC) during service propulsion system (SPS) thrusting. Pitch and Yaw axis control is achieved by TVC DAP generated commands to the SPS engine gimbal servos to maintain the thrust vector through the center of gravity of the vehicle. The TVC DAP also accepts angular rate commands from the steering program to position the thrust vector along a desired thrust direction.

Roll axis attitude and rate control during powered flight is accomplished by the TVC Roll DAP. Its function is strictly to provide attitude hold about the roll axis of the spacecraft by means of RCS thrusters.

ENTRY

The entry autopilot provides attitude control of the command module (CM) from separation from the service module (SM) to deployment of the drogue chutes. The DAP has an extra-atmospheric phase and an atmospheric phase. The extra-atmospheric phase provides three-axis spacecraft control for the trajectory segment prior to 0.05 g and accepts attitude commands from entry guidance to orient the S/C for the onset of 0.05 g. The atmospheric phase provides attitude control, after 0.05 g about entry roll or about the vector direction of S/C velocity relative to the air mass, to steer the S/C along the entry trajectory. The DAP accepts steering commands from the entry guidance programs.

SATURN TAKEOVER

In the event of a Saturn instrumentation unit (IU) fail, a capability is provided for the CMC to issue angular rate steering commands to the IU autopilot. CMC takeover of Saturn control, which is accomplished by means of the LV GUIDANCE switch, may be an automatic or manual steering mode.

The manual or stick mode is available by keying V46E, which terminates computation of automatic mode attitude errors. Discrete rate commands, based on erasable parameters, are initiated by means of the RHC and transmitted to the IU autopilot.

DAP DATA

The DAP registers containing the variable parameters which determine DAP selection and desired DAP performance are accessible by keying V48E.

CSM DAP CONTROL

V48E
Flashing V04 N46

	A	B	C	D	E
Register 1:	CONFIG	XTAC	XTBD	DB	RATE

CONFIG - Vehicle Configuration

- 0 = No DAP is requested
- 1 = CSM alone
- 2 = CSM and LM
- 3 = SIVB, CSM and LM (SIVB control)
- 6 = CSM and LM (ascent stage only)

XTAC - X-Translations Using Quads AC

- 0 = Do not use AC
- 1 = Use AC

XTBD - X-Translations Using Quads BD

- 0 = Do not use BD
- 1 = Use BD

DB - Angular Deadband for Attitude Hold and Automatic Maneuvers

- 0 = ± 0.5 degree
- 1 = ± 5.0 degrees

RATE - Rotational Rate for RHC in HOLD or AUTO Mode and for Automatic Maneuvers.

- 0 = 0.05 deg/s
- 1 = 0.2 deg/s
- 2 = 0.5 deg/s
- 3 = 2.0 deg/s

Register 2:	AC Roll	Quad A	Quad B	Quad C	Quad D
-------------	---------	--------	--------	--------	--------

AC Roll - Roll-Jet selection

- 0 = Use BD roll Quads
- 1 = Use AC roll Quads

A, B, C, D - Quad fails

- 0 = Quad has failed
- 1 = Quad operational

Flashing V06 N47

Register 1: CSM weight in pounds

Register 2: LM weight in pounds

Flashing V06 N48

Register 1: Pitch-trim gimbal offset, in 1/100 degree

Register 2: Yaw-trim gimbal offset, in 1/100 degree

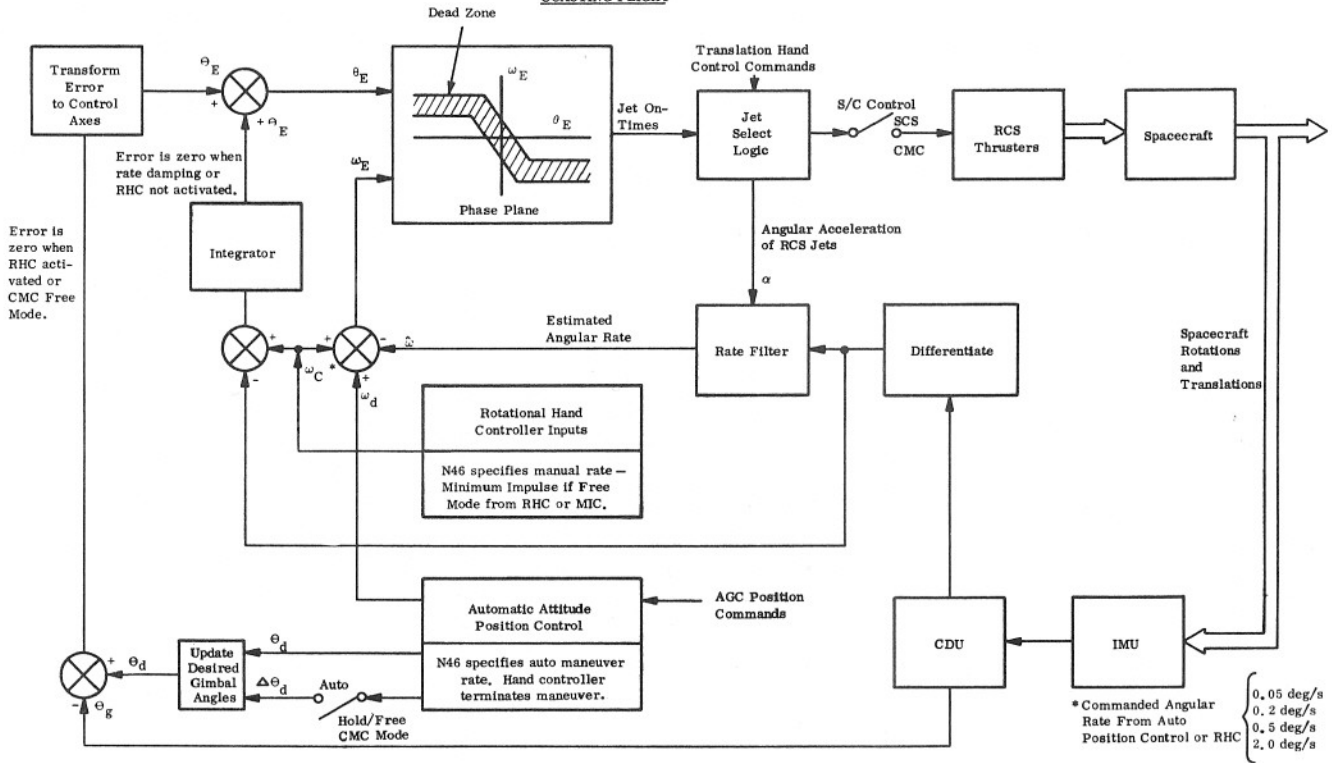
AUTOMATIC MODE

- Automatic three-axis rotation.
- Manual three-axis rotation and translation
- Attitude hold to program or manual defined attitude.
- Automatic rate damping.

ATTITUDE HOLD MODE

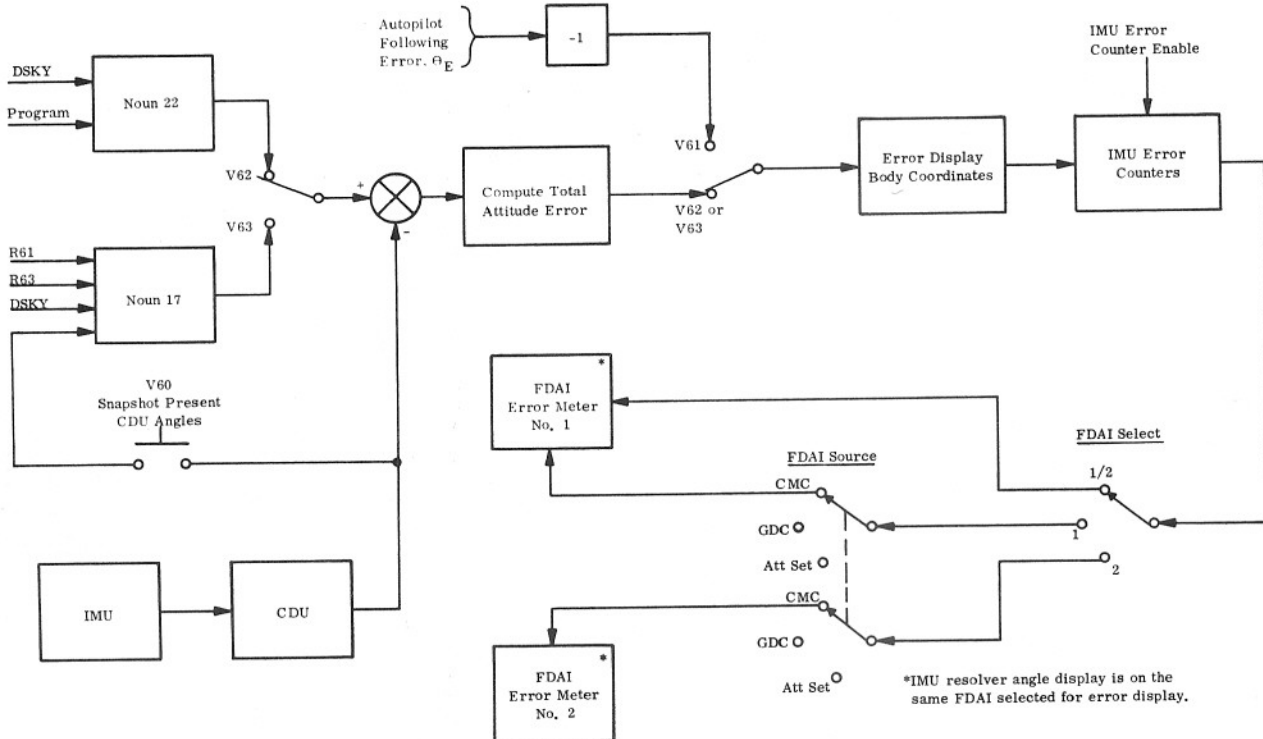
- Manual three-axis rotation and translation.
- RHC produces a rotational rate as specified by N46 while out of detent.
- Attitude hold to attitude selected via hand controller.
- Automatic rate damping.

**CSM DIGITAL AUTOPILOT
COASTING FLIGHT**



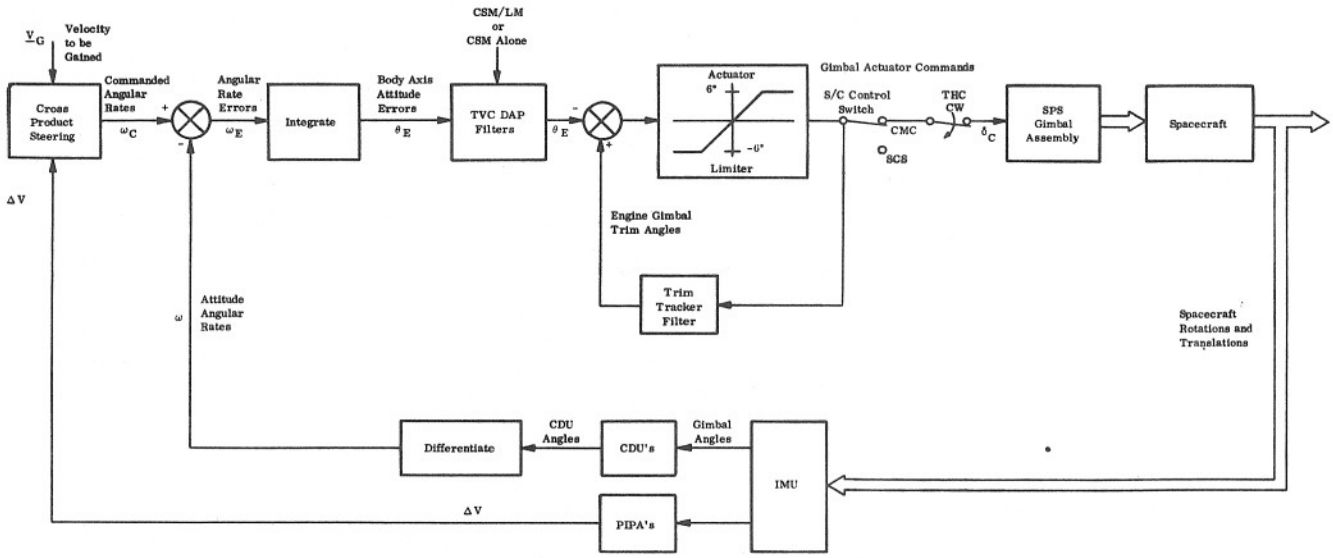
SC-18

**CSM DIGITAL AUTOPILOT
FDAI ATTITUDE ERROR DISPLAY**



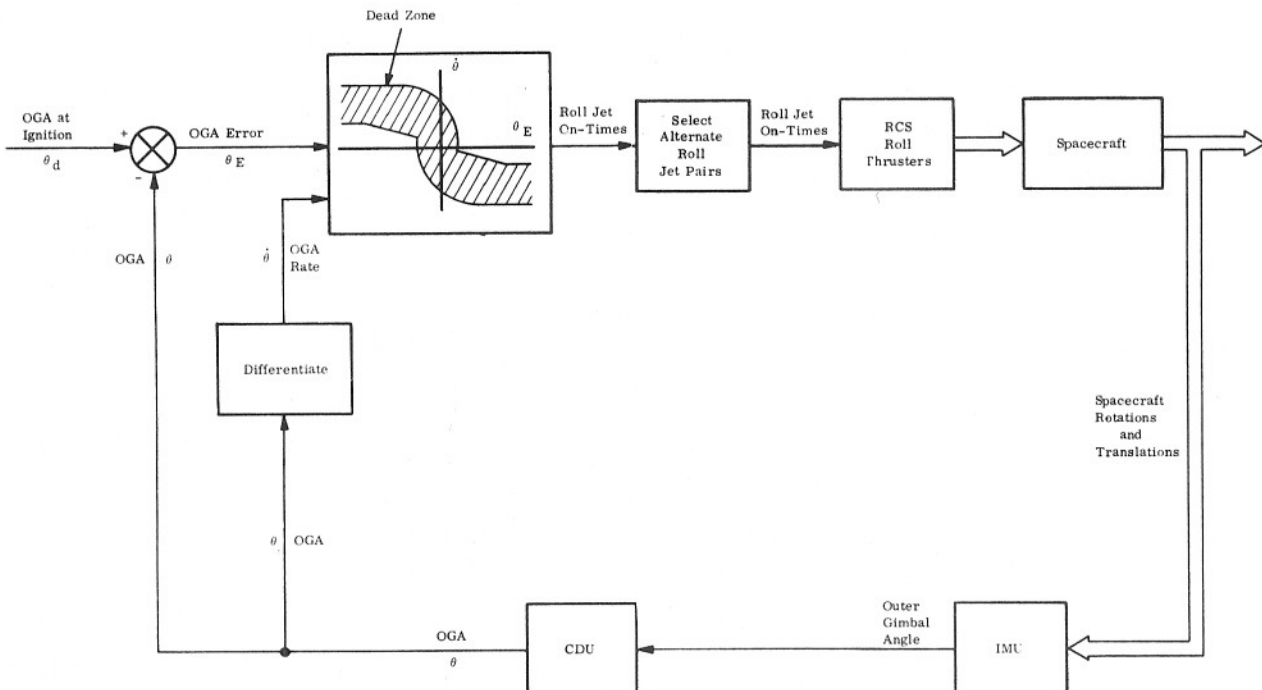
SC-19

**CSM DIGITAL AUTOPILOT
POWERED FLIGHT THRUST VECTOR CONTROL**



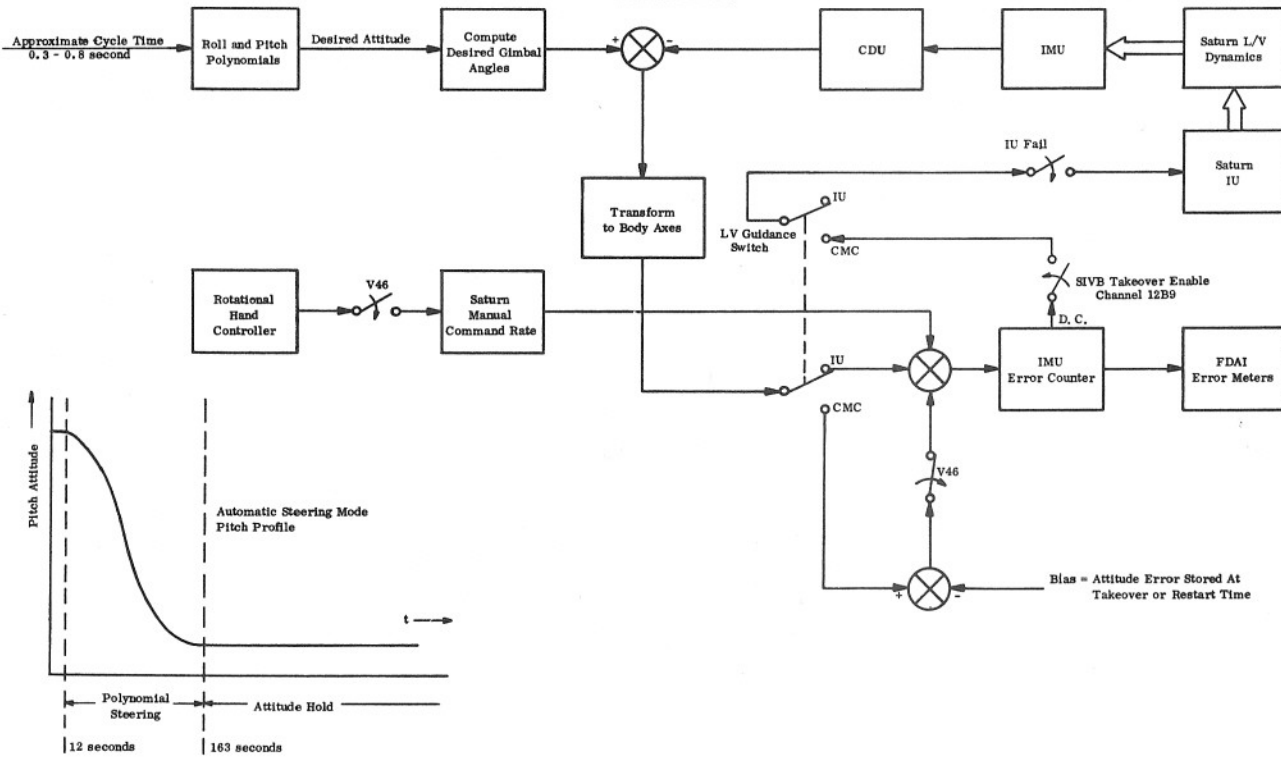
SC-20

**CSM DIGITAL AUTOPILOT
POWERED FLIGHT ROLL ATTITUDE CONTROL**



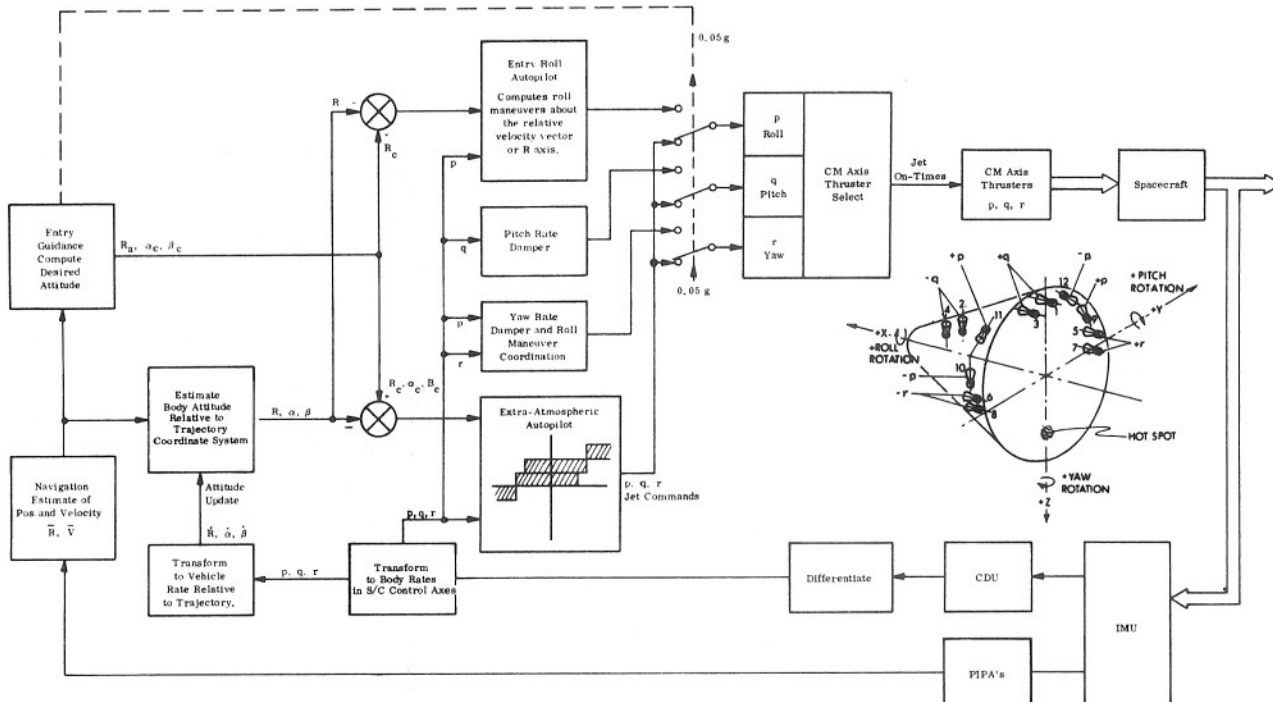
SC-21

**CSM DIGITAL AUTOPILOT
CMC TAKEOVER OF SATURN**



SC-22

**CSM DIGITAL AUTOPILOT
ENTRY**



SC-23

LM DAP CONTROL

Flashing V04 N46

	A	B	C	D	E
Register 1:	CONFIG	ACC	ACA	-DB	RATE

CONFIG - Vehicle Configuration

- 1 = Ascent stage only
- 2 = Ascent and descent stages
- 3 = LM and CSM docked

ACC - Acceleration Code

- 0 = Two-jet translation (RCS System A)
- 1 = Two-jet translation (RCS System B)
- 2 = Four-jet translation (RCS System A - minimum impulse)
- 3 = Four-jet translation (RCS System B - minimum impulse)

ACA - ACA Scaling

- 0 = Docked (4 deg/s, max. rate)
- 1 = Normal (20 deg/s, max. rate)

DB - Deadband

- 0 = 0.3 degree
- 1 = 1.0 degree
- 2 = 5.0 degrees

RATE - Maneuver Rate (Automatic Mode)

- 0 = 0.2 deg/s
- 1 = 0.5 deg/s
- 2 = 2.0 deg/s
- 3 = 10.0 deg/s

Flashing V06 N47

- Register 1: LM weight in pounds
- Register 2: CSM weight in pounds

Flashing V06 N48

- Register 1: Pitch-trim engine gimbal angle, in 0.01 degree
- Register 2: Roll-trim engine gimbal angle, in 0.01 degree

AUTOMATIC MODE

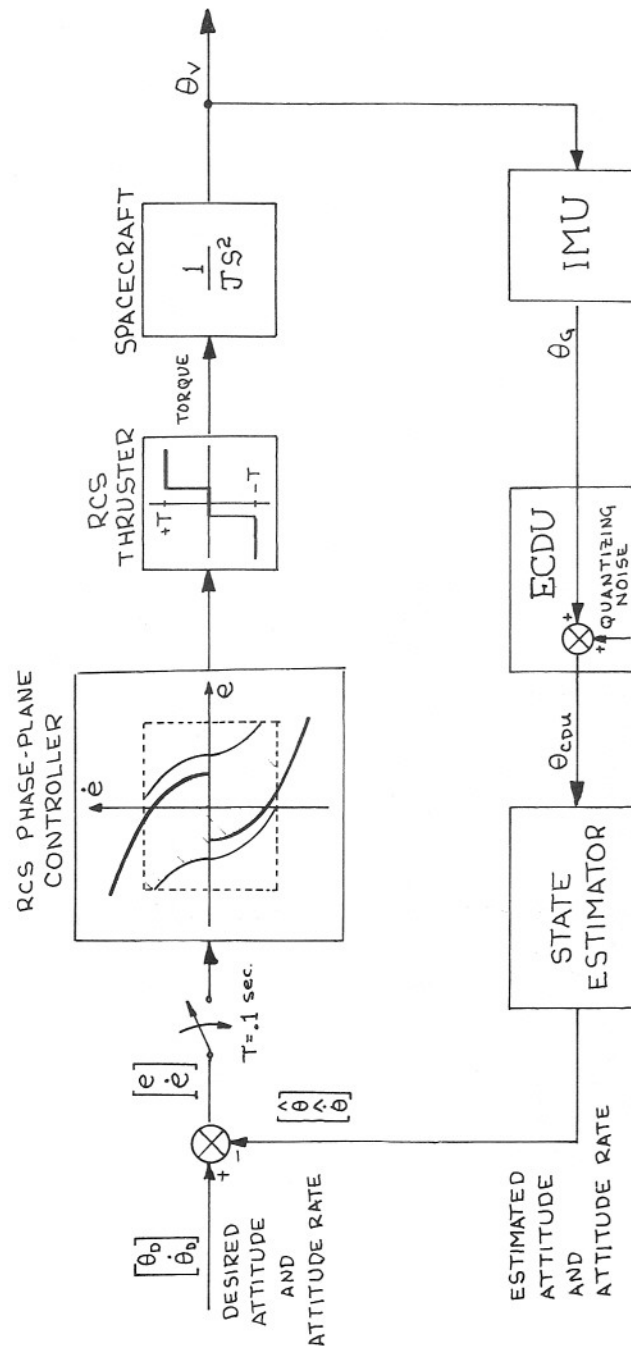
1. Automatic three-axis rotation and translation.
2. Manual three-axis translation.
3. Manual X-axis rate command (inhibited in LPD phase)
4. Attitude hold to program defined attitude
5. Automatic rate damping

ATTITUDE HOLD MODE

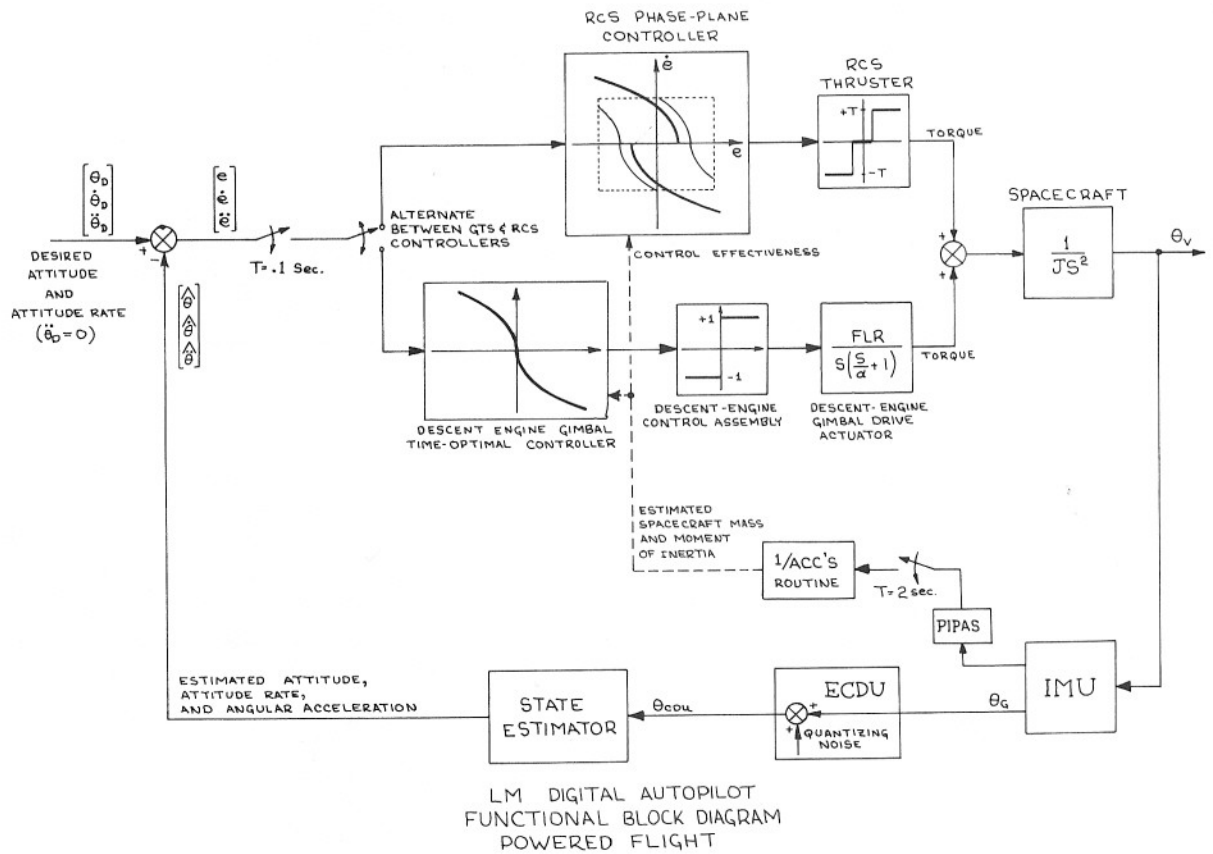
1. Manual three-axis translation.
2. Manual three-axis rate command using V77
3. Manual minimum impulse command using V76.
4. Attitude hold to attitude selected via hand controller
5. Automatic rate damping

V77 - Used to provide a manual rate command. Commanded rotational rate is proportional to hand controller (ACA) deflection. Maximum commanded rotational rate is either 4 deg/s or 20 deg/s as chosen in DAP Data Load routine.

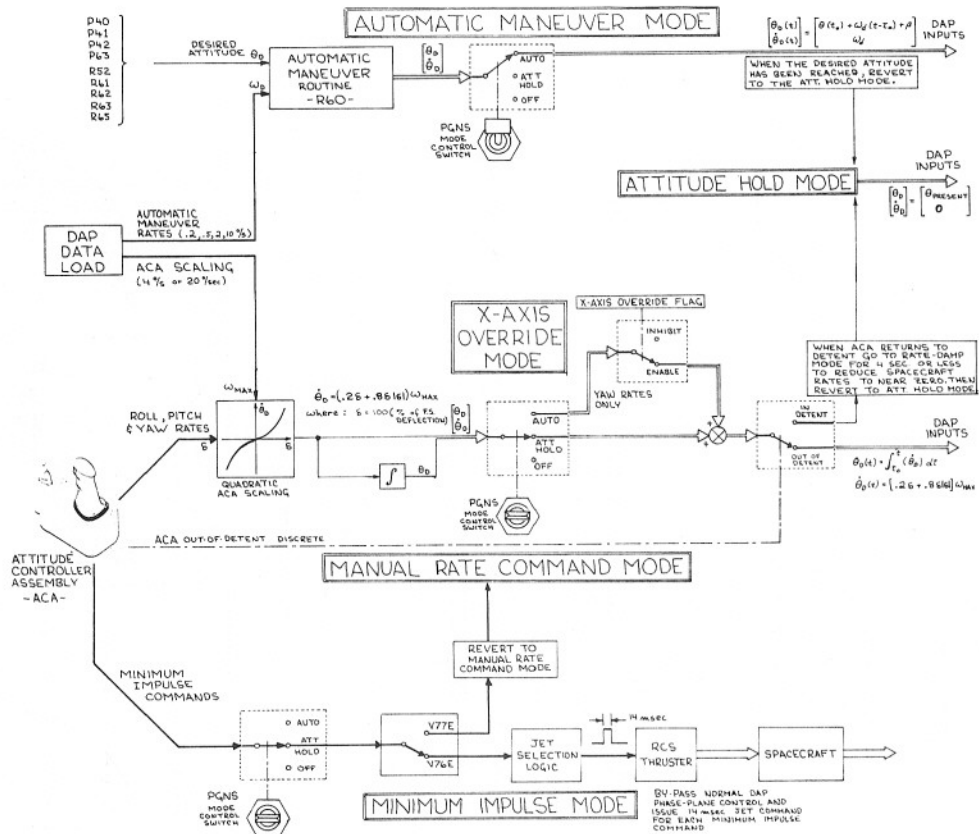
V76 - Used to provide a minimum impulse command. Releases Attitude Hold mode and allows vehicle to drift freely. One impulse is produced for each hand controller (ACA) deflection greater than 2.5 degrees.



LM DIGITAL AUTOPILOT
FUNCTIONAL BLOCK DIAGRAM
COASTING FLIGHT



SC-26



SC-27

PHASE-PLANE FUNDAMENTALS

The equations describing spacecraft attitude errors and attitude rate errors in the phase plane are derived as follows:

ATTITUDE RATE ERRORS

$$\dot{e}(t) = \dot{\theta}(t) - \dot{\theta}_D(t) \quad (1)$$

where

- $\dot{e}(t)$ = Attitude rate error
- $\dot{\theta}_D(t)$ = Desired attitude rate
- $\dot{\theta}_D(t)$ (desired rate is constant over sample interval)
- $\dot{\theta}(t)$ = Actual spacecraft rate
- $\dot{\theta}(t) = \dot{\theta}(t_0) + a(t - t_0)$
- (a = control + disturbance acceleration and is assumed constant over sample interval)

$$\therefore \dot{e}(t) = \underbrace{\dot{\theta}(t_0) - \dot{\theta}_D(t_0)}_{\dot{e}_0} + a(t - t_0) \quad (2)$$

ATTITUDE ERRORS

$$e(t) = \theta(t) - \theta_D(t) \quad (3)$$

where

- $e(t)$ = Attitude error
- $\theta_D(t)$ = Desired spacecraft attitude
- $\theta_D(t) = \theta_D(t_0) + \dot{\theta}_D(t_0)(t - t_0)$
- (desired attitude should be integral of desired rate for proper phase-plane control)
- $\theta(t)$ = Actual spacecraft attitude
- $\theta(t) = \theta(t_0) + \dot{\theta}(t_0)(t - t_0) + \frac{1}{2} a (t - t_0)^2$

$$\therefore e(t) = \underbrace{\theta(t_0) - \theta_D(t_0)}_{e_0} + \underbrace{[\dot{\theta}(t_0) - \dot{\theta}_D(t_0)]}_{\dot{e}_0} (t - t_0) + \frac{1}{2} a (t - t_0)^2 \quad (4)$$

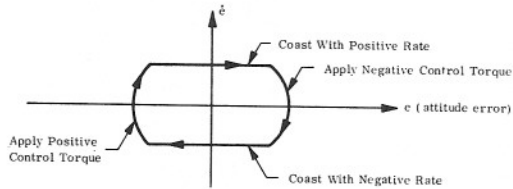
Equations 2 and 4 above can be combined to eliminate $(t - t_0)$ and solve for the phase plane equation of $e(t)$.

$$e(t) = e_0 + [\dot{e}(t)^2 - \dot{e}_0^2] / 2a \quad \text{[for constant torque]} \quad (5)$$

or

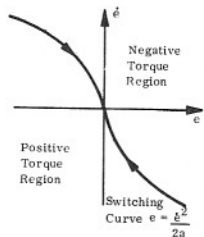
$$\dot{e}(t) = \dot{e}_0 = \text{constant} \quad \text{[for zero torque]} \quad (6)$$

Equation 5 describes a parabola that goes in a clockwise phase plane direction for negative applied torque and in a counterclockwise direction for positive torque. Equation 6 describes a straight line going to the right for positive initial rates and to the left for negative initial rates. Equations 5 and 6 together describe a typical limit cycle trajectory that travels in a clockwise direction about the origin of the error plane as shown below.



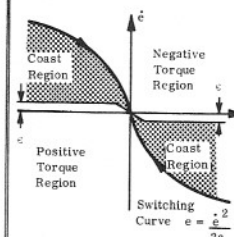
OPTIMAL PHASE PLANE CONTROLLERS (double integral plant with maximum control acceleration = a)

TIME-OPTIMAL CONTROLLER



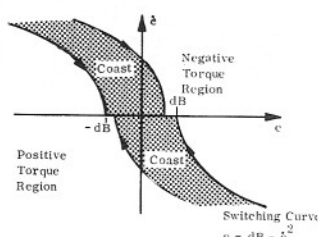
PURPOSE:
To bring e and \dot{e} to zero in minimum time.

FUEL-OPTIMAL CONTROLLER



PURPOSE:
To bring e and \dot{e} to zero with minimum fuel consumption. (ϵ must be diminishingly small for minimum fuel. This drastically increases response time.)

COMBINATION CONTROLLER



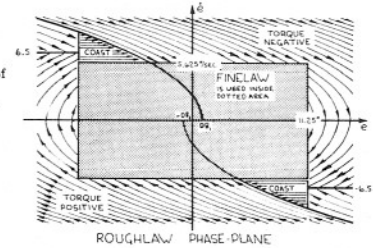
PURPOSE:
To achieve a satisfactory mix of time and fuel optimal control and at the same time minimize the number of RCS jet firings by establishing coasting deadbands.

LM UNDOCKED PHASE-PLANE

The purpose of the LM Undocked Phase-Plane logic is to compute and issue commands to the RCS thrusters in order to null spacecraft attitude and attitude rate errors. The thruster commands are in the form of a signed time, where the sign indicates the sense of rotation (positive or negative) and the time is the duration of thrusting required to reach the phase-plane switching boundary. In order to achieve these ends, the phase-plane is divided into two principal regions, ROUGHLAW and FINELAW.

ROUGHLAW

The ROUGHLAW phase-plane logic is used for coarse control of the spacecraft when the phase-plane errors are greater than 11.25 degrees and/or 5.625 degrees/second. The division between the positive and negative thrusting regions is shown in the diagram on the right.

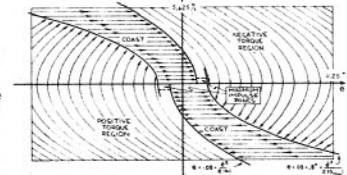


FINELAW

The FINELAW phase-plane logic is used whenever the phase-plane errors are less than 11.25 degrees and 5.625 degrees/second. The FINELAW phase-plane is configured differently for powered and coasting flight to compensate for offset accelerations caused by the DPS or APS engine not thrusting through the LM center of gravity.

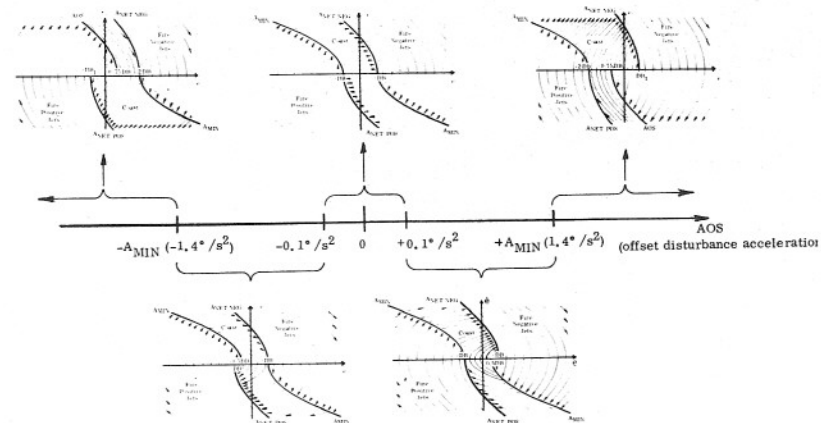
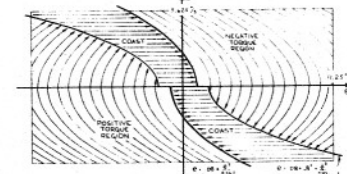
• DRIFTING FLIGHT

The drifting flight phase-plane is set up to achieve a minimum impulse limit cycle. When the phase-plane errors are in the minimum impulse zone, a 14-millisecond jet firing will be commanded. This should be sufficient to reverse the sign on \dot{e} and cause the spacecraft to drift back to the other deadband.



• POWERED FLIGHT

When the DPS trim gimbal system is operating during powered flight, the offset accelerations (AGS) will be nulled and the phase-plane looks like the drifting flight phase-plane except that the minimum impulse zones are eliminated. When the gimbal trim system (GTS) is not in operation or when AOS is too large to be nulled within 2 seconds, the phase-plane logic is set up according to the magnitude of AOS as shown below. The switching curves are then established based on AOS (disturbing acceleration), A_{MIN} (1.4 degrees/second²), $A_{NET NEG}$ (acceleration due to negative jets + AOS) or $A_{NET POS}$ (acceleration due to positive jets + AOS) and the established deadband (DB). The DB is selected as 0.3, 1, or 5 degrees dependent upon the mission programs and the LM astronaut.



LM/CSM DOCKED PHASE-PLANE

The purpose of the LM/CSM Docked autopilot is to provide backup attitude control of the docked configuration during drifting flight and docked DPS burns. Since this is a backup autopilot, simplicity is stressed rather than performance. Simplicity is achieved as follows.

- The same RCS phase-plane is used for all three control axes.
- Only two-jet thrusting is allowed for each axis.
- Jets are turned on or off at control sample intervals rather than computing the thrusting time required to reach the target switching point.
- The target switching parabolas are replaced by straight line segments.

The design objectives of the simplified autopilot are to:

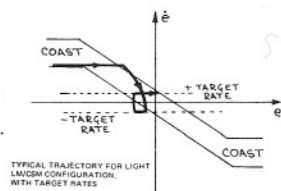
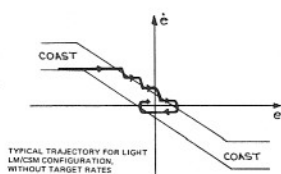
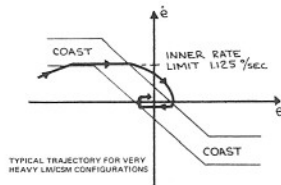
- Enable low rate attitude maneuvers or attitude hold during drifting flight.
- Enable RCS attitude control in conjunction with the GTS (Gimbal Trim System) during docked DPS burns.
- Reduce the probability of bending mode excitation in excess of docking-tunnel load criteria.
- Perform the above tasks assuming that the number of jet firings and RCS fuel usage are noncritical.

PHASE-PLANE MECHANIZATION

In order to achieve the above design objectives the LM/CSM Docked phase-plane employs rate-limiting, target rates, and jet inhibition logic in addition to the normal phase-plane logic.

RATE LIMITING

Rate limiting is incorporated to prevent the RCS jets from causing attitude rate errors exceeding 1.125 degrees/sec. This number was chosen to allow the heaviest configuration to reach a limit cycle with zero overshoots.

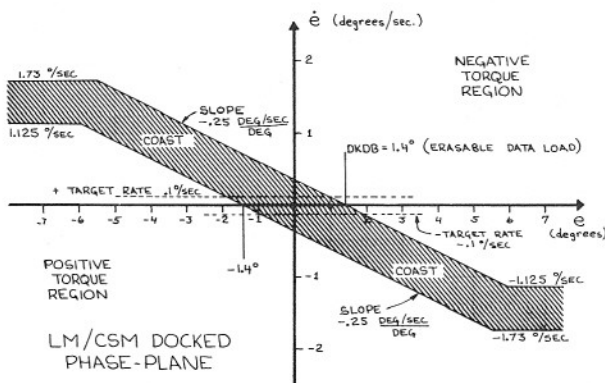


TARGET RATES

Target rates of +0.1 degree/second for positive thrusting and -0.1 degree/second for negative thrusting are incorporated during powered flight. (Target rate = 0 for drifting flight). This allows a steady state limit cycle to be reached in minimum time by bypassing the phase-plane logic when the jets are on and thrusting toward the target rate.

JET INHIBIT LOGIC

Jet inhibition logic operates to reduce bending mode excitation by preventing jet commands from reversing sign at a rapid rate. That is, if the jets are firing negative and the phase plane logic calls for positive jets, then the jet inhibition logic turns the jets off for a period of 1 second (0.4 second for the pitch axis) before they can be turned on again.



GTS ATTITUDE CONTROL SYSTEM

The Descent Propulsion System (DPS) engine is mounted on gimbals so that it can rotate about the LM pitch and roll axes thereby aligning the thrust vector with the LM center of gravity. This feature can be used for attitude control if the attitude error and error rates are small.

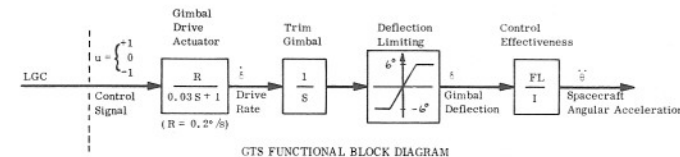
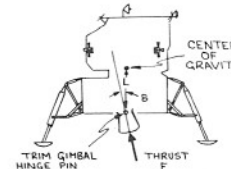
GIMBAL TRIM SYSTEM MECHANIZATION

The Gimbal Trim System (GTS) is controlled by the LGC which issues only on or off commands. The GTS responds to these commands by rotating the DPS engine about its hinge pin at a constant rate, R. The resulting deflection of the DPS engine produces an offset torque on the LM which can be used as the control acceleration. The relationship between this angular acceleration and DPS engine deflection is given by Equation 1.

$$\ddot{\theta} = \frac{T}{I} = \frac{FL\delta}{I} \quad (1)$$

where

- $\ddot{\theta}$ = spacecraft angular acceleration
- I = spacecraft moment of inertia
- T = torque due to DPS engine gimbaling
- = FL sin $\delta \approx FL\delta$ (small angle approximation)
- F = DPS engine thrust
- L = trim gimbal hinge pin to center of gravity distance
- δ = gimbal deflection



GTS CONTROL LAW

The GTS control law has two modes of operation: Acceleration Nulling and Attitude Control.

ACCELERATION NULLING MODE

The Acceleration Nulling mode is used when the RCS phase-plane errors lie outside the coast zone and the RCS jets are therefore controlling attitude. In this mode, a drive time is computed to drive the trim gimbal so that the thrust vector is aligned with the LM center of gravity.

$$T = 0.4 FLR/I$$

where

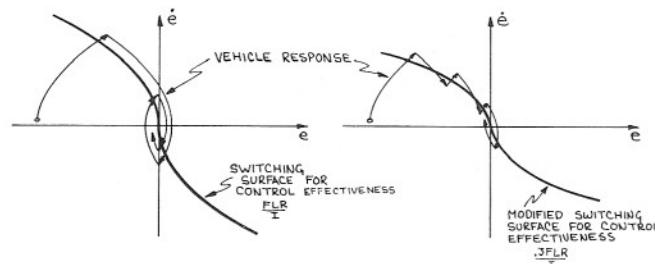
- T = drive time duration
- FLR/I = control effectiveness
- 0.4 = gain factor used to prevent overshoots in the control law.

ATTITUDE CONTROL MODE

The Attitude Control mode uses a "time-optimal" control law to control spacecraft attitude, attitude rate, and angular acceleration. The time-optimal control law was chosen for two reasons:

1. Fuel cost of the control effort is negligible because the DPS engine is on and thrusting and the electrical power required to drive the gimbal is very small.
2. Response time is an important consideration because of the fixed, slow (0.2 degree/second) gimbal drive rate.

The time-optimal control law is a three-dimensional phase-plane type controller designed to drive attitude error (e) and its first two derivatives (e-dot and e-double-dot) to zero simultaneously. The control law defines a switching surface and issues positive drive commands when the errors are on one side of the surface and negative drive commands on the other side of the surface. Control loop sampling rates and GDA risetime delays act to produce error limit cycles with high amplitudes and slow settle-out time. To compensate for this, a control effectiveness gain of 0.3 is introduced thereby modifying the switching surface.



EFFECT OF CONTROL EFFECTIVENESS GAIN ON ATTITUDE CONTROL RESPONSE

STATE ESTIMATOR

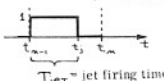
The purpose of the State Estimator is to compute spacecraft attitude, attitude rate, and angular acceleration based on the attitude information obtained from the CDU. The problem then is to estimate the state vector based on noisy measurements of one element of the state vector. The solution of the problem is patterned after Kalman filtering and uses a recursive filter with gains based on engineering considerations rather than statistical optimization.

STATE EQUATION (Continuous System)

$$\dot{x}(t) = [A(t)]x(t) + [B(t)]u(t) + m(t) \quad (1)$$

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \\ \dot{\alpha}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \omega(t) \\ \alpha(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_1(t) \\ 0 \end{bmatrix} \begin{bmatrix} u_j \\ u_g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ m_{CG} \end{bmatrix} \quad (2)$$

u_j, u_g are assumed constant over control sample interval



$T_{jet} = \text{jet firing time}$

STATE EQUATION (Discrete System)

$$x(t_n) = [\Phi(t_n, t_{n-1})]x(t_{n-1}) + \int_{t_{n-1}}^{t_n} [\Phi(t_n, \gamma)] [B(\gamma)] u(\gamma) d\gamma + \int_{t_{n-1}}^{t_n} [\Phi(t_n, \gamma)] m(\gamma) d\gamma \quad (3)$$

or:

$$x_m = [\Phi_{m-1}]x_{m-1} + [G_{m-1}]u_{m-1} + m_{m-1} \quad (4)$$

STATE TRANSITION MATRIX

$$[\Phi_{m-1}] = [\Phi(t_n, t_{n-1})] = \begin{bmatrix} 1 & t_n - t_{n-1} & (t_n - t_{n-1})^2/2 \\ 0 & 1 & t_n - t_{n-1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$T = t_n - t_{n-1}$

DRIVING MATRIX

$$[G_{m-1}] = \int_{t_{n-1}}^{t_n} [\Phi(t_n, \gamma)] [B(\gamma)] d\gamma = \int_{t_{n-1}}^{t_n} \begin{bmatrix} 1 & t_n - \gamma & (t_n - \gamma)^2/2 \\ 0 & 1 & t_n - \gamma \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ b_1(\gamma) & 0 \\ 0 & 1 \end{bmatrix} d\gamma \quad (6)$$

$$= \int_{t_{n-1}}^{t_n} \begin{bmatrix} b_1(\gamma)(t_n - \gamma) & (t_n - \gamma)^2/2 \\ b_1(\gamma) & t_n - \gamma \\ 0 & 1 \end{bmatrix} d\gamma = \begin{bmatrix} T_{jet}(T - T_{jet}/2) & T^2/6 \\ T_{jet} & T^2/2 \\ 0 & T \end{bmatrix} \quad (7)$$

MEASUREMENT EQUATION

$$\theta_m = \theta(t_n) + m_{CDU} \quad (m_{CDU} = \text{measurement noise, primarily CDU quantizing error}) \quad (8)$$

NOMENCLATURE

- $x(t)$ = state vector
- $u(t)$ = control or driving vector
- $m(t)$ = process noise vector
- $\theta(t)$ = spacecraft attitude
- $\omega(t)$ = spacecraft attitude rate
- $\alpha(t)$ = spacecraft angular acceleration
- u_j = angular acceleration due to RCS jets
- u_g = rate of change of angular acceleration due to GTS
- m_{CG} = spacecraft disturbance due to CG movement, nozzle erosion, propellant slosh, and so on
- θ_m = measured spacecraft attitude
- t_n, t_{n-1} = time of present and past state vector estimates, respectively
- T = control sample interval
- T_{jet} = duration of RCS jet firing

STATE ESTIMATOR (Cont)

STATE ESTIMATOR MECHANIZATION

The State Estimator is mechanized as a two-part process: State Vector Extrapolation and State Vector Updating.

State Vector Extrapolation

The state vector is extrapolated using Equation 4 where the state transition matrix $[\Phi_{n-1}]$ is defined in Equation 5 and the driving matrix $[G_{n-1}]$ in Equation 7. The process noise is assumed to be random with zero mean so that the best estimate of its effect on the state vector is zero.

$$\begin{aligned} \hat{\theta}'_m &= \hat{\theta}_{m-1} + T \hat{\omega}_{m-1} + T^2/2 \hat{\alpha}_{m-1} + T_{jet}(T - T_{jet}/2) u_j + T^3/6 u_g & (9) \\ \hat{\omega}'_m &= \hat{\omega}_{m-1} + T \hat{\alpha}_{m-1} + T_{jet} u_j + T^2/2 u_g & (10) \\ \hat{\alpha}'_m &= \hat{\alpha}_{m-1} + T u_g & (11) \end{aligned}$$

State Vector Updating

The state vector is updated based on the weighting functions and the measurement residual

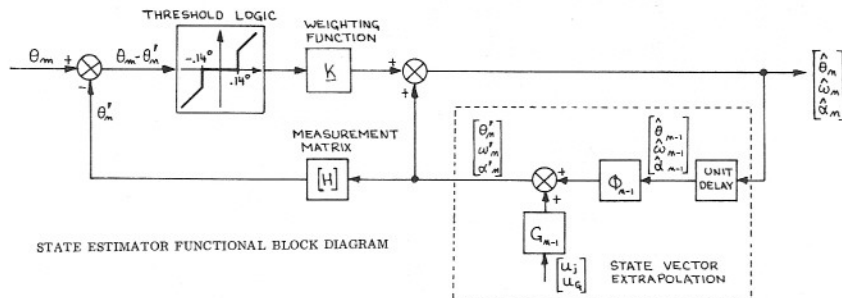
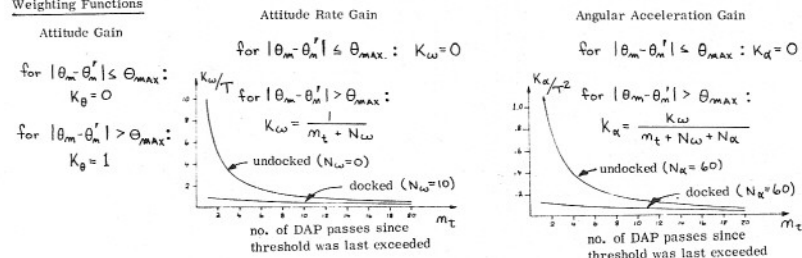
$$\begin{aligned} \hat{\theta}_m &= \hat{\theta}'_m + K_\theta (\theta_m - \theta'_m) & (12) \\ \hat{\omega}_m &= \hat{\omega}'_m + K_\omega / T (\theta_m - \theta'_m) & (13) \\ \hat{\alpha}_m &= \hat{\alpha}'_m + K_\alpha / T^2 (\theta_m - \theta'_m) & (14) \end{aligned}$$

The measurement residual is the difference between the actual measurement, θ_m , and the expected measurement, θ'_m . The expected measurement is equal to the extrapolated estimate of spacecraft attitude, because the measurement noise, m_{CDU} , is assumed to be random with zero mean and the best estimate of its effect on the measurement is therefore zero.

The weighting functions use a threshold logic to filter out CDU quantizing noise. This means that if the measurement residual is less than a preselected threshold, the weighting function is zero and the measurement residual is ignored. When the measurement residual is above the threshold, the gain is not zero and the measurement residual is used to update the state vector. In other words, small variations between the measured and expected attitudes are assumed to be due to CDU quantizing and large variations due to spacecraft maneuvering.

The weighting functions also vary as function of the time since the measurement residual last exceeded the threshold level.

Weighting Functions



1/ACCS ROUTINE

The 1/ACCS routine is scheduled by Servicer every 2 seconds during powered flight. The purpose of the 1/ACCS routine is to compute accelerations due to RCS jet firings, vehicle moments of inertia, and the vehicle center of gravity as functions of vehicle mass. The mass of the vehicle is computed by Servicer every 2 seconds to account for fuel consumed by the main thrusting engine. The mass computation is performed according to Equation 1 based on velocity changes measured by the PIPA's.

$$m(t_n) = m(t_{n-1}) \left(1 - \frac{\Delta V}{V_e} \right) \quad (1)$$

where

- $m(t_n)$ = vehicle mass at current time, t_n
- $m(t_{n-1})$ = vehicle mass at last time increment.
- ΔV = change in velocity as measured by the PIPA's between times t_{n-1} and t_n .
- V_e = exhaust velocity constant
- V_e (DPS) = 2,555,889 meters/second
- V_e (APS) = 3,630 meters/second
- V_e (RCS) = V_e (DPS) or V_e (APS)

The functions used by 1/ACCS to compute jet accelerations, moments of inertia, and center-of-gravity location are given below

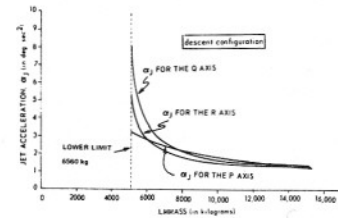
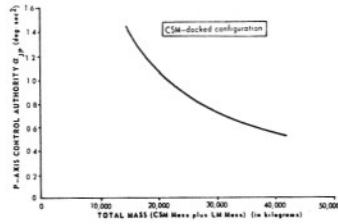
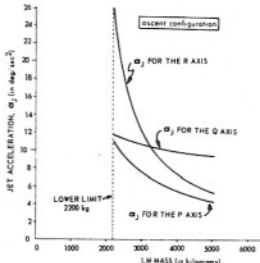
RCS Jet Accelerations

One-jet accelerations are computed as hyperbolic functions of vehicle mass.

$$a_j = \frac{a}{m/c} + b$$

where

- m = mass of vehicle (Equation 1 above)
- a, b, c = constants



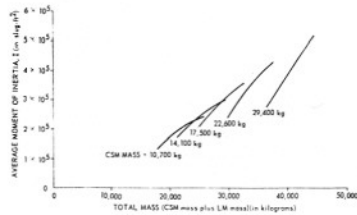
Vehicle Moment of Inertia

One-jet accelerations for the pitch and roll axes in the CSM/LM docked configuration are computed according to $a_j = T/I$. The torque, T , is assumed constant and the inertia, I , is computed as a quadratic function of the LM mass and the CSM mass.

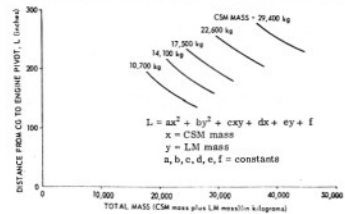
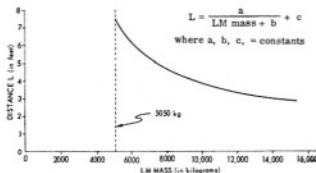
$$I = ax^2 + by^2 + cxy + dx + ey + f$$

where

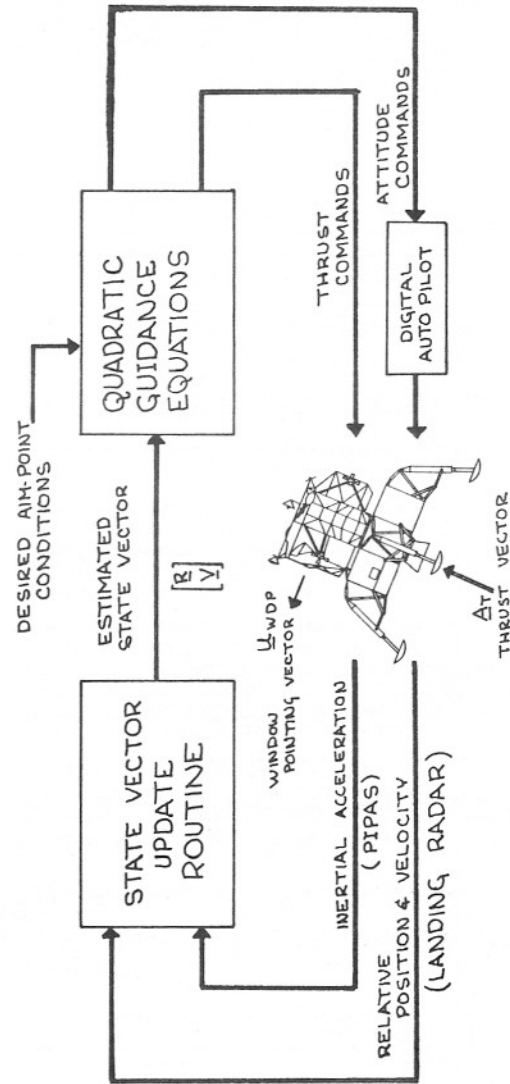
- I = moment of inertia about pitch or roll axes
- x = CSM mass
- y = LM mass
- a, b, c, d, e, f = constants



Hinge Pin to Center of Gravity Distance



LUNAR LANDING GUIDANCE & NAVIGATION



LUNAR LANDING NAVIGATION

NAVIGATION IS PERFORMED BY THE STATE VECTOR UPDATE ROUTINE WHICH USES A MODIFIED KALMAN FILTER TO ESTIMATE THE VEHICLE STATE VECTOR BASED ON PIPA & LANDING RADAR MEASUREMENTS.

LUNAR LANDING GUIDANCE

THE QUADRATIC GUIDANCE EQUATIONS GENERATE THRUST & ATTITUDE COMMANDS BASED ON THE PRESENT VEHICLE STATE VECTOR AND THE DESIRED AIM POINT CONDITIONS. THE THRUST & ATTITUDE COMMANDS ARE EXECUTED BY THE SPACECRAFT TO ACHIEVE THE DESIRED TRAJECTORY.

PRIMARY CONSTRAINTS ON LUNAR LANDING

THE TERMINAL POINT IS CONSTRAINED IN POSITION AND VELOCITY

$$\begin{aligned} \dot{r}(t_f) &= R_{LS} && \text{(LANDING SITE)} \\ \dot{v}(t_f) &\approx 0 && \text{(SOFT LANDING)} \end{aligned}$$



THE INITIAL POINT IS CONSTRAINED IN POSITION AND VELOCITY

$$\begin{aligned} r(t_0) &= r_0 && \text{= PRESENT POSITION} \\ v(t_0) &= v_0 && \text{= PRESENT VELOCITY} \end{aligned}$$

BASIC GUIDANCE SCHEME

EXPRESS LEM POSITION AS A TAYLOR SERIES IN TIME EXPANDED ABOUT THE TERMINAL TIME t_f .

$$\begin{aligned} r(t) &= r(t_f) + \dot{r}(t_f)(t-t_f) + \frac{\ddot{r}(t_f)}{2!}(t-t_f)^2 + \dots \\ &= r(t_f) + \dot{r}(t_f)(t-t_f) + a(t_f)\frac{(t-t_f)^2}{2} + \dots \end{aligned}$$

EXPRESS VELOCITY AND ACCELERATION AS THE FIRST AND SECOND DERIVATIVES OF POSITION.

$$\begin{aligned} v(t) &= v(t_f) + a(t_f)(t-t_f) + j(t_f)\frac{(t-t_f)^2}{2} + \dots \\ a(t) &= a(t_f) + j(t_f)(t-t_f) + \dots \end{aligned}$$

COMPUTE THRUST COMMANDS FROM THE ACCELERATION PROFILE AND GRAVITY.

$$\begin{aligned} a(t) - a_0 &= a(t_f) + j(t_f)(t-t_f) + \dots \\ \text{THRUST}(t) &= a(t) - g(t) \end{aligned}$$

CAPABILITY OF THE BASIC GUIDANCE SCHEME

THE FORM OF THE ACCELERATION PROFILE IS DETERMINED BY THE NUMBER OF TERMS INCLUDED IN THE TAYLOR SERIES EXPANSION OF $r(t)$.

ORDER OF EXPANSION

POSITION EQUATION

VELOCITY EQUATION

ACCELERATION PROFILE

	N=2	N=3	N=4
POSITION EQUATION	$r(t) = r_f + \dot{r}_f(t-t_f) + a_f \frac{(t-t_f)^2}{2}$	$r(t) = r_f + \dot{r}_f(t-t_f) + a_f \frac{(t-t_f)^2}{2} + j_f \frac{(t-t_f)^3}{6}$	$r(t) = r_f + \dot{r}_f(t-t_f) + a_f \frac{(t-t_f)^2}{2} + j_f \frac{(t-t_f)^3}{6} + s_f \frac{(t-t_f)^4}{24}$
VELOCITY EQUATION	$v(t) = \dot{r}_f + a_f(t-t_f)$	$v(t) = \dot{r}_f + a_f(t-t_f) + j_f \frac{(t-t_f)^2}{2}$	$v(t) = \dot{r}_f + a_f(t-t_f) + j_f \frac{(t-t_f)^2}{2} + s_f \frac{(t-t_f)^3}{6}$
ACCELERATION PROFILE	$a(t) = a_f$	$a(t) = a_f + j_f(t-t_f)$	$a(t) = a_f + j_f(t-t_f) + s_f \frac{(t-t_f)^2}{2}$
	<u>EFFECT OF INITIAL CONSTRAINTS ON THE ACCELERATION PROFILE</u>		
POSITION CONSTRAINT $r(t_0) = r_0, \dot{r}(t_0) = \dot{r}_0$	$a_f = \left[(r_0 - r_f) - \dot{r}_0(t_0 - t_f) \right] \frac{2}{(t_0 - t_f)^2}$	$a_f = \left[(r_0 - r_f) - \dot{r}_0(t_0 - t_f) - j_0 \frac{(t_0 - t_f)^3}{6} \right] \frac{6}{(t_0 - t_f)^3}$	$a_f = \left[(r_0 - r_f) - \dot{r}_0(t_0 - t_f) - j_0 \frac{(t_0 - t_f)^3}{6} - s_0 \frac{(t_0 - t_f)^4}{24} \right] \frac{24}{(t_0 - t_f)^4}$
VELOCITY CONSTRAINT $v(t_0) = v_0, j(t_0) = j_0$	$a_f = (v_0 - \dot{r}_f) \frac{1}{(t_0 - t_f)}$	$a_f = \left[(v_0 - \dot{r}_f) - j_0 \frac{(t_0 - t_f)^2}{2} \right] \frac{1}{(t_0 - t_f)}$	$a_f = \left[(v_0 - \dot{r}_f) - j_0 \frac{(t_0 - t_f)^2}{2} - s_0 \frac{(t_0 - t_f)^3}{6} \right] \frac{1}{(t_0 - t_f)}$
	2 EQUATIONS / 2 UNKNOWN a_f, t_{f0}	2 EQUATIONS / 3 UNKNOWN a_f, j_f, t_{f0}	2 EQUATIONS / 4 UNKNOWN a_f, j_f, s_f, t_{f0}
COMMENTS :	a_f & t_{f0} ARE UNIQUELY DETERMINED BY INITIAL CONSTRAINTS	- 1 DEGREE OF FREEDOM - j_f CAN BE PRESPECIFIED, THEN a_f & t_{f0} ARE UNIQUELY DETERMINED	- 2 DEGREES OF FREEDOM - a_f & j_f CAN BE PRESPECIFIED, THEN s_f & t_{f0} ARE UNIQUELY DETERMINED.

SECONDARY CONSTRAINTS OF LUNAR LANDING

- REACH TERMINAL CONDITIONS WITH HIGH ACCURACY
- VERTICAL DESCENT FOR LAST 200 ft. AT A RATE OF -5 ft/sec.
- LANDING SITE VISIBILITY FOR 200 SEC. BEFORE TOUCHDOWN

MODIFICATIONS TO BASIC GUIDANCE SCHEME

THE FOLLOWING MODIFICATIONS WERE MADE TO THE BASIC QUADRATIC GUIDANCE SCHEME IN ORDER TO ACHIEVE THE SECONDARY CONSTRAINTS OF LUNAR LANDING

- THE DESIRED TRAJECTORY IS REDEFINED ON EVERY ITERATION CYCLE (2 SEC.) SUCH THAT THE INITIAL CONDITIONS ($r(t_0), v(t_0)$) ARE THE CURRENT BEST ESTIMATE OF VEHICLE STATE AS DETERMINED BY THE STATE VECTOR UPDATE ROUTINE. THIS ENABLES PIN-POINT ACCURACY BY ELIMINATING CUMULATIVE ERRORS IN THE IMPLEMENTATION OF THE BASIC SCHEME.
- THE LANDING SEQUENCE IS DIVIDED INTO THREE PHASES EACH WITH ITS OWN FINAL CONDITIONS OR AIM POINTS.

1. BRAKING PHASE

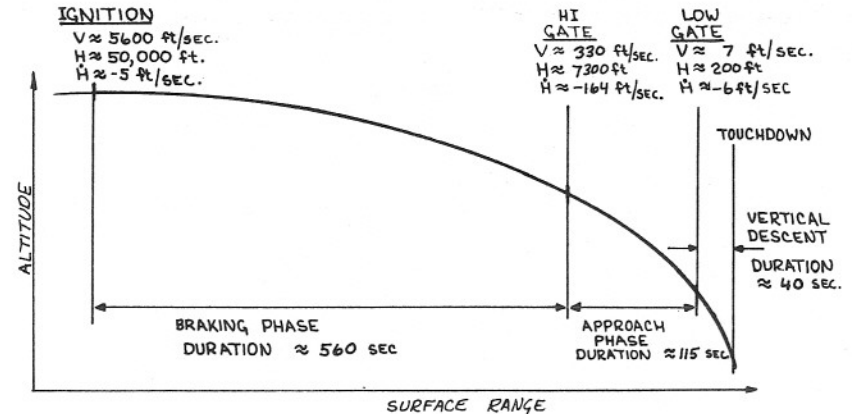
THIS PHASE BRAKES THE LEM DOWN FROM ORBITAL VELOCITY AND IS TARGETED SUCH THAT THE LANDING SITE VISIBILITY CONSTRAINTS CAN BE MET DURING THE VISIBILITY PHASE.

2. VISIBILITY PHASE (APPROACH PHASE)

THIS PHASE IS FLOWN WITH THE LEM ORIENTED SUCH THAT THE LANDING SITE IS VISIBLE FOR LANDING SITE REDESIGNATION. TARGETING IS CHOSEN TO YIELD THE PROPER INITIAL CONDITIONS FOR VERTICAL DESCENT CONSISTENT WITH THE VISIBILITY CONSTRAINTS.

3. VERTICAL DESCENT PHASE

THIS PHASE REDUCES THE SPACECRAFT HORIZONTAL VELOCITY TO ZERO AND VERTICAL VELOCITY TO ≈ 5 ft/sec. QUADRATIC GUIDANCE IS NOT USED.



DERIVATION OF QUADRATIC GUIDANCE LAW

EXPRESS THE POSITION, VELOCITY & ACCELERATION EQUATIONS IN MATRIX FORM

$$\begin{bmatrix} Q(t) \\ \dot{N}(t) \\ \Gamma(t) \end{bmatrix} = \begin{bmatrix} T_{go}^2/2 & T_{go} & 1 & 0 & 0 \\ T_{go}^3/6 & T_{go}^2/2 & T_{go} & 1 & 0 \\ T_{go}^4/24 & T_{go}^3/6 & T_{go}^2/2 & T_{go} & 1 \end{bmatrix} \begin{bmatrix} S_f \\ \dot{N}_f \\ Q_f \\ N_f \\ \Gamma_f \end{bmatrix}$$

WHERE: $T_{go} = t - t_f$

REARRANGE THE MATRIX EQUATION SUCH THAT $Q(t)$, S_f & \dot{N}_f , THE DEPENDANT VARIABLES, ARE EXPRESSED IN TERMS OF $\dot{N}(t)$, $\Gamma(t)$, Q_f , N_f & Γ_f , THE INDEPENDANT VARIABLES.

$$\begin{bmatrix} 1 & -T_{go}^2/2 & -T_{go} \\ 0 & -T_{go}^3/6 & -T_{go}^2/2 \\ 0 & -T_{go}^4/24 & -T_{go}^3/6 \end{bmatrix} \begin{bmatrix} Q(t) \\ S_f \\ \dot{N}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & T_{go} & 1 & 0 \\ 0 & -1 & T_{go}^2/2 & T_{go} & 1 \end{bmatrix} \begin{bmatrix} \dot{N}(t) \\ \Gamma(t) \\ Q_f \\ N_f \\ \Gamma_f \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} C & D \\ E & F \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix}$$

A = CG + DH
B = EG + FH

$$\begin{bmatrix} A-CG & D \\ -EG & -B+FH \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} 0 & D \\ -I & F \end{bmatrix} \begin{bmatrix} B \\ H \end{bmatrix}$$

SOLVE THE ABOVE EQUATION FOR $Q(t)$, S_f & \dot{N}_f .

$$\begin{bmatrix} Q(t) \\ S_f \\ \dot{N}_f \end{bmatrix} = \begin{bmatrix} 6/T_{go} & -12/T_{go}^2 & 1 & 6/T_{go} & 12/T_{go}^2 \\ 24/T_{go}^3 & -72/T_{go}^4 & 12/T_{go}^2 & 48/T_{go}^3 & 72/T_{go}^4 \\ -6/T_{go}^2 & 24/T_{go}^3 & -6/T_{go} & -18/T_{go}^2 & -24/T_{go}^3 \end{bmatrix} \begin{bmatrix} \dot{N}(t) \\ \Gamma(t) \\ Q_f \\ N_f \\ \Gamma_f \end{bmatrix}$$

$$[X]A = [Y]B$$

$$A = [X]^{-1}[Y]B$$

QUADRATIC ACCELERATION EQUATION

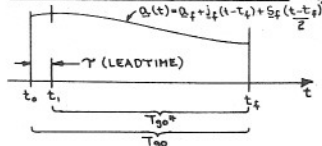
$$Q(t) = Q_f + \dot{N}_f(t-t_f) + S_f(t-t_f)^2$$

WHERE:

$$\dot{N}_f = -24 \frac{(\Gamma_f - \Gamma_o) - \frac{6}{T_{go}^2} (N_o + 3N_f) - \frac{6}{T_{go}^2} Q_f}{T_{go}^3}$$

$$S_f = \frac{72}{T_{go}^4} (\Gamma_f - \Gamma_o) + \frac{24}{T_{go}^3} (N_o + 2N_f) + \frac{12}{T_{go}^2} Q_f$$

LEADTIME COMPENSATION



\dot{N}_f & S_f ARE EVALUATED AT t_0 . $Q(t)$ IS EVALUATED AT t_1 TO YIELD LEADTIME COMPENSATION.

$$Q(t_1) = Q_f + \dot{N}_f T_{go}^* + S_f T_{go}^{*2}$$

$$= \left(-24 \frac{T_{go}^*}{T_{go}^3} + 36 \frac{T_{go}^{*2}}{T_{go}^4}\right) (\Gamma_f - \Gamma_o) + \left(-18 \frac{T_{go}^{*3}}{T_{go}^6} + 24 \frac{T_{go}^{*2}}{T_{go}^5}\right) N_f + \left(-6 \frac{T_{go}^{*3}}{T_{go}^2} + 12 \frac{T_{go}^{*2}}{T_{go}^3}\right) N_o + \left(6 \frac{T_{go}^{*3}}{T_{go}^2} - 6 \frac{T_{go}^{*2}}{T_{go}} + 1\right) Q_f$$

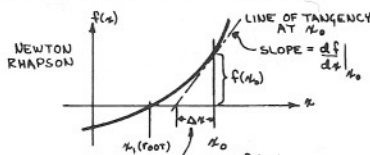
TIME-TO-GO COMPUTATION

TIME TO GO IS COMPUTED TO SATISFY THE REQUIRED FINAL VALUE OF DOWN-RANGE JERK.

FROM THE ABOVE EQUATIONS:

$$\dot{N}_f = \frac{24}{T_{go}^3} (\Gamma_o - \Gamma_f) - \frac{6}{T_{go}^2} (N_o + 3N_f) - \frac{6}{T_{go}^2} Q_f$$

THIS EQUATION IS SOLVED FOR T_{go} BY A NEWTON-RAPHSON ITERATION TECHNIQUE.



TO FIND N_1 :

INITIAL GUESS = N_o
NEW APPROXIMATION = $N_o - \Delta N$

TO FIND TIME-TO-GO:

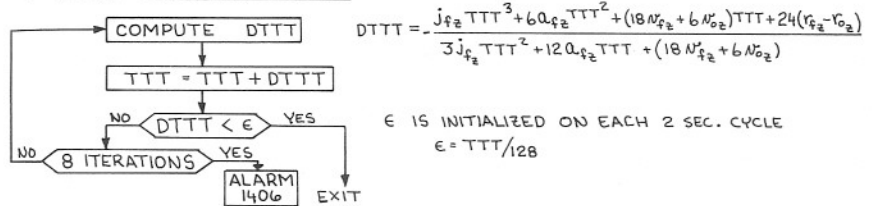
INITIAL GUESS = TTT
NEW APPROXIMATION = TTT - ΔTTT

$$\Delta TTT = \frac{j_{fz} TTT^3 + 6Q_{fz} TTT^2 + 6(N_{fz} + 3N_{oz}) TTT - 24(\Gamma_{fz} - \Gamma_{oz})}{3j_{fz} TTT^2 + 12Q_{fz} TTT + 6(N_{fz} + 3N_{oz})}$$

BASIC MECHANIZATION OF QUADRATIC GUIDANCE (P63, P64)

THE FOLLOWING SEQUENCE OF COMPUTATIONS IS PERFORMED DURING EACH TWO SECOND GUIDANCE CYCLE.

1. COMPUTE TIME-TO-GO



2. COMPUTE ACCELERATION COMMAND

$$Q_g = \left(-24 \frac{T_{go}^*}{T_{go}^3} + 36 \frac{T_{go}^{*2}}{T_{go}^4}\right) (\Gamma_f - \Gamma_o) + \left(-18 \frac{T_{go}^{*3}}{T_{go}^6} + 24 \frac{T_{go}^{*2}}{T_{go}^5}\right) N_f + \left(-6 \frac{T_{go}^{*3}}{T_{go}^2} + 12 \frac{T_{go}^{*2}}{T_{go}^3}\right) N_o + \left(6 \frac{T_{go}^{*3}}{T_{go}^2} - 6 \frac{T_{go}^{*2}}{T_{go}} + 1\right) Q_f - g$$

WHERE:

Γ_o } PRESENT VEHICLE POSITION & VELOCITY FROM STATE-VECTOR UPDATE ROUTINE

ERASABLE DATA LOAD AIM-POINTS

Q_f } DESIRED AIM-POINT CONDITIONS FOR P63 or P64.
 N_f } (ERASABLE LOAD)

	P63	P64
Γ_f (ft)	$\begin{bmatrix} -3118.35 \\ 0 \\ -11741.44 \end{bmatrix}$	$\begin{bmatrix} 158.5 \\ 0 \\ -27.35 \end{bmatrix}$
N_f (ft/sec)	$\begin{bmatrix} -196.46 \\ 0 \\ -166.75 \end{bmatrix}$	$\begin{bmatrix} -3.53 \\ 0 \\ +0.25 \end{bmatrix}$
Q_f (ft/sec ²)	$\begin{bmatrix} -.718 \\ 0 \\ -8.302 \end{bmatrix}$	$\begin{bmatrix} .0717 \\ 0 \\ -.589 \end{bmatrix}$
j_{fz} (ft/sec ³)	-1.512×10^{-2}	4.317×10^{-2}

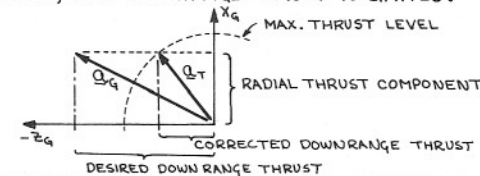
$T_{go} = TTT$ AS DETERMINED ABOVE

$T_{go}^* = TTT - \text{LEADTIME}$
(LEADTIME = 2.2 SEC ERASABLE LOAD)

g = GRAVITY VECTOR

RADIAL GUIDANCE

IF THE DESIRED ACCELERATION IS GREATER THAN MAX. ENGINE THRUST, THEN DOWNRANGE THRUST IS LIMITED.



BASIC MECHANIZATION (CONT.)

3. ALIGN THE GUIDANCE COORDINATE FRAME

THE GUIDANCE FRAME IS ALIGNED SO THAT THE CROSS-RANGE (Y-COMPONENT) OF JERK IS ZERO AT PHASE TERMINUS. THE Y-COMPONENT OF \dot{f}_c IS GIVEN BY THE QUADRATIC GUIDANCE EQUATIONS.

$$\dot{f}_{cy} = -\frac{6}{T_{90}^2} (N_{0y} + 3N_{fy}) - \frac{24}{T_{90}^3} (r_{fy} - r_{0y}) - \frac{6}{T_{90}} a_{fy}$$

THE Y-COMPONENTS OF THE AIM-POINT VECTORS ARE SPECIFIED AS ZERO: $r_{fy} = N_{fy} = a_{fy} = 0$

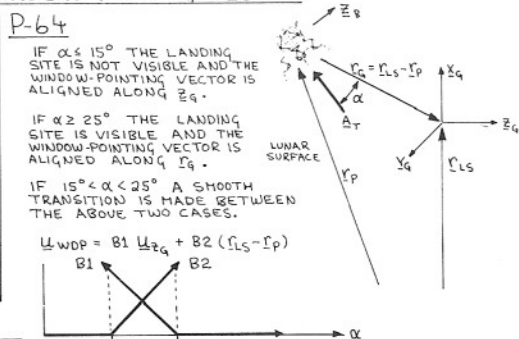
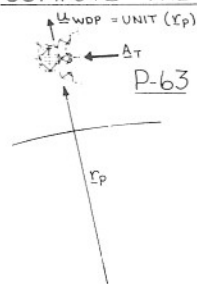
$$\therefore 0 = -\frac{6}{T_{90}^2} N_{0y} + \frac{24}{T_{90}^3} r_{0y}$$

$$\text{OR: } r_{0y} - \frac{T_{90}}{4} N_{0y} = 0 \Rightarrow (r_0 - N_0 \frac{T_{90}}{4})_y = 0$$

THUS, THE GUIDANCE FRAME IS ALIGNED WITH ITS Y-AXIS PERPENDICULAR TO THE VECTOR $(r_0 - N_0 \frac{T_{90}}{4})$

$$\begin{aligned} \underline{u}_{xg} &= \text{UNIT}(r_{Ls}) \\ \underline{u}_{yg} &= \text{UNIT}(r_{Ls} \times (r_0 - N_0 \frac{T_{90}}{4})) \\ \underline{u}_{zg} &= \underline{u}_{yg} \times \underline{u}_{xg} \end{aligned}$$

4. COMPUTE THE WINDOW POINTING VECTOR



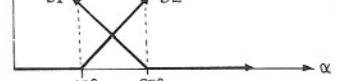
P-64

IF $\alpha \leq 15^\circ$ THE LANDING SITE IS NOT VISIBLE AND THE WINDOW-POINTING VECTOR IS ALIGNED ALONG \underline{e}_g .

IF $\alpha \geq 25^\circ$ THE LANDING SITE IS VISIBLE AND THE WINDOW-POINTING VECTOR IS ALIGNED ALONG \underline{r}_g .

IF $15^\circ < \alpha < 25^\circ$ A SMOOTH TRANSITION IS MADE BETWEEN THE ABOVE TWO CASES.

$$\underline{u}_{wdp} = B1 \underline{u}_{zg} + B2 (r_g - r_p)$$



$$\alpha = (\text{UNIT}(r_g) \times \underline{u}_{xgp}) \cdot \underline{u}_{yg}$$

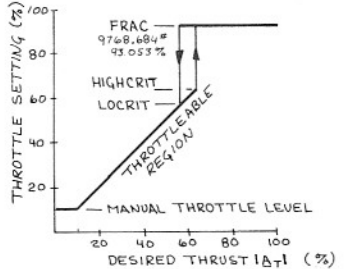
= ANGLE BETWEEN r_g & A_T NORMALIZED FOR ANY CROSS-RANGE THRUSTING.

SPACE-CRAFT ORIENTATION
 THE X-BODY AXIS IS ALIGNED ALONG THE DESIRED THRUST VECTOR A_T .
 THE Z-BODY AXIS IS ALIGNED IN THE PLANE CONTAINING A_T & \underline{u}_{wdp}

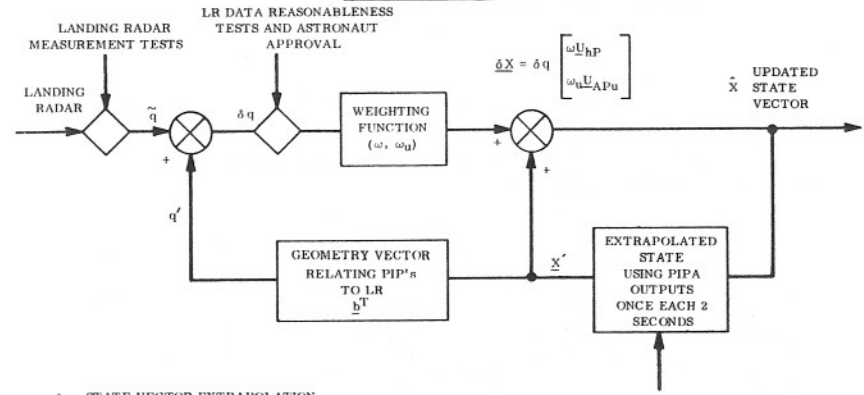
5. COMPUTE THRUST MAGNITUDE

ACTUAL THROTTLE SETTINGS ARE LIMITED BY ENGINE CONSIDERATIONS TO $\approx 93\%$ OF FULL SCALE OR LESS THAN 63%.

ERASABLE DATA LOAD PARAMETERS
 LOCRIT 5985 LBS = 57%
 HIGHCRIT 6615 LBS = 63%



LUNAR LANDING - STATE VECTOR UPDATE RECURSIVE FILTER BLOCK DIAGRAM



STATE VECTOR EXTRAPOLATION

$$\dot{X}' = \begin{bmatrix} \dot{X}_p \\ \dot{Y}_p \end{bmatrix} = \begin{bmatrix} X_{n-1} + (t_n - t_{n-1}) (V_{n-1} + \frac{\Delta \tilde{V}_p}{2} + \frac{E_{n-1} \Delta t}{2}) \\ Y_{n-1} + \Delta \tilde{V}_p + (E_p + g_{n-1}) \frac{\Delta t}{2} \end{bmatrix}$$

P = PLATFORM FRAME

STATE VECTOR UPDATE

$$\dot{X} = \begin{bmatrix} \dot{X}_p \\ \dot{Y}_p \end{bmatrix} + \begin{bmatrix} \delta q_h \omega_{hp} \\ \delta q_u \omega_{APu} \end{bmatrix}$$

CONDITIONS NECESSARY TO UPDATE STATE USING LR RANGE DATA:

- LANDING RADAR IS NOT BEING SWITCHED FROM POSITION NO. 1 TO POSITION NO. 2
- RANGE DATA MEASUREMENT TESTS ARE SATISFIED.
 DATA GOOD DISCRETE HAS BEEN PRESENT FOR 4 SECONDS OR MORE.
 LR RANGE SCALE HAS NOT BEEN CHANGED WITHIN LAST SECOND.
- MEASUREMENT RESIDUAL (δq) IS WITHIN SPECIFIED LIMITS $|\delta q| \leq \text{DELQFIX}^* + 0.25 (q')$ (only in P64)
- ASTRONAUT APPROVAL FOR UPDATING HAS BEEN GIVEN (V57).

CONDITIONS NECESSARY TO UPDATE STATE USING LR VELOCITY DATA:

- LANDING RADAR IS NOT BEING SWITCHED FROM POSITION NO. 1 TO POSITION NO. 2.
- VELOCITY DATA MEASUREMENT TESTS ARE SATISFIED.
 DATA GOOD DISCRETE HAS BEEN PRESENT FOR AT LEAST 4 SECONDS.
- MEASUREMENT RESIDUAL IS WITHIN SPECIFIED LIMITS $|\delta q_u| \leq \text{VELBIAS}^* + 0.125 (V'_u - \omega_p \times r_p)$.
- ASTRONAUT APPROVAL FOR UPDATING HAS BEEN GIVEN (V57).

*DELQFIX = 100 ft
 *VELBIAS = 2.5 ft/s

STATE VECTOR EXTRAPOLATION

State vector extrapolation is accomplished by an Average G routine at 2-second intervals coincident with PIPA ΔV processing.

LM position vector (\underline{r}_p) is extrapolated assuming constant acceleration over the 2-second interval

$$\underline{r} = \underline{r}_0 + \underline{v}_0 \Delta t + \frac{1}{2} \underline{a} \Delta t^2$$

$$\underline{r}_p = \underline{r}_{n-1} + \underline{v}_{n-1}(t_n - t_{n-1}) + \frac{\Delta \tilde{\underline{v}}_p}{2} \Delta t + \frac{\underline{g}_{n-1}}{2} \Delta t^2$$

where

\underline{r}_{n-1} = position vector (\underline{r}_p) at end of previous interval

\underline{v}_{n-1} = velocity vector (\underline{v}_p) at end of previous interval

$\Delta \tilde{\underline{v}}_p$ = accumulated PIPA ΔV pulses during 2-second interval

\underline{g}_{n-1} = lunar gravitational acceleration at end of previous interval

LM velocity vector (\underline{v}_p) is extrapolated using PIPA ΔV pulses and the average gravitational acceleration over the 2-second interval

$$\underline{v} = \underline{v}_0 + \underline{a} \Delta t$$

$$\underline{v}_p = \underline{v}_{n-1} + \frac{\Delta \tilde{\underline{v}}_p}{2} + \left[\frac{\underline{g}_{n-1} + \underline{g}_p}{2} \right] \Delta t$$

where

\underline{v}_{n-1} = velocity (\underline{v}_p) at end of previous interval

$\Delta \tilde{\underline{v}}_p$ = accumulated PIPA ΔV pulses over 2-second interval

\underline{g}_{n-1} = lunar gravitational acceleration at end of previous interval

\underline{g}_p = lunar gravitational acceleration at end of present interval

$$\underline{g}_p = \frac{-\mu M}{r_p^3} \underline{r}_p$$

In addition to the state vector update, the following terms are computed

Altitude $h' = r_p - r_{LS}$

where

r_p = magnitude of position, \underline{r}_p

r_{LS} = magnitude of landing site, \underline{r}_{LS}

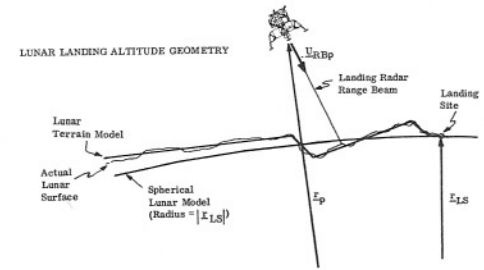
Velocity $\underline{v}' = \underline{v}_p$

Mass $m_n = m_{n-1} - \frac{|\Delta \tilde{\underline{v}}_p|}{V_e} m_{n-1}$ (V_e = Exhaust Velocity Constant)

Velocity Increment $\Delta V = \Delta V + \frac{|\Delta \tilde{\underline{v}}_p|}{m}$

UPDATE THE STATE VECTOR USING LR RANGE DATA

LUNAR LANDING ALTITUDE GEOMETRY



- Compute the estimated altitude

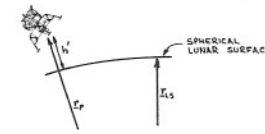
$$h' = |r_p| - |r_{LS}|$$

where

h' = estimated altitude

$|r_p|$ = magnitude of LM position vector

$|r_{LS}|$ = magnitude of Landing Site vector



- Compute the measured altitude

$$\tilde{h} = h_{LR} + h_T$$

$$h_{LR} = q^* (\underline{U}_{RBp} \cdot \text{Unit } \underline{e}_p)$$

where

\tilde{h} = measured altitude

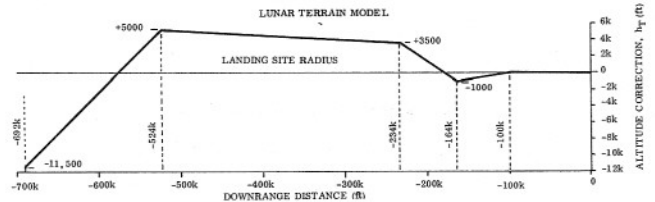
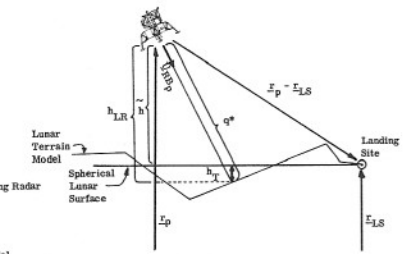
h_{LR} = altitude derived from LR range data

q^* = range to surface as measured by the Landing Radar

\underline{U}_{RBp} = unit vector along LR range beam

\underline{r}_p = spacecraft position vector

h_T = altitude correction from lunar terrain model



- Compute the measurement residual

$$\Delta h = \tilde{h} - h'$$

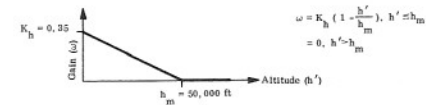
where

\tilde{h} = measured altitude derived from LR slant range

h' = estimated altitude

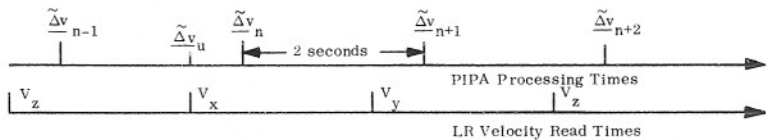
- Update the position vector using the precomputed gain and measurement residual

$$\underline{r}_p = \underline{r}_p + \omega \Delta h \underline{u}_{hp}$$



UPDATE THE STATE VECTOR USING LR VELOCITY DATA

The Landing Radar has three velocity components. They are used (one during each 2-second interval) to update state according to the time line shown below.



$$\tilde{q}_u = \begin{bmatrix} \tilde{v}_x \\ \tilde{v}_y \\ \tilde{v}_z \end{bmatrix}$$

$$\underline{u}_{AP_u} = [C_{PB_u}]^1 \underline{u}_x \quad \underline{u}_{AP_u} = [C_{PB_u}]^1 \underline{u}_y \quad \underline{u}_{AP_u} = [C_{PB_u}]^1 \underline{u}_z$$

- Compute the measurement residual corresponding to the time that the velocity data is read (t_u).

$$\delta q = \tilde{q}_u - q'_u$$

$$\tilde{q}_u = \text{LR velocity component read at time } t_u$$

$$q'_u = \text{estimated component of LM relative velocity in the direction of } \tilde{q}_u$$

$$q'_u = (\underline{v}'_u - \omega_p \times \underline{r}_p) \cdot \underline{u}_{AP_u}$$

where

$$\omega_p \times \underline{r}_p = \text{velocity of lunar surface}$$

$$\underline{u}_{AP_u} = \text{unit vector in direction of LM velocity data}$$

$$\underline{v}'_u - \omega_p \times \underline{r}_p = \text{velocity of LM relative to lunar surface}$$

$$\underline{v}'_u = \text{estimated LM velocity at time } t_u$$

$$\underline{v}'_u = \underline{v}_{n-1} + \Delta \tilde{\underline{v}}_u + \underline{g}_{n-1} (t_u - t_{n-1})$$

where

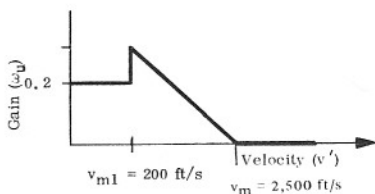
$$\underline{v}_{n-1} = \text{LM velocity at end of previous update cycle}$$

$$\Delta \tilde{\underline{v}}_u = \text{PIPA } \Delta V \text{ read at time of LR velocity data } t_u$$

$$\underline{g}_{n-1} = \text{lunar gravitational acceleration at end of previous cycle}$$

- Update the LM velocity vector at time t_n using measurement residual and extrapolate velocity (\underline{v}_p)

$$\underline{v}_p = \underline{v}_p + \omega_u \delta q \underline{u}_{AP_u}$$

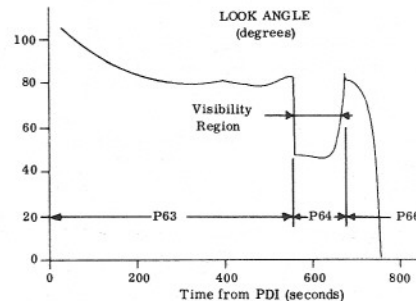
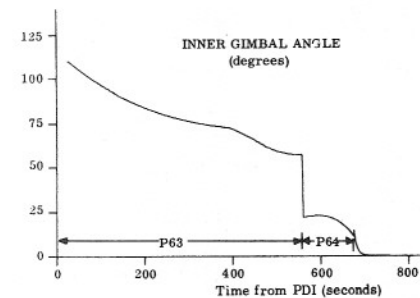
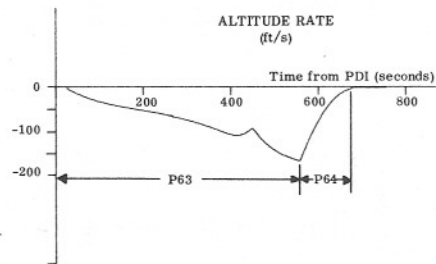
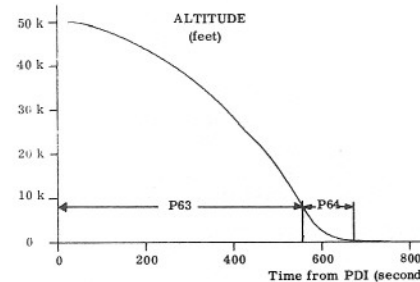
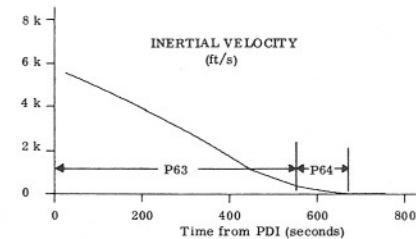
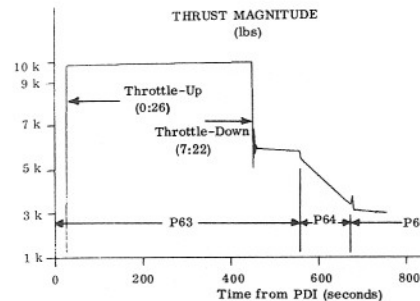


$$\omega_u = 0, v' > v_m$$

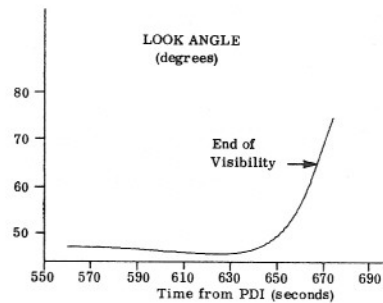
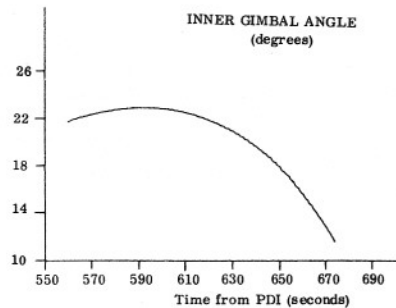
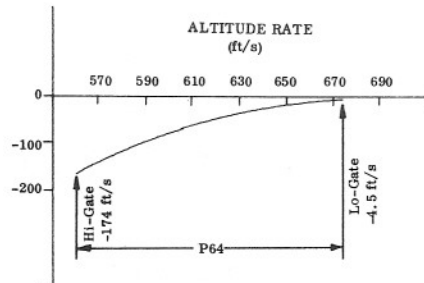
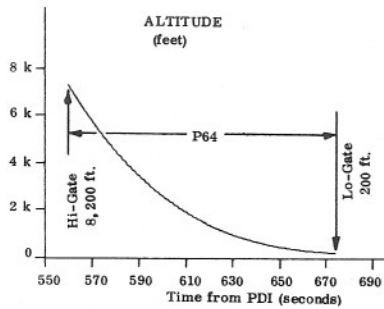
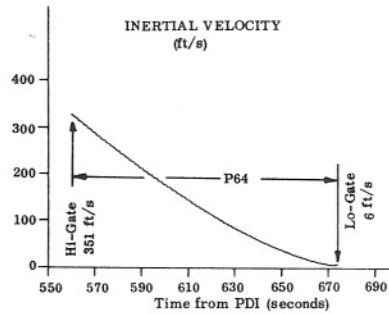
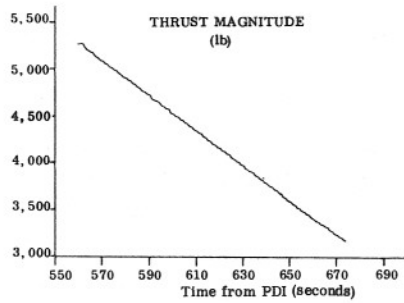
$$\omega_u = 0.3 (1 - v'/v_m), v_{m1} \leq v' \leq v_m$$

$$\omega_u = 0.2, v' < v_{m1}$$

LUNAR LANDING PARAMETERS (P63, P64) APOLLO 16



LUNAR LANDING PARAMETERS (P64)
APOLLO 16



A. ORBITAL RELATIONSHIPS

Assuming a spherical planet, the equation of motion for a satellite is given by

$$\frac{d^2 \vec{r}}{dt^2} = - \frac{\mu}{r^3} \vec{r} \quad (1)$$

where

$$\mu = (M_{\text{Planet}} + M_{\text{Satellite}}) G$$

G = gravitational constant

The solution of Equation 1 is:

$$r = \frac{h^2/\mu}{1 + e \cos f} \quad (2)$$

where

h = angular momentum of the satellite

e = eccentricity of the orbit

f = true anomaly

which is the polar equation of a conic. The conic will be an ellipse, a parabola, or a hyperbola. Treating the parabola as a special case of the ellipse and considering only the hyperbola and ellipse, the following relationships are obtained:

Angular Momentum	$\vec{h} = \vec{r} \times \vec{v} = r^2 \dot{f} = \text{constant}$
Velocity	$V^2 = \mu (2/r - 1/a)$
Apogee	$r_a = a (1 + e)$
Semilatus Rectum	$p = h^2/\mu$
Semimajor Axis	$a = r\mu / (2\mu - rv^2)$ Negative for hyperbola
Eccentricity	$e = (1 - h^2/\mu a)^{1/2}$
Perigee	$r_p = a (1 - e)$
True Anomaly	$\cos f = \frac{p}{re} - \frac{1}{e}, \sin f = \frac{h}{\mu re} \vec{r} \cdot \vec{v}$

Ellipse Only

Period	$P = 2\pi (a^3/\mu)^{1/2}$
Mean Motion	$n = \mu^{1/2}/a^{3/2}$
Mean Anomaly	$M = n (t - \tau)$
	$\tau = \text{time of perigee passage}$
Eccentric Anomaly	$E - e \sin E = M$ (Kepler's Equation)
	$\tan (E/2) = [(1 - e)/(1 + e)]^{1/2} \tan (f/2)$

Hyperbola Only

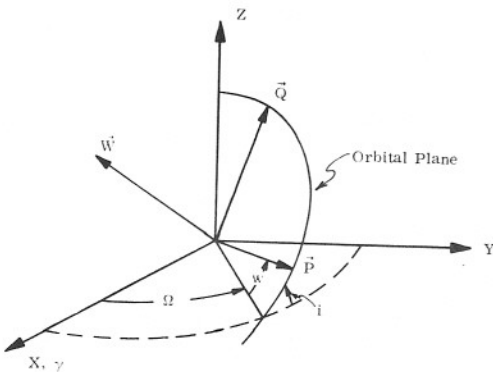
Mean Motion	$\gamma^2 = \mu/a^3$
Mean Anomaly	$M = \gamma (t - \tau)$
	$\tau = \text{time of perigee passage}$
Eccentric Anomaly	$H - e \sinh H = M$
	$\tanh (H/2) = [(e - 1)/(e + 1)]^{1/2} \tan (f/2)$

From these relationships, given an initial position and velocity vector, the orbit and orbit parameters are uniquely determined.

B. THE ORBIT IN SPACE

The orbit in space is defined by an orthogonal set of axes along perigee, the semilatus rectum, and the angular momentum vector. The ordered set of right hand rotations (Euler angles) to achieve this orientation from the earth-centered-inertial (ECI) frame are illustrated in Figure 1 and given by the expression

$$\begin{bmatrix} P \\ Q \\ W \end{bmatrix}_{\text{Orbit}} = [R_Z(w)] [R_X(i)] [R_Z(\Omega)] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{ECI}} = [A] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{ECI}} \quad (3)$$



where

- Ω = longitude of ascending node
- i = angle of incidence
- w = argument of perigee

Figure 1. The Orbit in Space

The angles Ω , i , and w are generally unknowns; therefore, the elements in [A] must be evaluated by some other means. To this end, Equation 3 is expressed as follows:

$$\begin{bmatrix} P \\ Q \\ W \end{bmatrix} = \begin{bmatrix} P_X & P_Y & P_Z \\ Q_X & Q_Y & Q_Z \\ W_X & W_Y & W_Z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{ECI}} = [B] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{ECI}} \quad (4)$$

The unit vector along perigee (that is, P_X, P_Y, P_Z) can be determined by Equation 141, Page 20 of Battin as follows:

$$\vec{P} = \frac{1}{\mu e} \left[\left(v^2 - \frac{\mu}{r} \right) \vec{r} - (\vec{r} \cdot \vec{v}) \vec{v} \right] \quad (5)$$

where

- v = absolute magnitude of velocity
- r = absolute magnitude of the radius vector
- \vec{v} = inertial velocity
- \vec{r} = radius vector

This relationship is obtained as follows:

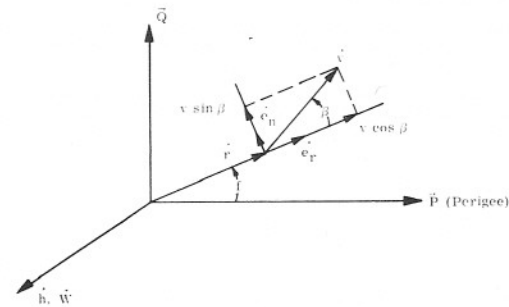


Figure 2

$$\vec{P} = \vec{e}_r \cos f - \vec{e}_n \sin f \quad (6)$$

$$(1) \vec{e}_r = \frac{\vec{r}}{r}; \vec{e}_h = \frac{\vec{h}}{h}; \vec{e}_n = \frac{\vec{h}}{h} \times \frac{\vec{r}}{r}$$

$$(2) r = \frac{h^2/\mu}{1+e \cos f}; \cos f = \frac{h^2}{\mu e} r^{-1} - \frac{1}{e}; \sin f = \frac{h}{\mu e} \dot{r}$$

$$\text{but } \dot{r} = v \dot{e}_r = v \cos \beta = \frac{\vec{r} \cdot \vec{v}}{r}$$

$$(3) \sin f = \frac{h}{\mu e} \frac{\vec{r} \cdot \vec{v}}{r}$$

Combining 1, 2, and 3 into Equation 6

$$\vec{P} = \frac{\vec{r}}{r} \left[\frac{h^2}{\mu e r} - \frac{1}{e} \right] - \left(\frac{\vec{h}}{h} \times \frac{\vec{r}}{r} \right) \left[\frac{h}{\mu e} \frac{\vec{r} \cdot \vec{v}}{r} \right]$$

By substituting

$$h = r v \sin \beta, \cos \beta = \frac{\vec{r} \cdot \vec{v}}{r v}$$

and adding the factor

$$r v^2 \cos^2 \beta \vec{e}_r - r v^2 \cos^2 \beta \vec{e}_r$$

the above equation reduces to

$$\vec{P} = \frac{1}{\mu e} \left[\left(v^2 - \frac{\mu}{r} \right) \vec{r} - (\vec{r} \cdot \vec{v}) (\nu \cos \beta \vec{e}_r + v \sin \beta \vec{e}_n) \right]$$

but

$$\vec{v} = v \cos \beta \vec{e}_r + v \sin \beta \vec{e}_n$$

therefore

$$\vec{P} = \frac{1}{\mu e} \left[\left(v^2 - \frac{\mu}{r} \right) \vec{r} - (\vec{r} \cdot \vec{v}) \vec{v} \right]$$

Since \vec{h} is constant, the unit vector along \vec{h} (that is, W_X, W_Y, W_Z) is determined given any \vec{r} and corresponding \vec{v} in the orbit. That is

$$\vec{W} = \frac{\vec{h}}{h} = \frac{\vec{r} \times \vec{v}}{r v \sin \beta} \quad (7)$$

Having determined \vec{P} and \vec{W} , the remaining unit vector, \vec{Q} , is calculated as follows:

$$\vec{Q} = \vec{W} \times \vec{P} \quad (8)$$

By using Equations 6, 7, and 8 the matrix [B] is determined. That is, the orbit is defined in inertial space. From Equations 3 and 4, [A] = [B]. By equating elements and by using the constraint $0^\circ < i < 180^\circ$, the Euler angles Ω , i , and w are determined.

C. ORBIT DETERMINATION

The problem of orbit determination can be stated as follows: Given an initial \vec{r}_1 and corresponding \vec{v}_1 vector expressed in ECI coordinates, determine the position and velocity in ECI coordinates at some time, t_2 .

From what has been presented so far, the approach would be to determine if the conic is a hyperbola or an ellipse. Then use the corresponding form of Kepler's equation to solve for the true anomaly and through the orbit-in-space transformation determine the position and velocity at t_2 .

This method has the undesirable feature of first determining if the conic is a hyperbola or an ellipse and requires two sets of equations.

A more unified approach as presented by Battin is the universal conic equations which are given by

$$\dot{\vec{r}}(t_2) = \left[1 - \frac{X^2}{r_1} C(X^2 \alpha_1) \right] \vec{r}_1 + \left[\Delta t - \frac{X^3}{\sqrt{\mu}} S(X^2 \alpha_1) \right] \vec{v}_1 \quad (9)$$

$$\dot{\vec{v}}(t_2) = \frac{\mu}{r_1} \left[\alpha_1 X^2 S(X^2 \alpha_1) - X \right] \vec{r}_1 + \left[1 - \frac{X^2}{r_2} C(X^2 \alpha_1) \right] \vec{v}_1 \quad (10)$$

where

$$C(X^2 \alpha_1) = \frac{1}{2!} - \frac{X^2 \alpha_1}{4!} + \frac{(X^2 \alpha_1)^2}{6!} - \dots$$

$$S(X^2 \alpha_1) = \frac{1}{3!} - \frac{X^2 \alpha_1}{5!} + \frac{(X^2 \alpha_1)^2}{7!} - \dots$$

$$\Delta t = t_2 - t_1$$

$$\alpha_1 = \frac{1}{a} = \frac{2}{r_1} - \frac{v_1^2}{\mu}$$

$$X = \frac{E_2 - E_1}{\alpha_1} \quad \text{for ellipse}$$

$$X = \frac{H_2 - H_1}{\alpha_1} \quad \text{for hyperbola}$$

The parameter X which is required for Equations 9 and 10 is determined as follows: An initial guess for X is given by

$$X = X_1 + S \left[1 - F_3 S(1 - 2 F_3 S) - \frac{1}{6} \left(\frac{1}{r_1} - \alpha_1 \right) S^2 \right] \quad (11)$$

where

$X_1 = 0$ for the first iteration and X for subsequent iterations

$$F_3 = \frac{\vec{r}_1 \cdot \vec{v}_1}{2 r_1 \mu}$$

$$S = \frac{\mu}{2 r_1} (t_2 - t_1)$$

Having an initial guess for X, an improved value is obtained by a Newton-Raphson iteration scheme as follows:

$$X_{n+1} = X_n - \frac{F(X_n)}{F'(X_n)} \quad (12)$$

where

$$F(X_n) = \left[\frac{\vec{r}_1 \cdot \vec{v}_1}{\sqrt{\mu}} X_n^2 C(X_n^2 \alpha_1) + (1 - r_1 \alpha_1) X_n^3 S(X_n^2 \alpha_1) + r_1 X_n \right] - \sqrt{\mu} \Delta t$$

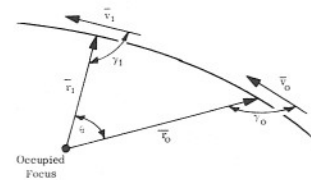
$$F'(X_n) = \frac{\vec{r}_1 \cdot \vec{v}_1}{\sqrt{\mu}} \left[X_n - \alpha_1 X_n^3 S(X_n^2 \alpha_1) \right] + \left[(1 - r_1 \alpha_1) X_n^2 C(X_n^2 \alpha_1) + r_1 \right]$$

The iteration scheme of Equation 12 continues until $X_{n+1} - X_n$ is sufficiently small. The good initial guess provided by Equation 11 limits the iteration to two or three in most cases. The value for X_{n+1} is then substituted into Equations 9 and 10, thereby specifying \vec{r}_2 and \vec{v}_2 in ECI space. Following the universal conic method just described, the position and velocity vectors referred to ECI space can be determined for any time t during freefall.

The conic section concept utilized for navigation requires the computation of trajectory segments under a wide variety of constraints. Depending on the constraints involved, the set of universal equations will solve

- 1 - Kepler's Problem given r_0 , v_0 , and Δt , solve for r_1 , v_1 .
- 2 - Lambert's Problem given r_0 , r_1 , and Δt , solve for conic parameters.
- 3 - Time - Theta Problem given r_0 , v_0 , and θ , solve for Δt .
- 4 - Time - Radius Problem given r_0 , v_0 , and r_1 , solve for Δt .

where:



ψ = true anomaly measured from pericenter

Δt = flight time from r_0 to r_1

p = semilatus rectum

a = $1/\alpha$, a = semimajor axis

γ = flight path angle

θ = true anomaly difference ($t_1 - t_0$)

The equation set used for solution of the above problem is

$$13. \quad \mu \Delta t = \sqrt{p} \cot \gamma_0 X^2 C(\theta \alpha^2) + (1 - r_0 \alpha) X^3 S(\theta \alpha^2) + r_0 \alpha$$

$$14. \quad \cot \frac{\theta}{2} = \frac{r_0}{p} \frac{1 - \theta \alpha^2 S(\theta \alpha^2)}{C(\theta \alpha^2)} - \cot \gamma_0$$

$$15. \quad \frac{p}{r_0} = \frac{1 - \cos \theta}{\frac{r_0}{r_1} - \cos \theta + \sin \theta \cot \gamma_0}$$

$$16. \quad \frac{r}{\mu} v^2 = 2 + r \alpha$$

$$17. \quad r \alpha = 2 - \frac{p}{r} (1 - \cot^2 \gamma)$$

$$18. \quad \cot \frac{\theta}{2} = \frac{\frac{r_0}{r_1} \cot \gamma_1}{\left(1 - \frac{r_0}{r_1}\right)} - r_0 \neq r_1$$

$$19. \quad \frac{p}{r_0} \left(\frac{r_0}{r_1} \right) \left(\frac{1 - \cos^2 \gamma_1}{1 - \cos^2 \gamma_0} \right) - \left(1 - \cos^2 \gamma_0 \right)$$

$$20. \quad r_1 = -1 - \frac{X^2}{r_0} C(\theta \alpha^2), \quad r_0 = -\Delta t - \frac{X^3}{\mu} S(\theta \alpha^2), \quad v_0$$

$$21. \quad v_1 = \frac{\mu}{r_0 r_1} \left[\theta \alpha^3 S(\theta \alpha^2) - X, \quad r_0 + 1 + \frac{X^2}{r_1} C(\theta \alpha^2) \right], \quad v_0$$

$$22. \quad \cot \gamma = \frac{r - \mu}{r \mu}$$

Therefore:

- 1 - Kepler's Solution. Compute r_1, θ using Equation 16; compute γ using Equation 14 and $p = \frac{r_0 v_0^2}{\mu}$; compute r_1 and v_1 using Equations 20 and 21.
- 2 - Lambert's Solution. Guess θ and γ_1 ; compute p/r_0 and r_1, θ using Equations 15 and 17; compute γ using Equation 14 iteratively; compute Δt using Equation 13. If Δt disagrees with the desired Δt , adjust θ, γ_1 and repeat.
- 3 - Time - Theta Solution. Compute $\cot \gamma_0, r_0 \alpha$ and p/r_0 using Equations 22, 16, and 17; solve for γ using Equation 14 iteratively; compute Δt using Equation 13.
- 4 - Time - Radius Solution. Compute $\cot \gamma_0, r_0 \alpha, p/r_0$, and $\cot \gamma_1$ using Equations 22, 16, 17, and 17; compute $\cot \frac{\theta}{2}$ using Equation 18; compute θ using Equation 14 iteratively; compute Δt using Equation 13.

D. DISTURBANCE ACCELERATIONS

So far the planet in question has been assumed to be spherical which is not the case for either the earth or the moon. Nor have the effects of the sun and the moon on the earth or vice versa been considered. Therefore, Equation 1 must be modified as follows

$$\frac{d^2 \vec{r}}{dt^2} + \frac{\mu}{r^3} \vec{r} = \vec{a}_d \quad (23)$$

where

\vec{a}_d is the disturbance acceleration due to

1. oblateness of the earth or the nonspherical shape of the moon depending on which reference body is used,
2. the effects of the sun,
3. the effects of the secondary body on the primary body; that is, effects of moon on earth if earth is primary body and vice versa.

Analytical expressions for \vec{a}_d are given in R-577, Section 5. Since \vec{a}_d is small in comparison to $\frac{\mu}{r^2}$, the two-body orbit given by Equation 1 is used as a reference or osculating orbit which is perturbed by \vec{a}_d . The actual position and velocity vectors are, therefore, given by

$$\begin{aligned} \vec{r} &= \vec{r}_c + \vec{\delta} \\ \vec{v} &= \vec{v}_c + \vec{\sigma} \end{aligned} \quad (24)$$

where

\vec{r}_c and \vec{v}_c are the position and velocity of the two-body conic solutions,

$\vec{\delta}$ and $\vec{\sigma}$ are the deviations from the two-body conic solutions of position and velocity, respectively.

Differentiating Equation 24 and substituting into Equation 23 gives the following expression for the differential acceleration

$$\frac{d^2 \vec{\delta}}{dt^2} = \frac{\mu}{r_c^3} \left[\left(1 - \frac{r_c^3}{r^3} \right) \vec{r} - \vec{\delta} \right] + \vec{a}_d \quad (25)$$

subject to the initial conditions

$$\vec{\delta}(t_0) = 0, \quad \frac{d\vec{\delta}}{dt}(t_0) = \vec{\sigma}(t_0) = 0$$

This method (Equation 25) is known as Encke's method of differential accelerations.

Since the coefficient of \vec{r} in Equation 25 requires the subtraction of nearly equal quantities, prohibitive errors are introduced by solving Equation 25 in its present form. This difficulty can be overcome by making the substitution

$$\left(1 - \frac{r_c^3}{r^3} \right) = \frac{\vec{r}}{r^2} \left(1 + \frac{\rho^2}{1 + \rho} \right) \left[(\vec{r}_c + \vec{r}) \cdot \vec{\delta} \right] \quad \text{where } \rho = r_c/r$$

Equation 25, therefore, becomes

$$\frac{d^2 \vec{\delta}}{dt^2} = \frac{\mu}{r_c^3} \left\{ \frac{\vec{r}}{r^2} \left(1 + \frac{\rho^2}{1 + \rho} \right) \left[(\vec{r}_c + \vec{r}) \cdot \vec{\delta} \right] - \vec{\delta} \right\} + \vec{a}_d \quad (26)$$

Equation 26 can be solved by any number of numerical integration schemes. The method used in the Apollo Guidance Computer (AGC) is the Nystrom method.

In order to maintain the efficiency of Encke's method of differential accelerations, $\vec{\delta}(t)$ must remain small. Therefore, a new osculating conic must be defined by the total position and velocity vectors $\vec{r}(t)$ and $\vec{v}(t)$ when $\vec{\delta}(t)$ reaches a predetermined limit. The process of selecting a new conic orbit from which to calculate deviations is called rectification.

To sum up, the position and velocity, during freefall, at time t_2 , given the position and velocity at time t_1 , are computed as follows:

1. Position and velocity in the osculating orbit at time t_2 are calculated according to Equations 9 and 10.
2. Deviations are then obtained by numerical integration of Equation 26.
3. A new conic from which to calculate deviations is defined each time the deviations ($\vec{\delta}(t)$) reach a predetermined limit.

E. DISTURBING FUNCTIONS AND THEIR APPLICABILITY

We define the following disturbance accelerations as applicable to Apollo:

- \vec{a}_{dE} = acceleration due to the nonspherical gravitational perturbations of the earth
- \vec{a}_{dM} = acceleration due to the nonspherical gravitational perturbations of the moon
- \vec{a}_{dq} = acceleration due to the secondary body on the primary body; that is, moon is secondary body when earth is used as reference and vice versa
- \vec{a}_{ds} = acceleration due to the sun

analytical expressions for which are given in R-577, Section 5.

The applicable disturbance accelerations and their region of applicability for Apollo are given in Figure 3.

R_{Earth}	=	3,441 nmi
R_f	=	3,660 nmi
$R_1 + R_2$	=	43,452 nmi
R_{Moon}	=	938 nmi
R_s	=	1,154 nmi
$R_5 + R_6$	=	8,690 nmi
R_{SPH}	=	34,761 nmi

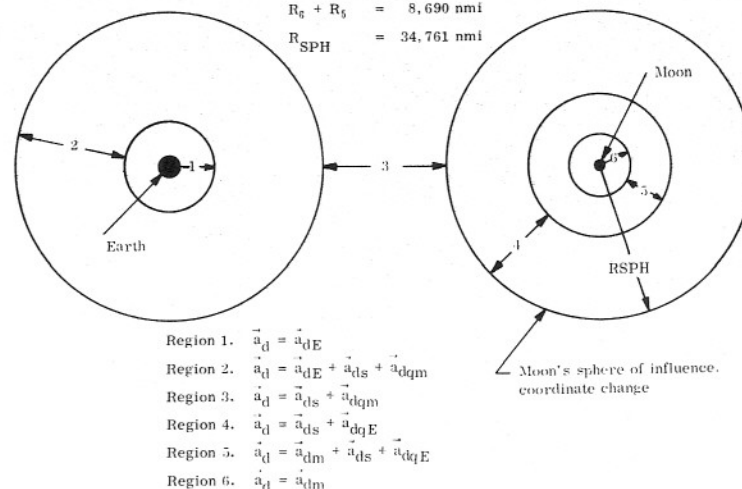
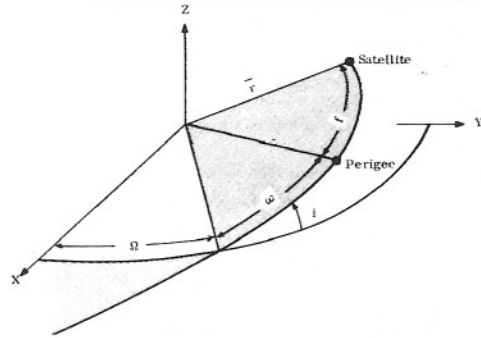


Figure 3. Apollo Disturbance Acceleration Regions

F. DISTURBANCE ACCELERATION/OSCULATING ELEMENTS



- a = Semimajor Axis (ft)
- e = Eccentricity
- i = Orbital Inclination
- f = Central Angle From Perigee
- r = Position (ft)
- Ω = Longitude of the Ascending Node
- ω = Argument of Perigee
- n = Mean Motion = $\sqrt{\frac{\mu}{a^3}}$ s⁻¹
- μE = 1.41×10^{17} ft³/s²
- μM = 1.73×10^{15} ft³/s²
- N, R, T = ft/s²

Acceleration Normal To Orbital Plane (N)

$$\frac{da}{dt} = \frac{de}{dt} = 0$$

$$\frac{di}{dt} = \frac{r \cos(\omega + f)}{na^2 \sqrt{1-e^2}} N$$

$$\frac{d\Omega}{dt} = \frac{r \sin(\omega + f)}{na^2 \sqrt{1-e^2} \sin(i)} N$$

$$\frac{d\omega}{dt} = -\cos(f) \frac{d\Omega}{dt} N$$

Acceleration Along Radial Line (R)

$$\frac{da}{dt} = \frac{2e \sin(f)}{n \sqrt{1-e^2}} R$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2} \sin(f)}{na} R$$

$$\frac{di}{dt} = \frac{dr}{dt} = 0$$

$$\frac{d\omega}{dt} = -\frac{\sqrt{1-e^2} \cos(f)}{nae} R$$

Acceleration Along Track (T)

$$\frac{da}{dt} = \frac{2a \sqrt{1-e^2}}{nr} T$$

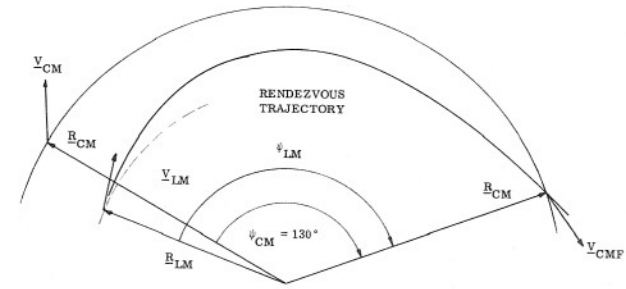
$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na^2 e} \left[\frac{a^2 (1-e^2)}{r} - r \right] T$$

$$\frac{di}{dt} = \frac{dr}{dt} = 0$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae} \left(\frac{2 + e \cos(f)}{1 + e \cos(f)} \right) \sin(f) T$$

G. TPI TARGETING

The TPI targeting for the Apollo 16 mission is accomplished using the equations and techniques stated below utilizing the following information:



T₁ - time at which the TPI maneuver is to be performed

R_{LM} - LM position vector at T₁

V_{LM} - LM velocity vector at T₁

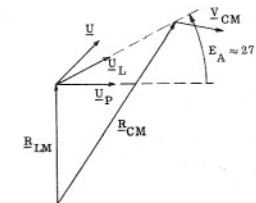
R_{CM} - CSM position vector at T₁

V_{CM} - CSM velocity vector at T₁

ψ_{CM} - central angle CSM will traverse between TPI and rendezvous (130° nominally)

Targeting Procedure:

1. Compute the elevation angle of the LM-CSM LOS above the LM horizontal plane at the time of TPI.



$$E_A = \cos^{-1} [\underline{U}_L \cdot \underline{U}_P \text{SGN} (\underline{U}_P \cdot \underline{U} \times \underline{R}_{LM})]$$

where:

$$\underline{U}_L = \text{unit} [\underline{R}_{CM} - \underline{R}_{LM}]$$

$$\underline{U} = \text{unit} [\underline{R}_{LM} \times \underline{V}_{LM}]$$

$$\underline{U}_P = \text{unit} \left[\frac{ \underline{U}_L \cdot (\underline{U}_L \cdot \underline{R}_{LM}) \underline{R}_{LM} }{ | \underline{R}_{LM} |^2 } \right]$$

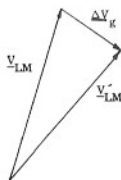
if E_A < 0° define E_A = 2π - E_A

2. Compute the time (t_{21}) required for the CSM to travel through the central angle ψ_{CM} using the Time-Theta method described on page SC-51.
3. Update the state vector of the CSM to the time of rendezvous ($\Delta T = t_{21}$) using Kepler's method as described on page SC-51 obtaining (\underline{R}_{CMF} , \underline{V}_{CMF}).
4. Compute the central angle (ψ_{LM}) through which the LM must traverse between its position at TPI and the CSM's position at rendezvous.

$$\psi_{LM} = \text{SGN} (\underline{R}_{LM} \times \underline{R}_{CMF} \cdot \underline{U}) \cos^{-1} (\underline{R}_{LM} \cdot \underline{R}_{CMF})$$

$$\text{if } \psi_{LM} < 0^\circ \text{ define } \psi_{LM} = \psi_{LM} + 2\pi$$

5. Using the Time-Theta method, determine the flight path angle γ required of the LM at the end of the TPI burn such that the time required by the LM to traverse the central angle of ψ_{LM} is the same as t_{21} .
6. Calculate the velocity to be gained.



$$\underline{\Delta V}_g = \underline{V}'_{LM} - \underline{V}_{LM}$$

where:

$$\underline{V}'_{LM} = \sqrt{\frac{P}{R_{LM}}} (\cot \gamma \cdot \text{unit } \underline{R}_{LM} + \underline{U}_N \times \text{unit } \underline{R}_{LM})$$

P = semilatus rectum of the TPI orbit

$$\underline{U}_N = \text{unit} [\text{unit } \underline{R}_{CMF} \times \text{unit } \underline{R}_{LM}]$$

The velocity to be gained is burned out using the APS and Lambert Aimpoint Guidance.