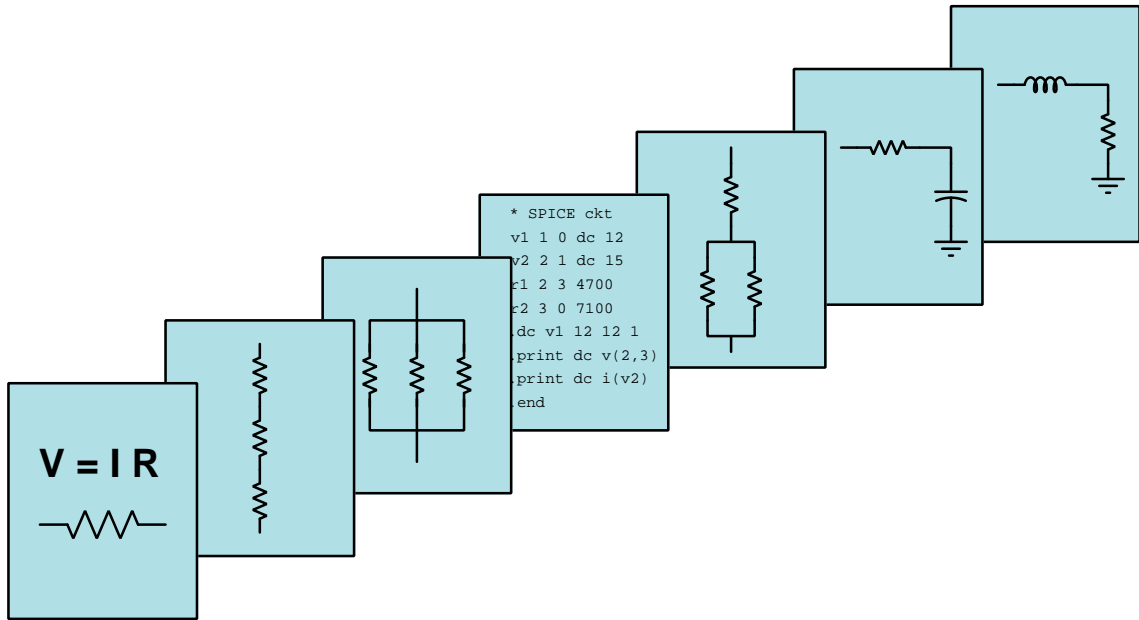


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



BASIC PRINCIPLES OF RADIO

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Chapter 1

Introduction

1.1 Recommendations for students

Few technologies have revolutionized culture like *radio*: the intentional production of electromagnetic waves in space for the purpose of communicating information. These waves are the stuff of light, X-rays, and some other forms of radiation, but the waves used in radio applications are much lower in frequency than any of those.

Generating radio waves is easy: just connect an AC source to a pair of open-ended wires splayed apart from each other. The parasitic capacitance and inductance of those wires forms an LC resonant circuit with a very high resonant frequency. When energized at or near that resonant frequency, waves of electromagnetic energy will dissipate from those wires and travel outward through open space.

The theoretical basis for radio waves is that a varying electric field will produce a varying magnetic field, and vice-versa, even in the absence of any conductive structures. Thus, a fast-changing electric field produces its own time-varying magnetic field, which re-generates the electric field, and so on. These waves travel at the speed of light, and are able to induce voltages and currents within distant conductive objects where they may be received as signals.

Important concepts related to radio include **capacitance**, **inductance**, the **Conservation of Energy**, **electric fields**, **magnetic fields**, **sources** versus **loads**, **tank circuits**, **resonance**, inductive and capacitive **reactance**, the **Second Law of Thermodynamics**, **parasitic** properties, **transmission lines**, **fundamental** and **harmonic** frequencies, **electromagnetic radiation**, **Maxwell's equations**, **antennas**, **transmitters** and **receivers**, **wavelength** versus **frequency**, and **filters**.

Here are some good questions to ask of yourself while studying this subject:

- How might you design and conduct an experiment similar to one of those performed by Heinrich Hertz but using more modern test instruments? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?
- How is energy stored within a capacitor?

- How is energy stored within an inductor?
- What is reactance, and how does it differ from resistance?
- What is resonance, and what conditions are necessary for it to take place?
- Why do tank circuit oscillations inevitably decay rather than continue forever?
- What conditions are necessary to prevent reflected signals in a transmission line?
- How do harmonic frequencies relate to fundamental frequencies?
- How may we generate an electromagnetic wave using primitive equipment?
- How may we intercept an electromagnetic wave so that it becomes detectable?
- How do electric and magnetic fields interact to sustain one another and produce an electromagnetic wave?
- What does “wavelength” mean, and how does it relate to frequency?
- What role does the FCC play in regulating radio communication within the United States?
- How can a transmission line resonate?
- In what ways do the resonant behavior of a tank circuit and a transmission line differ?
- How do antennas work to emit electromagnetic radiation?
- What do Maxwell’s equations tell us about electric and magnetic fields?
- How did Heinrich Hertz demonstrate the existence of electromagnetic waves?

1.2 Challenging concepts related to radio

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Parasitic effects** – phenomena such as resistance, capacitance, and inductance don't just exist where we want them to, but in fact always exist everywhere they can. All conductors (except for superconductors) have some amount of resistance, all conductors separated by an insulating gap exhibit capacitance, and all conductors possess inductance by virtue of their ability to produce a magnetic field with the passage of current. Often these "parasitic" effects are small enough to ignore, but in some applications such as radio circuitry their presence is in fact absolutely necessary for the system to work at all!
- **Conceptualizing the progression of a voltage or current wave** – some very important rules learned in DC and low-frequency AC circuits no longer apply in high-frequency and "pulse" circuits, namely that the effects of electricity are instantaneous across all portions of an electric circuit. In fact, they are not truly instantaneous, but rather propagate at (nearly) the speed of light. Thus, the establishment of a voltage or current at some given location in a circuit requires a certain amount of *time* following the application of a power source. In the most literal sense, those DC "rules" were not entirely true, but they were *close enough to being true* that they sufficed for DC and other low-frequency circuits. Certainly, basic circuit analysis would have been far more complicated for us if we had to view the circuit from the perspective of a wave traveling down the wires of that circuit. Instead, the rules learned for DC circuits represent the *final, steady-state conditions* of those circuits. Transmission lines are a case where we must come to terms with the limits of those rules and consider the circuit's behavior during the fleeting moments prior to reaching its steady-state condition.
- **Maxwell's equations** – the existence of electric and magnetic fields within empty space, with no conductors nearby, was a discovery made by James Clerk Maxwell in the late 1800's, and is as nonintuitive today as it was then. RF systems with their extremely high frequencies manifest effects related to detached electric and magnetic fields that are all but undetectable in lower-frequency systems.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

- **Outcome** – Apply the concept of parasitic effects to antenna structures

Assessment – Identify where parasitic inductance and capacitance exist in a monopole antenna; e.g. pose problems in the form of the “Quarter-wave antenna” Conceptual Reasoning question.

Assessment – Identify where parasitic inductance and capacitance exist in a dipole antenna; e.g. pose problems in the form of the “Dipole antenna” Conceptual Reasoning question.

- **Outcome** – Relate frequency to wavelength for RF signals

Assessment – Calculate frequency for a given wavelength and vice-versa for radio waves traveling through empty space; e.g. pose problems in the form of the “Frequencies and wavelengths” Quantitative Reasoning question.

- **Outcome** – Independent research

Assessment – Read and summarize in your own words reliable historical documents on the development of radio technology.

Chapter 2

Case Tutorial

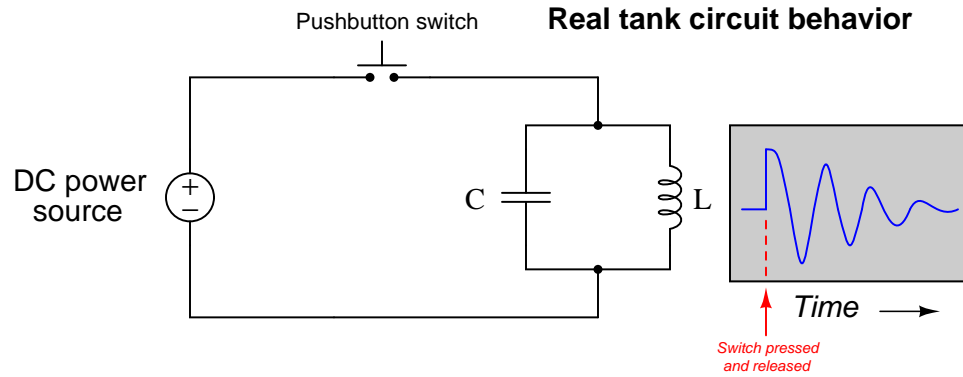
The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: inductor-capacitor resonance

A battery, inductor¹, and capacitor may be used to build a simple circuit demonstrating the phenomenon of resonance. The parallel combination of inductance (L) and capacitance (C) form what is commonly known as a *tank circuit*.

In an ideal tank circuit, oscillations will continue forever after initial energization, but in a real tank circuit dissipative losses in the circuit will cause the oscillations to subside over time:

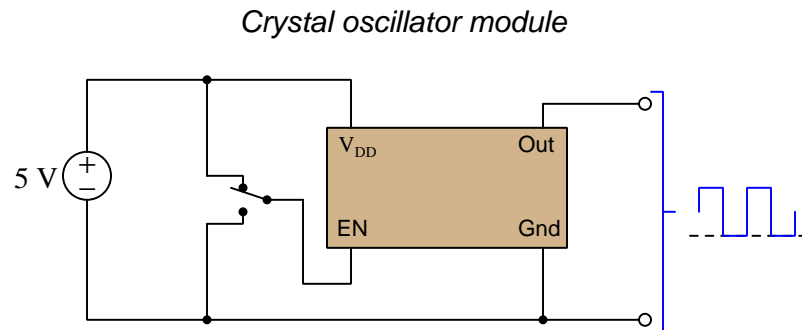


Oscillations following the release of the pushbutton switch are measurable using an oscilloscope connected in parallel with the inductor and capacitor. These oscillations will be short-lived, and may consist of just a few cycles before they completely dissipate, therefore the oscilloscope will need to be configured for “single-sweep” mode to capture and hold the image for viewing.

¹The windings of a step-down power transformer generally work well for this inductance. Feel free to experiment with using different windings to see their effects!

2.2 Example: extremely simple AM radio transmitter

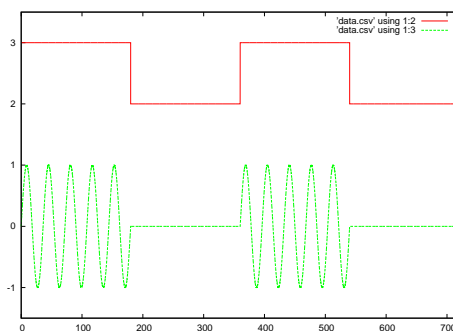
A commonly-available semiconductor integrated circuit is a crystal-based oscillator module, often packaged in a “metal can” enclosure. These ICs contain a transistor oscillator circuit with a precision quartz crystal as the frequency reference, the output signal consisting of a square wave at standard logic level voltages (e.g. 5 Volts “high” and 0 Volts “low” when the oscillator module is powered by a 5 Volt DC source):



1 MHz is a common frequency for these oscillator modules, and this is equivalent to 1000 kHz: a frequency near the middle of the commercial AM (“Amplitude Modulation”) broadcast radio range. This means such an oscillator connected to a length of wire (at the output terminal) will broadcast RF at 1000 kHz when powered and enabled. Connecting the “ground” pin to actual Earth ground will also help the antenna function better.

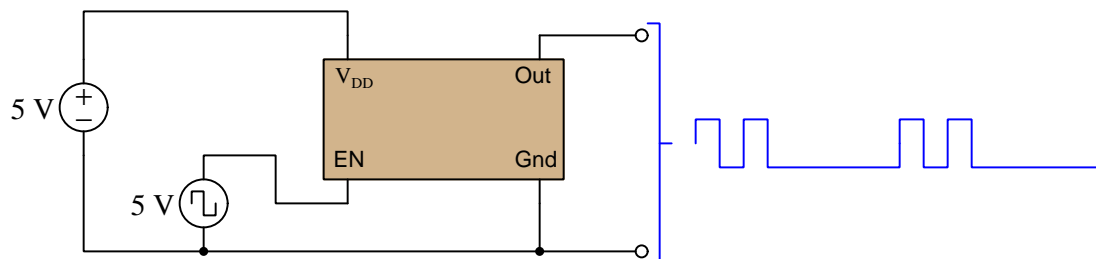
Since the output is a square wave rather than a sine wave, it is rich in harmonic frequencies. This means a 1 MHz oscillator will also broadcast at 3 MHz, 5 MHz, 7 MHz, ad infinitum. Obviously, this is not ideal, and as such it should never be used as the basis for any permanent radio transmitter. For experimental uses, though, it should be fine with an antenna short enough to limit its effective range to the work area you are in.

However, this crude RF transmitter as shown will not actually convey any *information* to an AM radio receiver unit because the 1000 kHz signal is unmodulated. That is to say, an AM radio receiver will not produce any sound when tuned to this transmitter’s frequency because there is no audio information impressed on its 1000 kHz “carrier” signal. In order to amplitude-modulate the carrier signal with audio information we must apply an audio-frequency control voltage to the oscillator’s “enable” pin to turn the RF signal on and off. The result will look something like this, the low-frequency (red) square-wave signal being the audio tone we wish to broadcast and hear over the AM receiver and the high-frequency (green) square-wave signal being the RF (1 MHz):



We could, of course, flip the toggle switch back and forth *really fast* to make an audible noise at the receiver, but this is impractical for producing tones. A better plan is to connect a low-frequency square-wave oscillator to the crystal oscillator’s “enable” pin like so:

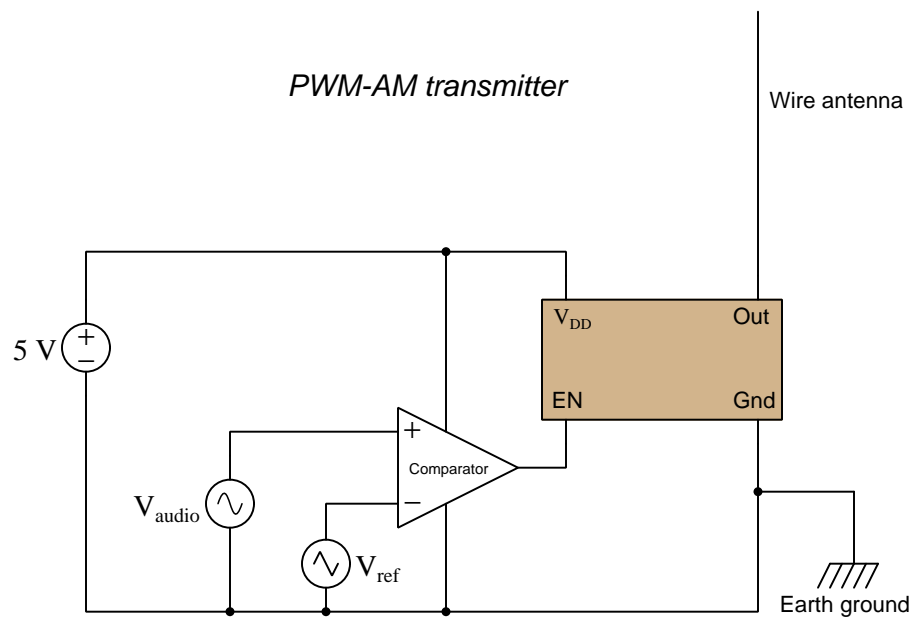
Simple amplitude-modulation



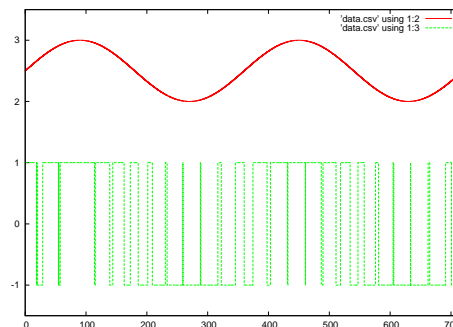
The wavelength of a 1 MHz RF signal is approximately 300 meters, and so a quarter-wave “whip” or “monopole” antenna consisting of a wire connected to the oscillator’s output pin would have to be 75 meters in length to be fully effective. However, we likely do not *want* this to be maximally effective out of concern for interfering with anyone’s radio reception of legitimate 1000 kHz radio broadcast stations, and so a wire antenna shorter than 75 meters will actually be a good thing for an experimental transmitter such as this. In fact, I recommend starting with a very short wire, and making that wire longer only if necessary to obtain any reception at all on a nearby AM receiver.

2.3 Example: slightly-more-sophisticated AM radio transmitter

Building on the previous circuit, we may broadcast more than just square-wave audio tones by pulse-width modulating the crystal oscillator module’s “enable” input pin, the PWM signal having a pulse frequency well above the audio range (many tens of kHz). Our low-frequency audio signal will adjust the duty cycle of the PWM signal, which conveys the full analog information through to the antenna while still respecting the digital (0/5 Volt) input signal requirements of the crystal oscillator module:



The PWM signal should look something like this, shown here in contrast to a pure sinusoidal audio signal:



This PWM signal, in turn, modulates the oscillator’s 1 MHz carrier signal so that you end up

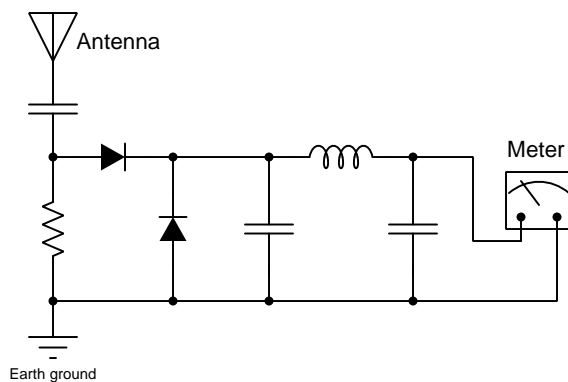
with a pulsed (amplitude-modulated) signal at the antenna which may be received at 1000 kHz on the standard broadcast AM band.

The crystal oscillator module will transmit the full PWM information, but the cutoff of the AM radio receiver's audio amplifier circuitry will naturally filter out the PWM reference frequency, leaving only the audio tone to be heard by the listener.

2.4 Example: simple field strength indicator

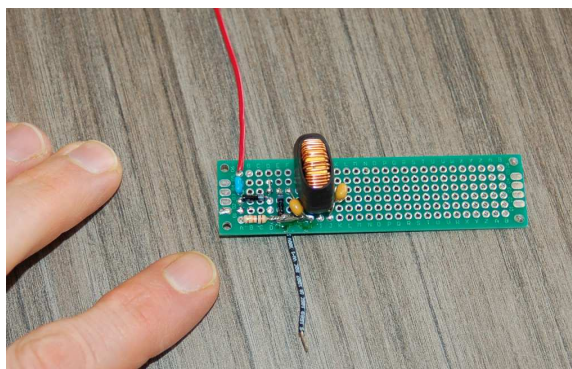
A simple tool for performing qualitative RF field strength measurements is a *field strength meter*. While more sophisticated field-strength meters exist capable of providing quantitative measurements (e.g. electric field strength displayed in units of *Volts per meter*), a non-calibrated meter is still very useful for tasks such as determining if the transmitter is functioning at all, measuring relative field strength at different locations from a transmitting antenna, etc.

The basic idea of an electric field strength indicator is a rectifier and low-pass filter network to translate the radio-frequency AC voltage intercepted by an antenna, and drive a DC meter with that signal:

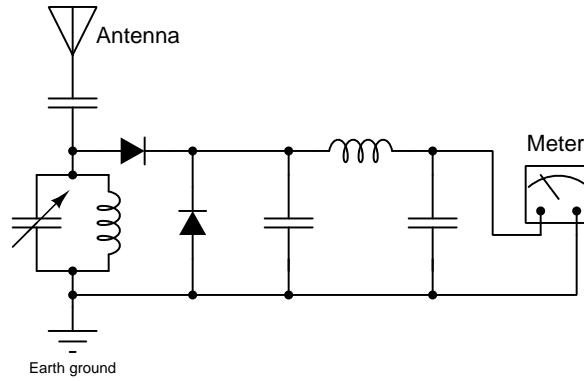


Diodes used in this circuit should be either germanium or hot-carrier (i.e. Schottky diodes) for low forward voltage drop. A model 1N270 (germanium) diode would be a good choice for this circuit. The higher forward voltage drop of a silicon diode will decrease the circuit's sensitivity to weak electric fields.

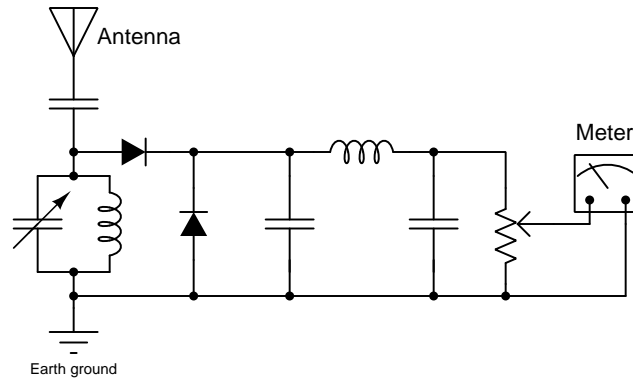
If a general-purpose multimeter is used for indication, all other components may easily fit onto a "prototype" printed circuit board as shown below:



The circuit may be made more selective to a particular frequency by replacing the resistor with a tuned LC (“tank”) circuit:



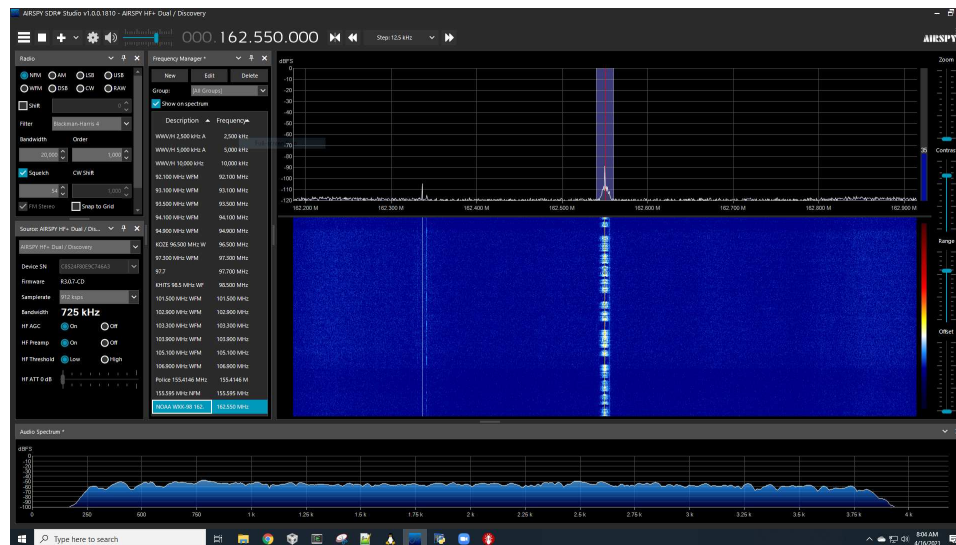
A practical addition to a simple field-strength indicator circuit is a potentiometer between the low-pass filter network and the meter, to provide means to “de-sensitize” the meter when measuring fields near a powerful transmitting antenna.



2.5 Example: SDR spectrum displays

A modern technology for monitoring radio transmissions is something called *Software-Defined Radio* or *SDR*. This is a radio receiver circuit that connects to a digital computer, sending that computer a fairly raw stream of data representing signals received by the antenna, and leaving it up to software algorithms in that computer to de-modulate and otherwise de-code those signals to reveal intelligible information. One of the prominent features of SDR is its ability to display the spectrum of received radio signals, both as a frequency-domain plot and something called a *waterfall* display.

A screenshot of SDR software showing both the frequency-domain plot and waterfall display is appears below. The frequency-domain plot has a vertical axis calibrated in dBFS (decibels of full-scale) and a horizontal axis calibrated in Hertz. The waterfall plot is immediately below the frequency-domain plot, and is currently a background of blue with some lighter-colored vertical stripes. It shares the same horizontal scale as the frequency-domain plot (Hertz) but its vertical axis is a slow-moving time scale and signal intensity is represented by *color*:



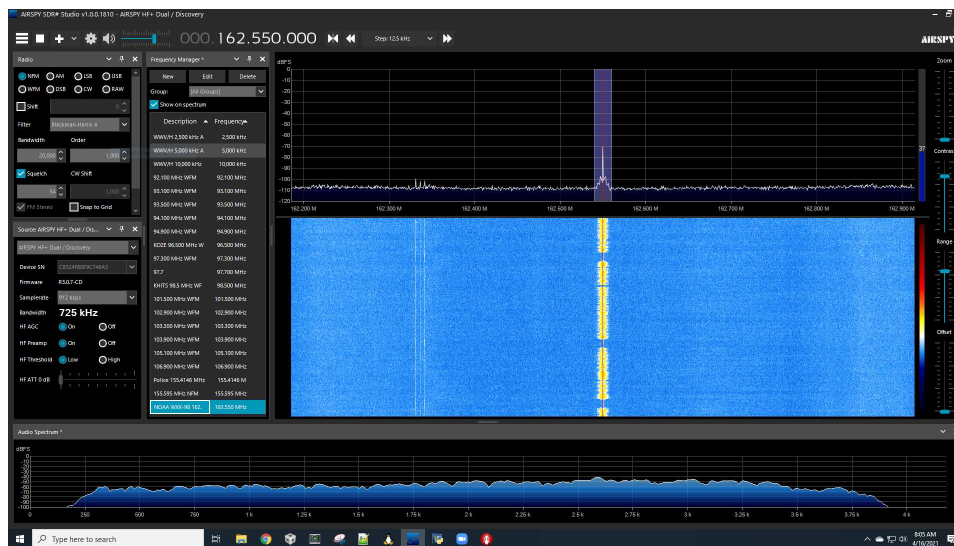
In this screenshot we see the SDR software centered on a signal peak representing a local NOAA² weather broadcast transmitter at a frequency of 162.55 MHz. Like all frequency spectrum plots, signal amplitude is shown by the height of the plot, in this case the carrier frequency signal measuring just above -90 dBFS. The red vertical line and blue-grey color band overlaid on the frequency-domain plot show the SDR software's center (tuning) frequency and the bandwidth of its digital filter for selecting this particular station. The waterfall display shows a colorized representation of signal amplitude, that colorize plot slowly “falling” down over a period of about 20 seconds from the top of the waterfall display to the bottom. This is why the middle stripe of color appears to be broken: the “breaks” represent periods where there is no audio signal and the NOAA transmitter

²In the United States, this is the National Oceanic and Atmospheric Administration, a federal agency tasked with ongoing study and reporting of terrestrial and aquatic conditions on Earth. Among its many functions is to provide regular weather reports and forecasts as part of its National Weather Service, this particular radio transmitter being one of those services.

outputs only a single carrier frequency at 162.55 MHz; the wider colored sections represent periods where speech occurred and the transmitter's signal occupied a wider slice of the frequency domain.

For this screenshot an extremely short antenna was connected to the input connector of the SDR unit, and as a result all signals were fairly weak. This included the *noise floor*, shown in the frequency-domain plot at about -120 dBFS and represented in the waterfall display as a solid blue background.

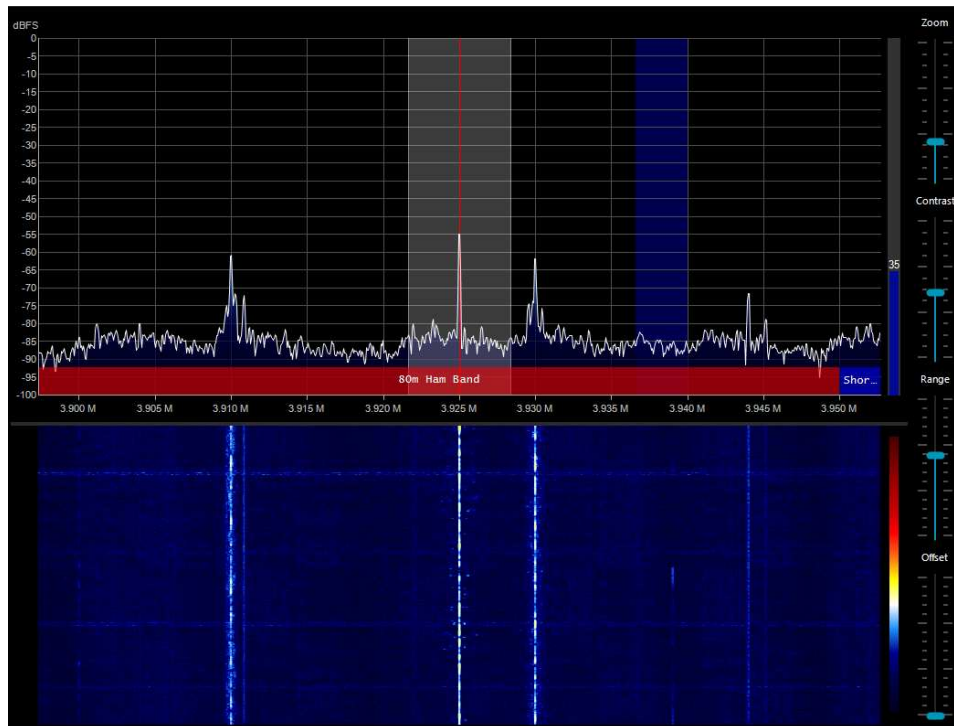
In this next screenshot I show the same SDR radio unit with SDR software locked into the same NOAA radio transmitter at 162.55 MHz. However, this time I touched the short antenna wire with my fingers, using my body as an antenna to capture more signal for the receiver to process. As a result we see all signals on the spectrum become stronger, with the noise floor moving up to approximately -110 dBFS:



Correspondingly, the waterfall display is uniformly brighter than before, that brightness representing increased signal strength across the displayed spectrum. Not only is the background a lighter shade of blue (with speckles representing peaks of random noise from the noise floor), but the NOAA station's signal is trending toward yellow and red.

In both examples we also see an audio spectrum display at the bottom of the SDR software display, representing the audio signal's frequency spectrum resulting from demodulation of the radio station's RF (radio-frequency) signal. As you can see, this audio spectrum extends from 0 Hz to 4 kHz, which is wide enough for typical human speech.

In this next screenshot we see an interesting phenomenon recorded by the SDR’s waterfall display, a series of lighter-blue horizontal lines every so often:



Recall that the coloring of a waterfall display represents signal strength, with dark representing weak signal (or noise floor) and light representing strong signal. This is why the four peaks on the spectrum display correspond to four matching light-colored vertical lines on the waterfall display: the slowly-scrolling waterfall display results in each of these peaks tracing its own light-colored line on the waterfall. What *doesn't* seem to match the spectrum display, however, are the multiple *horizontal* lighter-blue lines we see on the waterfall display. These represent brief moments in time where the entire “noise floor” elevated to a higher signal-strength level. What could cause that to happen?

The answer to this question is a series of *lightning strikes* happening at that time. Bolts of lightning, of course, are short-duration impulses of high electrical energy. Since each lightning bolt’s time duration is extremely brief, each lightning bolt is a nearly-perfect *delta impulse function* which is mathematically equivalent to a broad range of simultaneous frequencies. The SDR receiver displays each of these broadband noise bursts for what it is, resulting in a record of horizontal lines in the waterfall display.

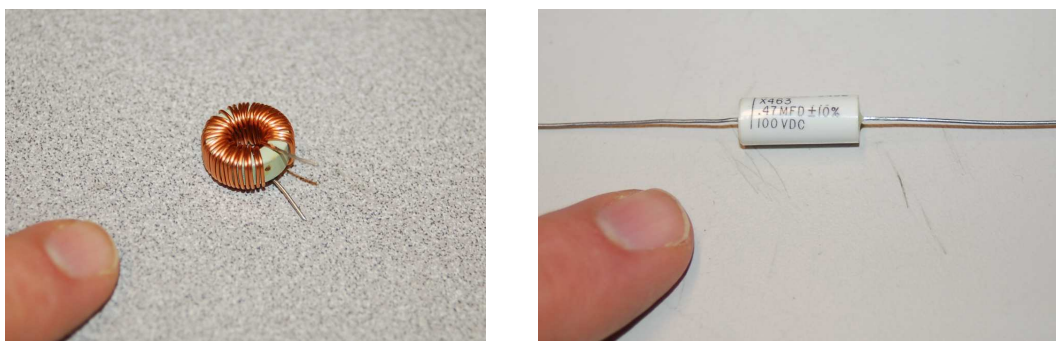
Chapter 3

Tutorial

3.1 Review of inductance and capacitance

Inductance and *capacitance* refer to modes of electrical energy storage: inductance being the storage of energy in a magnetic field associated with current passing through a conductor, and capacitance being the storage of energy in an electric field associated with voltage existing between two distinct conductors. These phenomena take purposeful form in the construction of *inductors* and *capacitors*, respectively. Inductors typically consist of coils of wire wrapped around a magnetic core material, while capacitors consist of metal surfaces¹ separated by an electrical insulator called a *dielectric*.

A small inductor (left) and small capacitor (right) appear in the photographs below:

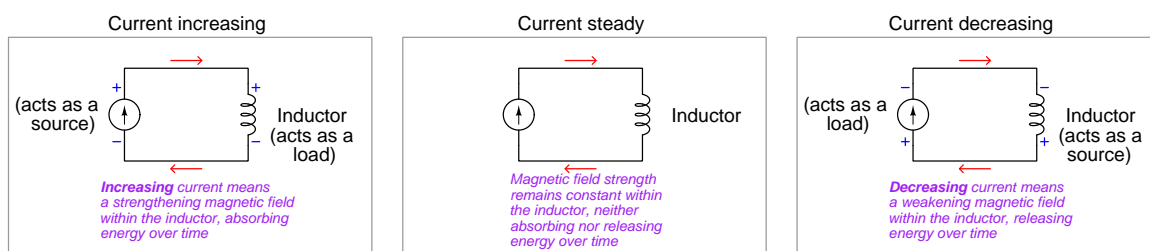


The amount of energy stored in any inductor's magnetic field is a function of current through the inductor's coil, since the strength of that magnetic field varies directly with current. Similarly, the amount of energy stored in any capacitor's electric field is a function of voltage between the capacitor's plates, since the strength of that electric field varies directly with voltage. Energy, as we know, is always conserved, which means it cannot simply appear or vanish without being transferred *from* or *to* somewhere. It follows logically, then, that neither the amount of current through an inductor nor the amount of voltage across a capacitor may vary without energy transfer from or to some other portion of an electric circuit.

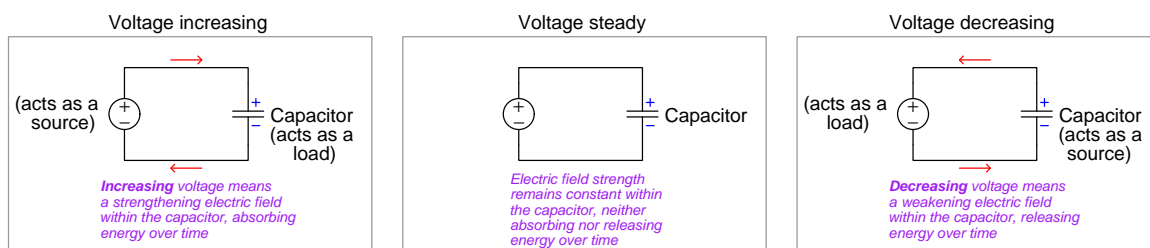
¹Typically the "plates" of a capacitor are sheets of thin metal foil wrapped into cylinders to conserve space.

Energy transfer to or from any electrical component requires that component act as either a *source* or a *load*, and these roles are defined by the relationship between voltage polarity and current direction for that component.

Inductors store energy in magnetic fields, and these fields relate directly to current. Therefore, in order for an inductor to absorb energy from the circuit it must function as a load (i.e. electric charge carriers entering at higher energy and leaving at lower energy, from + to -), and in order to release energy it must function as a source (i.e. charge carriers entering at lower energy and leaving at higher energy, from - to +). Zero voltage across an ideal² inductor means a condition of stasis where the magnetic field (and therefore stored energy) neither accumulates nor depletes:



Capacitors store energy in electric fields, and these fields relate directly to voltage. Therefore, in order for a capacitor to absorb energy from the circuit it must function as a load (i.e. electric charge carriers entering at higher energy and leaving at lower energy, from + to -), and in order to release energy it must function as a source (i.e. charge carriers entering at lower energy and leaving at higher energy, from - to +). Zero current through an ideal³ capacitor means a condition of stasis where the electric field (and therefore stored energy) neither accumulates nor depletes:







Note which parameters reverse and which maintain the same orientation for each of these energy-storing components. For inductors, current remains flowing in the same direction and voltage polarities reverse as the inductor switches roles as source versus load; for capacitors, voltage remains in the same polarity and current reverses direction as the capacitor switches roles as source versus load. *In each case, the parameter directly related to the amount of energy stored in the component is the one conserved – precisely because energy must be conserved – while the other parameter reverses as needed to facilitate energy transfer into or out of the component.*

²An ideal inductor being made of superconducting wire having no resistance to create its own voltage drop.

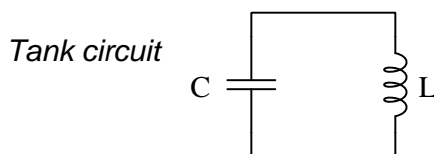
³An ideal capacitor being one with no "leakage" resistance through its dielectric to pass current.

When we compare and contrast the properties of inductance versus capacitance, we see a consistent symmetry between the two, and from this we may see how inductance and capacitance are *complementary properties* of electric circuits:

<p style="text-align: center;">Capacitance (C)</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>NEMA symbol</p> </div> <div style="text-align: center;">  <p>IEC symbol</p> </div> </div>	<p style="text-align: center;">Inductance (L)</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>NEMA symbol</p> </div> <div style="text-align: center;">  <p>IEC symbol</p> </div> </div>
An <i>electric field</i> is created by a <i>voltage</i> existing between two conductors	A <i>magnetic field</i> is created by a <i>current</i> passing through a conductor
Capacitance is enhanced by the area of the conductors, and a narrow spacing between them	Inductance is enhanced by the length of the conductor, and forming a coil shape with multiple turns
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$
Acts as a <i>load</i> when <i>voltage increases</i> over time ; current flows into + terminal and out of – terminal	Acts as a <i>load</i> when <i>current increases</i> over time ; voltage drop + where current enters and – where current exits
Maintains a constant voltage when passively storing energy	Maintains a constant current when passively storing energy
Acts as a <i>source</i> when <i>voltage decreases</i> over time ; current flows out of + terminal and into – terminal	Acts as a <i>source</i> when <i>current decreases</i> over time ; voltage rise – where current enters and + where current exits
Current is able to <i>instantly</i> cease or reverse direction while voltage cannot; current switches direction when capacitor transitions between source and load	Voltage is able to <i>instantly</i> cease or reverse polarity while current cannot; voltage switches polarity when inductor transitions between source and load
The amount of current is proportional to the <i>rate</i> (speed) at which energy is being absorbed by or released from the electric field	The amount of voltage is proportional to the <i>rate</i> (speed) at which energy is being absorbed by or released from the magnetic field
Current is equal to the capacitance times the <i>rate-of-change</i> (speed) that voltage varies over time: $I = C \frac{dV}{dt}$	Voltage is equal to the inductance times the <i>rate-of-change</i> (speed) that current varies over time: $V = L \frac{dI}{dt}$

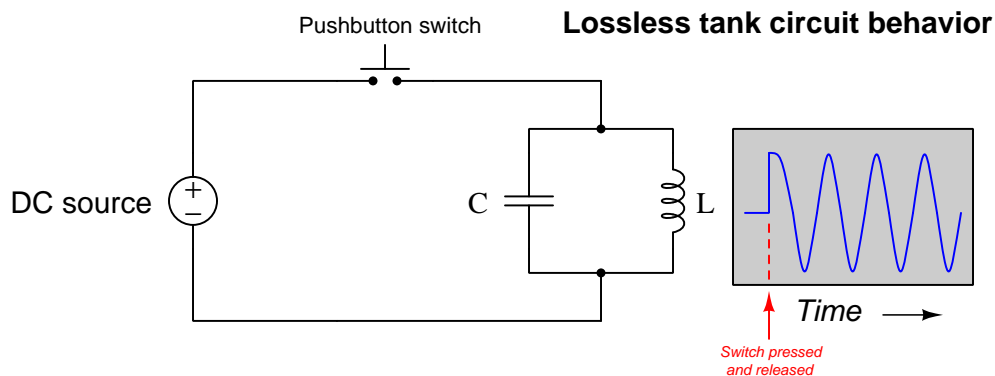
3.2 Tank circuits

If we directly connect a capacitor to an inductor, we find they naturally exchange energy back and forth. That is, if we initially energize one of the other of these two components, they take turns functioning as source and load. The result is that voltage and current both oscillate sinusoidally after initially energizing a capacitor-inductor network by some external source. We refer to this cyclic exchange of energy as *resonance*. The simplest resonant circuit possible is the so-called *tank circuit*, comprised of a single inductor connected to a single capacitor:

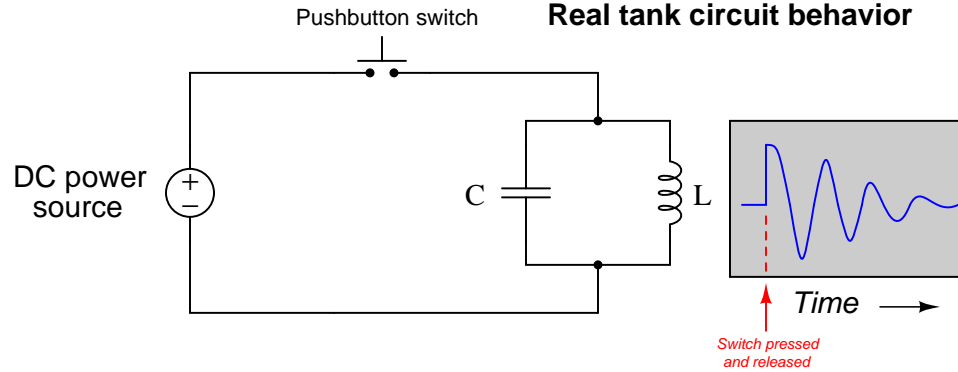


The natural frequency at which a tank circuit oscillates is given by the formula $f_r = \frac{1}{2\pi\sqrt{LC}}$, where f_r is the resonant frequency in Hertz, C is the capacitance in Farads, and L is the inductance in Henrys. This happens to be the same frequency where the capacitor's reactance equals the inductor's reactance ($\frac{1}{2\pi fC} = 2\pi fL$). If a tank circuit is energized by an AC source at its resonant frequency, these equal reactances cancel each other to make zero total impedance (in series) or infinite total impedance (in parallel).

A perfect tank circuit – devoid of electrical resistance and any other energy-dissipating characteristics (e.g. capacitor dielectric losses, inductor hysteresis losses) – would continue to oscillate forever once initially stimulated. That is, if initially “charged” with an impulse of DC voltage, the tank circuit would continue to produce AC voltage and current oscillations at its resonant frequency, at constant peak amplitude, forever:



Since real capacitors and inductors are not lossless, real tank circuits exhibit decaying-amplitude oscillations after initial “charging,” until no energy is stored in either the capacitor or the inductor:



Capacitive losses take the form of heat dissipation within the dielectric substance separating the capacitor plates. The electric field between the capacitor’s plates imparts forces on any polar⁴ molecules within that substance, thereby doing “work” on those molecules by forcing them back and forth as the electric field alternates. Though these forces and motions are extremely small, they are nevertheless drain some energy from the capacitor, converting that energy into heat.

Inductive losses are similar, but in the form of work done on ferromagnetic molecules in the core material as the magnetic field alternates polarity. Like dielectric heating, magnetic losses also drain energy from the inductor, converting it into heat.

Of course, the metallic plates and wire coils of capacitors and inductors, respectively, also contain resistance. Current passing through a resistance also dissipates energy in the form of heat, constituting another mechanism for energy loss (albeit much more significant in inductors than in capacitors owing to the long lengths of wire typical within inductors).

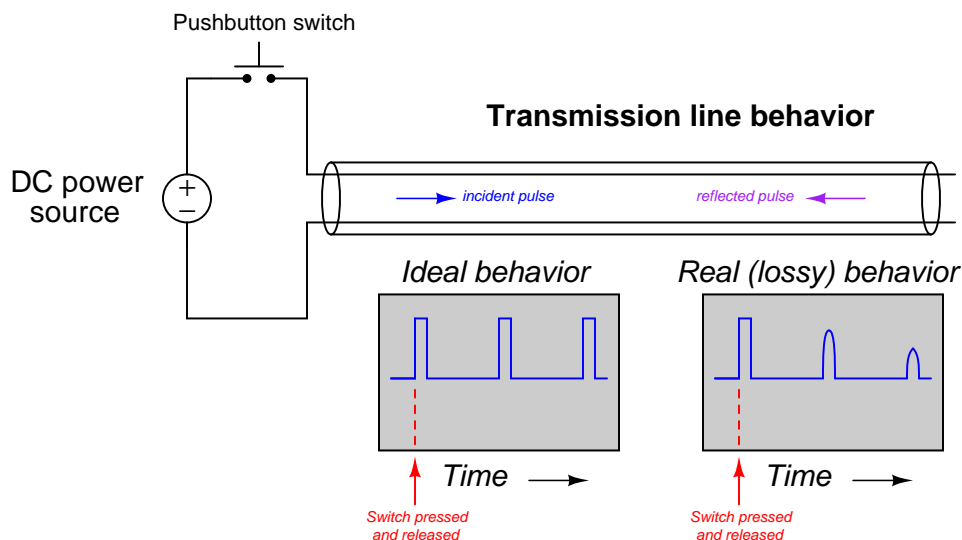
The combined effect of all these energy-loss mechanisms is that the oscillations of an unpowered tank circuit decay over time, until they cease completely. This is similar in principle to a pendulum gradually coming to a halt after being set in motion with a single push: air friction and mechanical friction in the pendulum’s bearing cause its oscillations to gradually diminish until all its energy has been lost – swing by swing – to heat. Similarly, a bell produces an audible tone when struck due to the combined effects of the bell’s mass and elasticity exchanging energy in kinetic and potential forms, but eventually the bell’s tone dampens until it is silent because its energy has been dissipated in the form of sound. In a more general sense this is the Second Law of Thermodynamics at work: every time energy converts from one form to another, some of that energy inevitably “spreads out” to become more diffuse.

⁴A “polar” molecule is one where the constituent atoms are bound together in such a way that there is a definite electrical polarity from one end of the molecule to the other. Water (H₂O) is an example of a polar molecule: the positively charged hydrogen atoms are bound to the negatively charged oxygen atom in a “V” shape, so the molecule as a whole has a positive side and a negative side which allows the molecule to be influenced by external electric fields. Carbon dioxide (CO₂) is an example of a non-polar molecule whose constituent atoms lie in a straight line with no apparent electrical poles. Interestingly, microwave ovens exploit the fact of water molecules’ polarization by subjecting food containing water to a strong oscillating electric field (microwave energy in the gigahertz frequency range) which causes the water molecules to rotate as they continuously orient themselves to the changing field polarity. This oscillatory rotation manifests itself as heat within the food.

3.3 Transmission lines and antennas

Capacitance and inductance, however, are not limited to discrete components called capacitors and inductors. Quite literally, *any* pair of conductors separated by an insulating medium exhibit capacitance because these conductors can serve as the end-points for an electric field. Similarly, *any* electrical conductor exhibits inductance because that conductor can carry an electric current which in turn will establish a magnetic field surrounding it. Electric and magnetic fields are fundamental features of our universe, and therefore exist in places we don't necessarily want or anticipate. It is for this reason that capacitance and inductance exist as *parasitic* properties of electrically conductive structures such as metal wires, printed circuit board traces, metal enclosure walls, etc.

For example, every multi-conductor cable is a *transmission line*, its distributed capacitance and inductance capable of storing energy. If a long, unterminated (i.e. no load connected to the far end) cable is “charged” by a pulse of applied DC voltage at one end, that pulse’s energy will “echo” back and forth along the length of the cable at a period dependent on the cable’s length and velocity factor⁵:



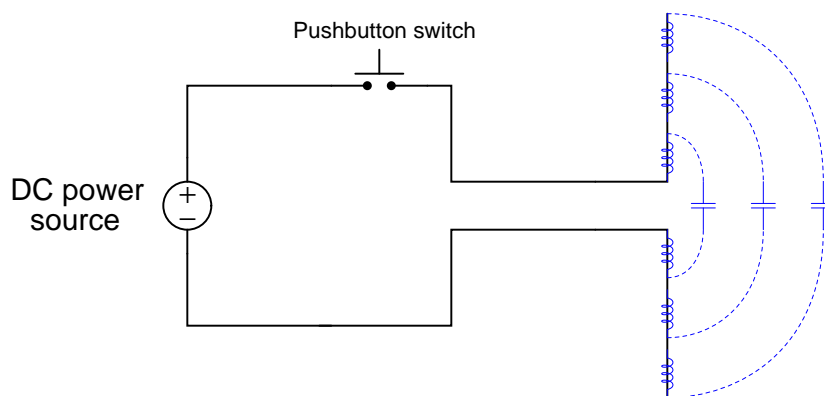
The ability for a transmission line to support periodic signal “echoes” means it may also resonate when energized by an AC power source, like a tank circuit. Unlike a tank circuit, however, a transmission line may resonate at more than one frequency: a “fundamental” frequency, or any whole-number multiple of that fundamental frequency (i.e. a *harmonic frequency*)⁶. For example,

⁵The *velocity factor* of a cable is the ratio of a pulse’s speed through that cable compared against the speed of light in a vacuum: 299792458 meters per second, or approximately 3×10^8 m/s. It is primarily a function of the dielectric permittivity of the cable’s conductor insulation.

⁶This “poly-resonant” behavior of transmission lines has an analogue in the world of music. “Wind” instruments such as trombones, flutes, trumpets, saxophones, clarinets, recorders, etc., are really nothing more than tubes with at least one open end. These tubes will acoustically resonate at specific frequencies when “excited” by turbulent air flow at one end. The lowest frequency such a tube will resonate at is its “fundamental” frequency, but increasing the turbulence of the incoming air flow will cause the tone to “jump” to some harmonic of that fundamental frequency. The fundamental frequency of the tube may be altered by changing the length of the tube (e.g. as in a trombone or

an unterminated transmission line with a length of 1 kilometer and a velocity factor of 0.7 has a round-trip echo time (period) of 9.53 microseconds, equivalent to a resonant frequency of 104.9 kHz. However, it will resonate equally well at exactly twice this fundamental frequency (209.8 kHz – the *second harmonic* of 104.9 kHz) as well as three times the fundamental frequency (314.8 kHz – the *third harmonic* of 104.9 kHz), etc. A simple LC tank circuit, by contrast, will only resonate at a single frequency.

If we were to alter our transmission line test circuit, splaying the two conductors apart from each other rather than running them alongside one another, it would also form another type of resonant circuit, owing to the distributed capacitance and inductance along the wires' lengths:



This special case of resonant circuit has some very interesting properties. First, its resonant frequency is quite high, because the distributed inductance and capacitance values are extremely small compared to the values of discrete components such as inductors and capacitors. Second, it is a very “lossy” circuit despite having little electrical resistance to dissipate energy, no solid insulating medium to incur dielectric losses, and no ferromagnetic core to suffer hysteresis losses. This special form of resonant circuit loses energy primarily to *electromagnetic radiation* rather than heat. In other words, the energy in the electric and magnetic fields leave the circuit and propagate through space in the form of *electro-magnetic waves*, or what we more commonly refer to now as *radio waves*: a series of electric and magnetic fields oscillating as they radiate away from the wires at the speed of light⁷.

Tank circuits and transmission lines do not radiate energy because they are intentionally designed to *contain* their fields: capacitors are designed to fully contain their electric fields, inductors are designed to fully contain their magnetic fields, and the fields within transmission lines are likewise contained. Our two-wire resonant circuit, by contrast, does just the opposite: its electric and magnetic flux lines spread through open space with no containment whatsoever. What we have built here is not a tank circuit nor a transmission line, but rather an *antenna*⁸: a structure designed

trumpet) or by opening ports along the tube’s length to alter its effective length (flute, saxophone, clarinet, recorder, etc.).

⁷In full disclosure, light itself is also an electromagnetic wave, albeit one of extremely high frequency. So are X-rays, gamma rays, cosmic rays, and certain other forms of radiation.

⁸An older term used by radio pioneers to describe antennas is *radiator*, which I personally find very descriptive. The word “antenna” does an admirable job describing the physical appearance of the structure – like antennas on an

to radiate electric and magnetic fields as waves into surrounding space.

3.4 Maxwell and Hertz

An interesting historical footnote is that this phenomenon of electric and magnetic field waves propagating through space was predicted theoretically before it was demonstrated experimentally. A Scottish physicist named James Clerk Maxwell made this astonishing theoretical prediction which he published in 1873. His equations were simplified and consolidated by the work of Oliver Heaviside, commonly expressed as follows:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

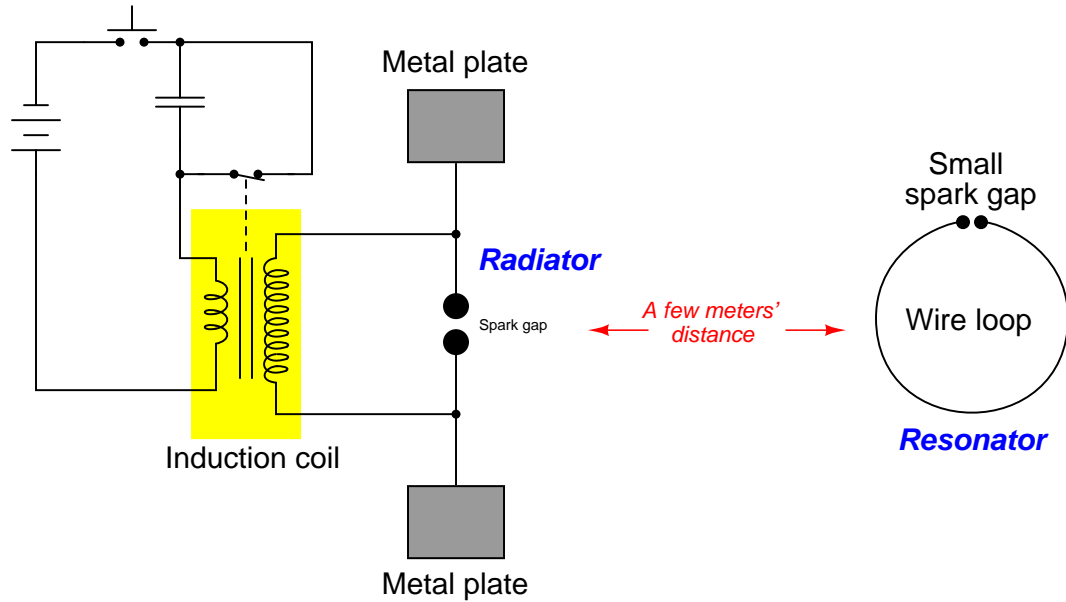
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

The last two equations hold the most interest to us with respect to electromagnetic waves. The third equation states that an electric field (\mathbf{E}) will be produced in open space by a varying magnetic flux ($\frac{d\Phi_B}{dt}$). The fourth equation states that a magnetic field (\mathbf{B}) will be produced in open space either by an electric current (I) or by a varying electric flux ($\frac{d\Phi_E}{dt}$). Given this complementary relationship, Maxwell reasoned, it was possible for a varying electric field to create a varying magnetic field which would then create another varying electric field, and so on. This cause-and-effect cycle could continue, ad infinitum, with fast-changing electric and magnetic fields radiating off into open space without needing wires to carry or guide them, or a tank circuit to sustain the oscillations. In other words, *these complementary fields self-sustain as they travel*, although they do become weaker due to their spreading out of a finite amount of energy over larger and larger volumes of space.

The Prussian Academy of Science offered a reward to anyone who could experimentally validate Maxwell's prediction, and this challenge was answered by Professor Heinrich Hertz at the Engineering College in Karlsruhe, Germany in 1887, eight years after Maxwell's death. Hertz constructed and tested a pair of devices: a "radiator" to produce the electromagnetic waves, and a "resonator" to receive them.

insect – but the word "radiator" actually describes its *function*, which is a far more useful principle for our purposes.

A simplified diagram showing Hertz's experimental device is shown here:



An “induction coil” (a buzzing device constructed of a self-interrupting relay and step-up transformer winding to generate a continuous pulsing waveform of high voltage) provided an extremely noisy (i.e. harmonically rich)⁹ AC signal to the radiator, while a spark gap at the resonator provided indication that the electromagnetic waves were captured and converted into voltage by the resonator wire.

Both the radiator and the resonator are antennas. The purpose of the transmitting antenna (radiator) is to take high-frequency AC power and radiate that power in the form of electromagnetic waves: self-sustaining electric and magnetic fields propagating outward into open space. The purpose of the receiving antenna is to capture those electromagnetic waves and convert them into an electrical signal (i.e. AC voltage and current). All antennas – from historical to the most modern – behave in fundamentally the same way: energize them with high-frequency AC electrical power, and they will radiate electromagnetic waves at that frequency; expose them to electromagnetic waves, and they will produce a very small AC electrical signal at the same frequency as the incident radiation.

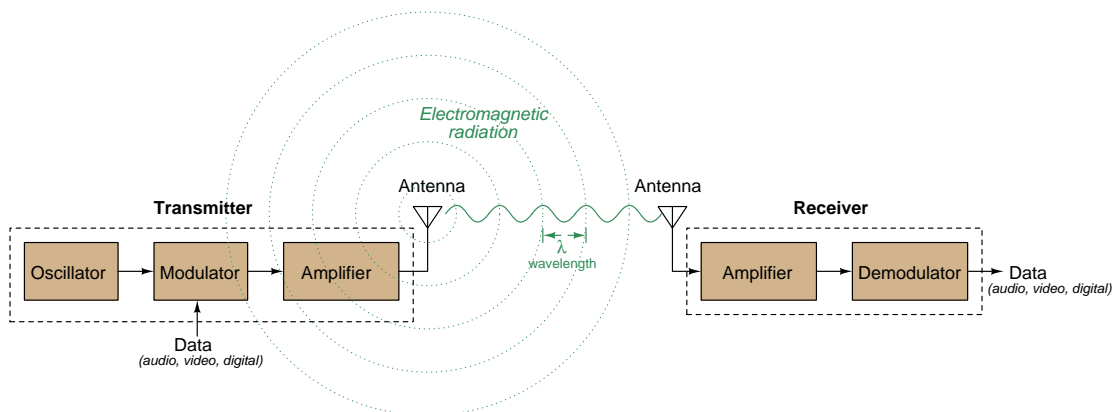
⁹A fundamental property of waves is that any wave-shape, no matter how complex, is mathematically equivalent to a series of perfectly sinusoidal waves of differing frequencies added together. Waveforms with fast rise/fall times, such as the pulse waveforms produced by switched induction coils, contain many such high-frequency “harmonic” signals, and it is these high-frequency signals that created the radiated electromagnetic waves in Hertz’s experiments. The induction coil’s secondary winding inductance combined with the parasitic capacitance of the radiator’s wires and plates provided the means to momentarily sustain some of these high-frequency oscillations long enough to sustain visible sparks at the resonator’s gap.

3.5 Applications for radio

Since the prediction and discovery of electromagnetic waves, their use has revolutionized society worldwide through such technological applications as *radio*, *television*, *wireless digital*, and *radar*:

- **Radio** – Electromagnetic waves transmitted from an antenna structure into space are received by one or more other antenna structures, and the resulting AC signals interpreted as information. The simplest form of radio is *carrier-wave* where the waves are sent in discrete on/off fashion, with the information encoded in a form such as *Morse code*. More complex forms of radio encode audio information onto the high-frequency waves, so that speech and music may be transmitted and received remotely.
- **Television** – Essentially, radio with video as well as audio information encoded onto the high-frequency electromagnetic waves.
- **Wireless digital** – The encoding of digital (binary) bits onto the high-frequency electromagnetic waves for the purpose of communicating nearly any form of information (e.g. audio, video, text, data files).
- **Radar** – A brief pulse of electromagnetic waves are transmitted from an antenna, travel to some remote object, reflect off of that object, and are received by the same antenna. The echo time and direction represent information about the location of that object relative to the antenna. Any shift in wave frequency between transmission and reception represents the object’s velocity relative to the antenna (the *Doppler effect*), in the same way that a bat discerns relative speed to a target by comparing the pitch of its emitted call against the received sonic echo.

A very simple block diagram shows essential components of a simplex (one-way) radio, television, or digital wireless system:



Two-way (duplex) communication requires that each station have both transmitting and receiving capability. In the case of radar there is only one station, designed to transmit a powerful burst of electromagnetic energy and then quickly switch into receiving mode to “listen” to the echoes from one or more objects reflecting those waves back to the antenna.

3.6 Wavelength

An important measure of radio-frequency (often abbreviated as *RF*) electromagnetic waves is something called *wavelength*. This is the physical distance traversed by the radiating wave in the timespan of one cycle (i.e. period) as it travels through space at the speed of light. Since the speed of light in open space is a fixed quantity (approximately 3×10^8 meters per second), wavelength and wave frequency are inversely related: the higher the frequency, the shorter the wavelength. This makes perfect conceptual sense: the higher a wave's frequency, the shorter its period, which means it won't travel as far in the span of one period.

We may mathematically express wavelength (symbolized by the lower-case Greek letter *lambda*, λ) as the product of the wave's period (t_p) and the speed of light in a vacuum (c):

$$\lambda = t_p c$$

Where,

λ = Wavelength, in meters

t_p = Period of wave, in seconds

c = Speed of light, 2.99792458×10^8 meters per second

Applying dimensional analysis to this, we see the units agree:

$$[\text{m}] = [\text{s}] \left[\frac{\text{m}}{\text{s}} \right]$$

If we prefer working in terms of wave *frequency* (f) rather than wave period, we may algebraically substitute the definition of period in terms of frequency ($t_p = \frac{1}{f}$) into the original equation:

$$\lambda = t_p c \quad t_p = \frac{1}{f}$$

$$\lambda = \frac{1}{f} c$$

$$\lambda = \frac{c}{f}$$

Some practical examples help us appreciate the scale of common RF wavelengths:

- An AM radio transmitter operating at 950 kHz broadcasts a wavelength of 315.57 meters.
- An FM radio transmitter operating at 103.1 MHz broadcasts a wavelength of 2.9078 meters.
- An amateur (“ham”) transmitter operating at a wavelength of 160 meters broadcasts a frequency of 1.8737 MHz

Wavelength has great practical importance in the field of RF communication because optimum antenna size is based on this dimension. The higher the operating frequency, the shorter the wavelength, and the smaller the antennas used.

3.7 Government regulation

Radio was a laboratory curiosity in the 19th century, a driving force of social, political, and national development in the 20th century, and today stands as an essential utility for most of the world. With such wide-spread use it becomes critically important to place limits on radio transmissions in order to avoid interfering with other peoples' radio communications. When multiple transmitters broadcast into open space, their electromagnetic waves overlap and may be distinguished from each other primarily by frequency. Thus, transmitters must broadcast at very precise and specific frequencies, while receivers must contain some means to *filter* the frequency(ies) of interest from all the other signals intercepted by the receiving antenna.

This technical limitation raises *regulatory* concerns as well. In order for a great many transmitting stations to successfully operate without interference within a common space, some method of *frequency allocation* must exist so that people know which frequencies they are allowed to broadcast on. Additionally, restrictions on transmitting power are necessary in order to limit the practical range of broadcasts and allow transmitters in different locations to use the same wavelength¹⁰. In the United States, the governing agency for this purpose is the *Federal Communications Commission* or *FCC*, an independent Federal agency entirely supported by fees collected by the agency. FCC-specific regulations are codified as Federal law in the United States under Title 47 of the Code of Federal Regulations (47 CFR) published by the Office of the Federal Register (OFR) and the Government Publishing Office (GPO), with that title being divided into “parts” and “subparts” addressing different concerns. For example, Part 15 of 47 CFR delineates regulations regarding “Radio Frequency Devices”. Additionally, the FCC itself publishes documents to aid in the understanding of these regulations, including technical bulletins from the FCC’s Office of Engineering and Technology (OET).

One of the major functions of the FCC is to grant *licenses* for operating radio transmitters, with some licenses being more restrictive than others. License holders must pay fees to the FCC in order to financially support the agency. Not only may licenses be granted to governmental agencies for purposes such as emergency dispatch services, and to private businesses such as telecommunication service providers, but private citizens may earn licenses to operated their own “amateur” radio transmitters (“Ham” radio). Certain portions of the radio-frequency *spectrum* are reserved for unlicensed public use (provided power output falls within specified maximum limits), such as Citizen’s Band (CB) radio, cellular telephones, and Wi-Fi data routers.

Another function of the FCC is to manage the inherently limited bandwidth of the electromagnetic spectrum. Since 1994 portions of the spectrum have been made available through an auction process managed by the FCC’s Wireless Telecommunications Bureau, whereby the highest bidder wins access. Prior to this open auction process, access to specific portions of the RF spectrum were granted by hearings or by lottery.

¹⁰Broadcast radio service (e.g. AM and FM) are an excellent example of this. Within the United States, for example, there are a great many radio and television transmitting stations sharing identical frequencies (wavelengths), but they do not interfere with each other because their power outputs are limited such that their geographic *ranges* do not overlap.

Violations of FCC regulations are punishable by law, as are other regulatory violations within the United States. In particular, students and hobbyists wishing to construct and test their own RF transmitters should become familiar with *FCC guidelines on low-power, non-licensed transmitters* to ensure compliance with intended experiments.

Chapter 4

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

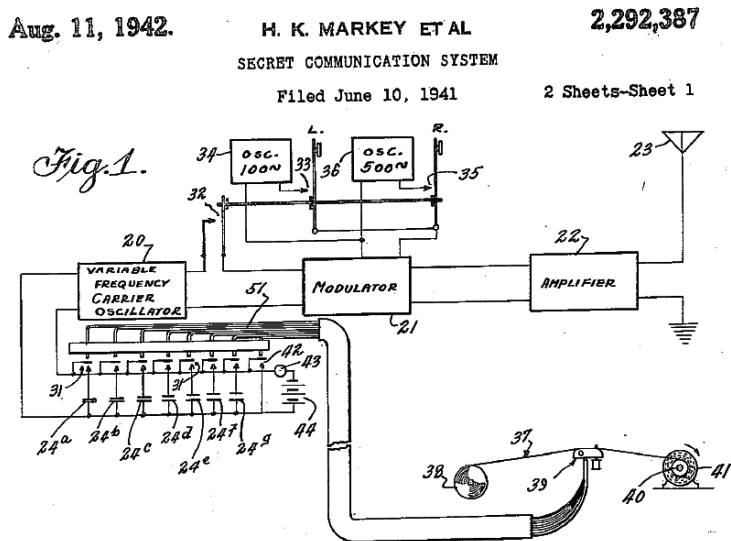
It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . [page xi]

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

4.1 Spread-spectrum radio patent

A popular method of fortifying the security of radio-based data communication is to broadcast the information over a range of different frequencies rather than on a single frequency where it may be more easily intercepted. The conceptually simplest method of spreading the information across a spectrum of frequencies (“spread-spectrum”) is to have the radio transmitter (and receiver) units “hop” between defined frequencies in the same pattern at the same times. An early conception of this idea took the form of a patent filed by George Antheil and the famous actress Hedy Lamarr (born under the name Hedwig Eva Maria Kiesler, with Markey being the surname of her husband at the time of the patent’s filing) in 1941.

In this patent – “Secret Communication System” (US patent number 2,292,387) filed 10 June 1941 and awarded 11 Aug 1942 – a radio transmitter circuit is shifted between different frequencies by a network of switched capacitors, these capacitors controlling the resonant frequency of the carrier oscillator. This radio-frequency oscillator signal is then modulated by one of two audio-frequency tones (100 Hz and 500 Hz), amplified, and sent to a broadcast antenna. Those two audio tones represent the information to be conveyed over the radio wave, while the “carrier” signal provides the high frequency necessary for effective electromagnetic-wave radiation at the antenna:

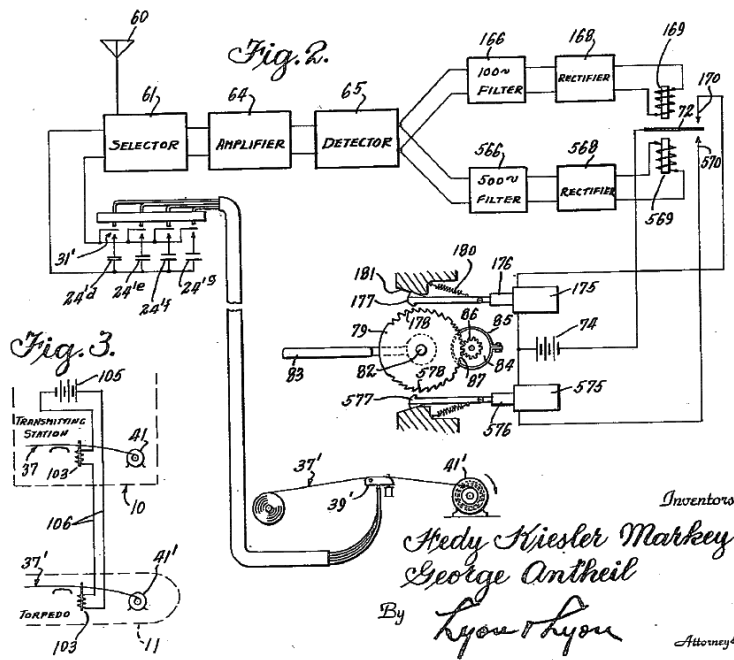


The oscillator’s pattern of frequency-changes is controlled by a paper tape moved between two spools, with holes punched in the tape in a manner not unlike that of a player-piano mechanism which served as this invention’s inspiration. Although “frequency-hopping” radio communication technology may be used for many purposes, these inventors intended it for the remote guidance of torpedoes to be used by the Allied forces during World War Two. In the inventors’ own words:

Briefly, our system as adapted for radio control of a remote craft, employs a pair of synchronous records, one at the transmitting station and one at the receiving station, which change the tuning of the transmitting and receiving apparatus from time to time, so that without knowledge of the records an enemy would be unable to determine at

what frequency a controlling impulse would be sent. Furthermore, We contemplate employing records of the type used for many years in player pianos, and which consist of long rolls of paper having perforations variously positioned in a plurality of longitudinal rows along the records. In a conventional player piano record there may be 88 rows of perforations, and in our system such a record would permit the use of 88 different carrier frequencies, from one to another of which both the transmitting and receiving station would be changed at intervals. Furthermore, records of the type described can be made of substantial length and may be driven slow or fast. This makes it possible for a pair of records, one at the transmitting station and one at the receiving station, to run for a length of time ample for the remote control of a device such as a torpedo. [page 1]

The receiver is shown below, a matching paper tape controlling the "selector" which tunes into the broadcast signals. So long as the transmitter and receiver tapes had matching sequences and were synchronized to each other, they would form a radio communication system suitable for conveying steering signals to a torpedo that would be difficult to jam:



An interesting feature of this design is that the transmitter was capable of “hopping” between more frequencies than the receiver was built to receive, and this was done purposely to permit the occasional broadcast of false signals which would further confound any attempt for an enemy to intercept or jam the radio transmissions:

It will be noted that whereas there are seven tuning condensers¹ 24 at the transmitting station, there are only four tuning condensers 24' at the receiving station. The extra three tuning condensers at the transmitting station provide three additional channels for the transmitter for which there are no corresponding channels at the receiver, to thereby permit the sending of false impulses to confuse the enemy. [page 3]

Any “frequency-hopping” spread-spectrum radio system must somehow manage to synchronize the frequency-hopping pattern between transmitter(s) and receiver(s) or else it simply will not function. To this end, the patent’s authors suggest a method for synchronizing the starting times of the two units’ tape-drive motors, illustrated in figure 3:

It is of course necessary that the record strips 37 and 37' at the transmitting and receiving stations, respectively, be started at the same time and in proper phase relation with each other, so that corresponding perforations in the two record strips will move over their associated control heads at the same time. We therefore provide an apparatus for holding both record strips in a starting position until the torpedo is fired, and for then simultaneously releasing both strips so that they can be moved at the same speed by their associated motors 41 and 41'. [page 3]

The long-term synchronization of two paper strips driven by electric motors may at first seem to be a challenging problem to solve, but this is obviated by the fact that a running torpedo is a short-lived machine. The transmitter and receiver tapes need only be synchronized with each other for a short run time, so slight differences in motor speed would not be as debilitating as one might assume.

¹“Condenser” is an obsolete term for *capacitor*.

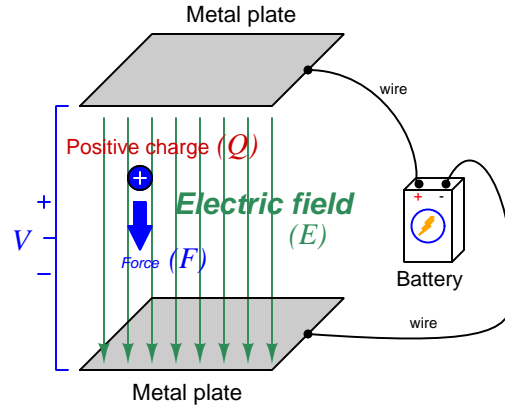
Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs¹)

\vec{E} = Electric field (Newtons per Coulomb)

The small “arrow” symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{d}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

¹One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,

\vec{E} = Electric field, or electric flux density (Newtons per Coulomb)

Φ_E = Electric flux (Newton-meter squared per Coulomb)

\vec{A} = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,

Φ_E = Electric flux (Newton-meter squared per Coulomb)

Q = Charge quantity (Coulombs)

ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field². As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

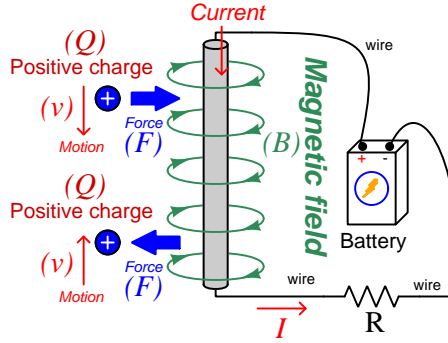


For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

²Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

5.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F , called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs³)

\vec{v} = Velocity of moving charge (meters per second)

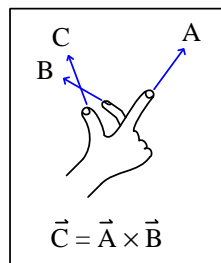
\vec{B} = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small “arrow” symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The “cross-product” (\times) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

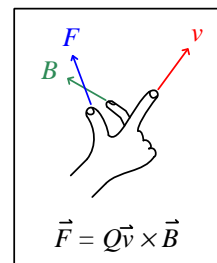
³One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the “right-hand rule” originates:

General "right-hand rule"
for vector cross-products

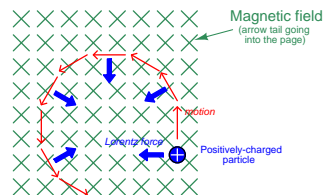


"Right-hand rule" specific
to the Lorentz force



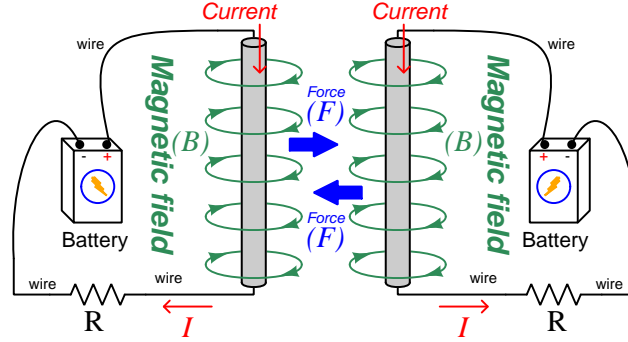
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector (\vec{v}), your middle finger in the direction of the magnetic field vector (\vec{B}), and your thumb in the direction of the force vector (\vec{F}). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current⁴ (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **T**humb points in the direction of the **t**hrust (i.e. force) acting upon the moving charge.

The Lorentz force’s effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



⁴Some textbooks speak of a “left-hand rule” which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically “electron flow” notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be “positive” and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

\vec{F} = Force exerted on both conductors (Newtons)

I = Current (Amperes)

\vec{l} = Length of wire (meters)

\vec{B} = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\text{C}] \left[\frac{\text{m}}{\text{s}} \right] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\text{C}}{\text{s}} \right] [\text{m}] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

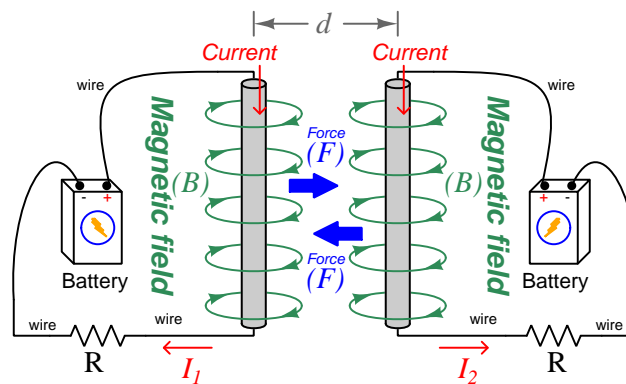
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract⁵.

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l \frac{\mu I_1 I_2}{2\pi d}$$



⁵That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

\vec{B} = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

\vec{A} = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss⁶.

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

⁶Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at <https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm>, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu AI}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most⁷ substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi\mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

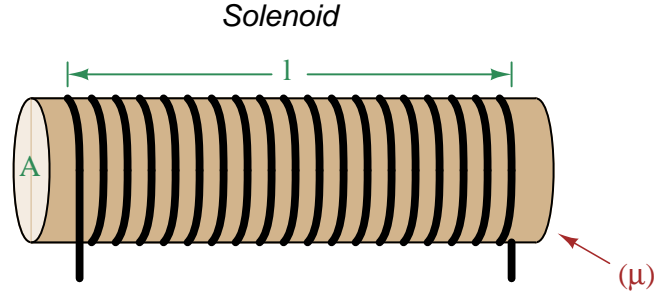
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

⁷Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long⁸ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

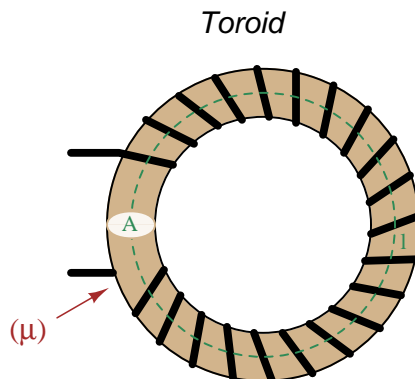
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A .

⁸These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet⁹ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

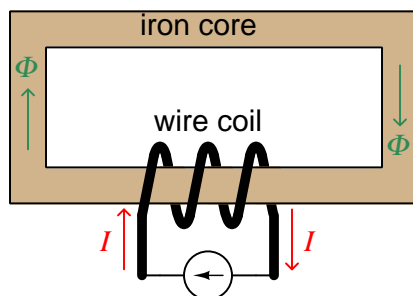
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \qquad B = \frac{\mu N I}{2\pi r}$$

⁹Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic “circuit” formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to “electromotive force” or “emf” often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current’s effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance (\mathfrak{R}) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material’s permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\mathfrak{R} = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	Symbol	SI unit	CGS unit
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	\mathfrak{R}	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the “raw” measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance (\mathfrak{R}) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the “normalized” measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	Conversion equivalence
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance (\mathfrak{R})	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

5.3 The Poynting vector

We know that electric fields (E) are always associated with a voltage (V), and magnetic fields (field strength B and field force H) with a current (I). Specifically, electric field strength is the quotient of voltage and distance between two points which is why electric fields are measured in units of *Volts per meter*. Similarly, magnetic field force around a straight current-carrying conductor is the quotient of current and circumference of the circular magnetic path (equal to 2π times the distance from the conductor) which is why magnetic field force is measured in units of *Amperes per meter*.

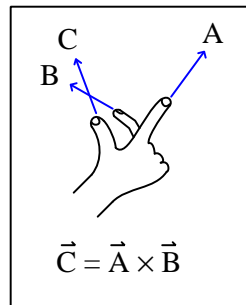
From Joule's Law we know that the rate of energy transfer (called power) whenever there is simultaneous voltage and current ($P = VI$). It stands to reason, then, that just as voltage multiplied by current yields power, an electric field multiplied by a magnetic field must similarly express power transferred by those two fields through space. That is, if $P = VI$, then the product of E and H should yield a power *intensity* which we shall label S :

$$P = VI \qquad \text{[Watts]} = \text{[Volts]}\text{[Amperes]}$$

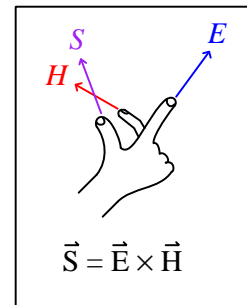
$$S = EH \qquad \text{[Watts/meter}^2\text{]} = \text{[Volts/meter]}\text{[Amperes/meter]}$$

Notice how the Joule's Law equation $P = VI$ involves scalar quantities, while the $S = EH$ equation necessarily involves spatial dimensions of distance (meters) and area (square meters). This is because both electric and magnetic fields are *vector* quantities having both magnitude and spatial direction. Power intensity (S) is the measure of how much power is flowing per square meter of area in open space. As it so happens, power intensity in Watts per square meter relates to electric and magnetic fields by the right-hand rule of vector cross-products, the vector expression of that power intensity being called the *Poynting vector* (\vec{S}) after its formulator, the English physicist John H. Poynting:

General "right-hand rule"
for vector cross-products



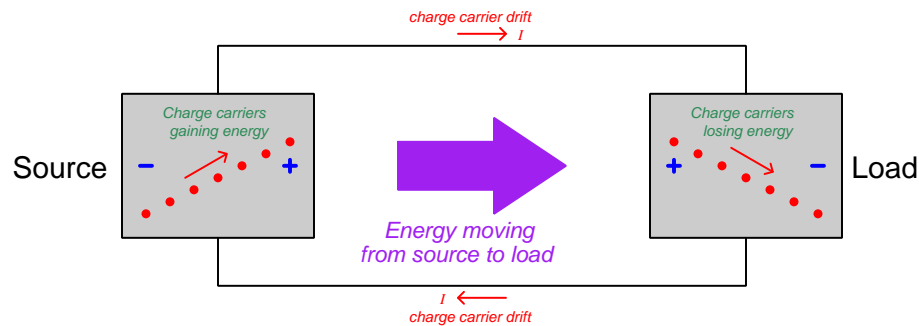
"Right-hand rule" specific
to the Poynting vector



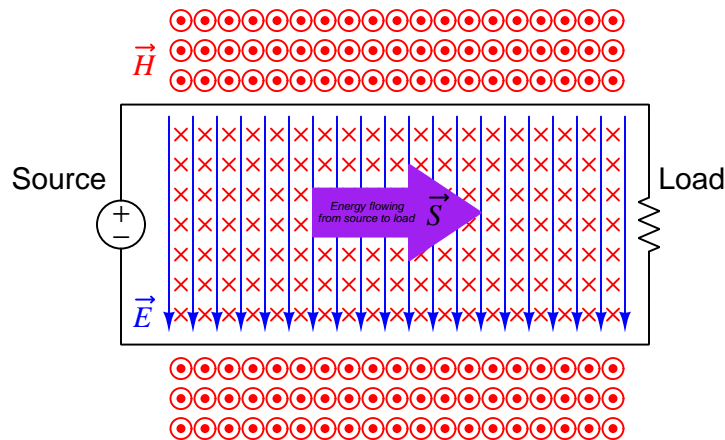
Poynting's contribution to the science of electromagnetism was expressing energy conservation and energy transfer in terms of electric and magnetic fields. That is, the electric and magnetic fields associated with any electrical *source* result in a flow of energy away from that source *through space*, and the electric and magnetic fields associated with any *load* result in a flow of energy toward that load *through space*. The direction of the Poynting vector shows the direction of that energy flow,

and the Poynting vector magnitude shows how many Watts of power flow per square meter of space.

This should be a rather startling conclusion to anyone familiar with voltage, current, and power in simple DC circuits. Beginning students of electricity regard energy transfer in simple circuits as the gain and loss of potential energy in mobile electric charges, those charges gaining energy as they pass through a source and losing energy as they pass through a load:

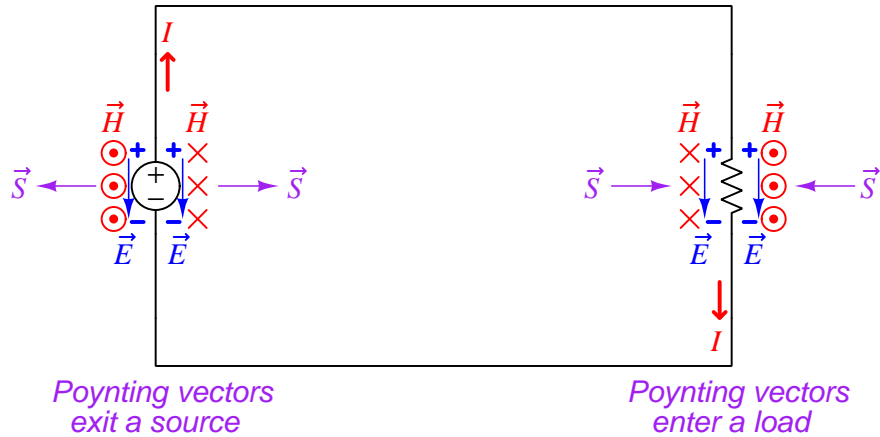


Though this model is conceptually correct, *so is Poynting's model of energy flowing through space*. In fact, if we re-draw this simple circuit illustration showing the electric and magnetic field vectors between the two wires, we can see the Poynting vector expressing that flow of energy from source to load¹⁰:



¹⁰Holding your right hand such that your index finger points downward (\vec{E}) and your middle finger into the page (\vec{H}), your thumb will point to the right (\vec{S}) from source to load.

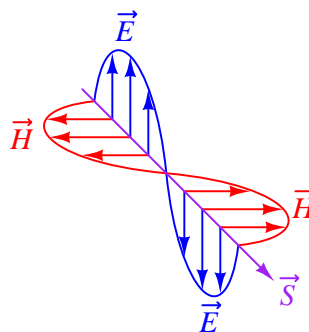
If we examine the source and load separately using the right-hand rule, we see how the Poynting vector in each case shows the proper direction of energy transfer:



It is well-worth your time to apply the right-hand rule to each of the four Poynting vectors shown above, to familiarize yourself with the use of this tool as a means to determine vector cross-product directions. For example, to the left of the source we see the electric field vector pointing down which means your index finger should point downward as well. The magnetic field vector on that side is coming out of the page (middle finger pointing toward you), and holding your right hand in that manner points your thumb to the left.

For simple DC circuits and circuit elements, Poynting's theory of power transfer may seem like a mere curiosity. It fails to explain anything we could not already explain using the model of potential energy gained and lost by mobile charge carriers, and it requires contortions of your right hand to properly align all the vectors. However, in some applications such as *radio* where energy radiates away from structures called *antennas* in the form of electromagnetic waves (oscillating electric and magnetic fields), Poynting's theory is the *only* suitable explanation for power transfer because there are no wires to convey mobile electric charges from source (transmitting antenna) to load (receiving antenna). Below we see a simplified representation of an electromagnetic wave consisting of two oscillating fields (shown here as an electric field oscillating vertically and a magnetic field oscillating horizontally), and if you use the right-hand rule in each half-cycle you will find that the Poynting vector consistently points in the same direction at the speed of light:

Electromagnetic wave



Based on the direction of the Poynting vector we see in this illustration, the source (transmitting antenna) would be located somewhere above and to the left of the drawn wave. There is no definite location for a load, as electromagnetic waves spread out in *all directions* away from the source. Our simple illustration merely shows a “cross-section” of a wave radiating *cylindrically* away from the transmitting antenna. The real electromagnetic wave (of which this illustration shows just a portion) would have an electric field resembling ripples on a pond oscillating vertically while radiating away in all horizontal directions from a single point, and have a magnetic field resembling a set of concentric circles oscillating rotationally in the horizontal plane while also radiating in all directions from that same point.

5.4 Maxwell's equations of electromagnetism

The Scottish physicist James Clerk Maxwell mathematically proved the interrelationships of electric and magnetic fields, publishing his results in the book *A Treatise on Electricity and Magnetism* in 1873. His equations were later simplified and consolidated by the work of Oliver Heaviside, commonly expressed in the following forms describing electric fields (\mathbf{E}) and magnetic fields (\mathbf{B}) within a vacuum:

$$\begin{aligned}\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\ \oint \mathbf{B} \cdot d\mathbf{A} &= 0 \\ \oint \mathbf{E} \cdot d\mathbf{s} &= -\frac{d\Phi_B}{dt} \\ \oint \mathbf{B} \cdot d\mathbf{s} &= \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}\end{aligned}$$

We will now explore each of these four equations, relating them to laws codified by other early electrical researchers, as well as describe how they relate to the phenomenon of *electromagnetic waves*.

5.4.1 Gauss's Law of electric fields

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The first of these equations simply describes electric fields as spreading out from a quantity of electric charge (Q). If we imagine a three-dimensional surface completely enclosing an electric charge (e.g. a soap bubble surrounding a charge), the total quantity of electric flux passing through that surface will be the product of the electric field (also known as the electric flux density, \mathbf{E}) and the enclosing area (\mathbf{A}). This is also known as *Gauss's Law of electric fields*.

5.4.2 Gauss's Law of magnetic fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

The second of these equations describes magnetic fields passing through a three-dimensional enclosing surface. If we imagine a magnet completely enclosed within a soap bubble, the sum total of magnetic flux passing out through that soap bubble's surface must equal the sum total of magnetic flux re-entering it somewhere else. Literally, this equation says the product of magnetic field (also known as magnetic flux density \mathbf{B}) and the enclosing area (\mathbf{A}) must be algebraically equal to zero. This is to say, magnetic fields must always exist as *closed loops*, or alternatively we may say there is no such thing as a magnetic monopole (i.e. a solitary North or South pole without its complement, or a *magnetic charge*). In this way magnetic fields are quite different from electric fields which radiate away from discrete electric charges without ever returning. This is also known as *Gauss's Law of magnetic fields*.

5.4.3 Faraday's Law of electromagnetic induction, extended

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

The third equation states that an electric field (\mathbf{E}) will be produced in open space by a varying magnetic flux ($\frac{d\Phi_B}{dt}$). Michael Faraday expressed this phenomenon in terms of voltage induced along the length of a conductor by a time-varying magnetic field, but Maxwell demonstrated mathematically that this same thing happens even in empty space!

5.4.4 Ampère's Circuital Law, extended

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

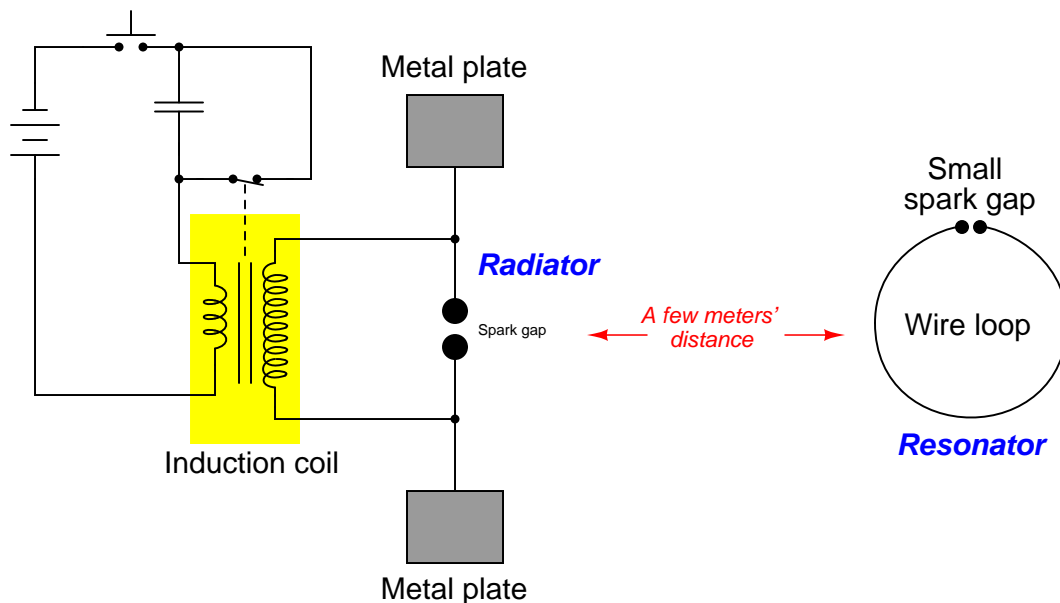
The fourth equation states that a magnetic field (\mathbf{B}) will be produced in open space either by an electric current (I) and/or by a varying electric flux ($\frac{d\Phi_E}{dt}$). The French scientist André-Marie Ampère described the relationship between current passing through a conductor and the magnetic field created around that conductor by the current, but once again Maxwell extended this concept to include magnetic fields generated in empty space. In empty space, where there are no moving electric charges (i.e. current, I) to generate a magnetic field, only the second term of the equation applies with a time-varying electric field substituting for actual electric current.

5.4.5 Proof of electromagnetic waves

Given the complementary relationship between time-varying electric and magnetic fields, Maxwell reasoned, it was possible for a varying electric field to create a varying magnetic field which would then create another varying electric field, and so on. This cause-and-effect cycle could continue, ad infinitum, with fast-changing electric and magnetic fields radiating off into open space without needing wires to carry or guide them, or a tank circuit to sustain the oscillations. In other words, *these complementary fields self-sustain as they travel*, although they do become weaker due to their spreading out of a finite amount of energy over larger and larger volumes of space.

The Prussian Academy of Science offered a reward to anyone who could experimentally validate Maxwell’s prediction, and this challenge was answered by Professor Heinrich Hertz at the Engineering College in Karlsruhe, Germany in 1887, eight years after Maxwell’s death. Hertz constructed and tested a pair of devices: a “radiator” to produce the electromagnetic waves, and a “resonator” to receive them.

A simplified diagram showing Hertz’s experimental device is shown here:



An “induction coil” (a buzzing device constructed of a self-interrupting relay and step-up transformer winding to generate a continuous pulsing waveform of high voltage) provided an extremely noisy (i.e. harmonically rich)¹¹ AC signal to the radiator, while a spark gap at the

¹¹A fundamental property of waves is that any wave-shape, no matter how complex, is mathematically equivalent to a series of perfectly sinusoidal waves of differing frequencies added together. Waveforms with fast rise/fall times, such as the pulse waveforms produced by switched induction coils, contain many such high-frequency “harmonic” signals, and it is these high-frequency signals that created the radiated electromagnetic waves in Hertz’s experiments. The induction coil’s secondary winding inductance combined with the parasitic capacitance of the radiator’s wires and plates provided the means to momentarily sustain some of these high-frequency oscillations long enough to sustain visible sparks at the resonator’s gap.

resonator provided indication that the electromagnetic waves were captured and converted into voltage by the resonator wire.

Both the radiator and the resonator are antennas. The purpose of the transmitting antenna (radiator) is to take high-frequency AC power and radiate that power in the form of electromagnetic waves: self-sustaining electric and magnetic fields propagating outward into open space. The purpose of the receiving antenna is to capture those electromagnetic waves and convert them into an electrical signal (i.e. AC voltage and current). All antennas – from historical to the most modern – behave in fundamentally the same way: energize them with high-frequency AC electrical power, and they will radiate electromagnetic waves at that frequency; expose them to electromagnetic waves, and they will produce a very small AC electrical signal at the same frequency as the incident radiation.

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly **SUMMARIZE THE TEXT** in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Electrical source

Electrical load

Electric field

Magnetic field

Capacitance

Inductance

Potential energy

Kinetic energy

Resonance

Second Law of Thermodynamics

Filter

Electromagnetic wave

Dielectric

Parasitic effect

Transmission line

Sinusoidal decomposition (i.e. Fourier's Theorem)

Fourier series

Fundamental frequency

Harmonic frequency

Antenna

Period

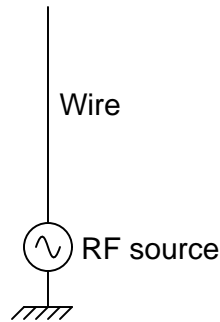
Frequency

Wavelength

Dimensional analysis

6.1.3 Quarter-wave antenna

Shown below is a simple *quarter-wave* or *monopole* antenna, comprised of a single wire projecting vertically from one terminal of an RF voltage source, the other terminal connected to earth ground:



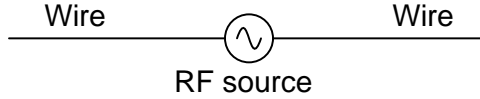
Re-draw this illustration, showing the equivalent inductance and capacitance exhibited by this antenna. Show these properties using actual inductor and capacitor symbols.

Challenges

- How would you expect the inductance and capacitance of this antenna to relate to its physical length? In other words, as you increase the length of an antenna, would its inductance increase or decrease? As the length increases, would its capacitance increase or decrease? What effect would an increased in antenna length have on the antenna's resonant frequency?

6.1.4 Dipole antenna

Shown here is a simple *dipole* antenna, comprised of two equal-length wires projecting from the terminals of an RF voltage source:



Re-draw this illustration, showing the equivalent inductance and capacitance exhibited by this antenna. Show these properties using actual inductor and capacitor symbols.

Challenges

- How would you expect the inductance and capacitance of this antenna to relate to its physical length? In other words, as you increase the length of an antenna, would its inductance increase or decrease? As the length increases, would its capacitance increase or decrease? What effect would an increased in antenna length have on the antenna's resonant frequency?

6.1.5 Antenna dissipation

Although radio transmitter antennae ideally possess only inductance and capacitance (no resistance), in practice they are found to be very dissipative. In other words, they tend to act as large *resistors* to the transmitters they are connected to. Explain why this is. In what form is the dissipated energy manifest (heat, light, or something else)?

Challenges

- Identify a common electrical or electronic component that dissipates energy in multiple forms.

6.1.6 Crystal goblet

A crystal goblet may be shattered if exposed to high-intensity sound. Less volume is required to shatter the goblet if the sound is at such a frequency that it *resonates* with the goblet's natural frequency. That is, there will be maximum transfer of energy to the goblet if the sound waves are transmitted at precisely the goblet's resonant frequency.

Explain how this phenomenon is analogous to the reception of radio waves.

Challenges

- Suppose you determine the optimum frequency to shatter a particular size and style of goblet. If you then wished to shatter a *larger* version of that same goblet, how would you adjust the sound to match to this new goblet size?

6.1.7 RF shielding

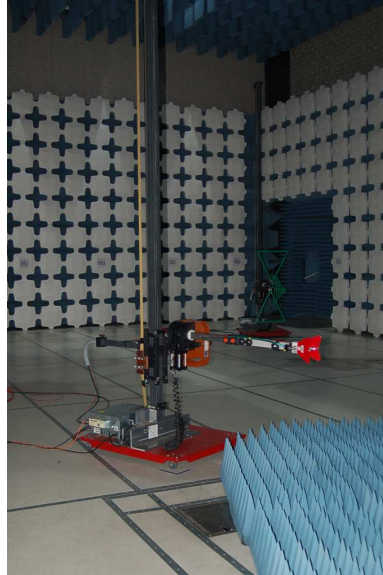
Radio waves may be blocked by any barrier capable of shielding electric *or* magnetic fields. Explain why this is.

Challenges

- How might an electronic device capable of radiating electromagnetic waves be shielded from doing so?

6.1.8 RF measurement chamber

Electronic devices must be tested for emissions of electromagnetic energy, and the best way to do this is inside of an *RF measurement chamber*. The following photograph shows the interior view of such a chamber, the metal object in the camera's view being an adjustable receiver antenna pointed at a device being tested (out of the camera's view):



Special panels mounted to the interior walls of an RF measurement chamber help scatter and absorb stray electromagnetic energy, in order to produce the “quietest” electromagnetic environment possible for the receiving antenna and for the device under test (DUT).

Identify any mitigations you can think of to eliminate all electromagnetic energy from this room except for that emitted by the DUT.

Challenges

- What advantage is there in making the receiving antenna's height and position easily adjustable?

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol⁻¹)

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^4$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared (m³/kg-s²)

Molar gas constant (R) = **8.314462618...** **Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin⁴** (W/m²·K⁴)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Frequencies and wavelengths

Calculate the following frequency (f) and wavelength (λ) values corresponding to the given value:

- $f = 35 \text{ GHz}$; $\lambda =$
- $f = 285 \text{ kHz}$; $\lambda =$
- $f = 9.22 \text{ MHz}$; $\lambda =$
- $f = 5.221 \times 10^5 \text{ Hz}$; $\lambda =$
- $\lambda = 35 \text{ meters}$; $f =$
- $\lambda = 844 \text{ meters}$; $f =$
- $\lambda = 5 \text{ millimeters}$; $f =$
- $\lambda = 4.925 \times 10^3 \text{ meters}$; $f =$

Challenges

- Explain why wavelength is a relevant parameter in radio work.

6.2.4 Frequency-wavelength calculator program

Take this code (written in the C programming language) and complete it so that it can convert radio-wave frequency into wavelength or vice-versa:

```
#include <stdio.h>

int main (void)
{
    int choice;
    float f;

    printf("Calculate either (1) wavelength or (2) frequency from the other: ");
    scanf("%i", &choice);

    if (choice == 1)
    {
        // Your code goes here
    }

    else
    {
        // Your code goes here
    }

    return 0;
}
```

Challenges

- Explain why wavelength is a relevant parameter in radio work.
- Write your program so that it includes metric prefixes as needed to most conveniently format the output.

6.2.5 Transmitter power and field strength

A simple formula relating approximate transmitted power to measured electric field strength, useful when estimating the RF power output by a radio transmitter at a certain distance away based on measurements of the E field, is as follows:

$$\frac{PG}{4\pi D^2} = \frac{E^2}{120\pi}$$

Where,

P = Transmitter output power, in Watts

G = Antenna gain (1 = theoretical “isotropic” antenna)

D = Distance between transmitter antenna and receiving instrument

E = Electric field strength detected by receiving instrument, in Volts per meter

The $4\pi D^2$ term is a calculation of spherical surface area, used to estimate how much the emitted RF energy “spreads out” from the antenna based on the assumption that it radiates away equally in all directions. The 120π term represents the characteristic impedance (Z_0) of free space (approximately 376.99Ω).

First, algebraically simplify this formula as much as possible to eliminate any unnecessary constant values.

Next, algebraically manipulate this formula to solve for P given a measured value E .

Lastly, estimate the following transmitter powers given these measured field strength values and distances:

- $E = 50$ microVolts per meter ; $D = 25$ meters ; Antenna gain = 1 ; $P =$
- $E = 420$ microVolts per meter ; $D = 10$ meters ; Antenna gain = 1 ; $P =$
- $E = 100$ milliVolts per meter ; $D = 3$ meters ; Antenna gain = 1 ; $P =$
- $E = 88$ milliVolts per meter ; $D = 15$ meters ; Antenna gain = 4 ; $P =$
- $E = 0.5$ Volt per meter ; $D = 70$ meters ; Antenna gain = 6 ; $P =$

Challenges

- If you double the distance between the transmitting antenna and the receiving instrument, what will this do to the measured field strength?

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 RF emissions from a computer PCB

Suppose a digital computer circuit with components soldered to a printed circuit board (PCB) keeps failing radiated emissions tests in the laboratory of the manufacturing center where it is being developed. You are tasked with mitigating these emissions so that this computer may be brought to market as a new product.

Explain how a *computer* of all things may unintentionally act as a *radio transmitter*.

Identify practical methods to mitigate the radiated emissions emanating from this computer.

Supposing you also determined this computer produces excessive *conducted emissions* (i.e. RF “noise” detectable on wiring leading to or from the computer). Identify practical methods to mitigate these emissions as well.

Challenges

- Identify some ways in which these emissions might be measured.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge, critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's Vim text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix/Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the `MODEL` modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

`SPICE` is to circuit analysis as `TEX` is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

gnuplot mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

30 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

10 October 2023 – added section title (“Review of inductance and capacitance”) to the Tutorial, and made minor edits to the C code in the “Frequency-wavelength calculator program” Quantitative Reasoning question.

2 June 2023 – added a Quantitative Reasoning question challenging students to complete a computer program written in C to convert between wavelength and frequency.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

8 March 2022 – minor edits to the Tutorial and Introduction chapters.

26 November 2021 – added a Technical Reference section on the Poynting vector.

18 November 2021 – added instructor note on wavelength formula to the “Frequencies and wavelengths” Conceptual Reasoning question.

12 October 2021 – divided Tutorial into sections, and made minor edits to the text.

29 September 2021 – added a new Historical References chapter with a section on early spread-spectrum radio technology, which is shared amongst other modules.

9 July 2021 – added a Case Tutorial section showing how to demonstrate resonance using an inductor, a capacitor, a battery, and an oscilloscope.

10 May 2021 – commented out or deleted empty chapters.

16-23 April 2021 – added a Case Tutorial section, and made minor edits to an existing section discussing the importance of modulation for a simple transmitter as well as ideal antenna length.

14 April 2021 – added more content to the Introduction chapter, and added a new Case Tutorial section.

18 November 2020 – added a new Conceptual Reasoning question.

16 November 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

26-27 July 2020 – added content to the Tutorial, as well as questions.

24 July 2020 – document first created.

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