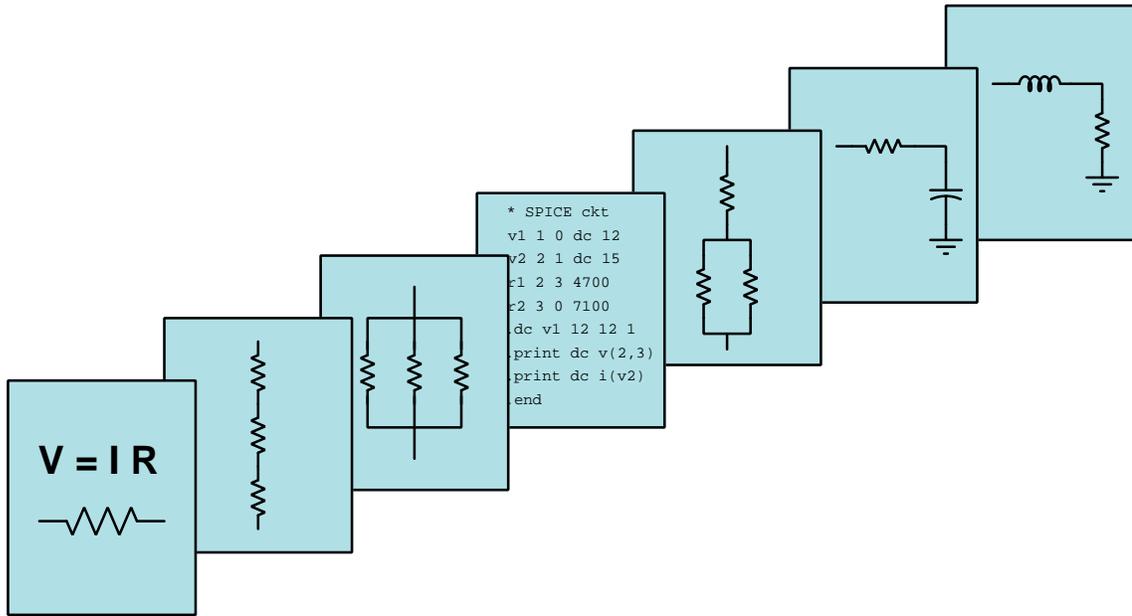


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



THÉVENIN'S AND NORTON'S THEOREMS

© 2019-2024 BY TONY R. KUPHALDT – UNDER THE TERMS AND CONDITIONS OF THE
CREATIVE COMMONS ATTRIBUTION 4.0 INTERNATIONAL PUBLIC LICENSE

LAST UPDATE = 23 OCTOBER 2024

This is a copyrighted work, but licensed under the Creative Commons Attribution 4.0 International Public License. A copy of this license is found in the last Appendix of this document. Alternatively, you may visit <http://creativecommons.org/licenses/by/4.0/> or send a letter to Creative Commons: 171 Second Street, Suite 300, San Francisco, California, 94105, USA. The terms and conditions of this license allow for free copying, distribution, and/or modification of all licensed works by the general public.

Contents

1	Introduction	3
1.1	Recommendations for students	3
1.2	Challenging concepts related to Thévenin's and Norton's Theorems	6
1.3	Recommendations for instructors	8
2	Case Tutorial	9
2.1	Example: Thévenizing series-parallel networks	10
2.2	Example: Thévenizing and Nortonizing a power supply	13
2.3	Example: improvised 3.3 Volt power supply	14
2.4	Example: Thévenizing a multi-source network	17
2.5	Example: Nortonizing a multi-source network	18
2.6	Example: Thévenizing an unbalanced bridge	19
3	Tutorial	21
3.1	Ideal versus real sources	21
3.2	Equivalent electrical networks	23
3.3	Thévenin's and Norton's theorems	27
3.4	Uses for Thévenin's and Norton's Theorems	32
3.4.1	Power supply design testing	33
3.4.2	Input/output impedances	35
3.5	Example: Voltage dropped by a varying load resistance	37
3.6	Example: Unbalanced bridge circuit	39
4	Derivations and Technical References	43
4.1	Derivation of Millman's Theorem	44
4.2	Time delay circuits and Thévenin's/Norton's Theorems	45
5	Animations	49
5.1	Animation of Thévenin's Theorem	50
6	Questions	117
6.1	Conceptual reasoning	121
6.1.1	Reading outline and reflections	122
6.1.2	Foundational concepts	123

6.1.3	Testing a “black box” voltage source	124
6.1.4	Testing a “black box” current source	125
6.1.5	Load lines of equivalent sources	126
6.1.6	Thévenizing a solar cell	129
6.2	Quantitative reasoning	130
6.2.1	Miscellaneous physical constants	131
6.2.2	Introduction to spreadsheets	132
6.2.3	Testing sources under load	135
6.2.4	Thévenin and Norton equivalents of a single-source network	136
6.2.5	Thévenin equivalent of an AC/DC power supply	137
6.2.6	Norton equivalent of an electric arc welder	138
6.2.7	Thévenizing a loaded voltage divider	140
6.2.8	Unbalanced bridge circuit	142
6.3	Diagnostic reasoning	143
6.3.1	Fault current of a battery bank	144
6.3.2	Variable-voltage power source	145
A	Problem-Solving Strategies	147
B	Instructional philosophy	149
C	Tools used	155
D	Creative Commons License	159
E	References	167
F	Version history	169
	Index	171

Chapter 1

Introduction

1.1 Recommendations for students

A *network theorem* is a useful tool for simplifying the analysis of some system, provable on the basis of more fundamental principles and/or theorems. Anyone familiar with the proofs and theorems of geometry (e.g. the Isosceles Triangle Theorem¹) has experienced the benefit of theorems, allowing rigorous analysis of geometric systems without having to constantly resort to low-level definitions and axioms of geometry. Similarly, network theorems are proven tools useful for quickly and easily analyzing complex electrical networks without being limited to fundamental rules such as Ohm's Law and Kirchhoff's Laws.

Thévenin's and Norton's Theorems are two such network theorems, and they find frequent application in electronic circuit analysis. The end-result of each is simple: any linear network, no matter how complex, may be reduced to an equivalent network consisting of one voltage source and one resistor (a Thévenin equivalent) or one current source and one resistor (a Norton equivalent). As with the Superposition Theorem, both Thévenin's and Norton's Theorems are limited to *linear* functions, which means components having stable resistance (R) over their operational ranges and directly-mathematical functions.

Important concepts related to these network theorems include **voltage sources**, **current sources**, **internal resistance**, the effects of **opens** versus **shorts**, **light** versus **heavy** load conditions, **Ohm's Law**, properties of **series** networks, properties of **parallel** networks, **bridge** networks, **linear** versus **non-linear** functions, **equivalent** electrical networks, and **substitution** of source resistances.

A problem-solving technique found throughout this text is the *thought experiment*, whereby we imagine altering some facet of an electrical network and analyze the effects of that alteration, for the purpose of better understanding the network. You may think of this as akin to a real experiment except that we apply known principles to a hypothetical condition rather than set up real components and take real measurements.

¹According to the Isosceles Triangle Theorem, any triangle having two sides of equal length must exhibit equal angles opposite to those two sides.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Another useful reading strategy is to write your own summary page of important principles, especially when those principles mirror each other. For this module I would recommend writing your own summaries of network Thévenization and Nortonization: the steps involved to perform each type of analysis.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate Thévenin's Theorem? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate Norton's Theorem? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to test whether or not two different electrical sources were equivalent to one another? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- In what ways do voltage sources differ from current sources?
- What is the definition of a “series” network?
- What is the definition of a “parallel” network?
- How much internal resistance does an ideal voltage source possess?
- How much internal resistance does an ideal current source possess?
- What makes a load either “heavy” or “light” from the perspective of the source supplying energy to it?
- How would you describe the procedure for “Thévenizing” a network, in your own words?
- How would you describe the procedure for “Nortonizing” a network, in your own words?
- How does either of these theorems help to simplify complex networks?
- What does it mean for two electrical networks to be considered “equivalent” to one another?
- What is a “linear” electrical component, and what are some practical examples of linear components?
- What is a “nonlinear” electrical component, and what are some practical examples of nonlinear components?

- Why is a five-resistor unbalanced bridge circuit impossible to analyze as a series-parallel network?
- Would the bridge circuit be easier to analyze, harder to analyze, or the same difficulty to analyze if it were balanced rather than unbalanced?
- Why is it permissible to “wire-bend” a circuit diagram to aid in its analysis?
- What do we replace a voltage source with when “disabling” it during Thévenin or Norton analysis, and why?
- What do we replace a current source with when “disabling” it during Thévenin or Norton analysis, and why?

1.2 Challenging concepts related to Thévenin's and Norton's Theorems

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Equivalent electrical networks** – the possibility that completely different networks of components may nevertheless *behave* identically from the perspective of two or more connection terminals is non-intuitive for many students new to electric circuits. However, there are many analogues in life that may be used to help explain. For example, a box containing an unknown assortment of items may have precisely the same weight as a different box containing something completely different from the first: i.e. from the perspective of weight sense at the handle when lifting each box, the boxes may behave the same even though their contents may be wholly different.
- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank “scratch” paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts! Both Thévenin's and Norton's Theorems embody this principle by reducing any collection of linear and bilateral electrical components to a single source and a single resistance behaving precisely the same as the original network from the perspective of two connection terminals.
- **Creative application of foundational concepts** – many students enter college-level study of electronics with an educational background stressing rote memorization at the expense of logical reasoning from trusted principles, and as such tend to find circuit analysis daunting where there is no single procedure or single formula always yielding the correct answer(s). The educational solution to this is for the instructor to *constantly* tie all new circuit concepts back to previously-learned principles and show the logical relationships between them, and then with every new circuit-analysis example challenge students to reason from those concepts on their own. When students construct a mental “toolbox” they may apply in a plurality of ways to any new challenge they encounter, they will be as free as any skilled mechanic who knows how and why each of their hand tools functions to freely select and use whichever of those tools is most appropriate at any given time. A word of caution for new instructors: *be very sparing in your direction on how to use those tools, instead engaging with students Socratically when they are “stuck” and cannot see a path to a solution!*

1.2. CHALLENGING CONCEPTS RELATED TO THÉVENIN'S AND NORTON'S THEOREMS⁷

Thévenin's and Norton's Theorems are particularly challenging because they involve *spatial reasoning* more so than other electrical analysis techniques. This means it is particularly important to determine exactly *where* the equivalent network “fits” within the original network as shown. A similar spatial-relations challenge is in identifying series and parallel connections while sources are disabled, and/or when load terminals are shorted together (e.g. finding the Norton current).

A helpful resource for the confused is the *Animations* chapter section showing Thévenization performed step-by-step on a series-parallel electrical network. As usual, the *Case Tutorial* chapter also contains completely-worked examples of networks being reduced to Thévenin and Norton equivalents.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter’s examples. *This is especially important with Thévenin’s and Norton’s Theorems. Do not be surprised when students plead to have you verbally explain everything to them rather than closely inspect the approaches described in the text. Remember that their intelligence is up to the task – always treat them as capable thinkers!*

- **Outcome** – Apply the concept of Thévenin’s Theorem to the simplification of electrical networks

Assessment – Reduce a given electrical network to a Thévenin equivalent; e.g. pose problems in the form of the “Thévenin and Norton equivalents of a single-source network” Quantitative Reasoning question.

Assessment – Reduce a given electrical network to a Thévenin equivalent; e.g. pose problems in the form of the “Thévenin equivalent of an AC/DC power supply” Quantitative Reasoning question.

- **Outcome** – Apply the concept of Norton’s Theorem to the simplification of electrical networks

Assessment – Reduce a given electrical network to a Norton equivalent; e.g. pose problems in the form of the “Thévenin and Norton equivalents of a single-source network” Quantitative Reasoning question.

Assessment – Reduce a given electrical network to a Norton equivalent; e.g. pose problems in the form of the “Norton equivalent of an electric arc welder” Quantitative Reasoning question.

Chapter 2

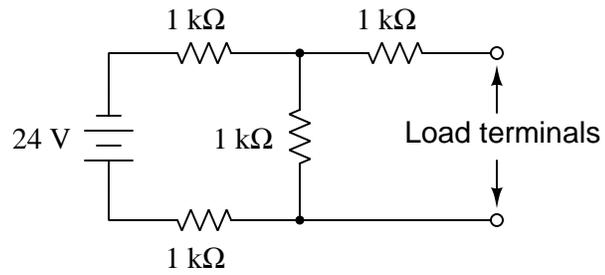
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

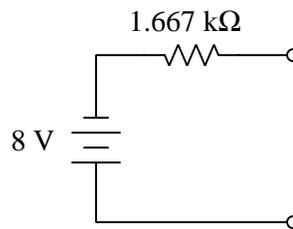
These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: Thévenizing series-parallel networks

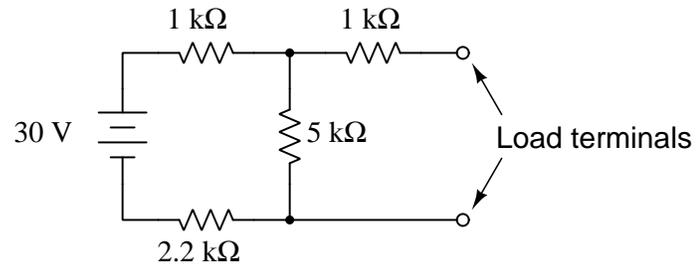
Original network #1:



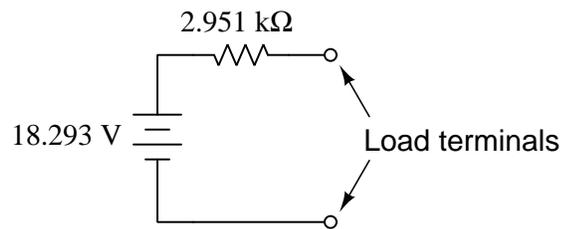
Thévenin equivalent network #1:



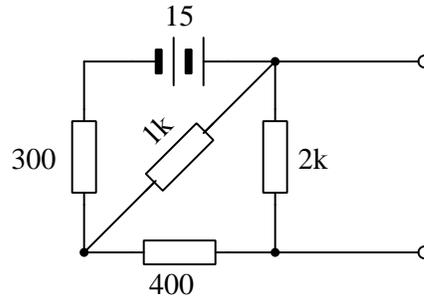
Original network #2:



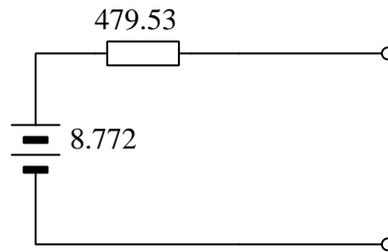
Thévenin equivalent network #2:



Original network #3:

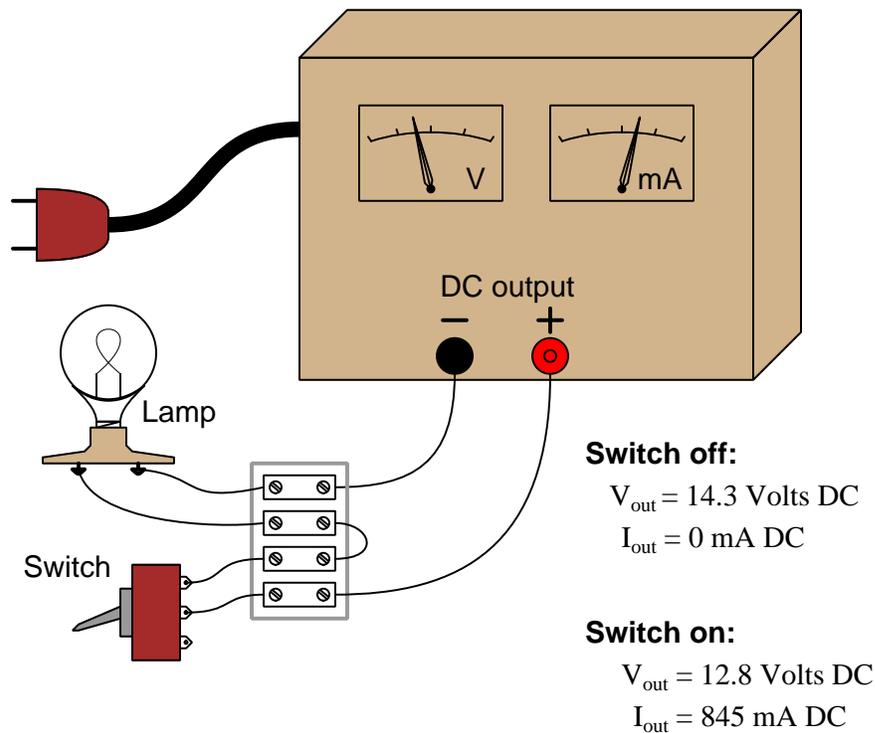


Thévenin equivalent network #3:



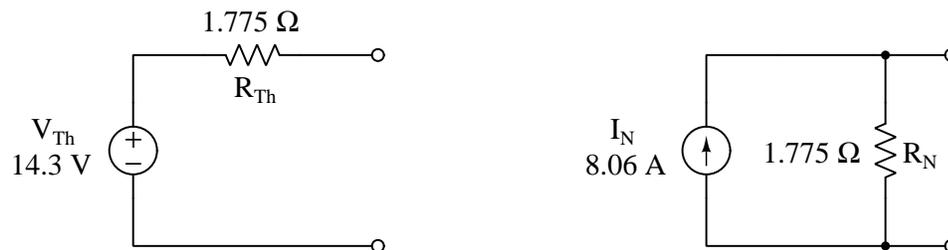
2.2 Example: Thévenizing and Nortonizing a power supply

Based on the following open-circuit and loaded tests of an AC-to-DC power supply (voltage being measured at the power supply's terminals), we may derive a Thévenin equivalent network:



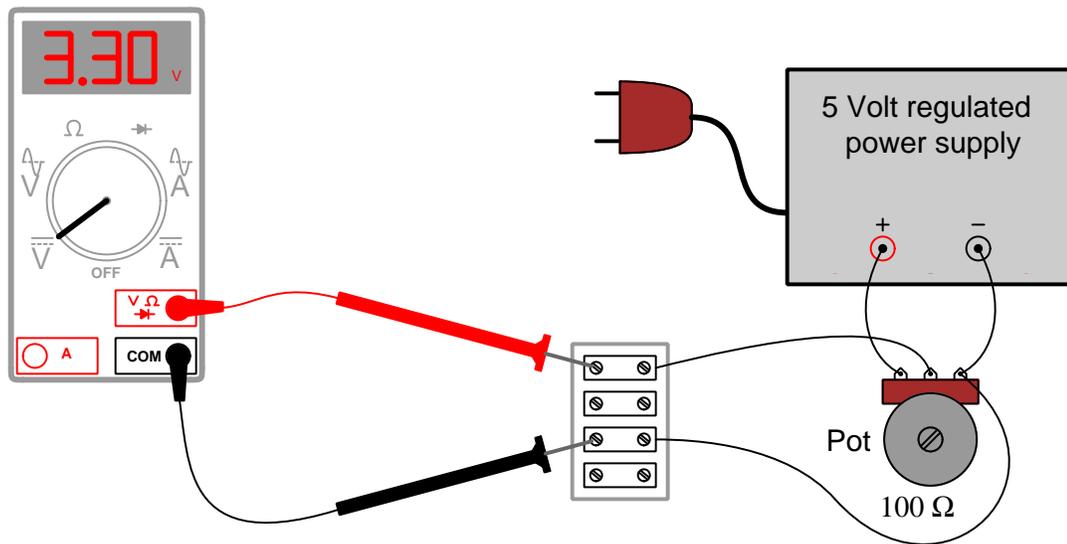
Thévenin voltage is simply the power supply's open-circuit voltage value of 14.3 Volts. By calculating how much that terminal voltage “sagged” under the 845 mA load (14.3 Volts – 12.8 Volts = 1.5 Volts of “sag”), we know how much voltage must be dropped across the Thévenin equivalent resistance at that amount of current. Ohm's Law then provides us with that R_{Th} value.

Thévenin and Norton equivalent networks:

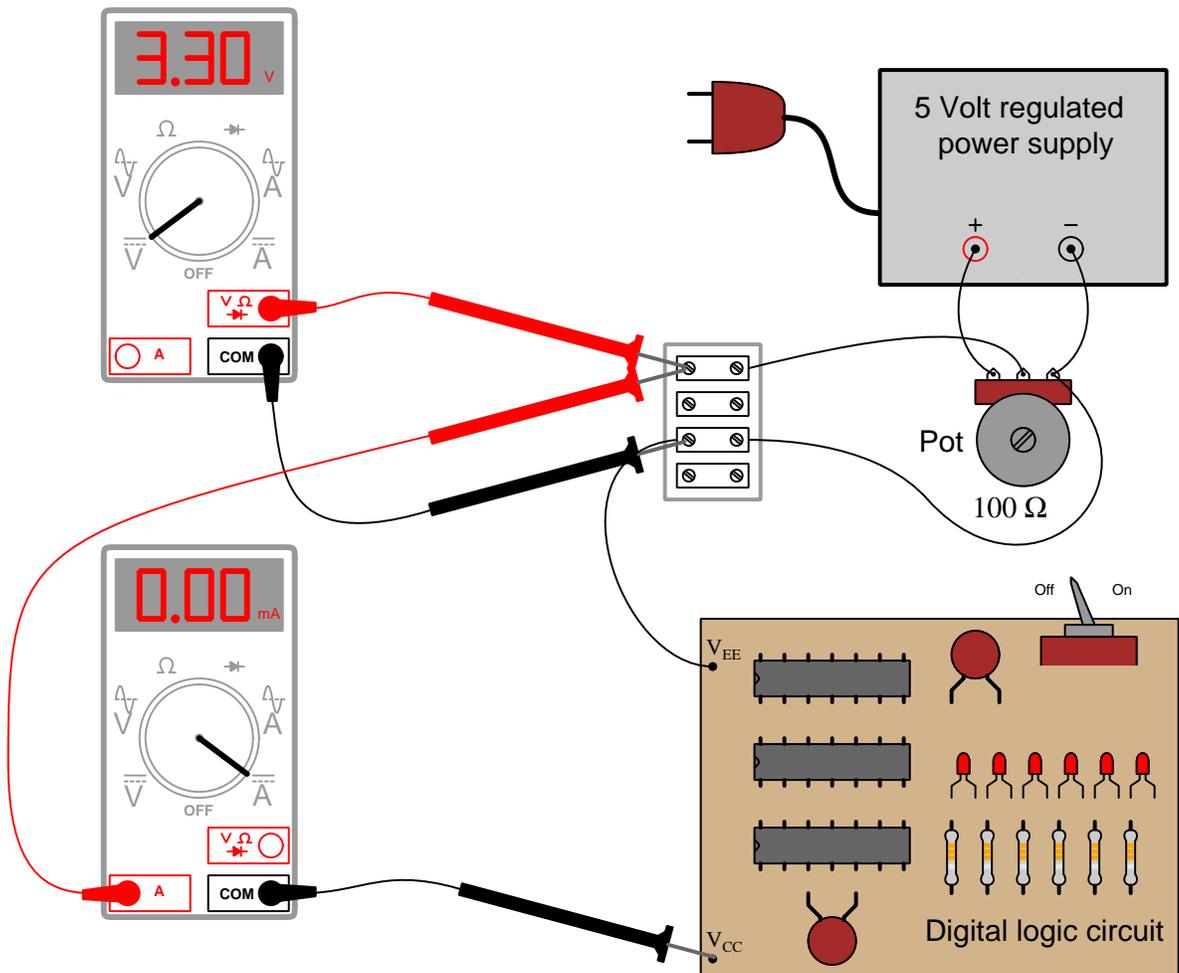


2.3 Example: improvised 3.3 Volt power supply

Suppose that a student builds a digital logic circuit designed to operate with a DC source voltage of 3.3 Volts, but they only have a 5.0 Volt DC power supply on hand. Improvising, the student connects a *potentiometer* to the output of the 5.0 Volt DC power supply terminals and carefully adjusts the knob to the 66% position where she measures 3.3 Volts between the two wires attached to the terminal block:



The student's plan is to connect the V_{CC} (+) and V_{EE} (-) DC terminals of her digital logic circuit to the same two terminal block terminals where the voltmeter connects. An ammeter installed in series between the digital circuit and the terminal block measures load current:



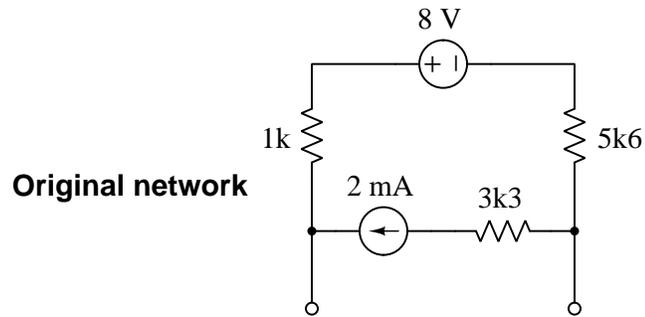
With both meters in place, the student will be able to monitor the stability of the DC power sent to the digital logic circuit as its current varies. The regulated 5.0 Volt power supply is designed to compensate for variations in load current, maintaining a stable 5 Volts for all loads within its rated capability. However, the resistance of the potentiometer will cause some voltage “sag” to occur at the digital circuit’s power terminals as its current varies. This is an inherent limitation of the improvised power supply circuit, achieving a reduced output voltage at the expense of less stability.

This table shows the results of the student's testing:

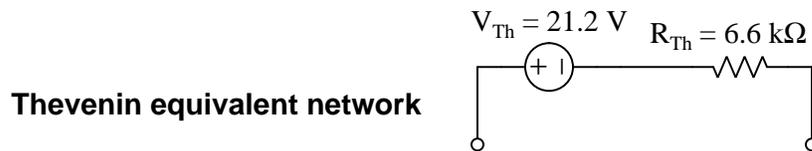
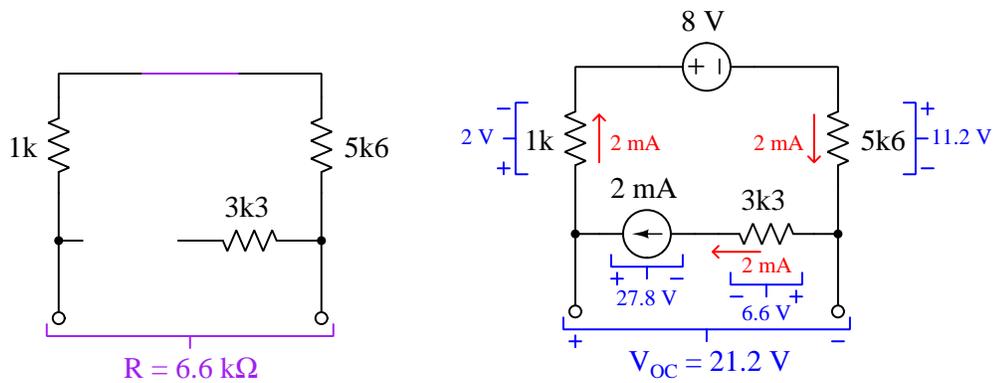
Switch	Current	Voltage
Off	0.00 mA	3.30 V
On	1.00 mA	3.28 V
On	2.00 mA	3.26 V
On	5.00 mA	3.19 V
On	7.40 mA	3.13 V
On	9.25 mA	3.09 V

Using a $100\ \Omega$ potentiometer to reduce 5.0 Volts down to 3.3 Volts is equivalent to an ideal 3.3 Volt voltage source with a series resistance value of $22.44\ \Omega$.

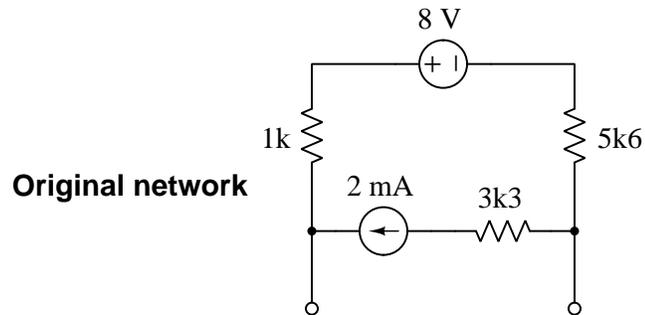
2.4 Example: Thévenizing a multi-source network



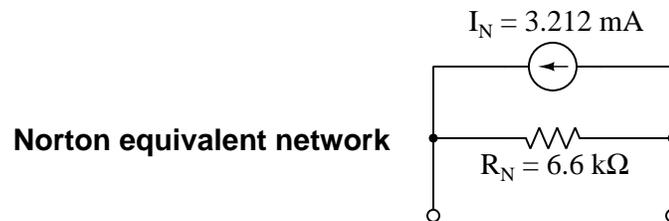
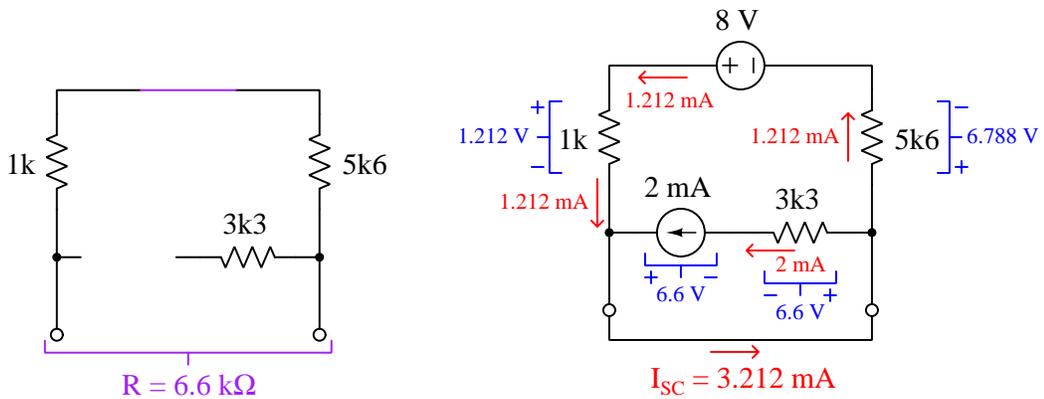
Calculating terminal resistance (left) and open-circuit terminal voltage (right):



2.5 Example: Nortonizing a multi-source network

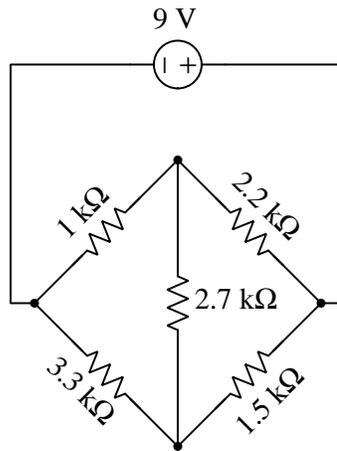


Calculating terminal resistance (left) and short-circuit terminal current (right):

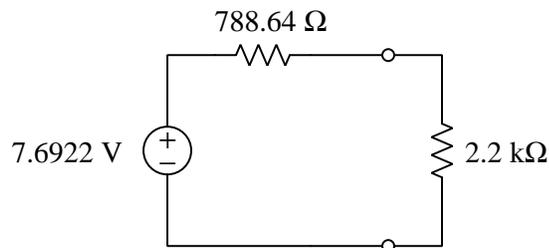


2.6 Example: Thévenizing an unbalanced bridge

Original network:

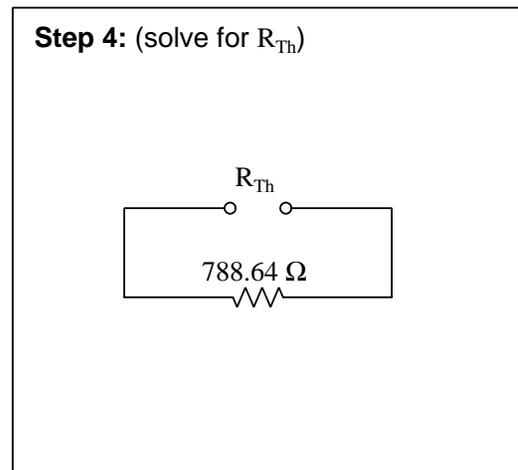
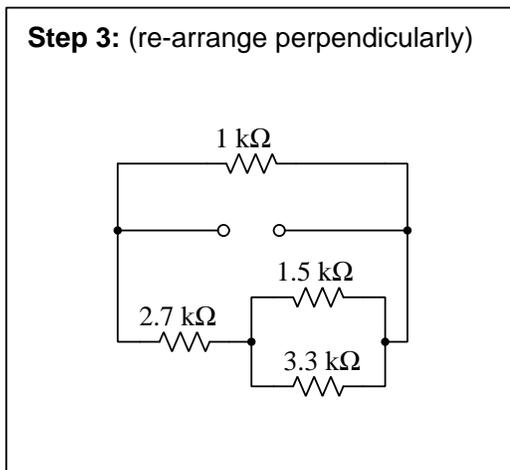
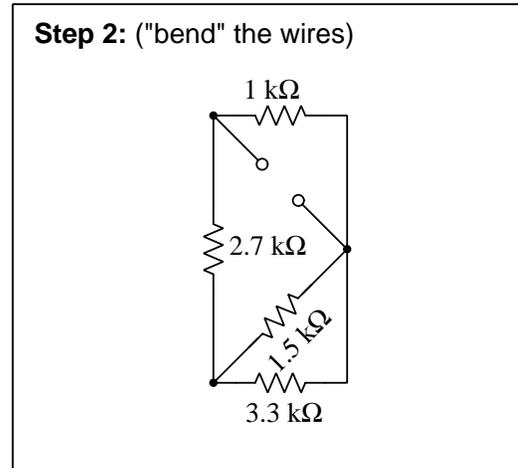
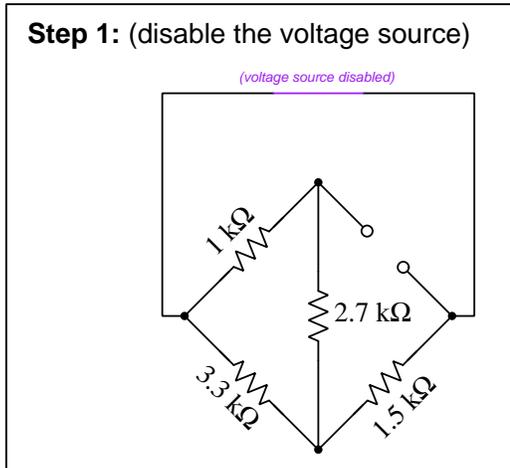


Thévenin equivalent network (from the perspective of the $2.2\text{ k}\Omega$ resistor as the load):



In this Thévenin equivalent network we may easily calculate the voltage across that $2.2\text{ k}\Omega$ load resistor as 5.6624 Volts , because the equivalent network is a simple series circuit. The bridge circuit, however cannot be reduced to a single resistance as we can do with standard series-parallel circuits. In this scenario we *must* use a network theorem (or some other advanced circuit-analysis technique) to determine any of the voltage or currents.

The most difficult part of this analysis is determining the Thévenin equivalent resistance (R_{Th}). Here is a step-by-step reduction of the bridge into a single R_{Th} value from the perspective of the terminals the $2.2\text{ k}\Omega$ resistor used to connect to:

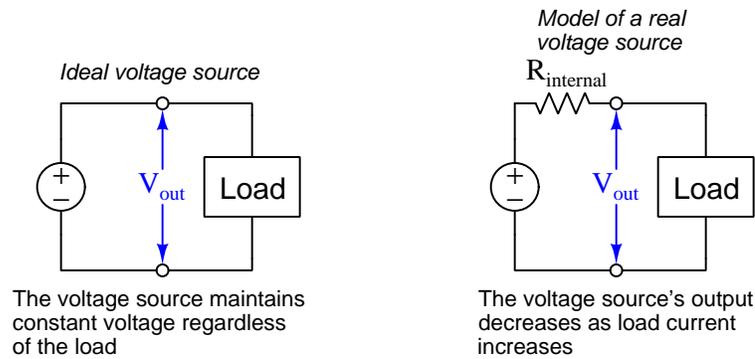


Chapter 3

Tutorial

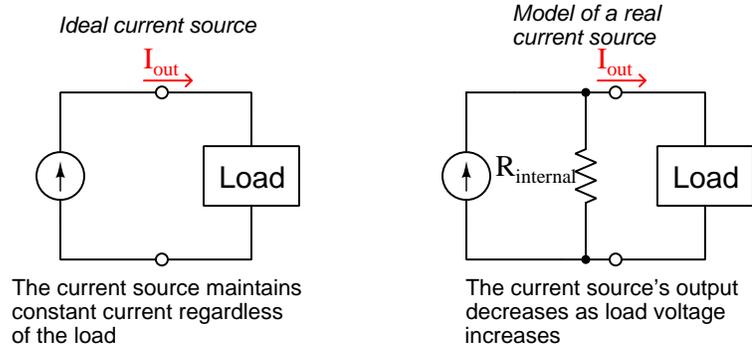
3.1 Ideal versus real sources

A perfect voltage source outputs constant voltage regardless of how much or little current passes through it: no matter the rate of electrical charge carriers flowing through a perfect voltage source, each charge carrier gains the exact same amount of energy from the source. Real voltage sources cannot maintain absolutely constant voltage over wide variations in current, and so we often model real voltage sources as ideal voltage sources connected in series with internal resistance:



The closer to ideal a voltage source is, the less its internal resistance. An ideal voltage source is considered to have zero internal resistance.

Similarly, a perfect current source outputs constant current regardless of how much or little voltage appears across its terminals: no matter how much energy is given to each electrical charge carrier as it passes through the source, the rate that charge carriers move through will remain constant. Real current sources cannot maintain absolutely constant current over wide variations in voltage, and so we often model real current sources as ideal current sources connected in parallel with internal resistance:



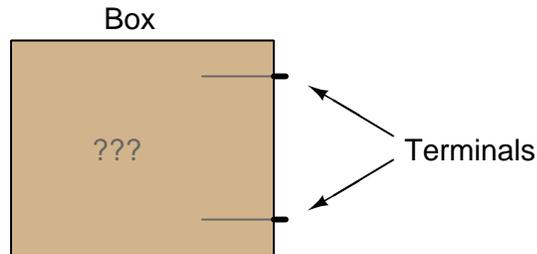
The closer to ideal a current source is, the more its internal resistance. An ideal current source is considered to have infinite internal resistance.

Real sources, both voltage and current, approach ideal behavior as their loads become lighter (i.e. dissipate less power) and deviate more from ideal behavior when their loads become heavier (i.e. dissipate more power). For a voltage source, a “light” load is one drawing little or no current, such as an open-circuit, because less current for any given voltage results in less power. For a current source, a “light” load is one dropping little to no voltage, such as a short-circuit, because less voltage for any given current also results in less power. If you examine the schematic models of real voltage and current sources previously shown, and imagine their respective minimum-power load conditions (i.e. open load for the voltage source and shorted load for the current source) you will see that in both cases the internal resistance becomes irrelevant. For the voltage source, a load drawing no current means that the source’s $R_{internal}$ drops no voltage, leaving full source voltage at the output terminals. For the current source, a load dropping no voltage means that the source’s $R_{internal}$ passes no current, leaving full source current at the output terminals.

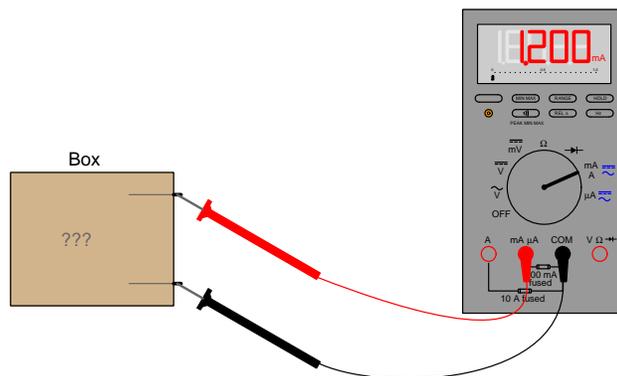
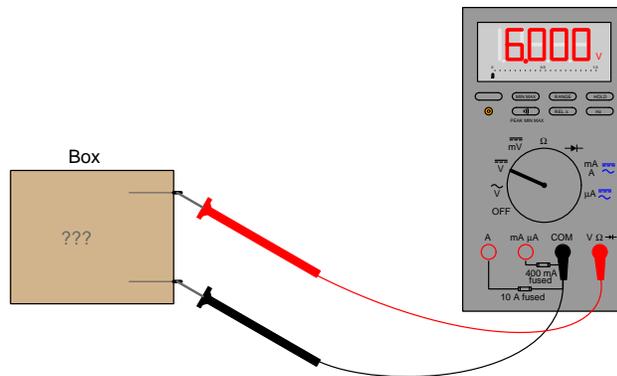
While the internal resistances of real voltage and current sources have many practical implications, the important point to recognize for this tutorial is that *voltage sources ideally have no internal resistance while current sources ideally have infinite internal resistance*. In particular, some electrical network theorems require this knowledge to apply, as one must consider the internal resistance of each source as a factor in the theorem.

3.2 Equivalent electrical networks

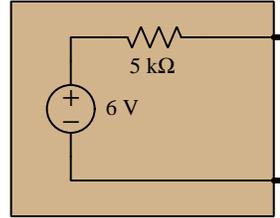
Suppose someone handed you a sealed wooden box with two metal terminals on its exterior. Inside this box, you are told, is a voltage source in series with a resistance connected to those two terminals:



Connecting a digital multimeter to those terminals, you decide to perform an open-circuit voltage test and a short-circuit current test to figure out what those internal component values are, measuring 6 Volts and 1.2 milliAmperes, respectively:

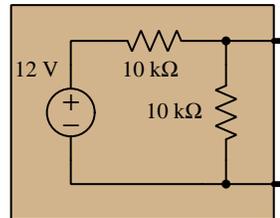


These two measurements provide enough information to identify the values of the voltage source and resistor we were told are both hidden inside the box:



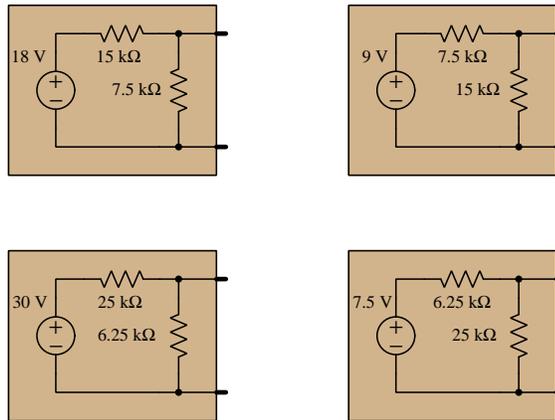
A voltmeter connected to the two terminals will directly sense the 6 Volt voltage source, since its negligible current draw ensures the box's series resistor will drop none of the source's voltage during the test. An ammeter connected to the same two terminals senses the 1.2 milliAmperes that the 6 Volt source is able to drive through the series 5 kiloOhm resistor. In other words, the open-circuit voltmeter test directly indicates the voltage source's value, while the short-circuit ammeter test indicates current which may be combined with the voltmeter measurement to yield the series resistance value by Ohm's Law ($R = \frac{V}{I}$).

Later, though, when we open up this box and look inside, we discover the person who gave it to us did not tell us the whole truth about the circuit inside. Rather than containing a single 6 Volt source and a single 5 kiloOhm resistor, we discover a 12 Volt source and two 10 kiloOhm resistors:



The fact that our hypothetical 6 Volt source and 5 kiloOhm series resistor would in fact behave identically to this more complex network comprised of a 12 Volt source and two 10 kiloOhm resistors means the two electrical networks are equivalent to one another. In other words, these two networks are *indistinguishable* from one another as measured from the perspective of their test terminals, and are guaranteed to behave identically to one another regardless of what we might connect between those terminals. Without peering into the box, there is absolutely no way for us to tell which of the equivalent networks is inside.

What is even more interesting is that other equivalent networks are possible¹. Consider the following networks which behave the same as the single 6 Volt source and series 5 kiloOhm resistor:

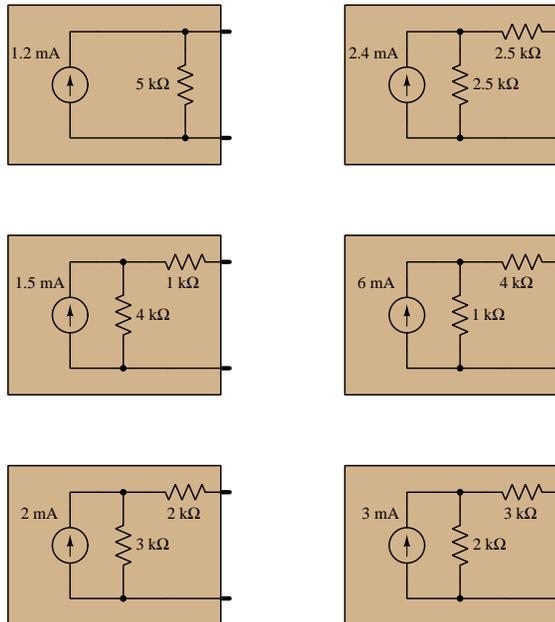


It is highly recommended as an *active reading* exercise to analyze each of these equivalent networks by calculating the amount of voltage between each network's open terminals as well as the amount of current that would flow if each network's terminals were directly "shorted" together with wire. In each case you should find 6 Volts (open-circuited) and 1.2 milliAmperes (short-circuited).

Each of these equivalent voltage-sourced networks behaves identically to the original "sealed box" circuit and to our simple single-source-single-resistor network. They are *all* equivalent from the perspective of their two test terminals, in that every one of them manifests a 6 Volt open-circuit voltage and a 1.2 milliAmpere short-circuit current at their terminals. If some arbitrary load resistance value were connected between their terminals, they would all power that load at identical voltage and current levels.

¹These examples represent just a few possible with (fairly) round-number component values. As it so happens, there exist an *infinite* number of possible voltage-source-and-two-resistor networks equivalent to our original 6 Volt and 5 kΩ network!

We may even construct equivalent networks using *current* sources rather than voltage sources:



As with the voltage-sourced equivalent networks, it is highly recommended as an *active reading* exercise to analyze each of these equivalent networks by calculating the amount of voltage between each network's open terminals as well as the amount of current that would flow if each network's terminals were directly "shorted" together with wire. In each case you should find 6 Volts (open-circuited) and 1.2 milliAmperes (short-circuited).

Just like the set of equivalent voltage-sourced networks shown previously, this set of current-sourced networks also behaves identically to the original "sealed box" circuit and to our simple single-source-single-resistor network. They are *all* equivalent from the perspective of their two test terminals, in that every one of them manifests 6 Volts when open-circuited and 1.2 milliAmpere when short-circuited, and when connected to some load resistance would all power that load at identical voltage and current levels.

The existence of electrically-equivalent networks means it is possible, at least in theory, to take most any complex electrical network and reduce it down to a much simpler network that behaves the same. In the following sections of this Tutorial we will explore two such theorems based on this principle of electrical equivalence, *Thévenin's Theorem* and *Norton's Theorem*, the only difference between the two theorems being that the former yields an equivalent network based on a single *voltage source* while the latter gives an equivalent network based on a single *current source*.

3.3 Thévenin's and Norton's theorems

Two network theorems particularly useful for analyzing complex circuits are *Thévenin's and Norton's Theorems*. Each of these may be stated quite simply:

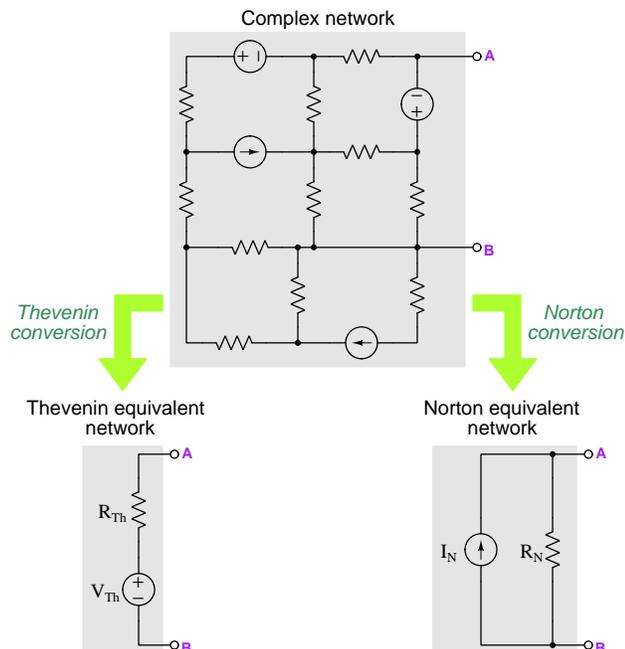
Thévenin's Theorem

Any linear network connected between two terminals may be modeled as an ideal voltage source in series with a single equivalent resistance.

Norton's Theorem

Any linear network connected between two terminals may be modeled as an ideal current source in parallel with a single equivalent resistance.

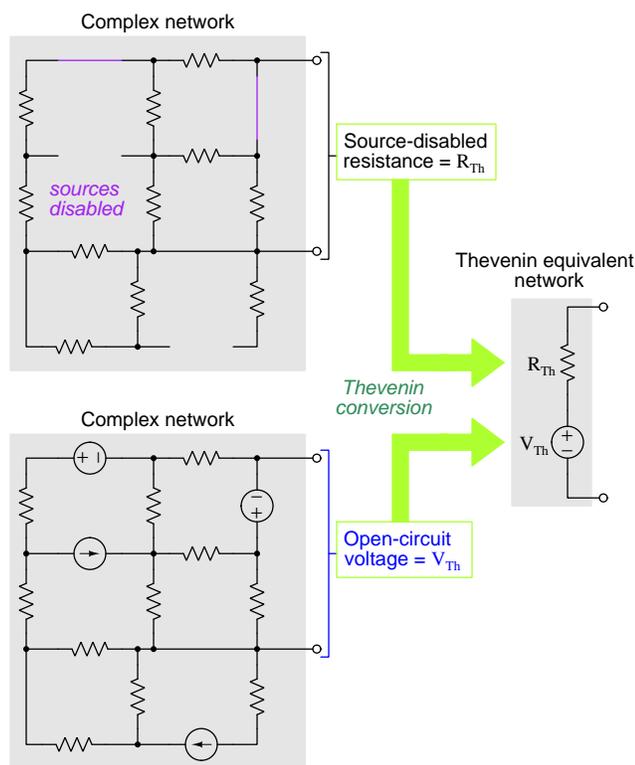
So long as all the components behave linearly², both Thévenin's and Norton's Theorems apply equally well to *any* network no matter how complex that network may be:



²For the purpose of this discussion, a “nonlinear” component is one where voltage and current are *not* directly proportional to each other at any portion of the component’s range. Another way to consider this characteristic would be to think of such components as having unstable resistance values. Not only are semiconductor PN junctions quite nonlinear, but so are some non-semiconductor devices such as incandescent lamps (where filament resistance changes with temperature, which in turn is affected by voltage and current). Some texts also add the condition of *bilateral symmetry* to the use of the Superposition Theorem, meaning that components must behave the same when energized in both polarities. However, the concept of linearity applied across a range of values both positive and negative covers this criterion. If a component does not exhibit bilateral symmetry, then it cannot be linear across its *entire* range of permitted values.

Converting a complex network into a Thévenin equivalent network consists of two steps. First, you will need to determine the complex network's open-circuit³ voltage between its two output terminals. This open-circuit output voltage will be the value of V_{Th} in the Thévenin equivalent network. Next, you will need to determine the complex network's resistance as measured from its output terminals and with all of its internal sources disabled and substituted with their equivalent resistances (i.e. voltage sources replaced by shorts and current sources replaced by opens). This resistance will be the value of R_{Th} in the Thévenin equivalent network.

Illustrating the “Thévenization” process:

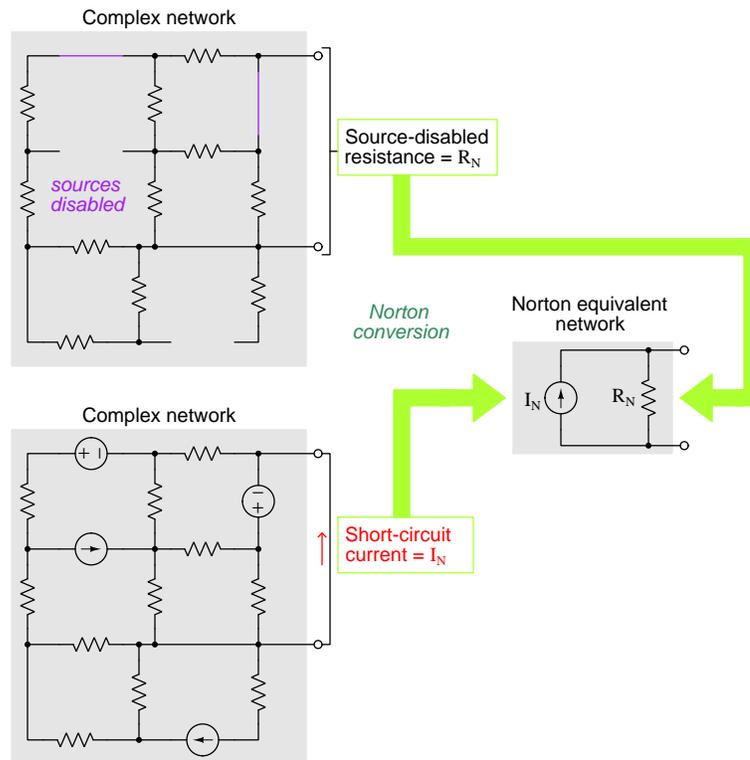


The rationale for these steps is easiest to understand by examining the Thévenin equivalent network. Once we accept the fact that there is such a thing as a Thévenin equivalent network for any complex linear-component network, whose behavior as measured from its output terminals is indistinguishable from the complex network's, it becomes easy to see why V_{Th} must be the open-circuit voltage and R_{Th} must be the resistance with V_{Th} replaced by a short. Open-circuiting the Thévenin equivalent network *must* result in V_{Th} being present between the output terminals because R_{Th} would drop zero voltage in an open-circuit (i.e. zero-current) state. Resistance between the output terminals likewise *must* be equal to R_{Th} if V_{Th} is disabled by a short. If the Thévenin equivalent behaves like this, then so must the complex network it is equivalent to.

³“Open-circuit” in this context means nothing else connects to the network's terminals but the network itself. For example, if the network happened to be powering a load from those two terminals, you would omit that load before determining voltage between those terminals.

Converting a complex network into a Norton equivalent network likewise consists of two steps. First, you will need to determine the complex network's short-circuit current between its two output terminals. This short-circuit output current will be the value of I_N in the Norton equivalent network. Next, you will need to determine the complex network's resistance as measured from its output terminals and with all of its internal sources disabled and substituted with their equivalent resistances (i.e. voltage sources replaced by shorts and current sources replaced by opens). This will be the Norton resistance value (R_N). You will note that this second step is identical to the step involved for calculating R_{Th} in a Thévenin equivalent network.

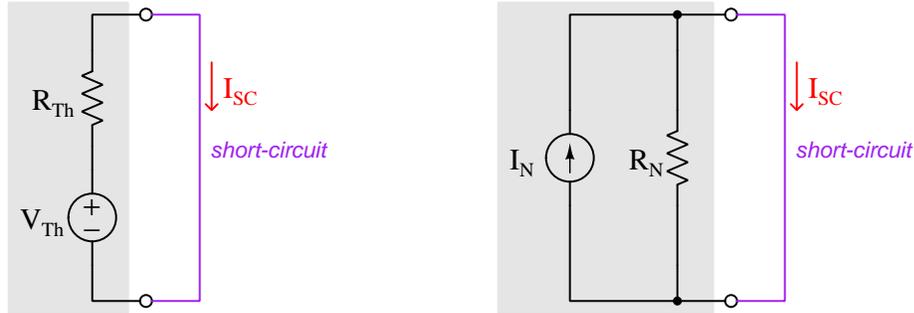
Illustrating the “Nortonization” process:



As with Théveninization, the rationale for the steps required to “Nortonize” a network is easiest to understand by examining the Norton equivalent network. Once we accept the fact that there is such a thing as a Norton equivalent network for any complex linear-component network, whose behavior as measured from its output terminals is indistinguishable from the complex network's, it becomes easy to see why I_N must be the short-circuit current and R_N must be the resistance with I_N replaced by an open. Short-circuiting the Norton equivalent network *must* result in I_N being present at the output terminals because the short would “shunt” the entirety of the source's current past R_N . Resistance between the output terminals likewise *must* be equal to R_N if I_N is disabled by an open. If the Norton equivalent behaves like this, then so must the complex network it is equivalent to.

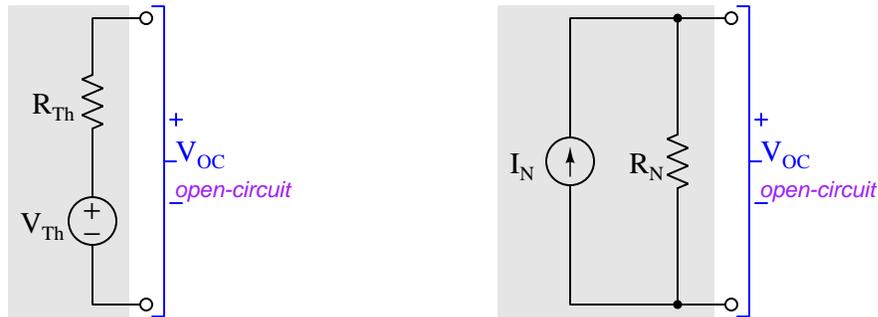
Not surprisingly, it is fairly easy to convert from any circuit's Thévenin equivalent to its corresponding Norton equivalent network, and vice-versa. A good way to understand this process is to apply the same “thought experiment” of shorting and opening the output terminals of each network that we performed in obtaining either a Thévenin or Norton equivalent from the original circuit. If a Thévenin and Norton network pair are to be equivalent to the same circuit (i.e. and therefore equivalent to each other), they must behave identically when short-circuited and when open-circuited.

First, let's imagine a Thévenin and Norton pair under short-circuit conditions:



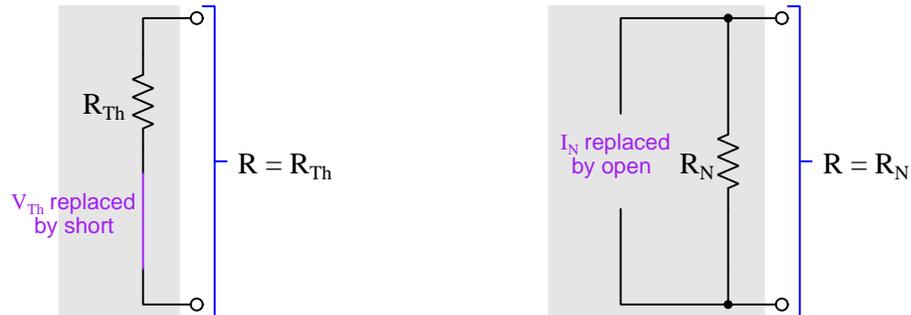
Calculating the short-circuit current (I_{SC}) in the Thévenin circuit is as simple as applying Ohm's Law ($I_{SC} = \frac{V_{Th}}{R_{Th}}$). In the Norton circuit, I_{SC} is simply equal to I_N because the short-circuit “jumper” wire bypasses all current around R_N . Therefore, the equivalent Norton current value must be equal to the Thévenin voltage divided by the Thévenin resistance ($I_N = \frac{V_{Th}}{R_{Th}}$).

Next, let's imagine a Thévenin and Norton pair under open-circuit conditions:



Calculating the open-circuit voltage (V_{OC}) in the Norton circuit is as simple as applying Ohm's Law ($V_{OC} = I_N R_N$). In the Thévenin circuit, V_{OC} is simply equal to V_{Th} because R_{Th} carries no current and therefore drops none of V_{Th} 's voltage, allowing all of V_{Th} to appear between the networks' terminals. Therefore, the equivalent Thévenin voltage value must be equal to the Norton current times the Norton resistance ($V_{Th} = I_N R_N$).

Converting between Thévenin and Norton internal resistance values is even simpler: *they are identical*. The equality between R_{Th} and R_N is evident if you disable each source (replacing it with its ideal resistance⁴) and imagine measuring resistance between the network terminals:



Both Thévenin and Norton equivalents are useful *tools* for analyzing complex electrical networks, and the choice between the two is arbitrary. If you prefer to think in terms of voltage sources, you are free to use Thévenin equivalents. If current sources are preferred, you are free to use Norton equivalents. And, if you end up wishing to switch from one type to the other, the conversion (as we have just seen) is very simple: Thévenin and Norton resistance values for equivalent networks are identical, and their respective voltage and current source values may be computed using Ohm's Law ($V_{Th} = I_N R_N$ and $I_N = \frac{V_{Th}}{R_{Th}}$).

As interesting as Thévenin's and Norton's Theorems may be, their utility may not be obvious. The practical value of any network theorem is to somehow *reduce the complexity of a given problem* for the purpose of making it easier to solve. In the next section we will explore this concept in more detail. In the sections following that, we will explore some specific example problems illustrating the power of Thévenin's and Norton's Theorems.

⁴Another way to conceptualize the disabling of a source is to consider what we would have to replace it with in order to ensure a "zero" condition. For example, in order to ensure a zero-voltage condition when disabling a voltage source, it makes sense that we would substitute a short for the voltage source since the guaranteed effect of a short is a condition of zero voltage. Similarly, in order to ensure a zero-current condition with disabling a current source, it makes sense we would replace that current source with an open since we know the guaranteed effect of an open is to halt current.

3.4 Uses for Thévenin's and Norton's Theorems

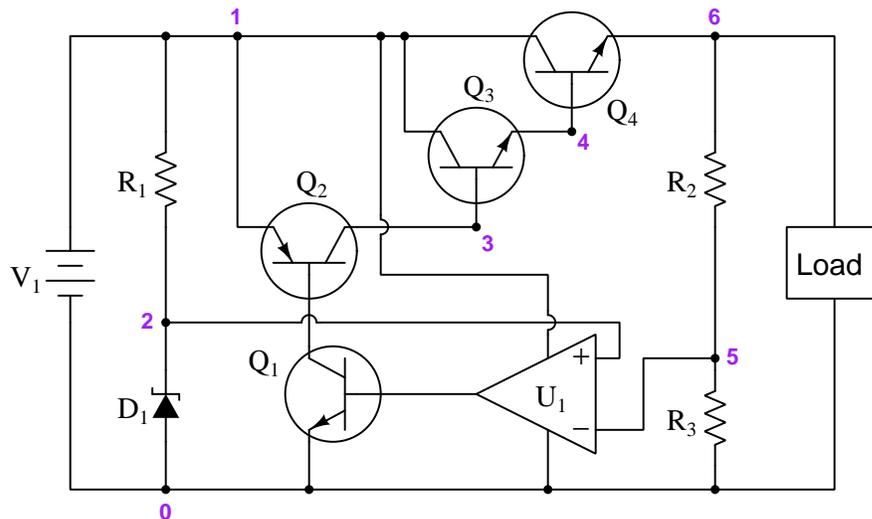
The real-world practicality of any theorem converting a complex network of electrical components into a simpler (equivalent) network may not be apparent at first. Sure, this might be useful for the kinds of convoluted resistor networks you see in a textbook, but how does this help us analyze *real* circuits? Here we will briefly discuss two important applications:

- Load-testing power supply designs
- Selecting components to achieve required input or output impedance

3.4.1 Power supply design testing

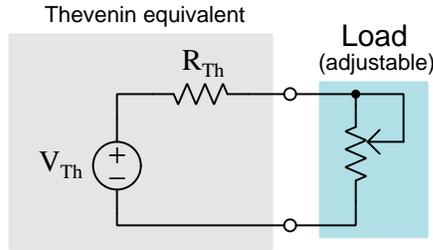
A *power supply* is any circuit designed to condition electrical power between a given source and a specific load, often performing such tasks as transforming power between different voltage and current levels and/or regulating voltage or current to prescribed values. Such circuits can be quite complex, containing many individual components, but at least for certain ranges of operation we may apply either Thévenin's Theorem or Norton's Theorem to reduce such a complex circuit to a single equivalent network consisting of a perfect voltage or current source and an internal resistance. If we apply one of these network theorems and thereby derive an equivalent network, we may then very quickly and easily run a series of calculations predicting how stable that power supply's output voltage and/or current will be for a range of load conditions.

The following schematic diagram shows a standard “pass”-style voltage regulator circuit, designed to take electrical power from any imperfect voltage source and deliver it at nearly constant voltage to a load whose resistance may vary substantially:



If we are careful to define the limits of operation for this circuit, it is possible to apply either Thévenin's or Norton's Theorem to it and arrive at an elementary network behaving very nearly the same. An ideal voltage source plus low-valued internal resistance makes sense to model a power supply circuit designed to maintain constant load resistance, making Thévenin's Theorem a natural fit for this problem.

Once we have determined⁵ the values of V_{Th} and R_{Th} in the equivalent network, it then becomes a trivial matter to explore how stable the circuit's output terminal voltage will be for varying load conditions because the power supply and attached load resistance is nothing more than a simple series circuit:



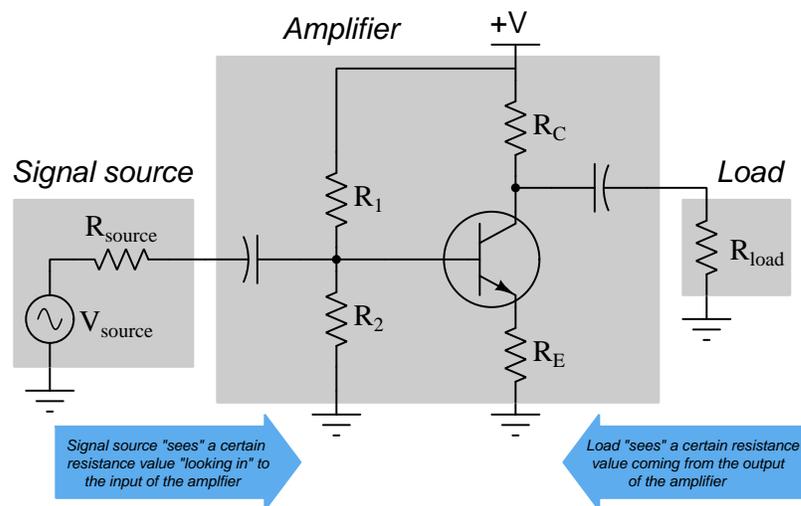
Rather than have to perform a long series of calculations over and over again to determine load voltage and current for different load resistance values applied to the original circuit, we may perform a long series of calculations *once* to derive the Thévenin equivalent network and then after that perform *very simple* calculations over and over again for each proposed load resistance value. Not only will this be less tedious for us, but it may also allow us to gain insights we might otherwise miss if we are cognitively burdened by the complexity of the original power supply circuit.

⁵Details of how to perform this determination on the voltage-regulator circuit shown will not be given here. For now we are merely exploring *why* these theorems are useful, not precisely *how* they apply to all scenarios. In order to explain how to apply Thévenin's or Norton's Theorem to this voltage regulator circuit, we would first have to understand how transistors, Zener diodes, and operational amplifiers function which is far beyond the scope of this Tutorial.

3.4.2 Input/output impedances

An important parameter for many electronic circuits is something called *impedance*, which if you have not yet studied this term is equivalent to *resistance* for the sake of this discussion. Any circuit receiving power or signal from an external source will present a load to that source as it extracts energy from it, and we may model that loading effect as though it were a single resistance (called the *input impedance* of that circuit). Similarly, any circuit delivering power to an external load will itself manifest some amount of internal resistance, which is the Thévenin or Norton equivalent *output impedance* of that circuit.

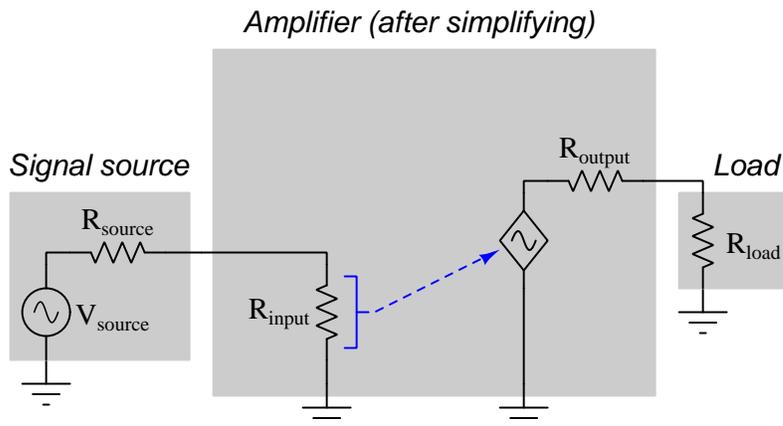
Below we see a schematic diagram of a single-transistor amplifier receiving an AC signal from a signal source and delivering an amplified version of that signal to a resistive load:



In order to operate at peak efficiency, the input impedance (resistance) of the amplifier must match the signal source's internal resistance to satisfy the Maximum Power Transfer Theorem. If we know the source's internal resistance value, it is possible to choose component values within the amplifier circuit to make its input impedance match. In other words, we treat the AC signal source as a Thévenin equivalent network and apply Thévenin/Norton reduction techniques to the amplifier to reduce its input circuitry to a single equivalent resistance, then determine what the amplifier's component values must be to yield that matching amount of input resistance.

Similarly, the amount of internal impedance (resistance) exhibited by the amplifier at its output is a function of its component values, and by applying Thévenin/Norton reduction techniques we may find out exactly what dictates this internal value too.

Here we see the simplified⁶ equivalent of the amplifier circuit, its input presenting a simple resistance “load” to the signal source, and its output transferring power to the load through an internal output resistance:

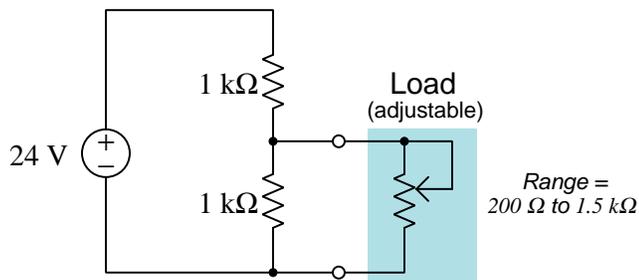


In summary, we find Thévenin’s and Norton’s Theorems to be useful tools to *model* complex networks as much simpler networks. This in turn allows us to assess in a much easier and intuitive fashion how the original (complex) network will perform when interfaced with other sources and loads.

⁶Again, details of how to perform this simplification from the original amplifier circuit to this equivalent will not be given here. For now we are merely exploring *why* these theorems are useful, not precisely *how* they apply to all scenarios. In order to explain how to apply either Thévenin’s or Norton’s Theorem to this amplifier circuit, we would have to also explain how transistors function.

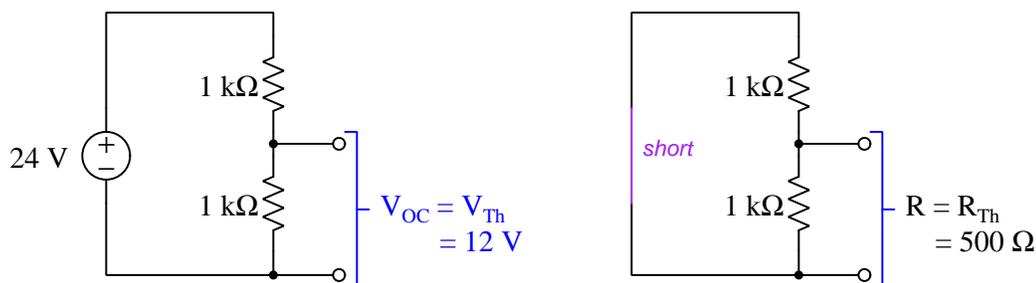
3.5 Example: Voltage dropped by a varying load resistance

For our first example we will consider a voltage divider circuit providing power to a load of varying resistance, the variable load represented by a potentiometer wired as a *rheostat*⁷:



In an unloaded condition, this voltage divider consisting of identical resistors and a 24 Volt source would output half of its internal source voltage (i.e. 12 Volts). However, we know that under loaded conditions the divider's output voltage will “sag” below the ideal value of 12 Volts. Our task is to determine how much that voltage sags as the load's resistance varies between $200\ \Omega$ and $1.5\text{ k}\Omega$.

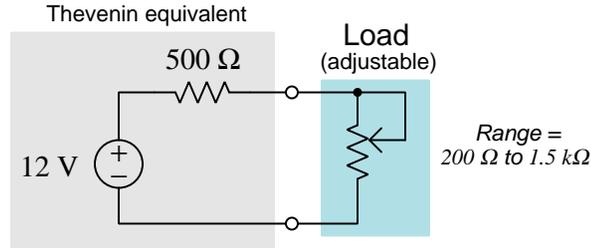
The most direct method for calculating the divider's output voltage under varying load conditions would be to analyze this as a series-parallel circuit for different values of R_{load} . However, Thévenin's Theorem provides a faster solution. Our first task is to “Thévenize” the voltage divider network to obtain an equivalent network that will behave the same as viewed from its output terminals:



From the open-circuit and disabled-source tests, we see the Thévenin equivalent network must consist of a 12 Volt source in series with a 500 Ohm resistance.

⁷A “rheostat” is simply a variable resistance. By connecting the wiper of the potentiometer to one of its outside terminals, it simply functions as a variable resistance.

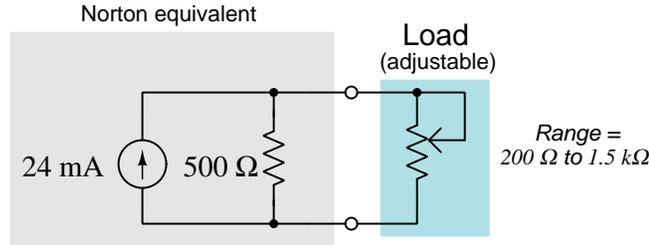
With this Thévenin equivalent in hand, our loaded voltage divider circuit becomes a *simple-series circuit* rather than an series-parallel circuit, thereby simplifying our task of computing load voltage at different load resistance varies:



Now, instead of having to perform series-parallel analysis for each and every load resistance value, we may simply use the voltage divider formula:

- $V_{Load} @ 200 \text{ Ohms} = (12 \text{ V}) \left(\frac{200 \Omega}{500 \Omega + 200 \Omega} \right) = 3.429 \text{ Volts}$
- $V_{Load} @ 1000 \text{ Ohms} = (12 \text{ V}) \left(\frac{1000 \Omega}{500 \Omega + 1000 \Omega} \right) = 8.000 \text{ Volts}$
- $V_{Load} @ 1500 \text{ Ohms} = (12 \text{ V}) \left(\frac{1500 \Omega}{500 \Omega + 1500 \Omega} \right) = 9.000 \text{ Volts}$

We could have just as easily reduced the original voltage divider circuit to a *Norton* equivalent instead⁸, and we would still have a circuit easier to analyze for multiple load resistance values than the original series-parallel network:



- $V_{Load} @ 200 \text{ Ohms} = (24 \text{ mA}) \left(\frac{1}{\frac{1}{500 \Omega} + \frac{1}{200 \Omega}} \right) = 3.429 \text{ Volts}$
- $V_{Load} @ 1000 \text{ Ohms} = (24 \text{ mA}) \left(\frac{1}{\frac{1}{500 \Omega} + \frac{1}{1000 \Omega}} \right) = 8.000 \text{ Volts}$
- $V_{Load} @ 1500 \text{ Ohms} = (24 \text{ mA}) \left(\frac{1}{\frac{1}{500 \Omega} + \frac{1}{1500 \Omega}} \right) = 9.000 \text{ Volts}$

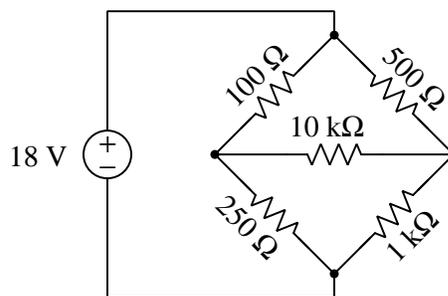
Bear in mind that Thévenin and Norton equivalents make *any* circuit with a varying load as easy to analyze as simple-series and simple-parallel networks, respectively.

⁸Since we already know V_{Th} and R_{Th} , we may quickly calculate the Norton equivalent values $I_N = \frac{V_{Th}}{R_{Th}} = 24 \text{ mA}$ and $R_N = R_{Th} = 500 \Omega$.

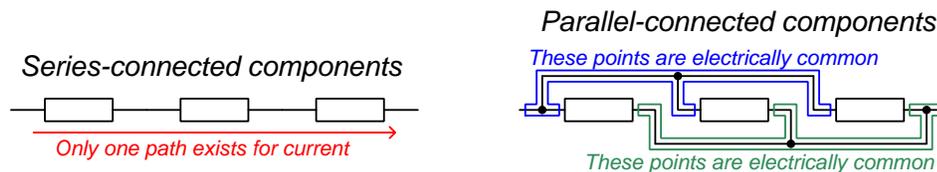
3.6 Example: Unbalanced bridge circuit

In the previous example we saw how either Thévenin's or Norton's Theorem could be used to simplify a series-parallel circuit into a simple-series or simple-parallel circuit (respectively) to facilitate modeling a variable load resistance. The resulting Thévenin or Norton equivalent network saved us the trouble of performing longer calculations, and therefore was a useful problem-solving strategy. In this example, however, we will see a more powerful application of Thévenin's and Norton's Theorems – one where the original circuit is *impossible* to analyze as a series-parallel network.

Consider the following *bridge* circuit, where each of the five resistance values are unique:



Not only are all resistors unequal in value, but their ratios are unequal as well. This fact makes the bridge circuit *unbalanced*⁹ and therefore challenging to analyze. As tempting as it may be to try to condense these five resistors into a single equivalent resistance representing a series-parallel network, we will find this task impossible because *no two resistors are in series or in parallel with each other*. Recall the respective definitions of series and of parallel, shown in the following illustrations:



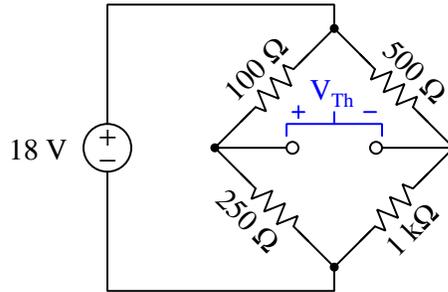
Series is defined as having only one path for current. *Parallel* is defined as the components all connected across the same two sets of electrically-common points. You will search in vain for any two resistors in the bridge circuit sharing just one path for current between them, or sharing the same two sets of electrically-common points. Bridge circuits are *not* series-parallel networks, and so we cannot use the techniques of series-parallel analysis to reduce any bridge circuit to a single equivalent resistance!

⁹Recall that a *balanced* bridge circuit has equal ratios between pairs of resistors on opposite sides of the bridge, resulting in zero voltage across the middle resistor (in this case, the 10 kΩ resistor). Balanced bridge circuits are easy to analyze because the lack of voltage across the middle resistance means there is no current flowing through that resistor, and therefore we may effectively regard that resistor as not being there at all (i.e. equivalent to an open). This is why balanced bridge circuits may be considered nothing more than a pair of two-resistor voltage dividers.

How does either Thévenin's or Norton's Theorem help us in this case? We may apply either theorem by arbitrarily selecting one of the resistors in the bridge circuit and call it our *load*, and then reducing the rest of the bridge circuit to a Thévenin or Norton equivalent. The reduction of the single source and four resistors to a single source and single resistor will greatly simplify our determination of voltage and/or current for that arbitrarily-selected resistor. This will become clear by example.

Suppose we wish to know the voltage and current for the 10 kΩ resistor in this bridge circuit. We may declare this resistor to be the load, then Thévenize or Nortonize the *rest* of the bridge circuit to arrive at an equivalent network of just one source and one resistance, and finally re-connect our 10 kΩ load to this equivalent network to solve for its voltage and current. The temporary removal of the 10 kΩ resistor from the original bridge circuit is the key to success: eliminating this resistor simplifies the bridge into a series-parallel network, for which we have applicable tools to analyze.

First, removing the 10 kΩ “load” resistor and solving for the Thévenin voltage (V_{Th}):



Kirchhoff's Voltage Law informs us that the voltage between the open terminals (V_{Th}) must be equal to the difference in voltage dropped across the two lower¹⁰ resistors.

$$V_{Th} = V_{250\Omega} - V_{1000\Omega}$$

$$V_{Th} = (18 \text{ V}) \left(\frac{250 \Omega}{100 \Omega + 250 \Omega} \right) - (18 \text{ V}) \left(\frac{1000 \Omega}{500 \Omega + 1000 \Omega} \right)$$

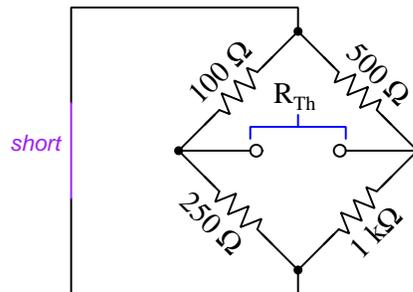
$$V_{Th} = 12.857 \text{ V} - 12 \text{ V}$$

$$V_{Th} = 0.857 \text{ V (+ on left)}$$

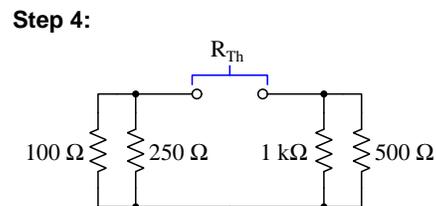
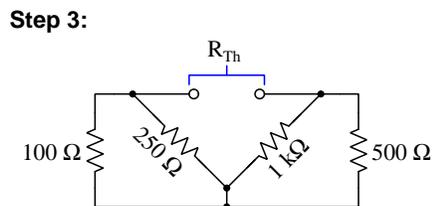
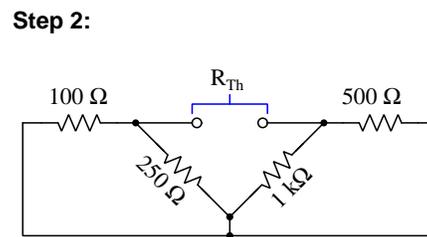
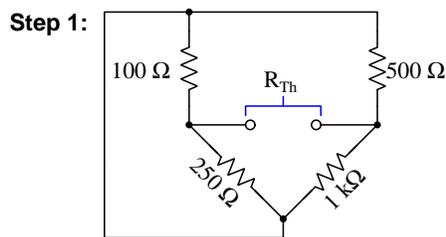
When we re-draw the circuit as a Thévenin equivalent connected to the 10 kΩ “load” resistor, the Thévenin equivalent will have a V_{Th} voltage value of 0.857 Volts.

¹⁰Also, equal to the difference in voltage dropped across the two upper resistors. The choice of which resistor-pair to use is arbitrary.

Next, we determine the Thévenin equivalent resistance of the four remaining resistors in the bridge circuit. To do this, of course, we must replace the 18 Volt source by a shorting wire:



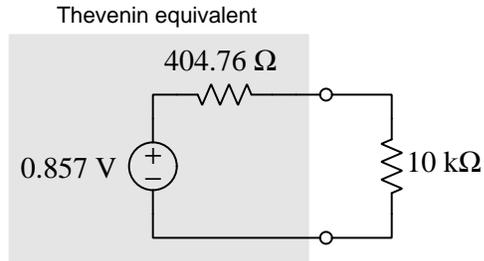
The result is a terminal-to-terminal resistance equal to the 100 Ω and 250 Ω resistors in parallel with each other, in series with the 500 Ω and 1 kΩ resistors in parallel with each other. This is perhaps the most confusing aspect of the Thévenization/Nortonization process for this bridge circuit, and it may be more easily grasped by viewing a sequence of “wire-bending” steps:



$$R_{Th} = \frac{1}{\frac{1}{100\ \Omega} + \frac{1}{250\ \Omega}} + \frac{1}{\frac{1}{1000\ \Omega} + \frac{1}{500\ \Omega}} = 404.76\ \Omega$$

When we re-draw the circuit as a Thévenin equivalent connected to the 10 kΩ “load” resistor, the Thévenin equivalent will have an internal R_{Th} resistance of 404.76 Ohms.

Re-drawing the circuit as a Thévenin equivalent plus the 10 kΩ “load” resistor:

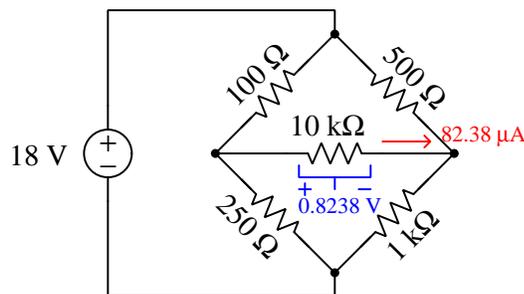


Now, it is a very simple matter indeed to calculate the 10 kΩ resistor’s voltage and current using the properties of series circuits and Ohm’s Law:

$$I = \frac{V_{Th}}{R_{total}} = \frac{0.857 \text{ V}}{404.76 \Omega + 10000 \Omega} = 82.380 \mu\text{A}$$

$$V_{load} = IR_{load} = (82.380 \mu\text{A})(10000 \Omega) = 0.8238 \text{ V}$$

Therefore, the 10 kΩ resistor in the original bridge circuit must drop 0.8238 Volts and pass 82.380 μA of current:



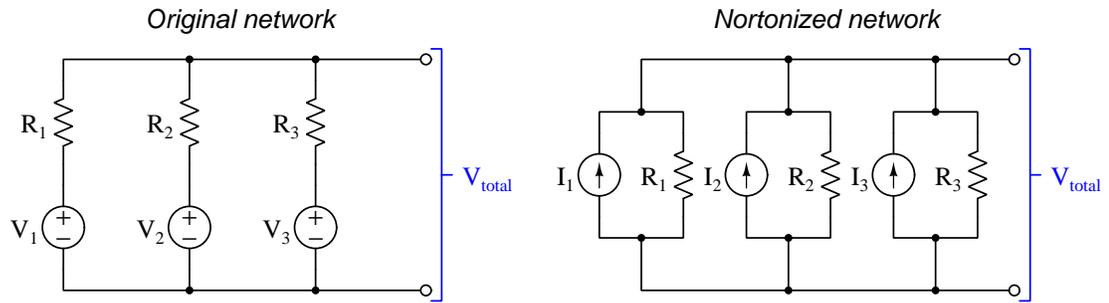
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Derivation of Millman's Theorem

Whenever we encounter a network of multiple voltage sources connected together in parallel, each with its own associated series-connected resistance, there is an easy way to calculate that parallel network's voltage value. This method treats each source-resistor pair as a Thévenin voltage source and then converts each into a corresponding Norton current source (current source and parallel-connected resistance). In the converted network all sources and resistors are in parallel with each other, which means all the current source currents simply add together and pass through a parallel-equivalent resistance value determined by the individual resistors:



Voltage across the Nortonized network is fairly simple to calculate, multiplying total current by the total (parallel-diminished) resistance in accordance with Ohm's Law ($V = \frac{I}{R}$):

$$V_{total} = \frac{I_1 + I_2 + I_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Expanding back to the original network with three voltage sources, we must re-write the numerator of the fraction to show each Norton-equivalent current as the quotient of a voltage source and its respective resistance:

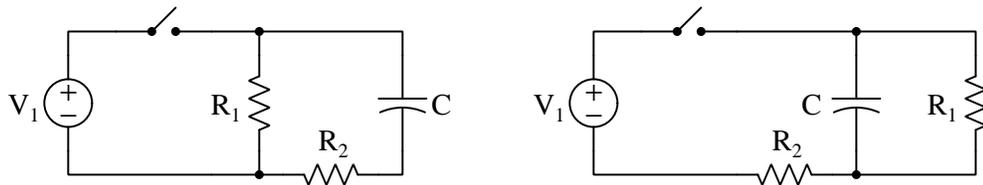
$$V_{total} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

That last equation is the mathematical expression of *Millman's Theorem*, useful for predicting voltage in any network comprised of parallel-connected voltage sources (with individual resistances).

4.2 Time delay circuits and Thévenin's/Norton's Theorems

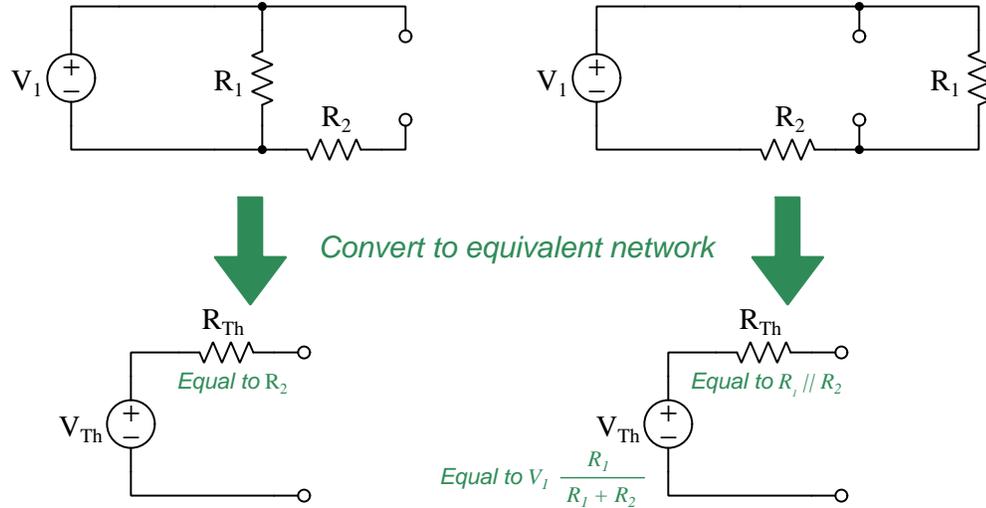
Circuits containing resistors along with capacitors or inductors typically result in voltage and current values that vary over time. Determining starting and final values for these parameters, as well as calculating the “time constant” ($\tau = RC = \frac{L}{R}$) can be challenging especially if the circuit in question consists of more than just a single resistor, single source, and single reactive (C or L).

Take for example the following two RC circuits, each one containing one capacitor but two resistors:



If the capacitor begins in a completely de-energized state with the switch in the open position, capacitor voltage for each circuit is guaranteed to begin at 0 Volts because energy stored in any capacitance always manifests as a voltage across that capacitance. When we close the switch, we would expect the capacitor in both circuits to begin energizing as they absorb energy from the voltage source V_1 , and this means their voltages will rise over time. However, what is not as clear upon first inspection is what the final voltage will be for either capacitor after the switch has been left in its closed state for a long time, as well as how we might calculate the time constant $\tau = RC$ given the fact each circuit has *two* resistors and not just one.

Obviously, an RC circuit containing just one resistor and one capacitor instead of two resistors and one capacitor would be simpler to analyze, so why don't we just apply either Thévenin's Theorem or Norton's Theorem to each circuit in order to reduce it to just a single source and single resistance? Choosing the capacitor to be the load, we will convert the rest of each circuit into its Thévenin equivalent:

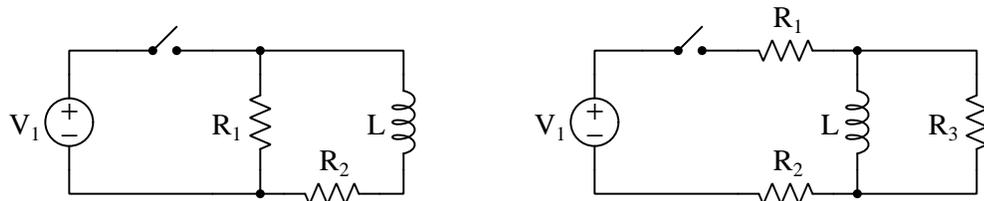


Recall that determining either the Thévenin or the Norton equivalent resistance of any complex network consists of disabling all sources in the original circuit and determining resistance as measured from load terminal to load terminal. In the left-hand circuit the disabled V_1 is replaced by a short which completely bypasses R_1 and leaves only R_2 remaining. In the right-hand circuit the disabled V_1 (again, replaced by a short) causes R_1 and R_2 to be in parallel with each other from the perspective of the two load terminals. Thus, the time constant for the left-hand circuit is simply $\tau = R_2 C$ while the time constant for the right-hand circuit will be $\tau = \frac{C}{\frac{1}{R_1} + \frac{1}{R_2}}$.

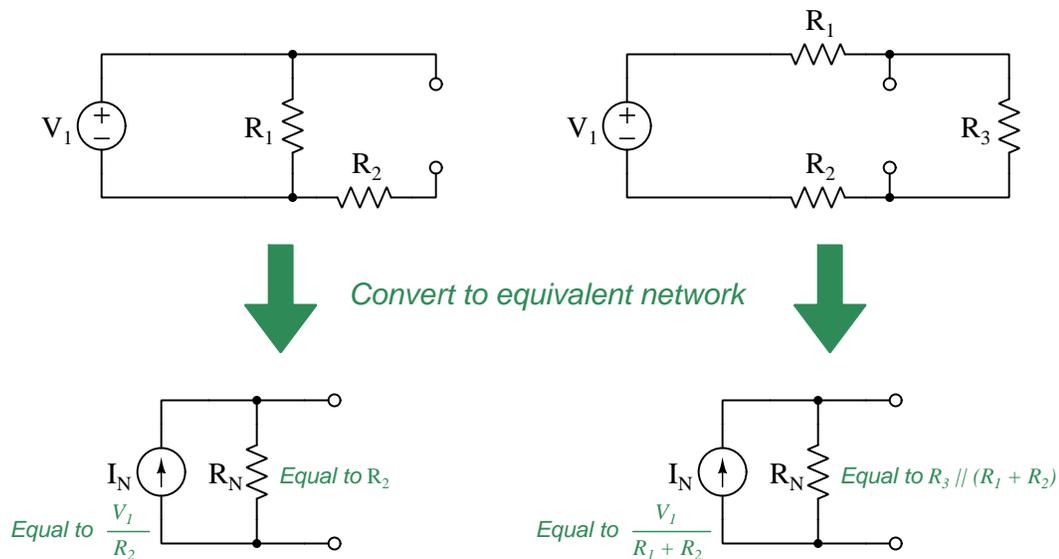
Recall that determining the Thévenin equivalent voltage of any complex network consists of calculating voltage from one load terminal to the other with all original sources active. Here we see that the left-hand circuit's open-circuit load terminal voltage must simply be V_1 , while the right-hand circuit's open-circuit terminal voltage will be the result of V_1 powering a two-resistor voltage divider.

Re-attaching the capacitor C to the load terminals of the Thévenin equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = R_{Th} C$ and the final capacitor voltage after full energization is V_{Th} .

Norton's Theorem, of course, works just as well as Thévenin's Theorem for this purpose. To illustrate, we will apply Norton's Theorem to a pair of resistor-inductor circuits to determine how multiple resistors affect the inductors' energization:



Removing the inductor from each circuit and converting the remaining networks into Norton equivalents:



The process for determining Norton equivalent resistance is exactly the same as it is for Thévenin equivalent resistance: simply disable all original sources and determine what an ohmmeter would register if connected between the two load terminals. In the left-hand circuit we see once again that R_1 is of no effect to the Norton equivalent value of R_N , while in the right-hand circuit R_N is a series-parallel combination of the three original resistors.

Recall that determining the Norton equivalent current of any complex network consists of calculating current passing through a shorting wire placed between the two load terminals with all the original sources active. In the left-hand circuit we see that this short-circuit current is the limited only by R_2 and so $I_N = \frac{V_1}{R_2}$. In the right-hand circuit we see only R_1 and R_2 serving to limit current, and so $I_N = \frac{V_1}{R_1 + R_2}$.

Re-attaching the inductor L to the load terminals of the Norton equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = \frac{L}{R_N}$ and the final inductor current after full energization is I_N .

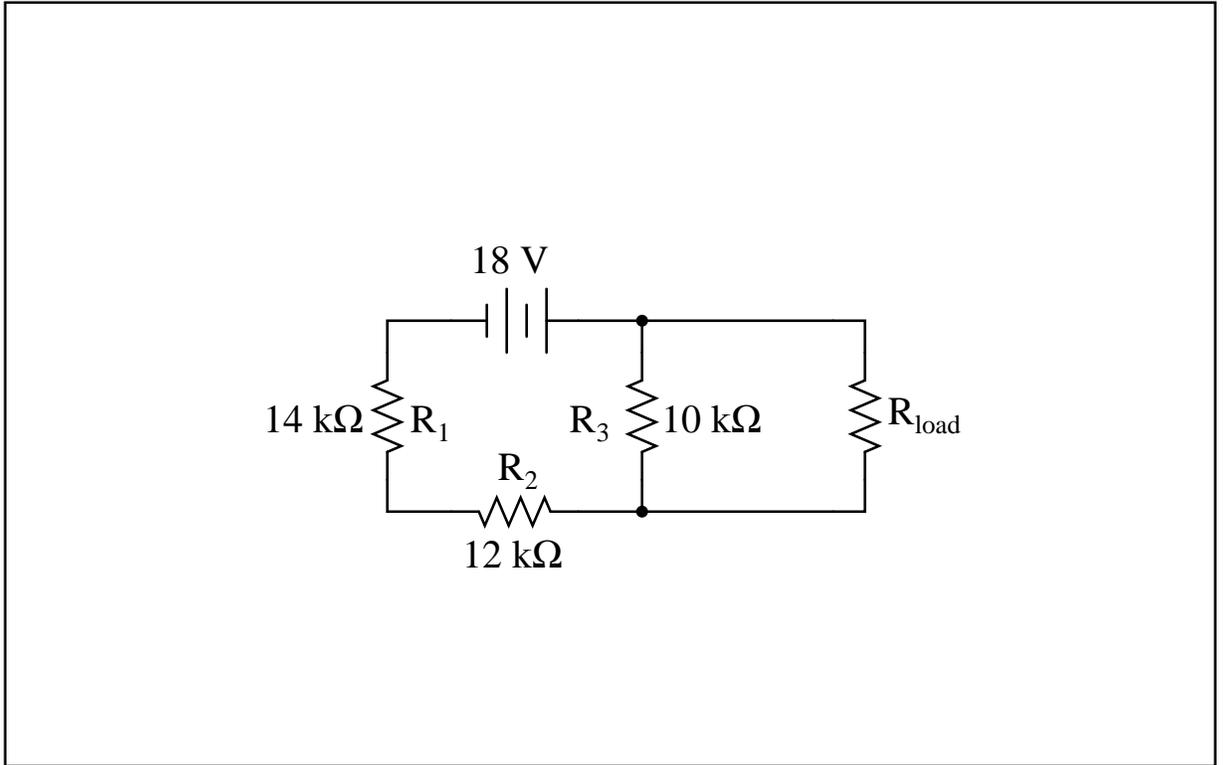
Chapter 5

Animations

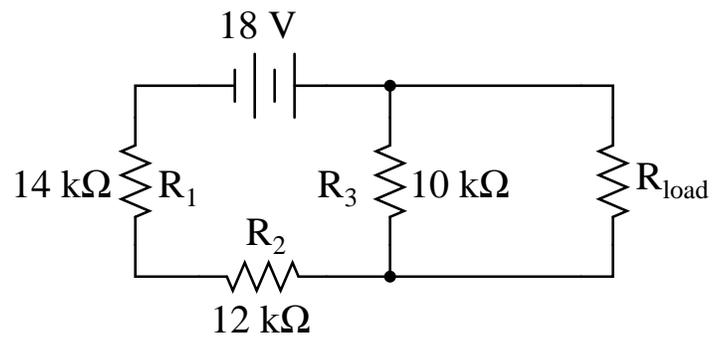
Some concepts are much easier to grasp when seen in *action*. A simple yet effective form of animation suitable to an electronic document such as this is a “flip-book” animation where a set of pages in the document show successive frames of a simple animation. Such “flip-book” animations are designed to be viewed by paging forward (and/or back) with the document-reading software application, watching it frame-by-frame. Unlike video which may be difficult to pause at certain moments, “flip-book” animations lend themselves very well to individual frame viewing.

5.1 Animation of Thévenin's Theorem

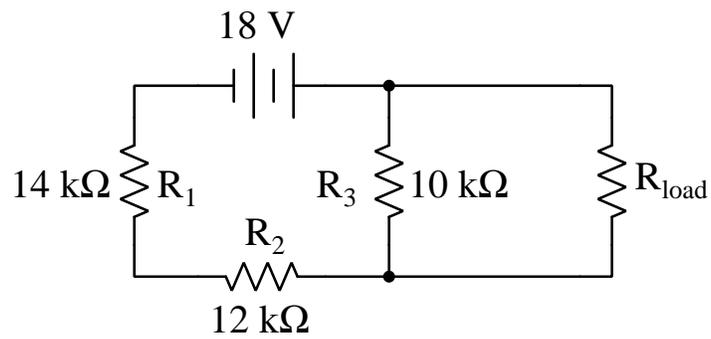
The following animation shows the steps involved in “Thévenizing” a circuit.



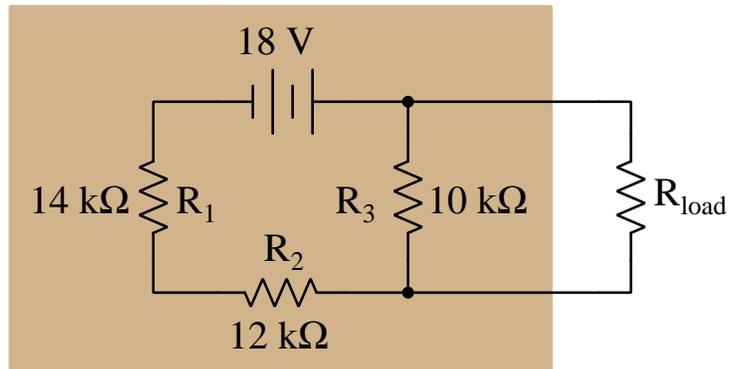
This is our original circuit:



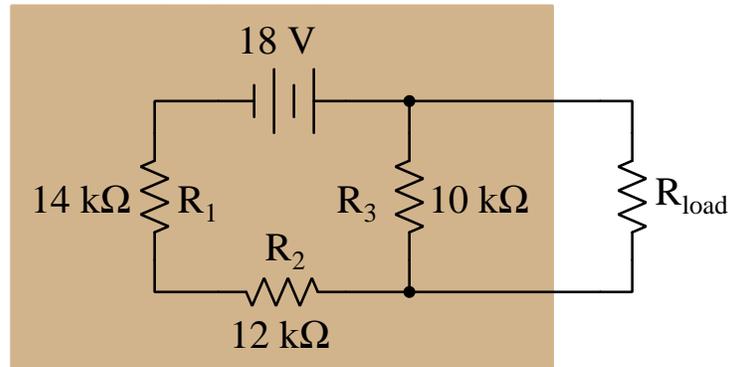
*We may use Thevenin's Theorem
to simplify this portion of the circuit . . .*



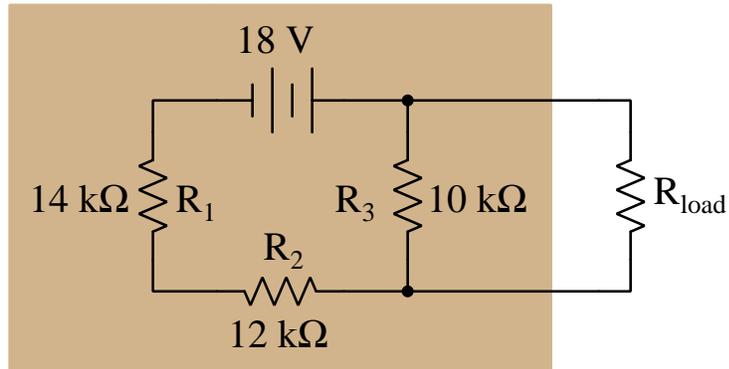
*We may use Thevenin's Theorem
to simplify this portion of the circuit . . .*

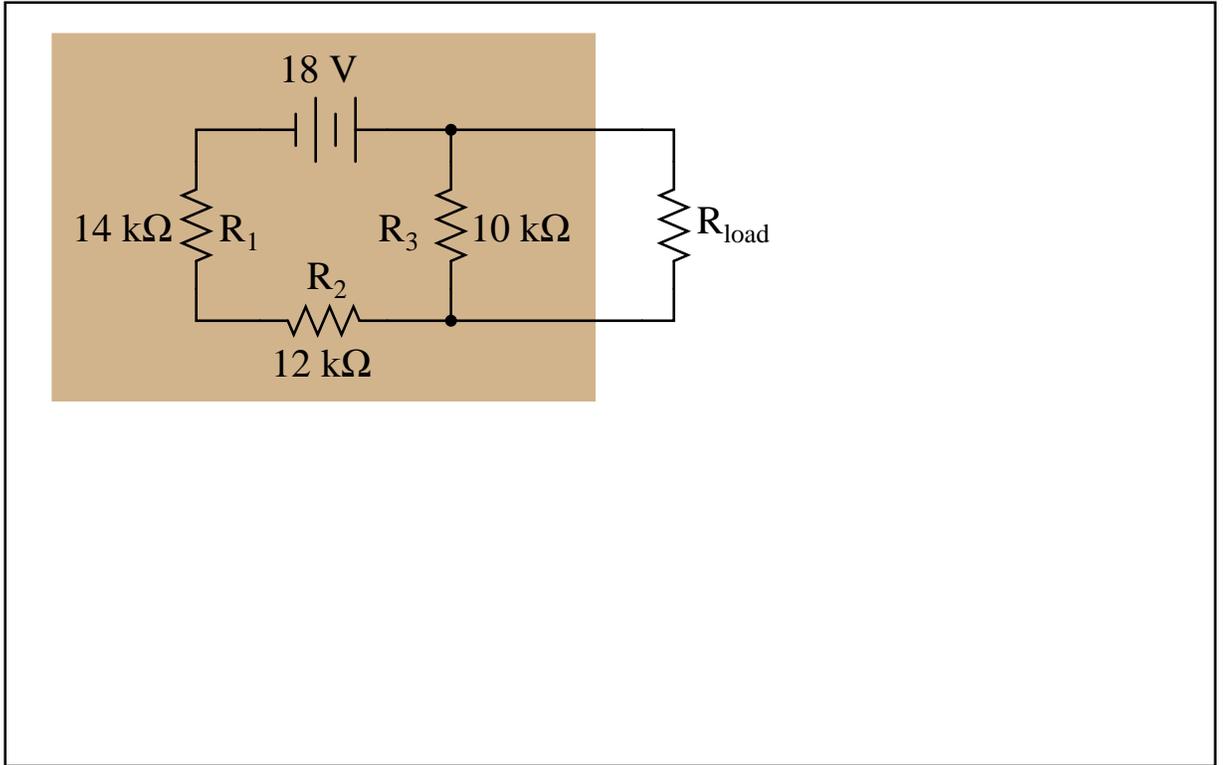


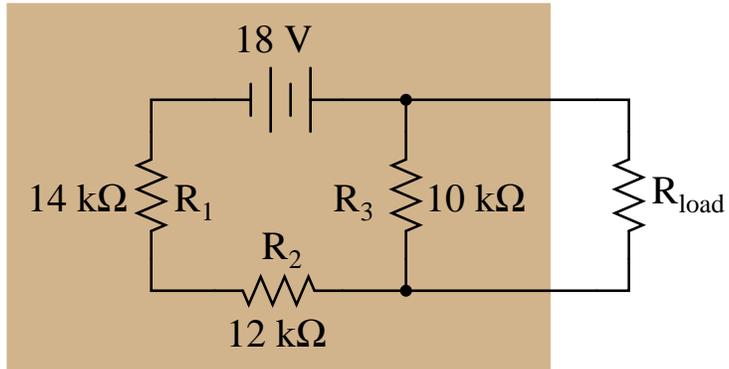
*We may use Thevenin's Theorem
to simplify this portion of the circuit . . .*



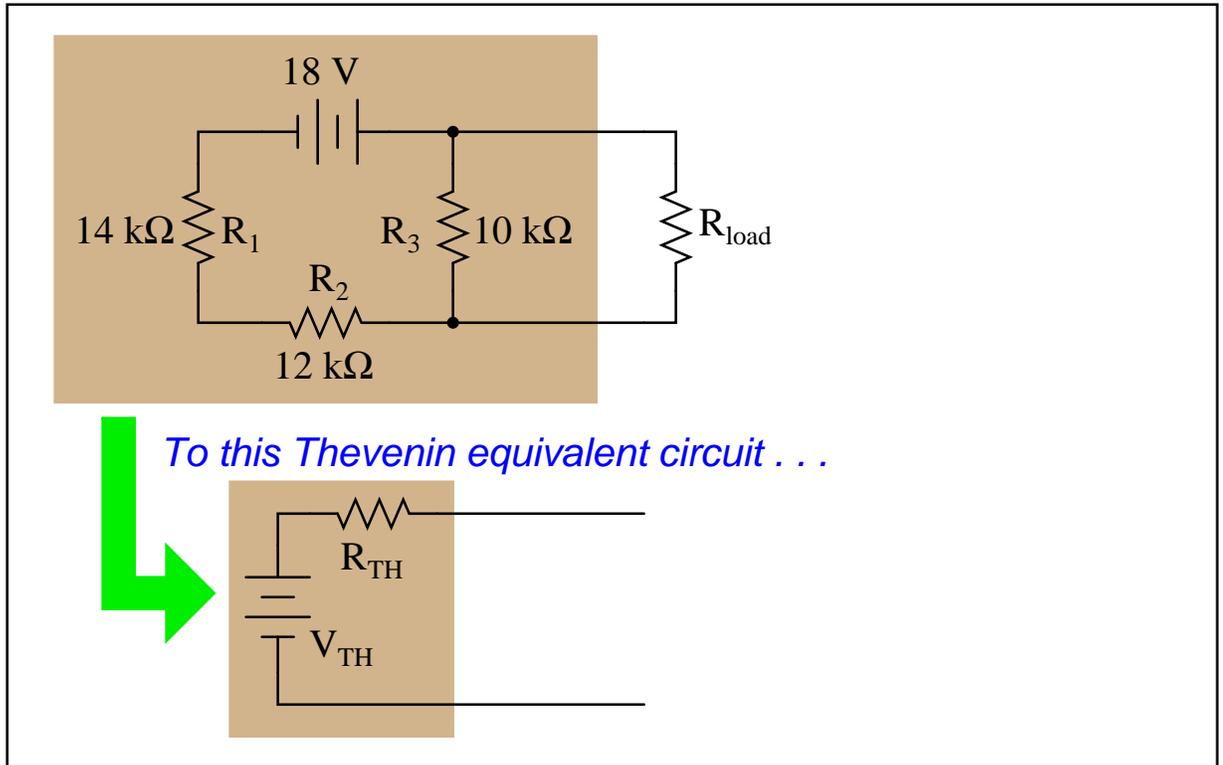
*We may use Thevenin's Theorem
to simplify this portion of the circuit . . .*

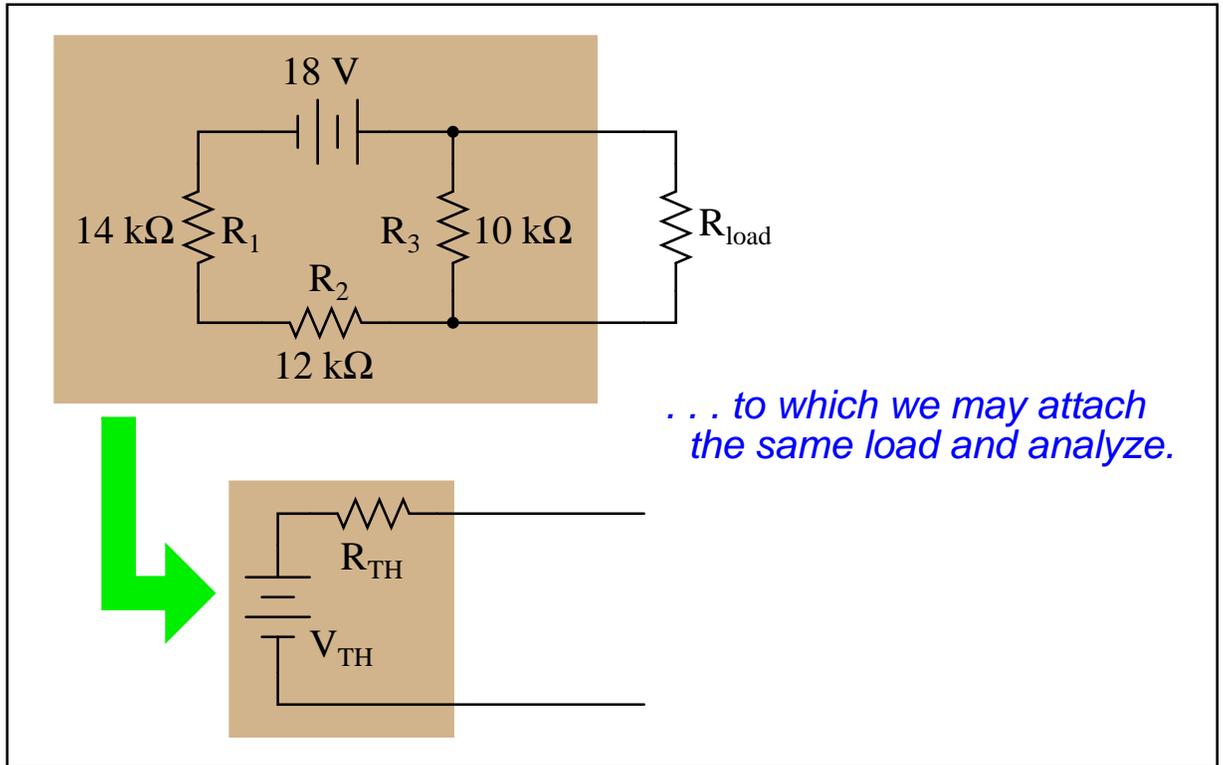


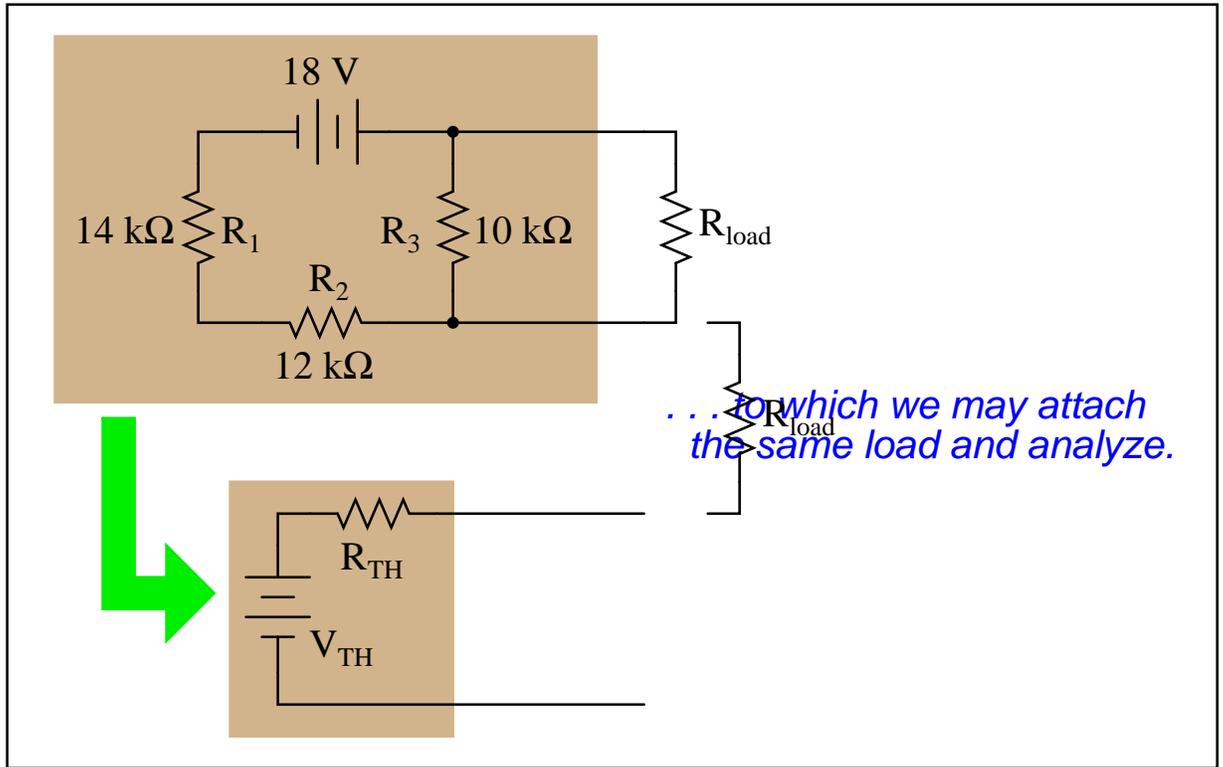


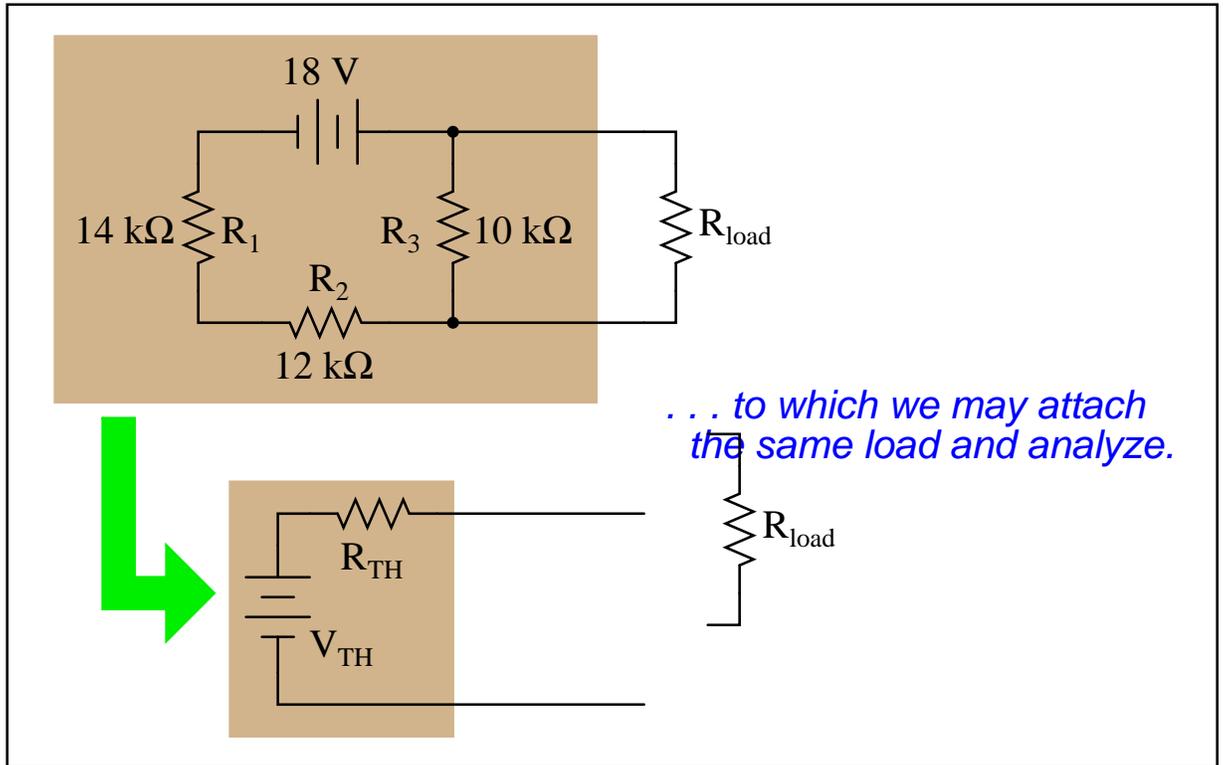


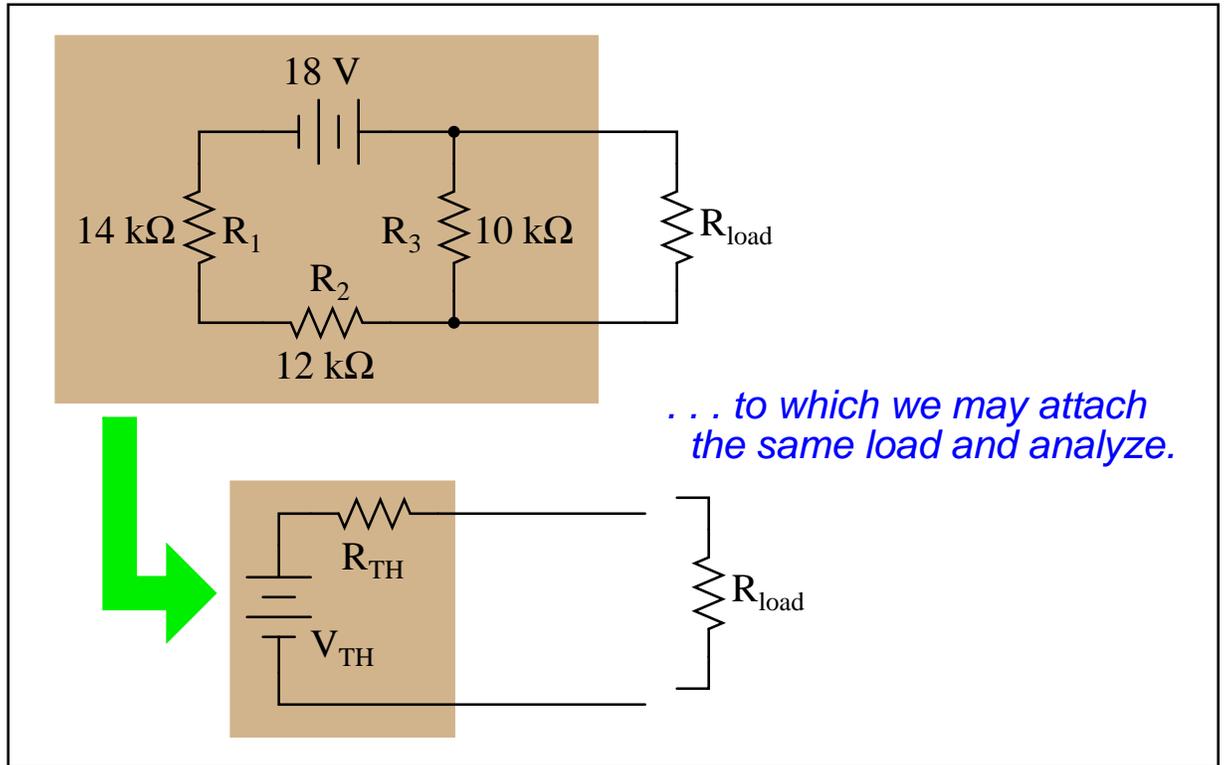
To this Thevenin equivalent circuit . . .

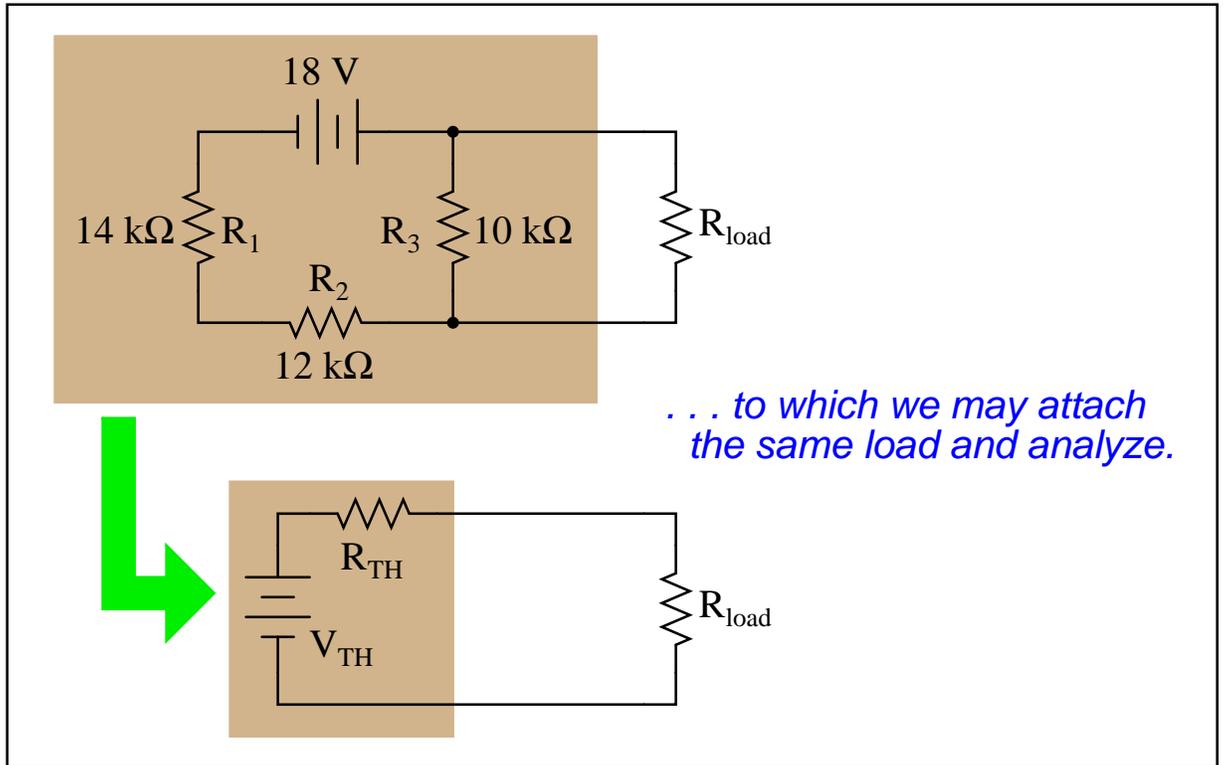


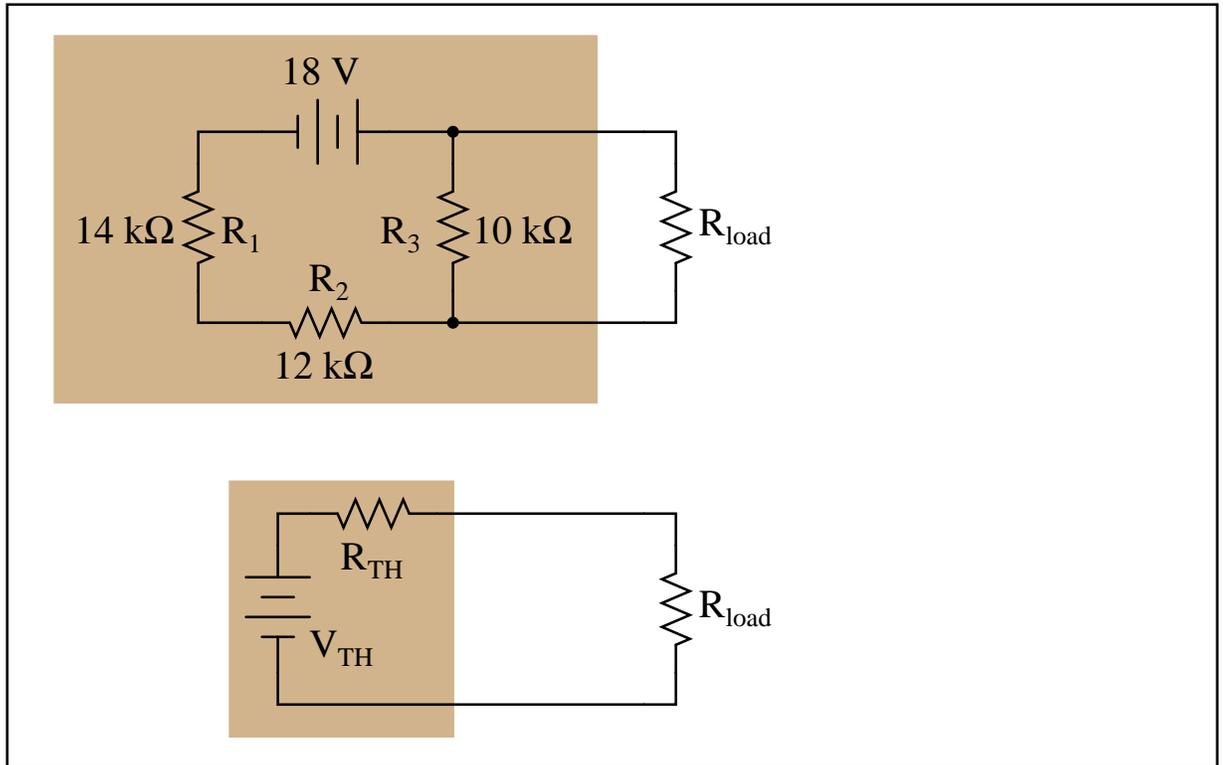


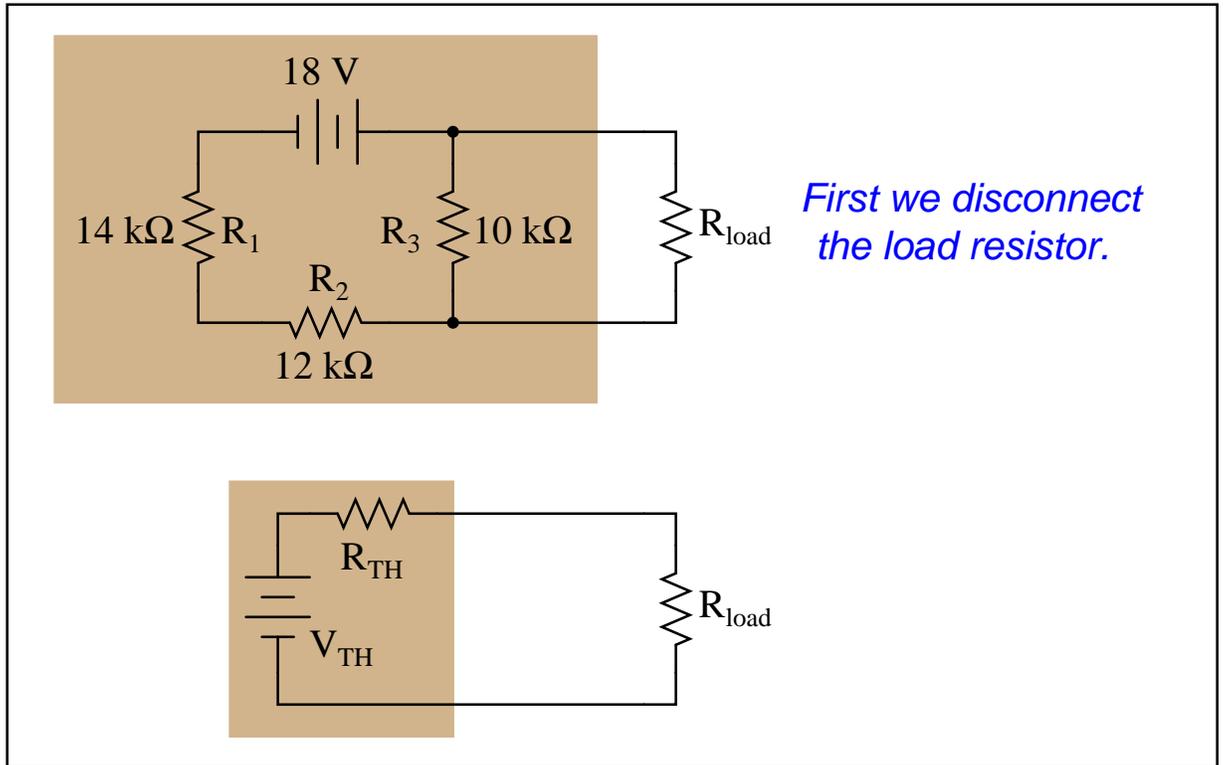


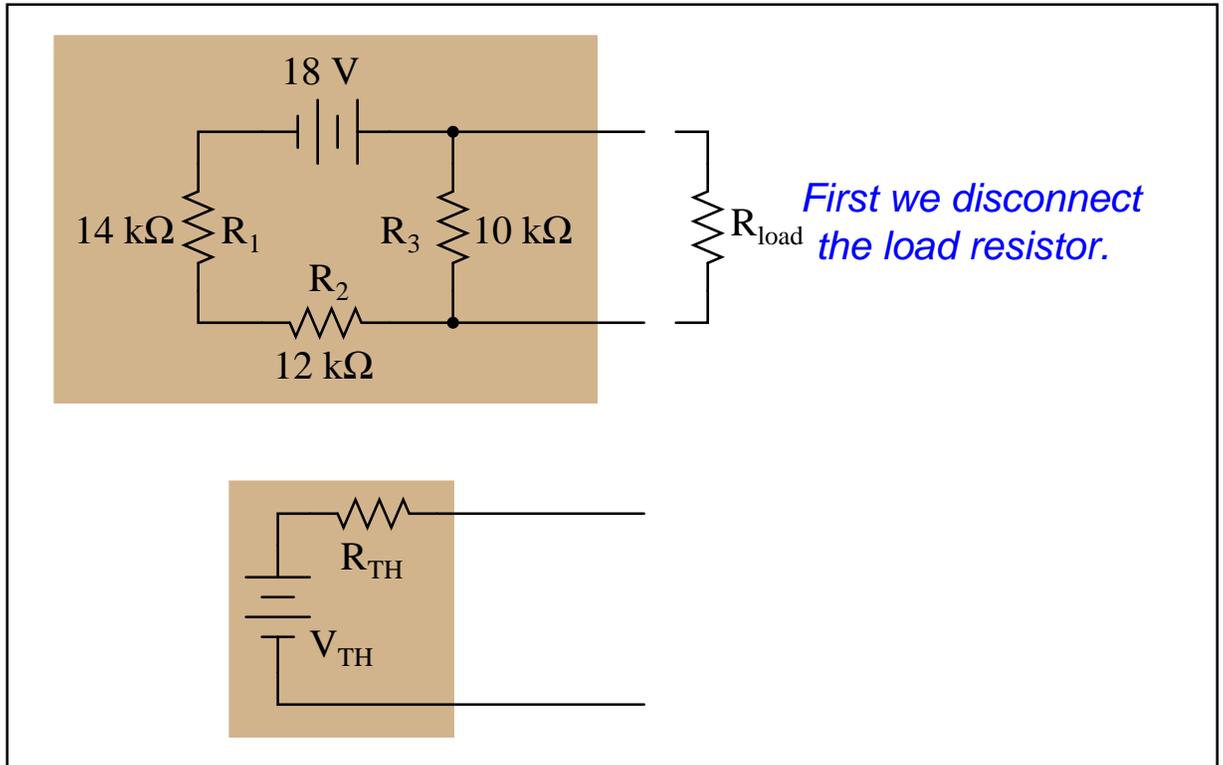


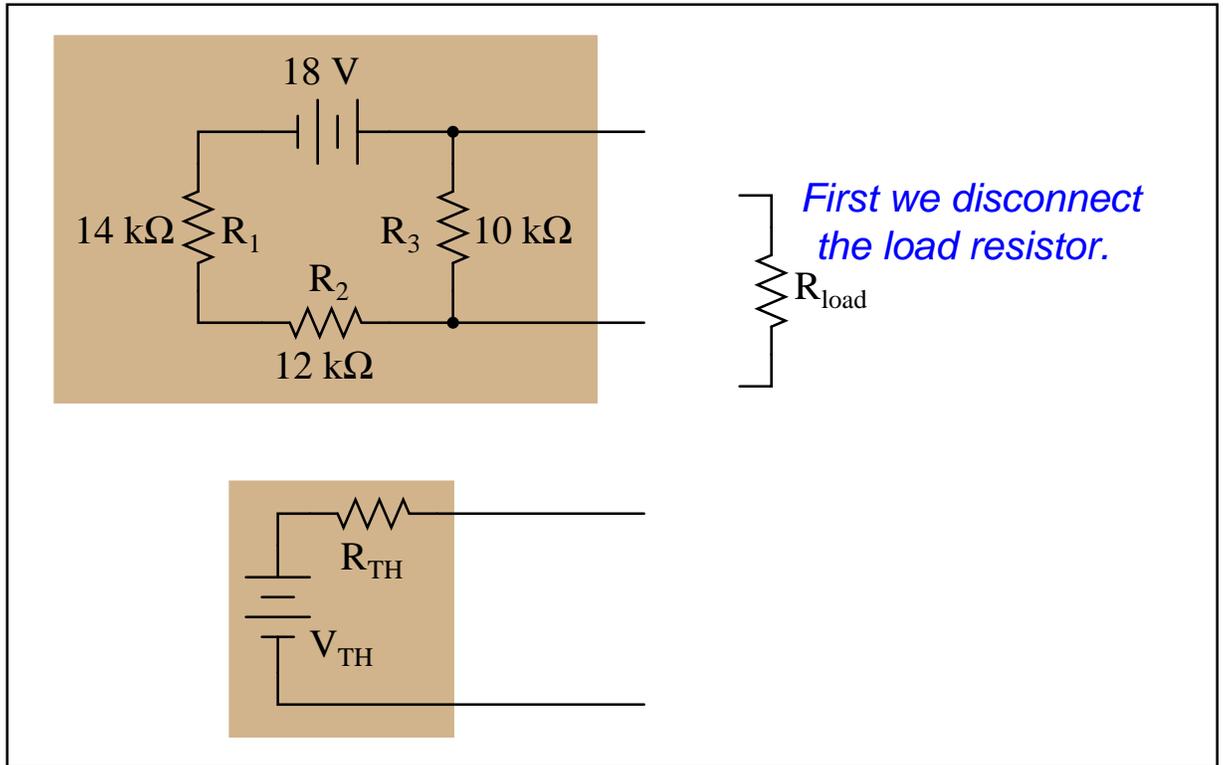


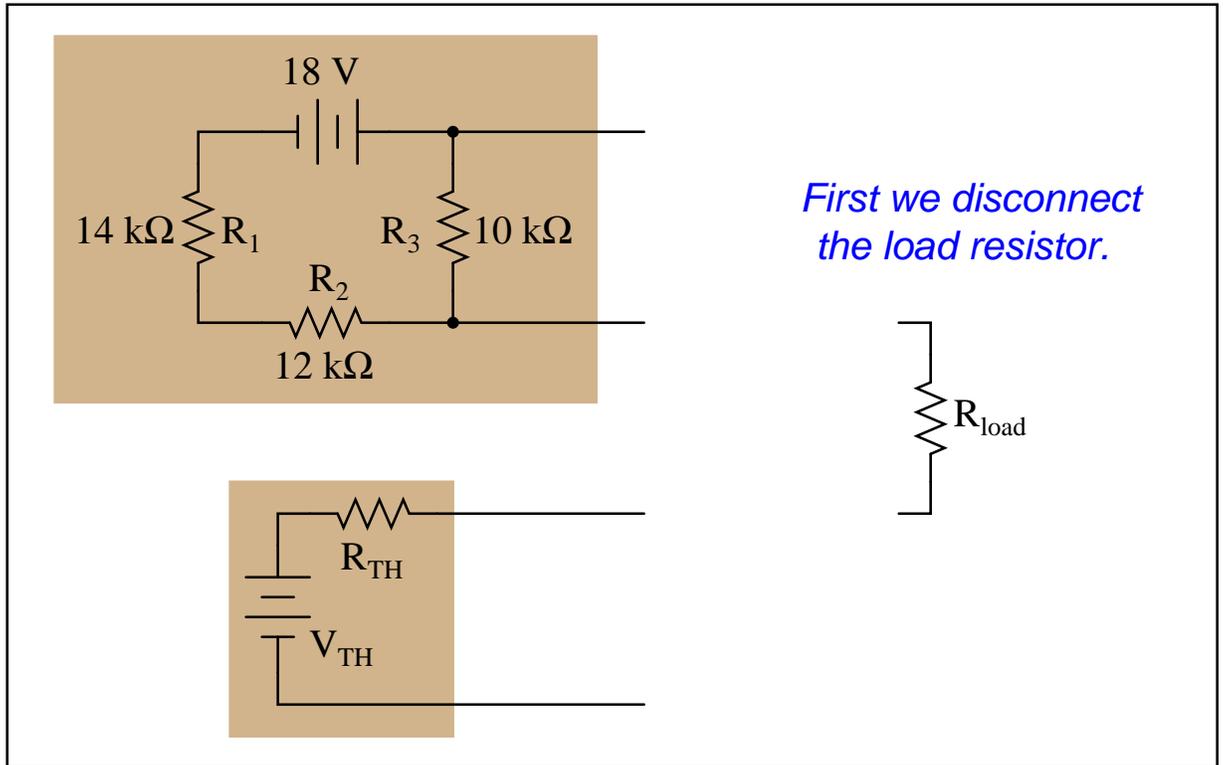


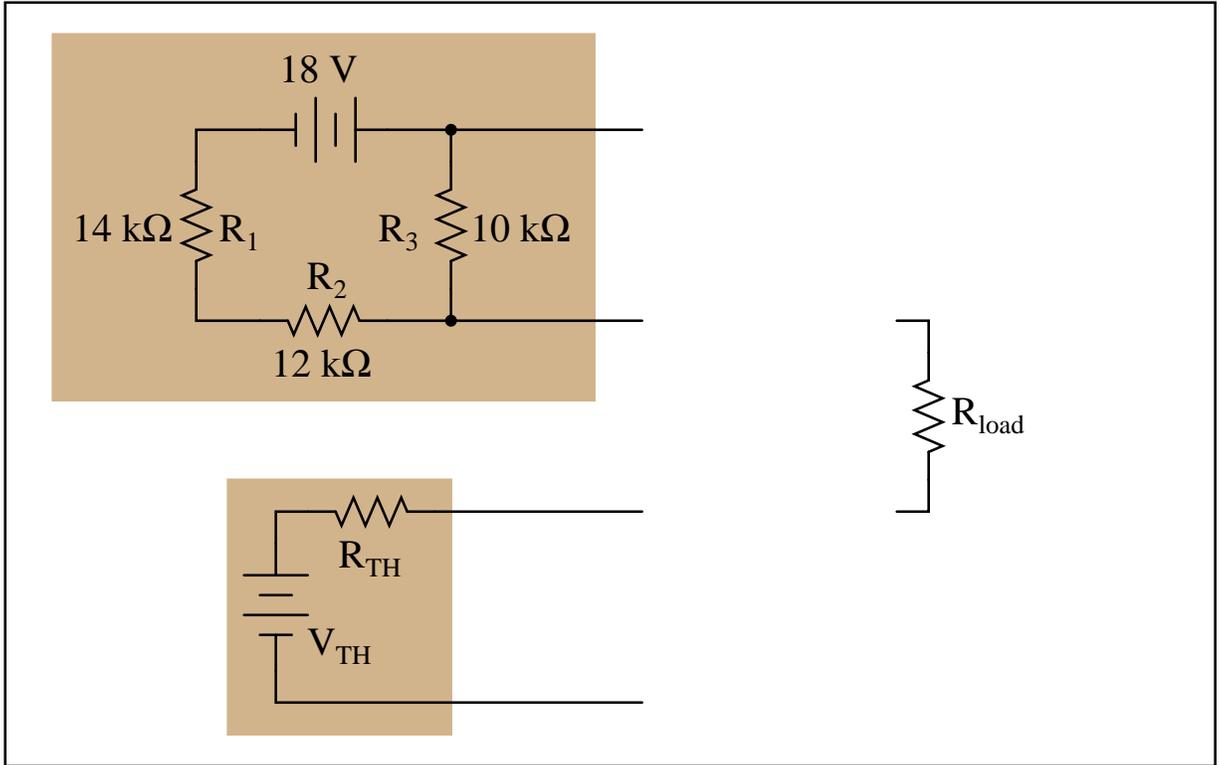


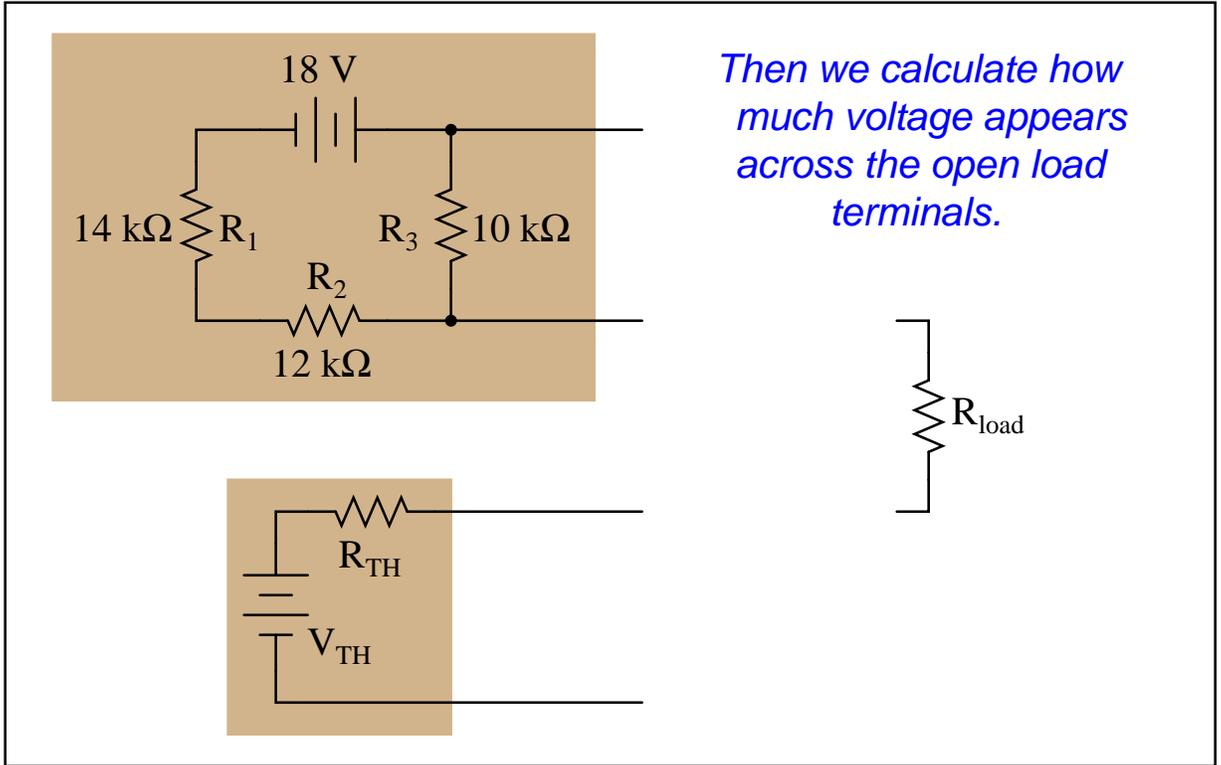


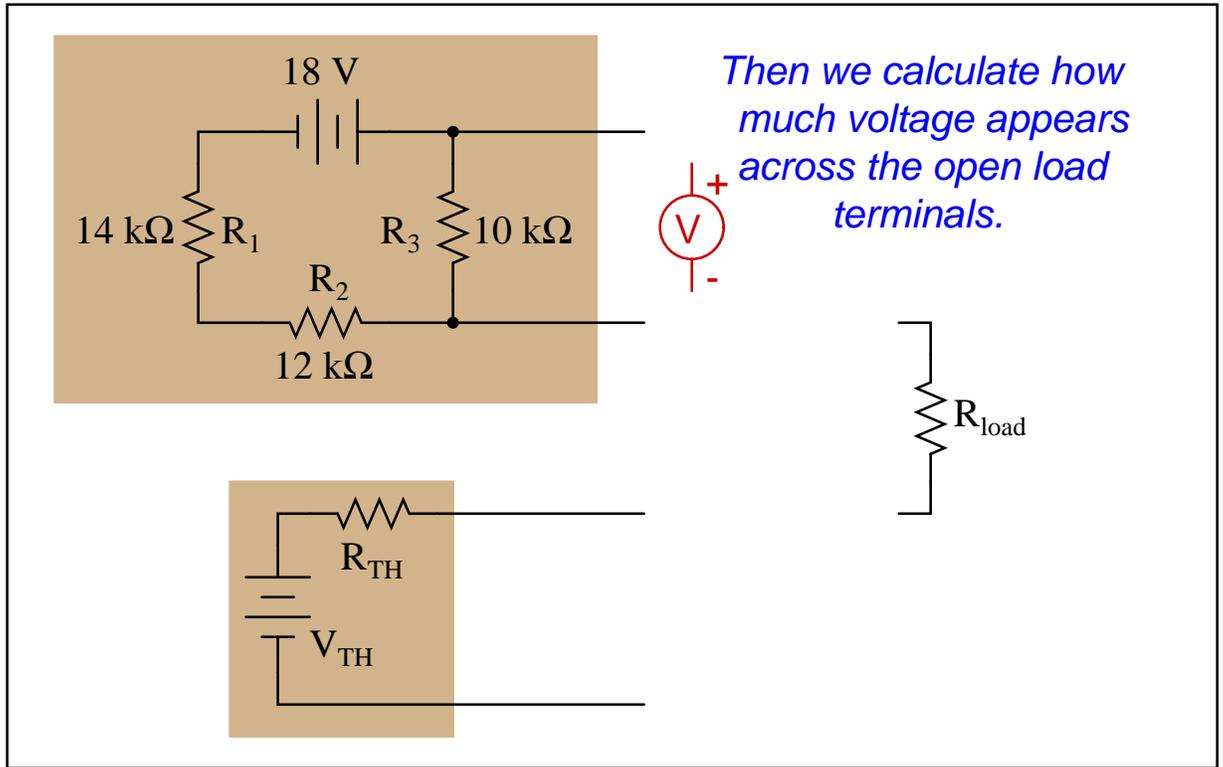


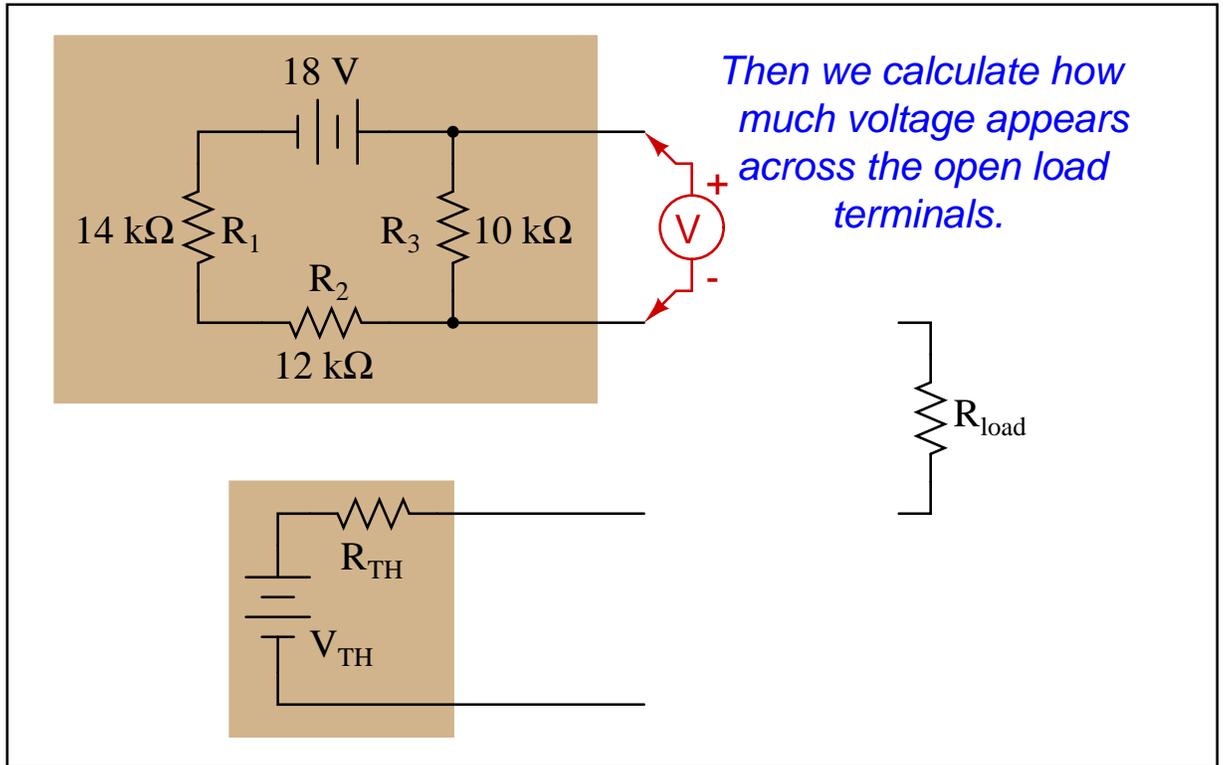


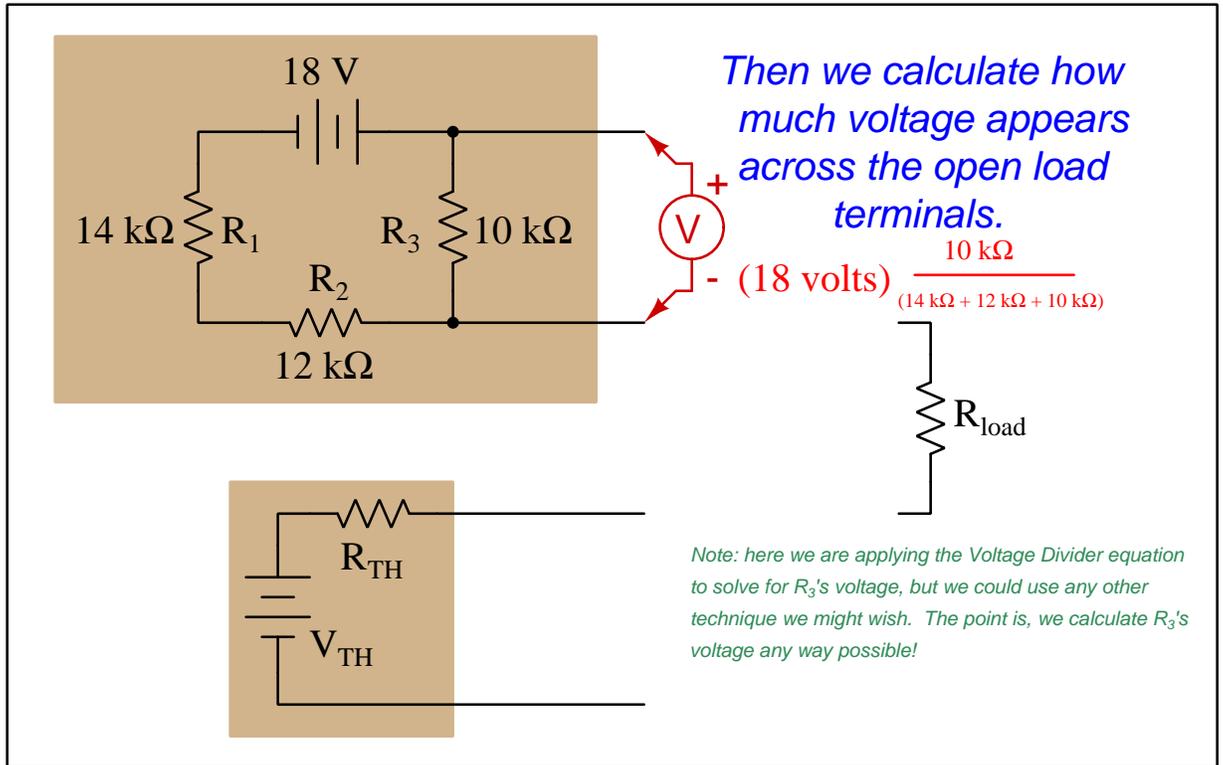


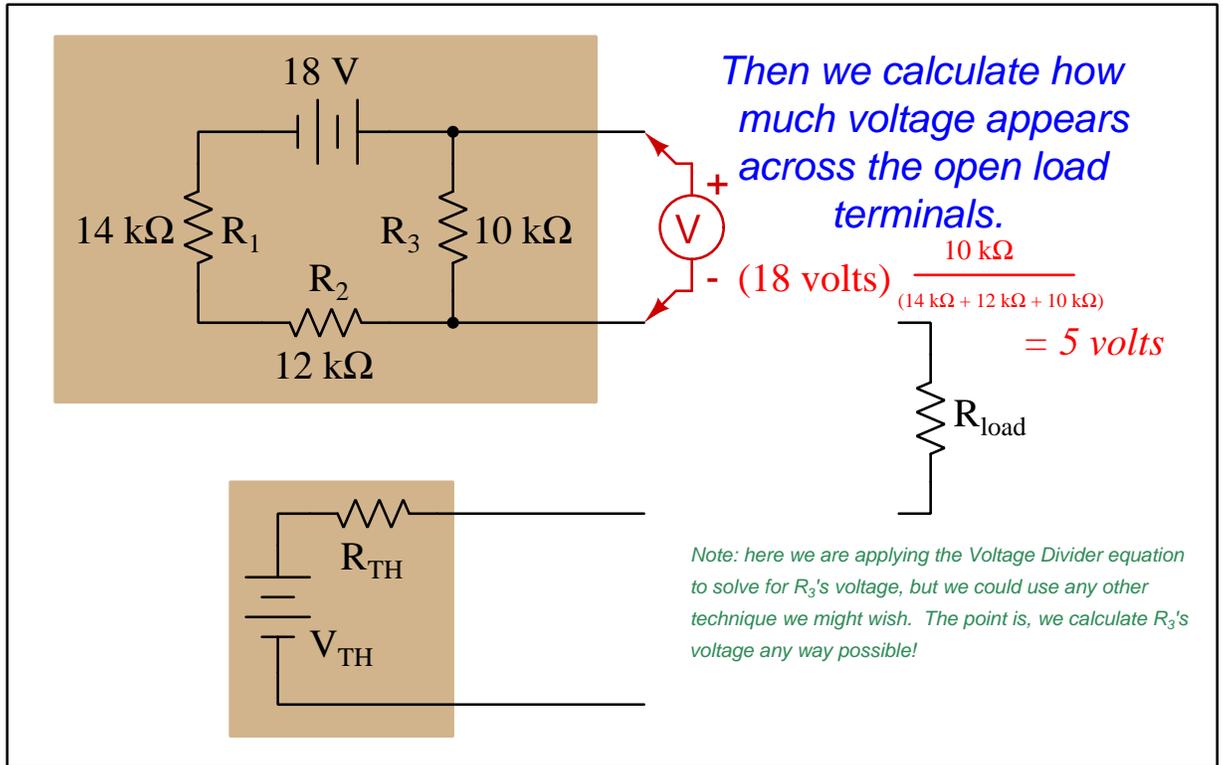


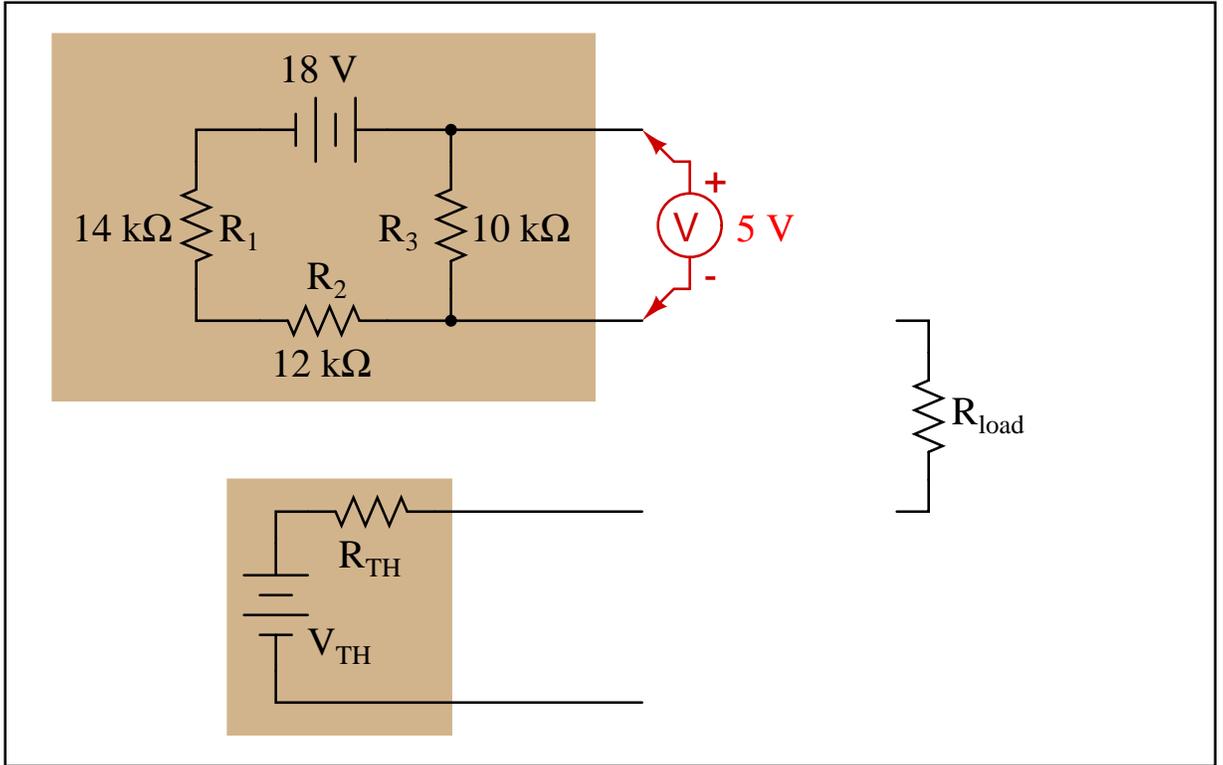


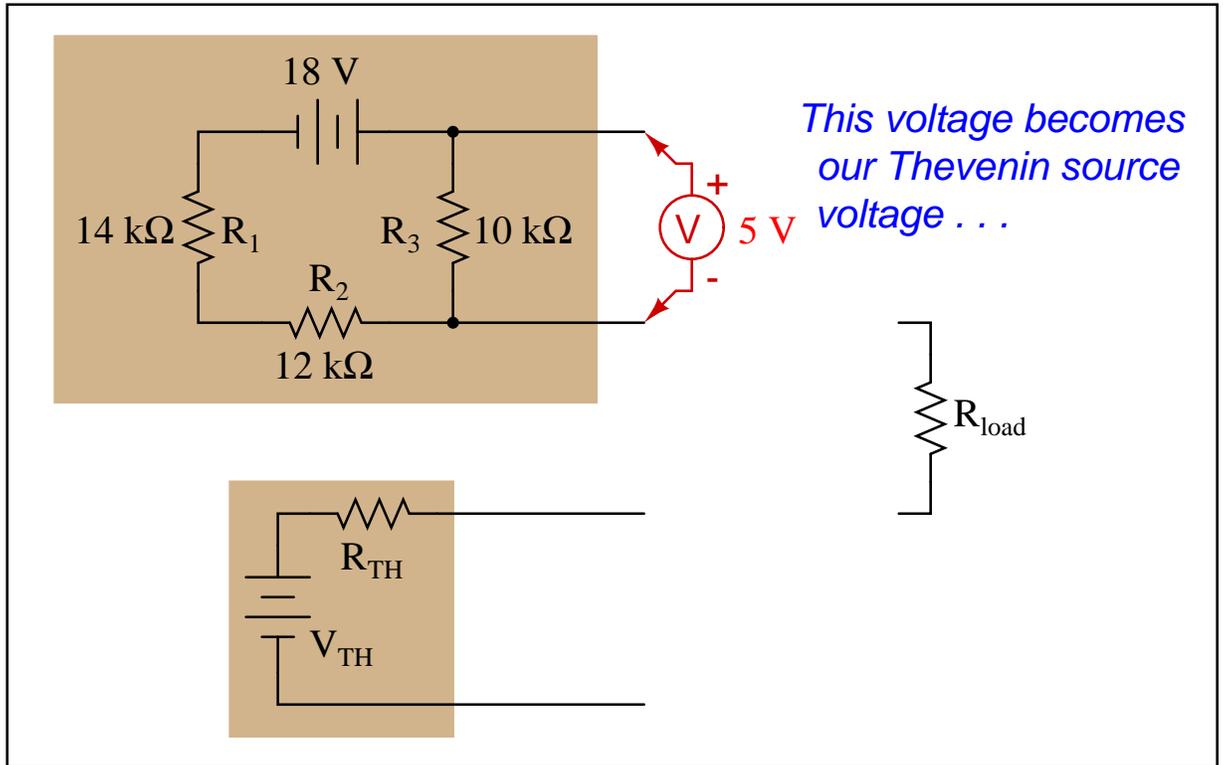


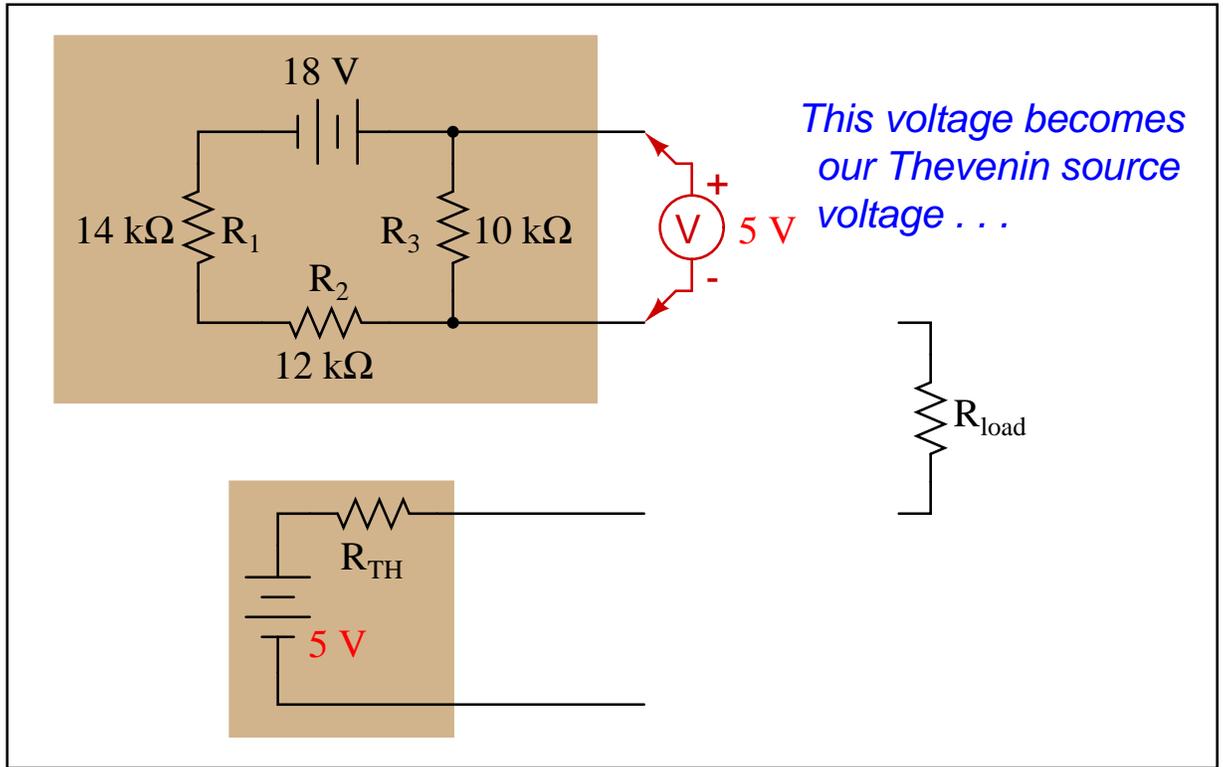


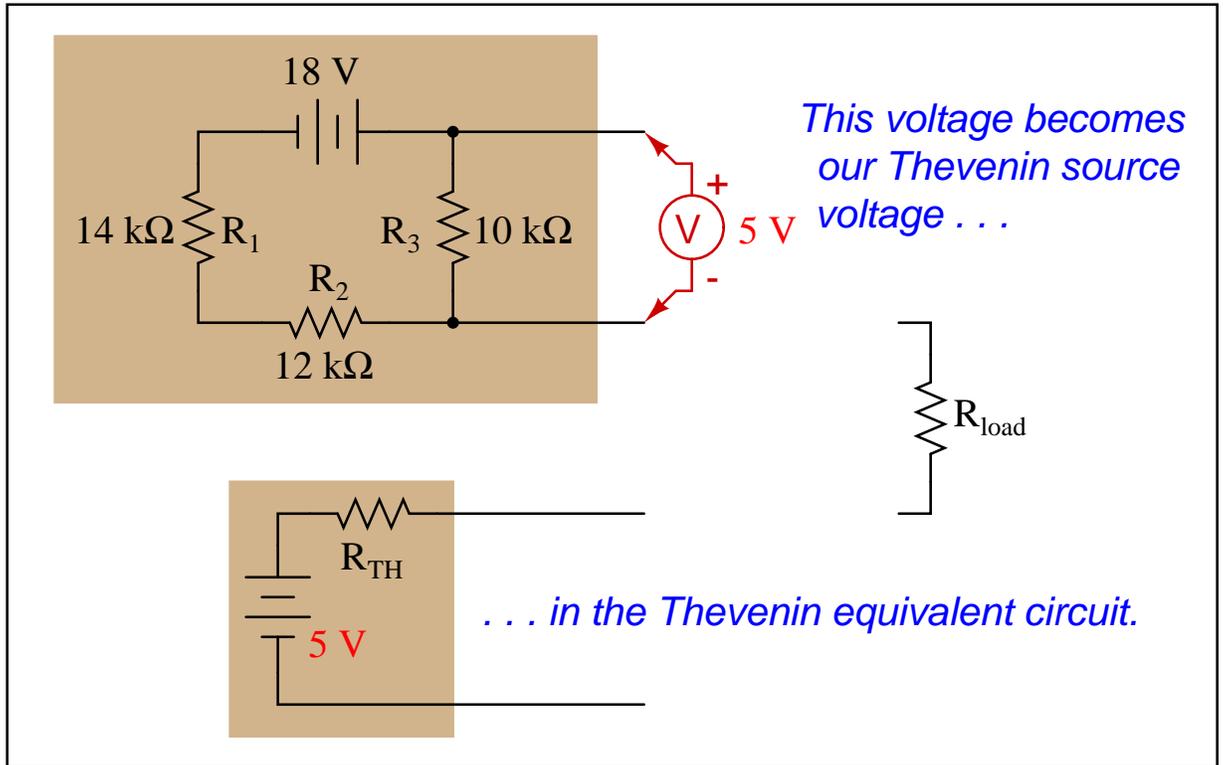


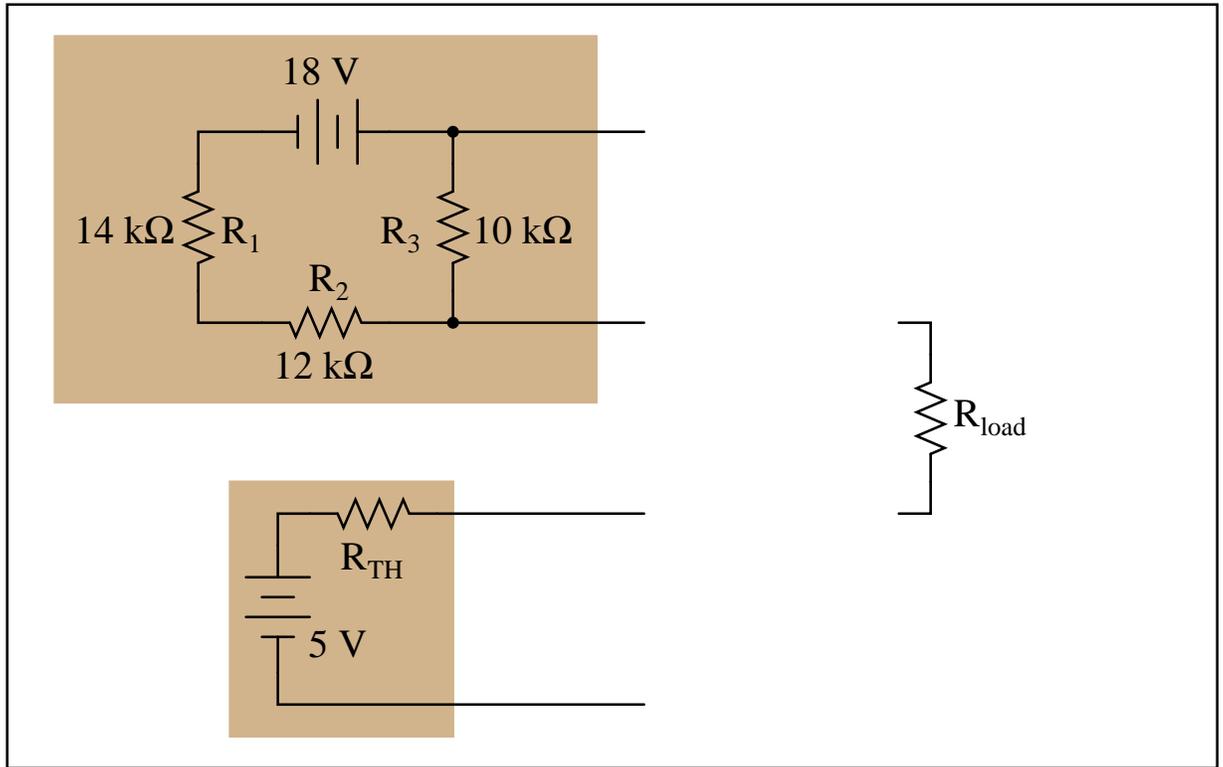


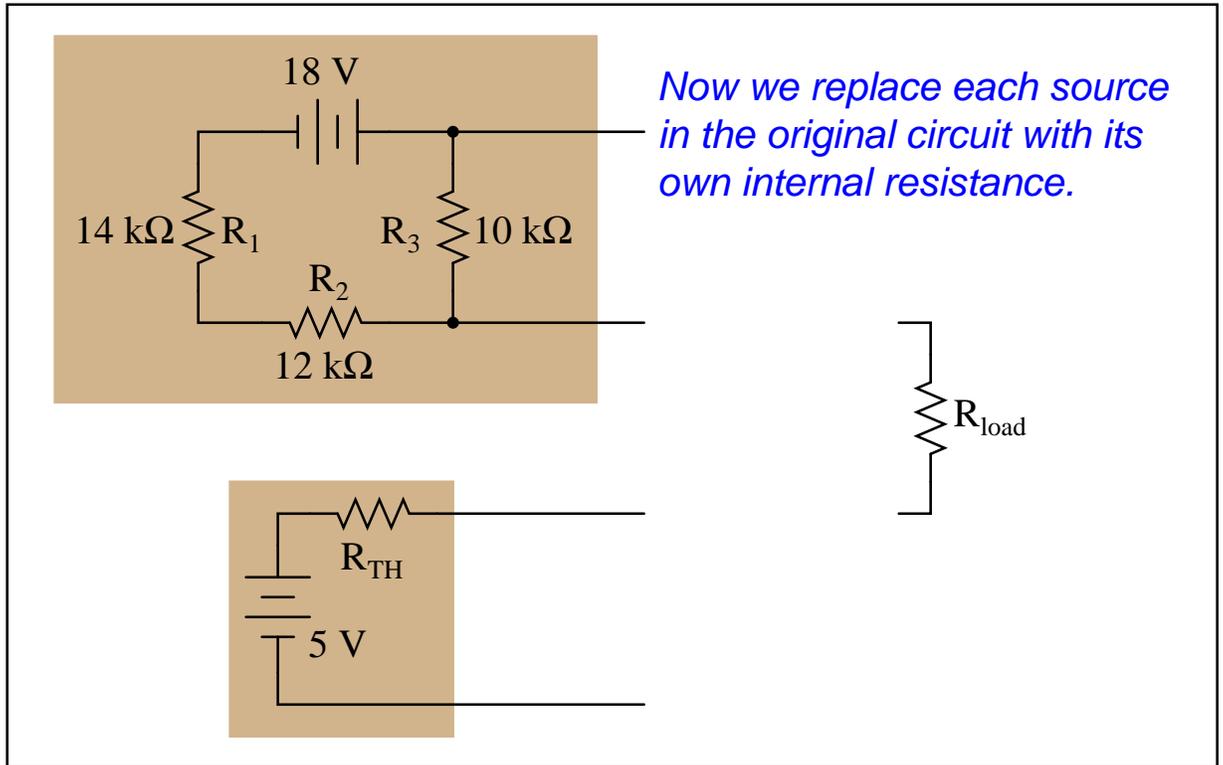


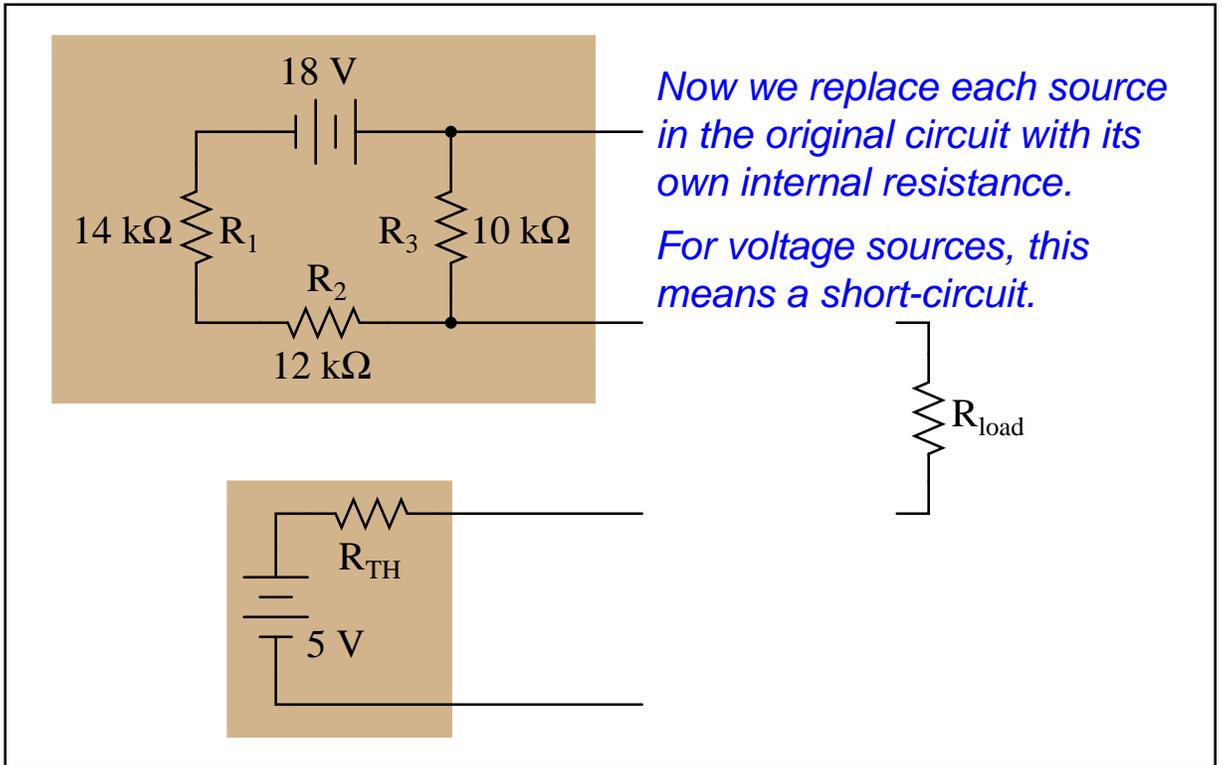


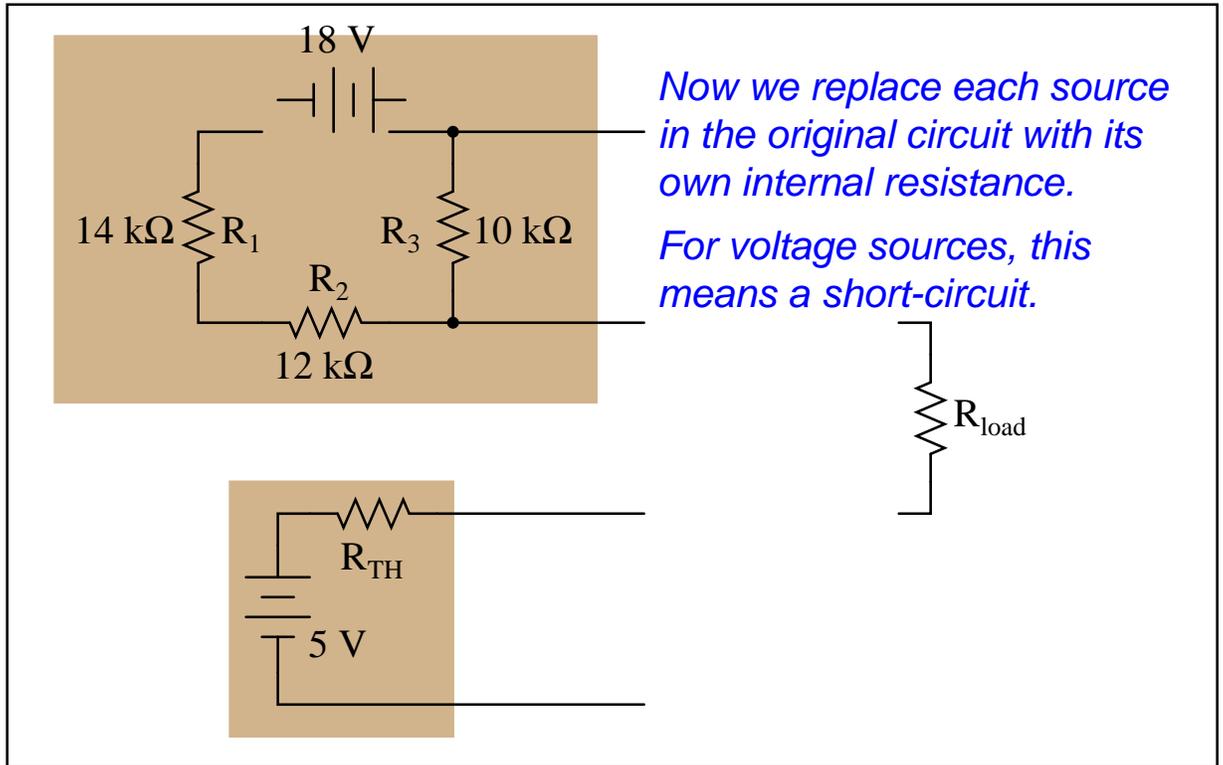


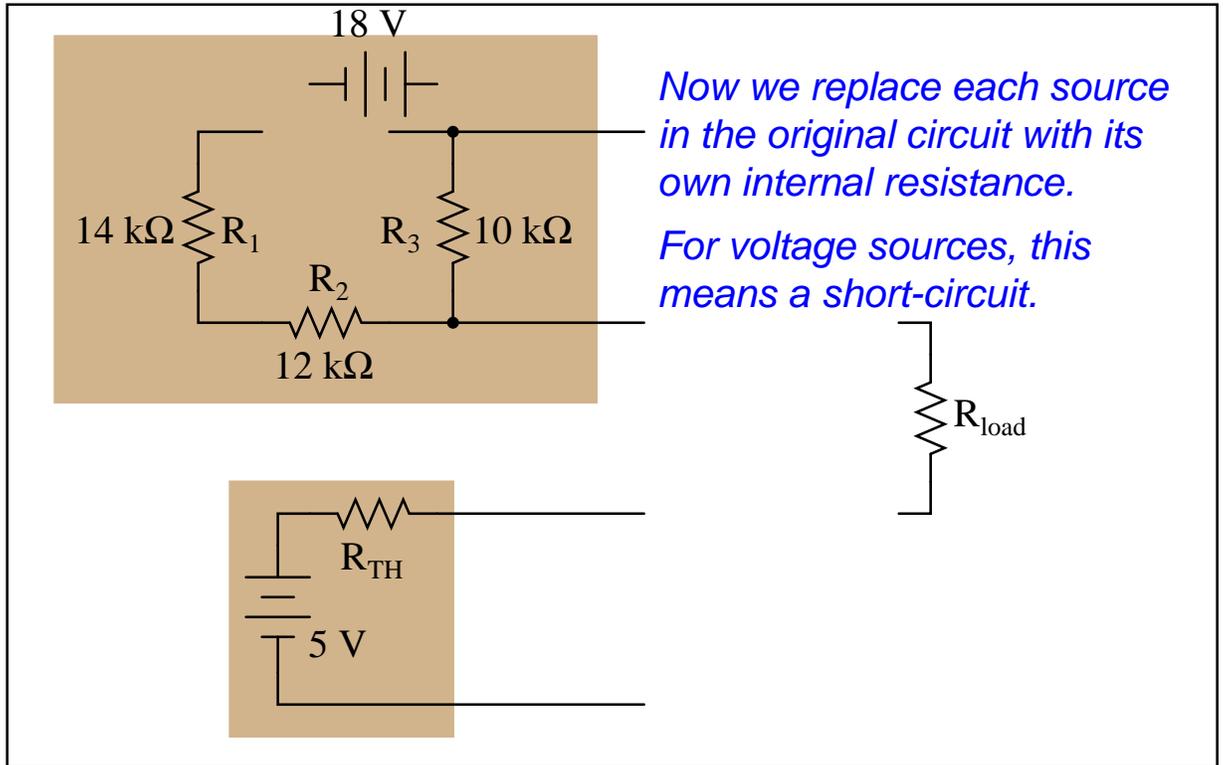


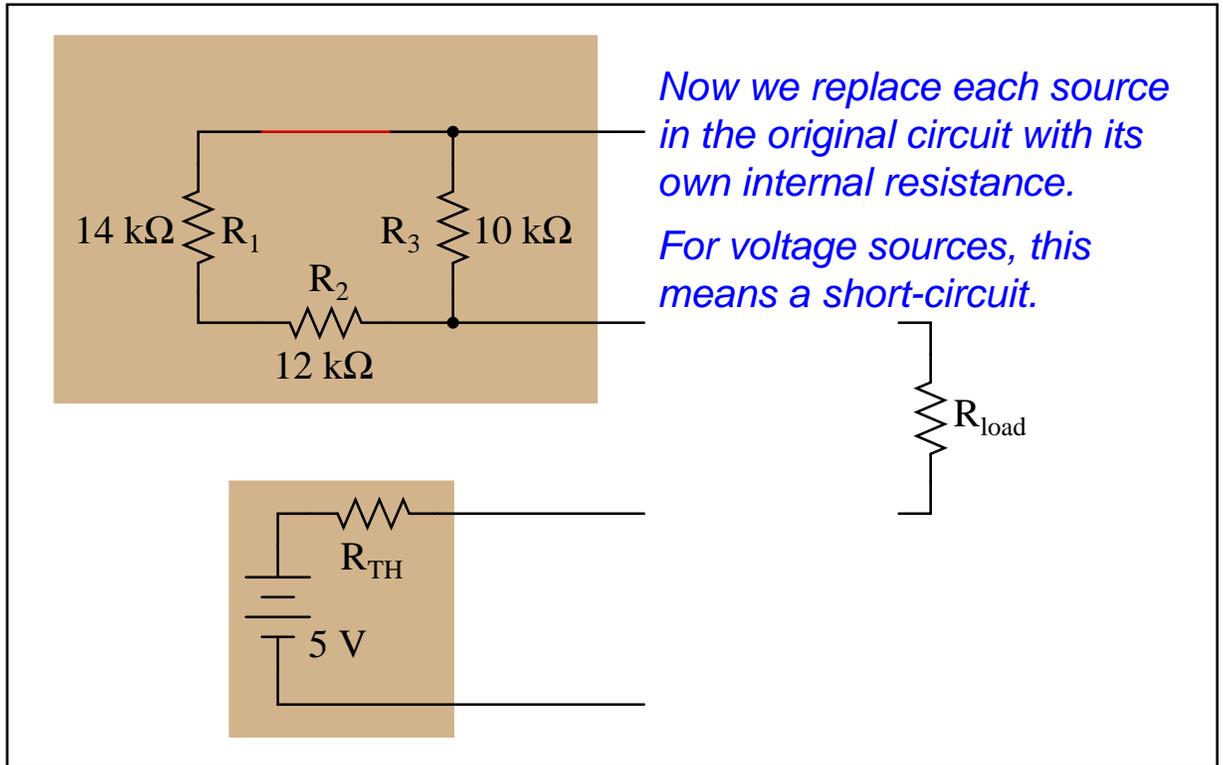


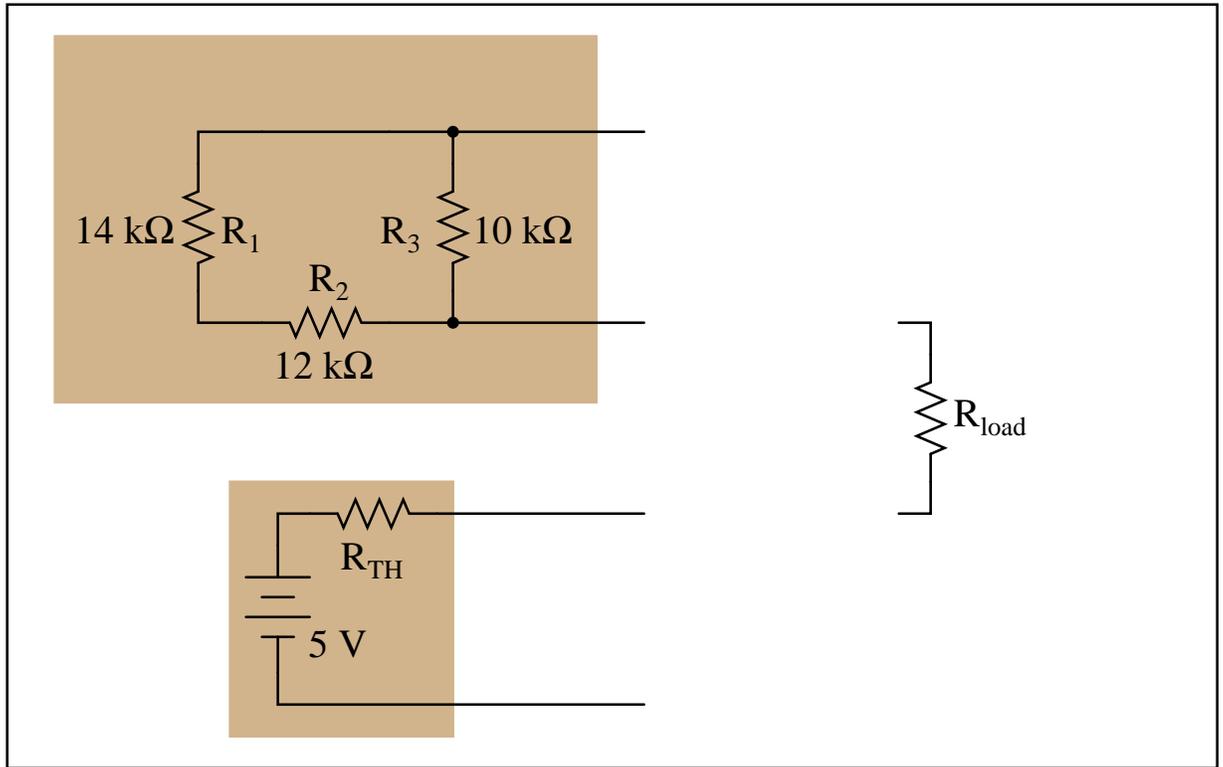


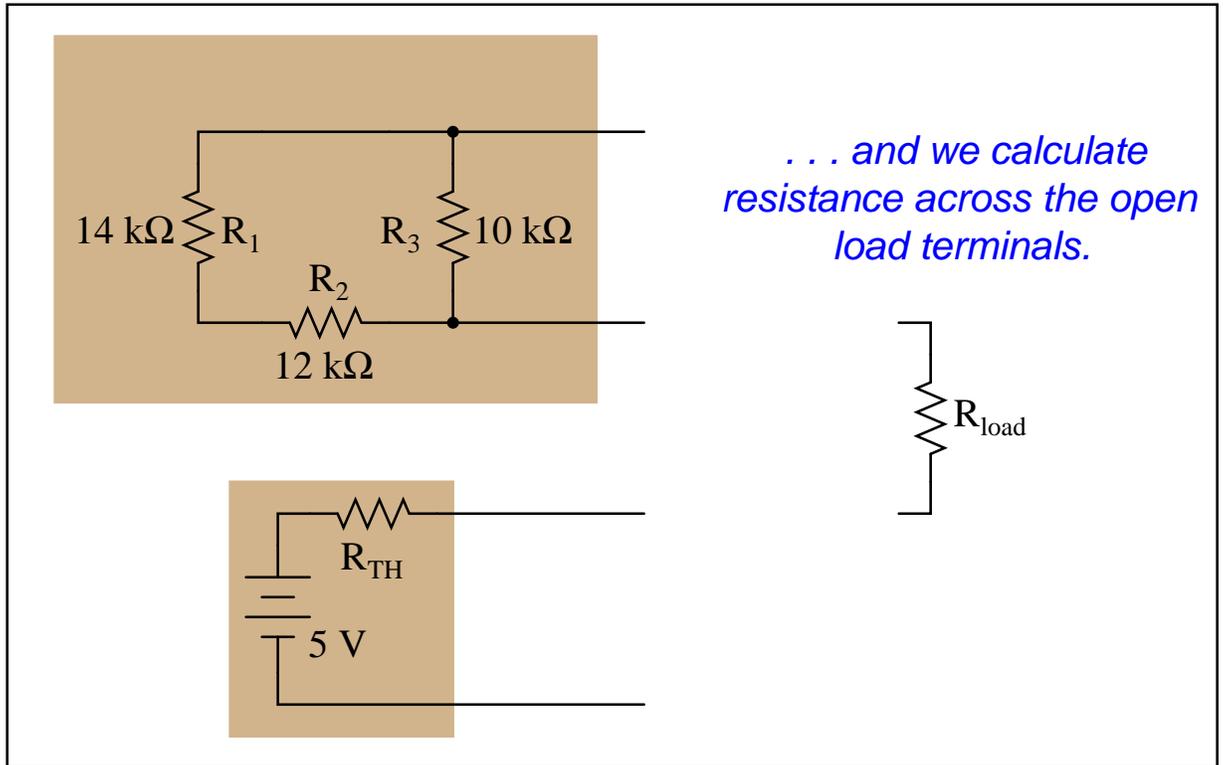


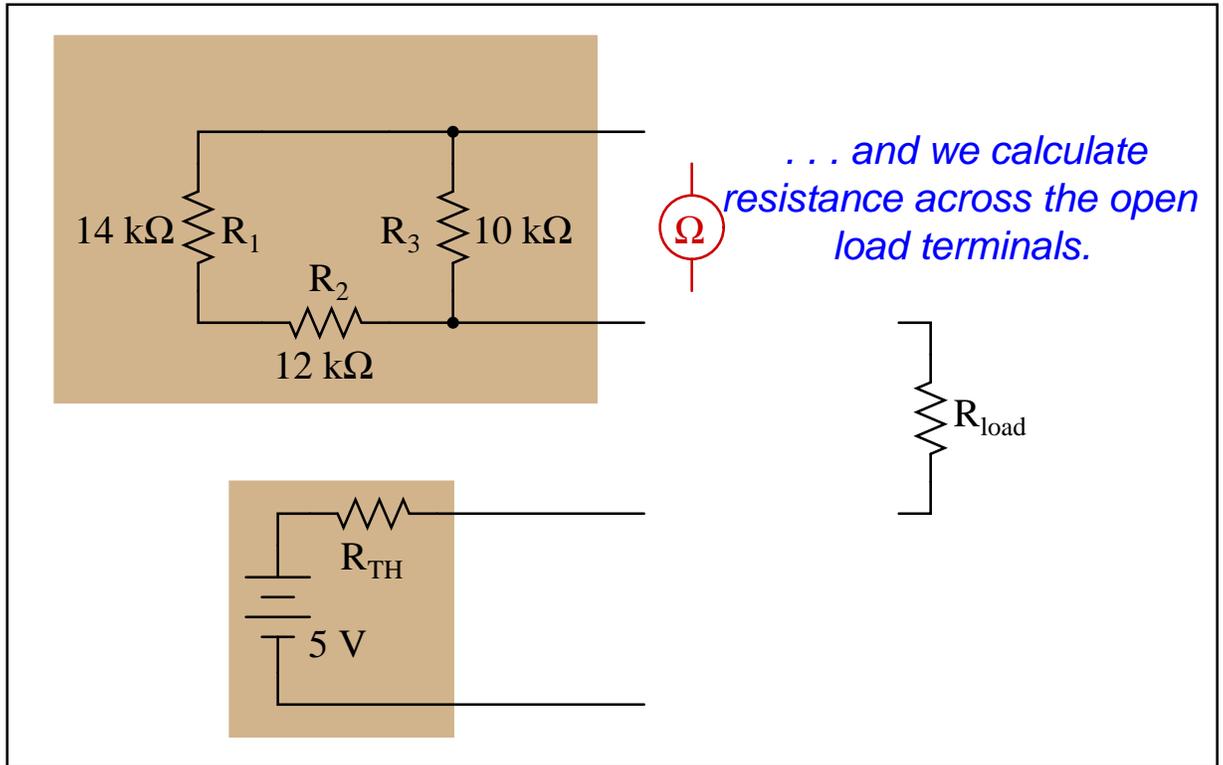


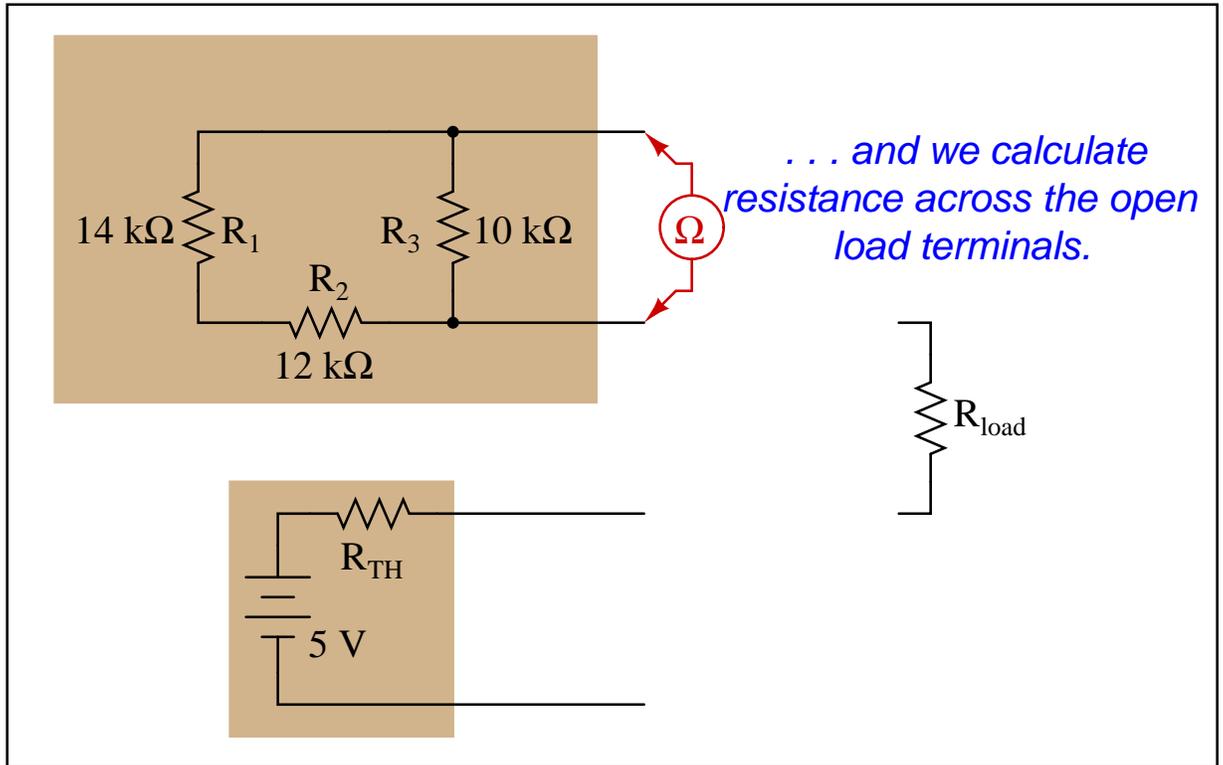


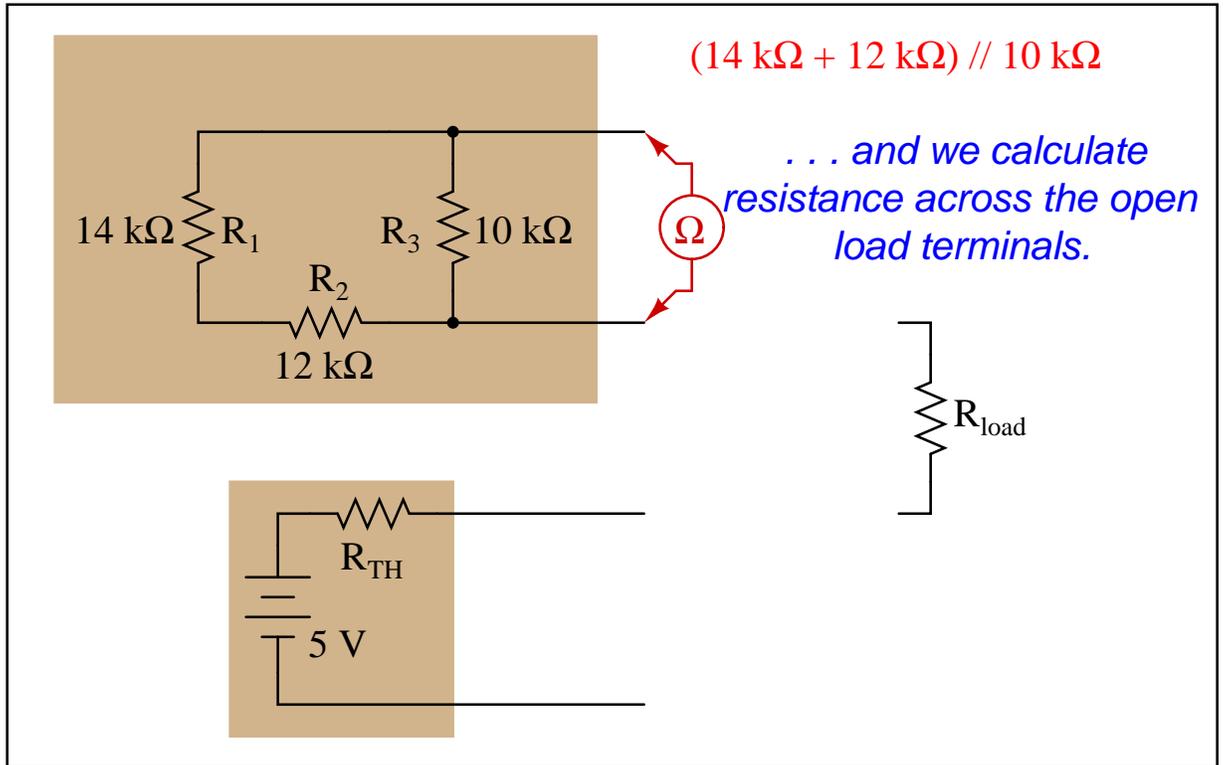


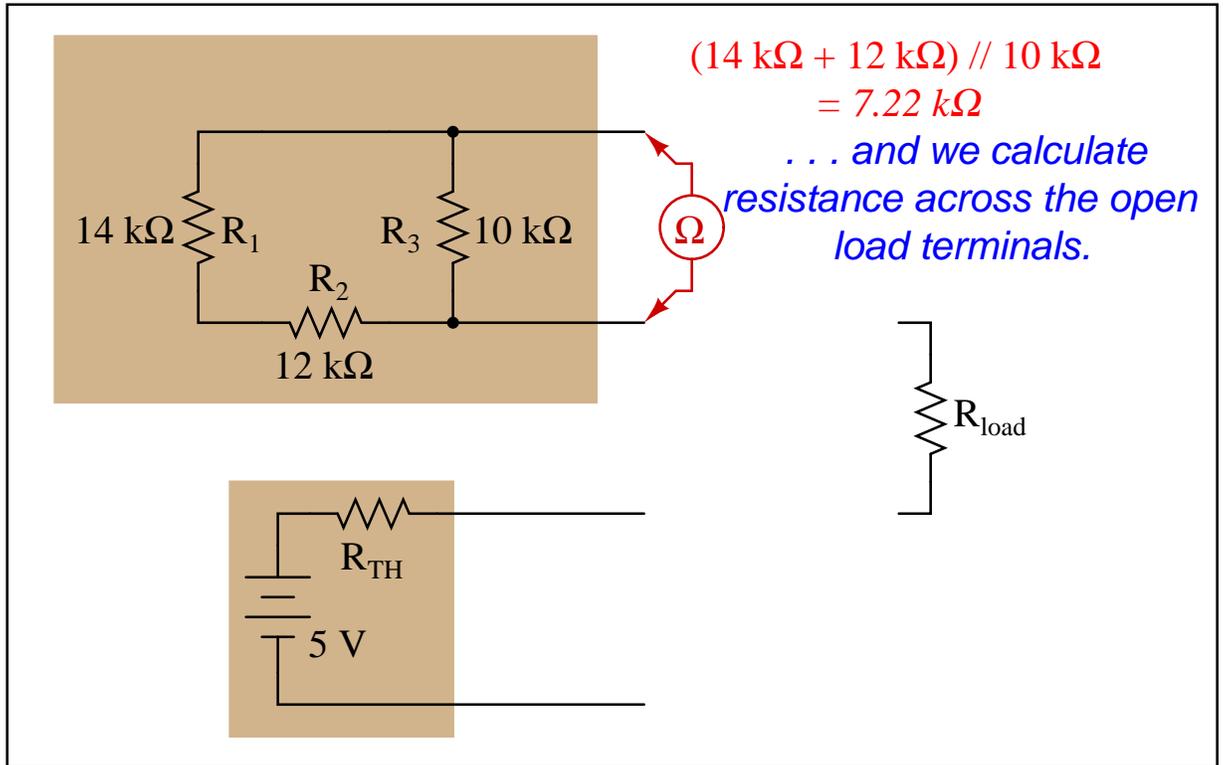


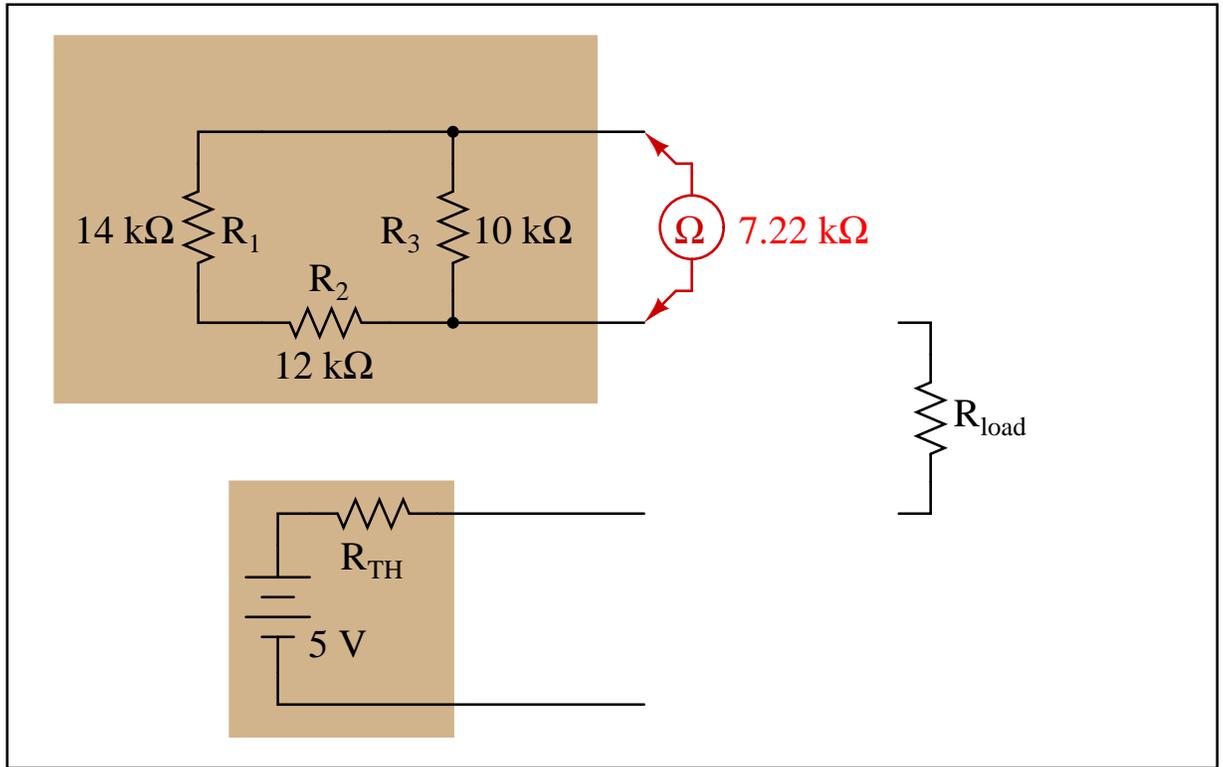


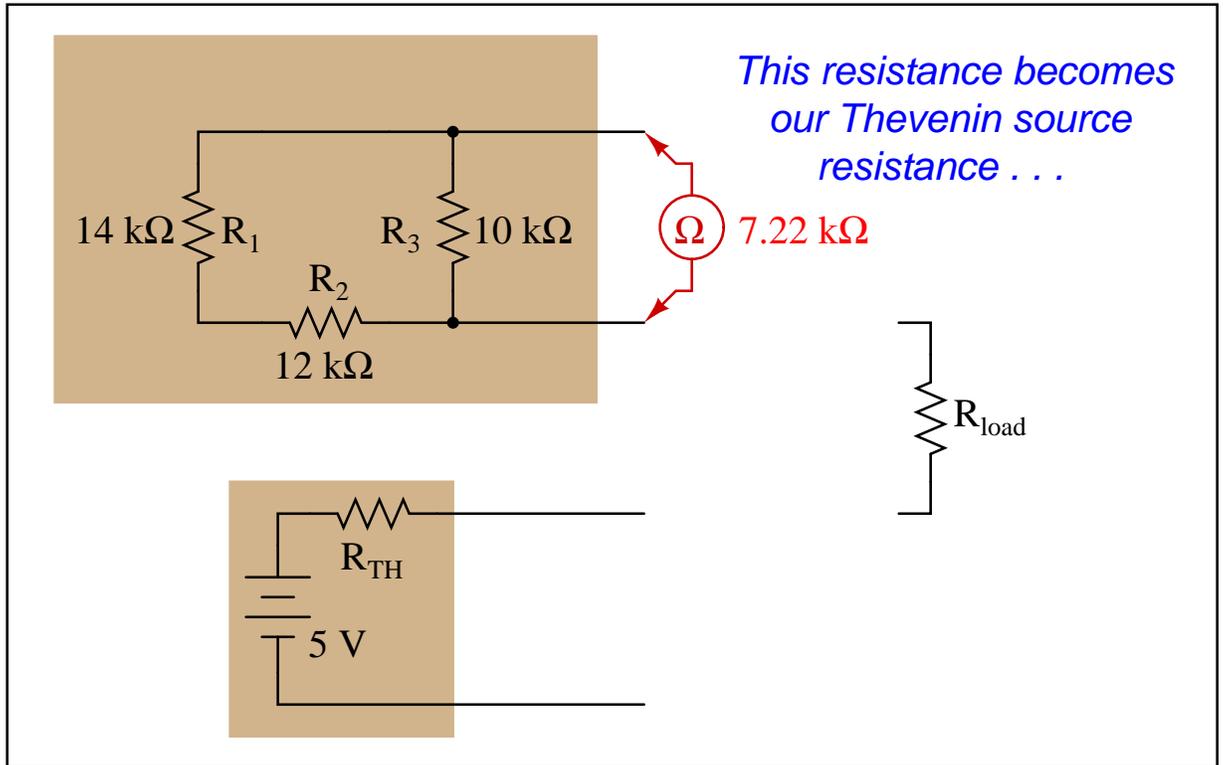


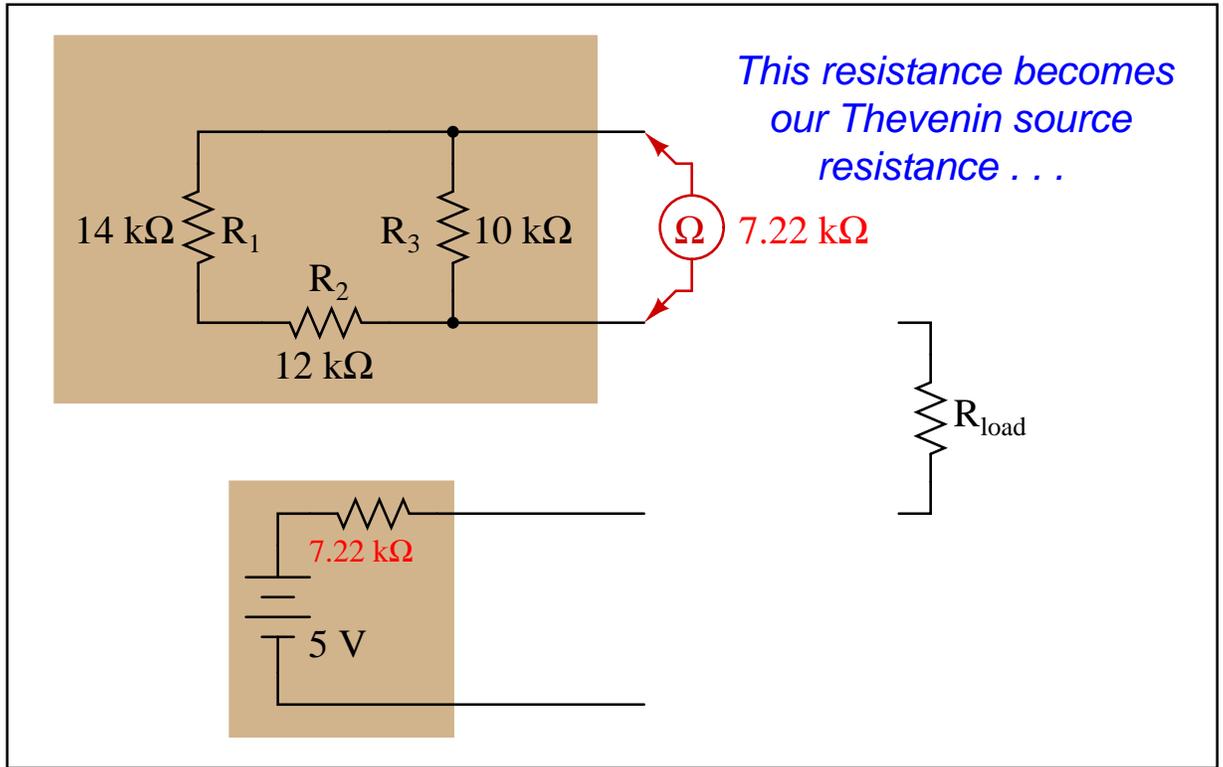


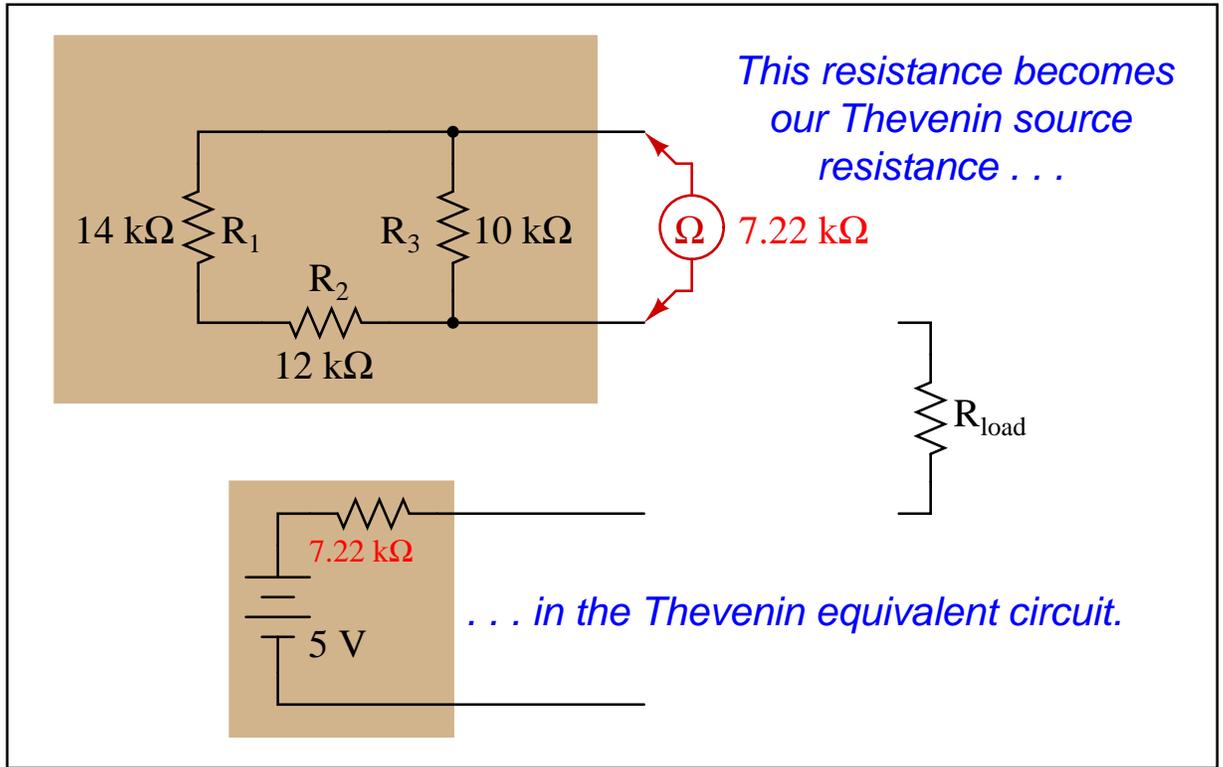


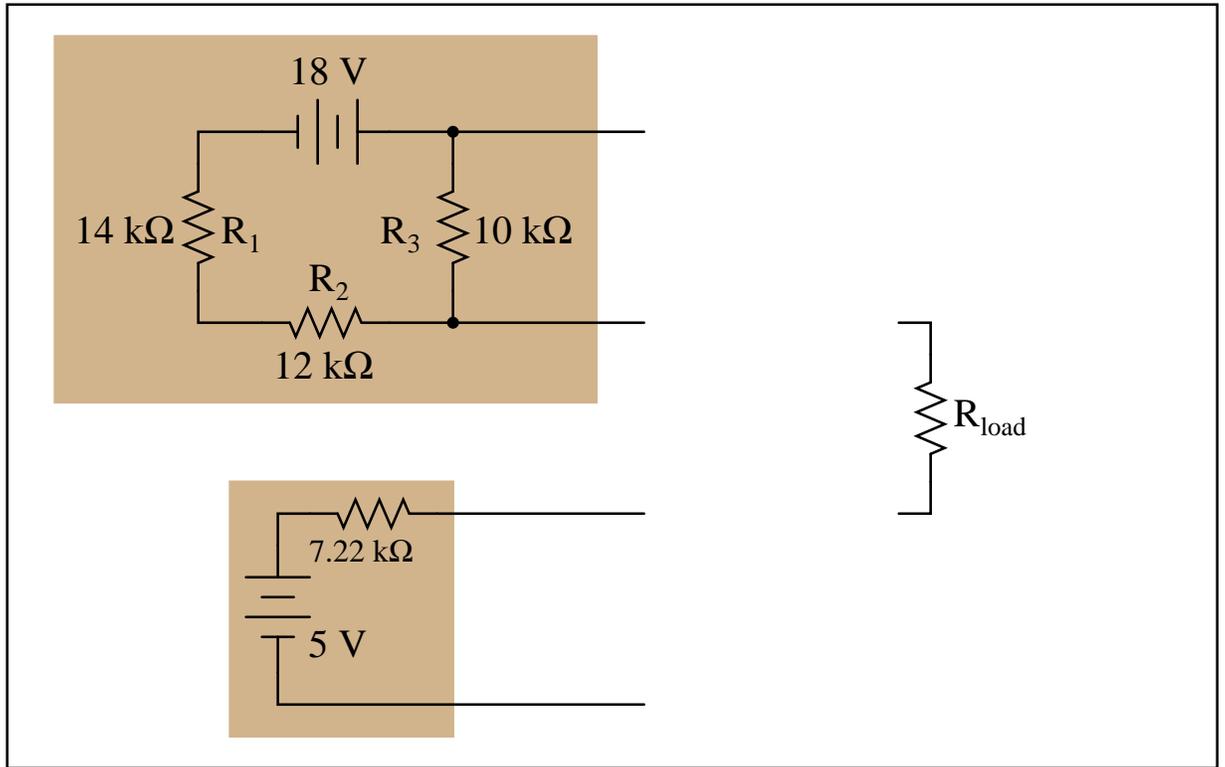


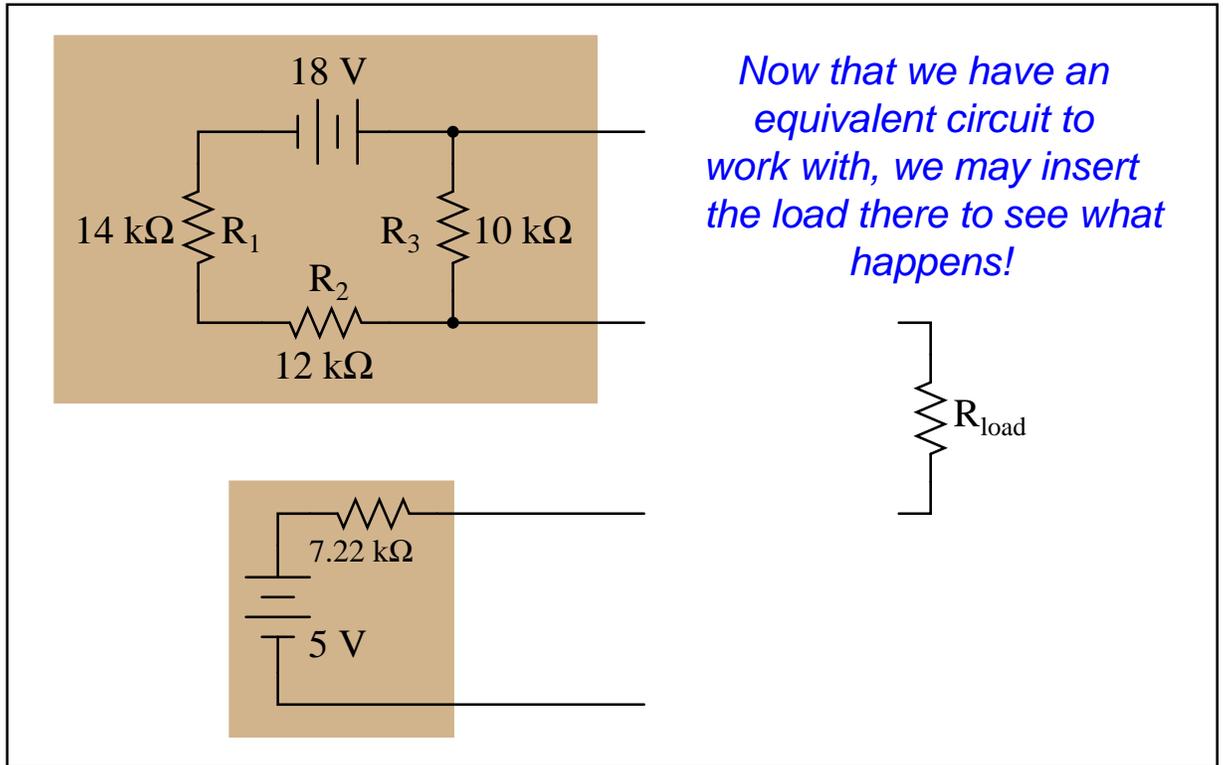


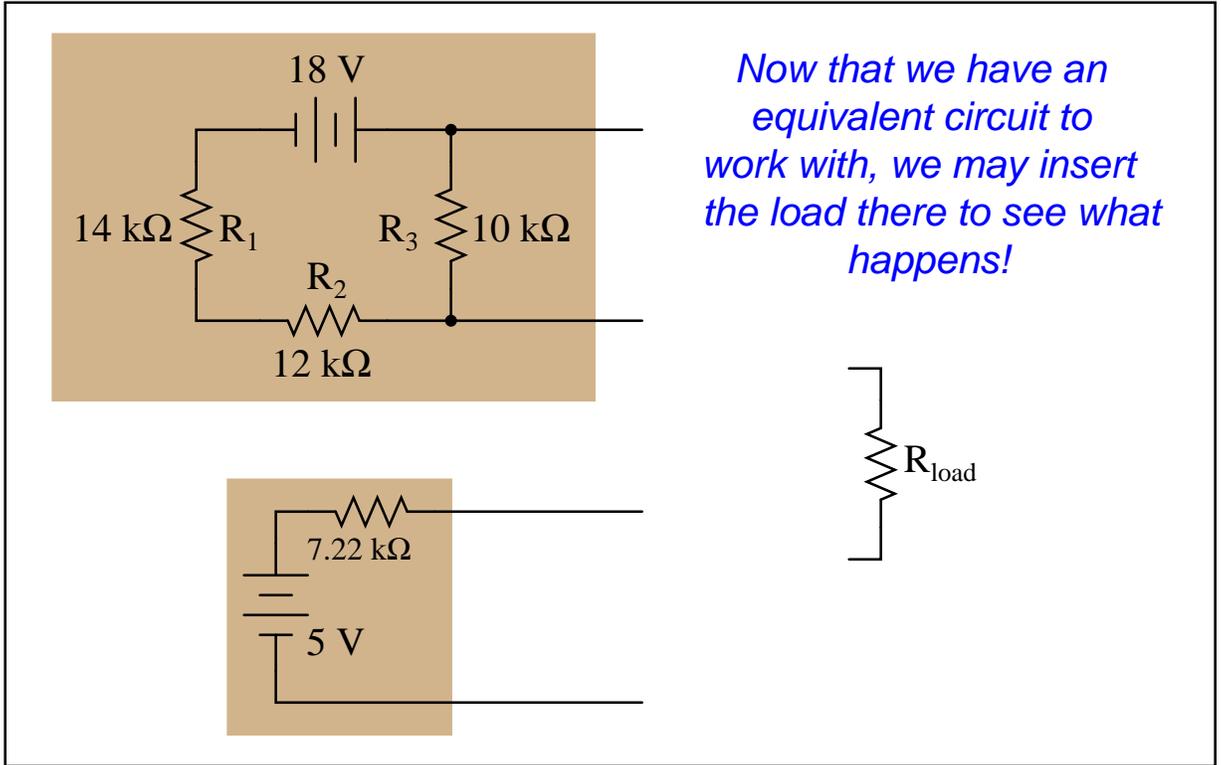


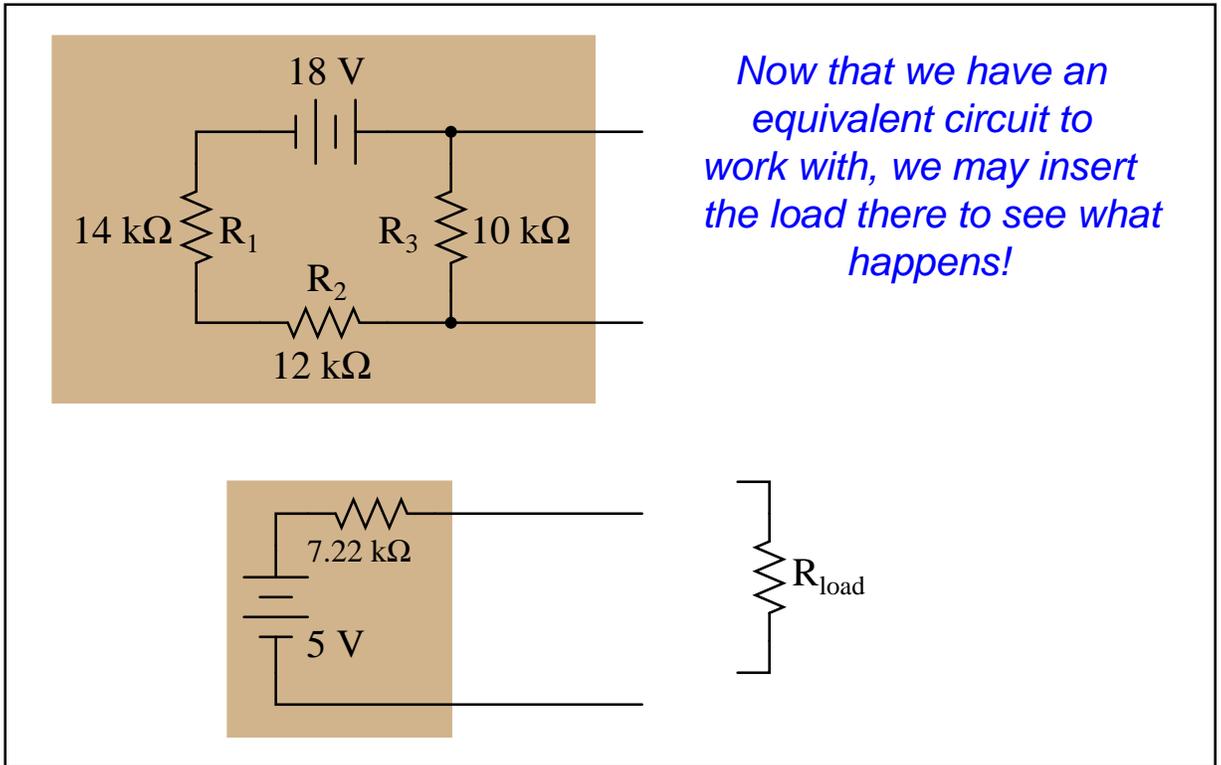


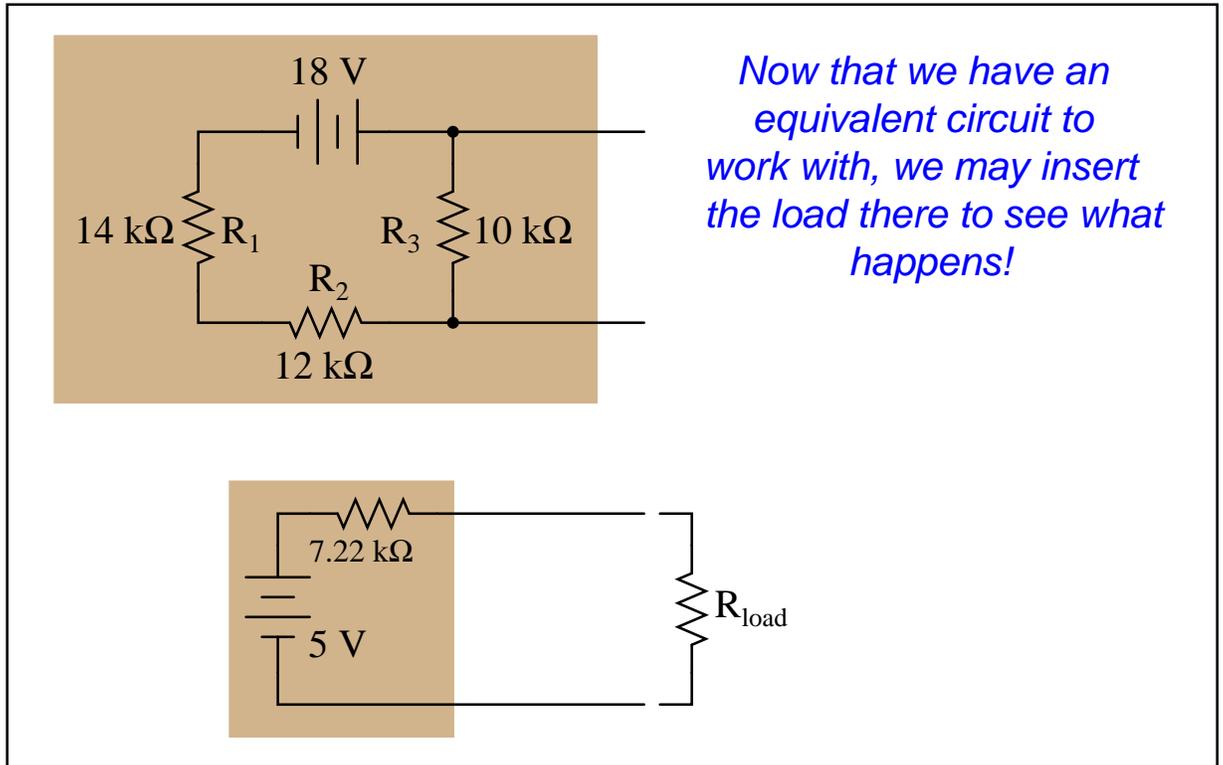


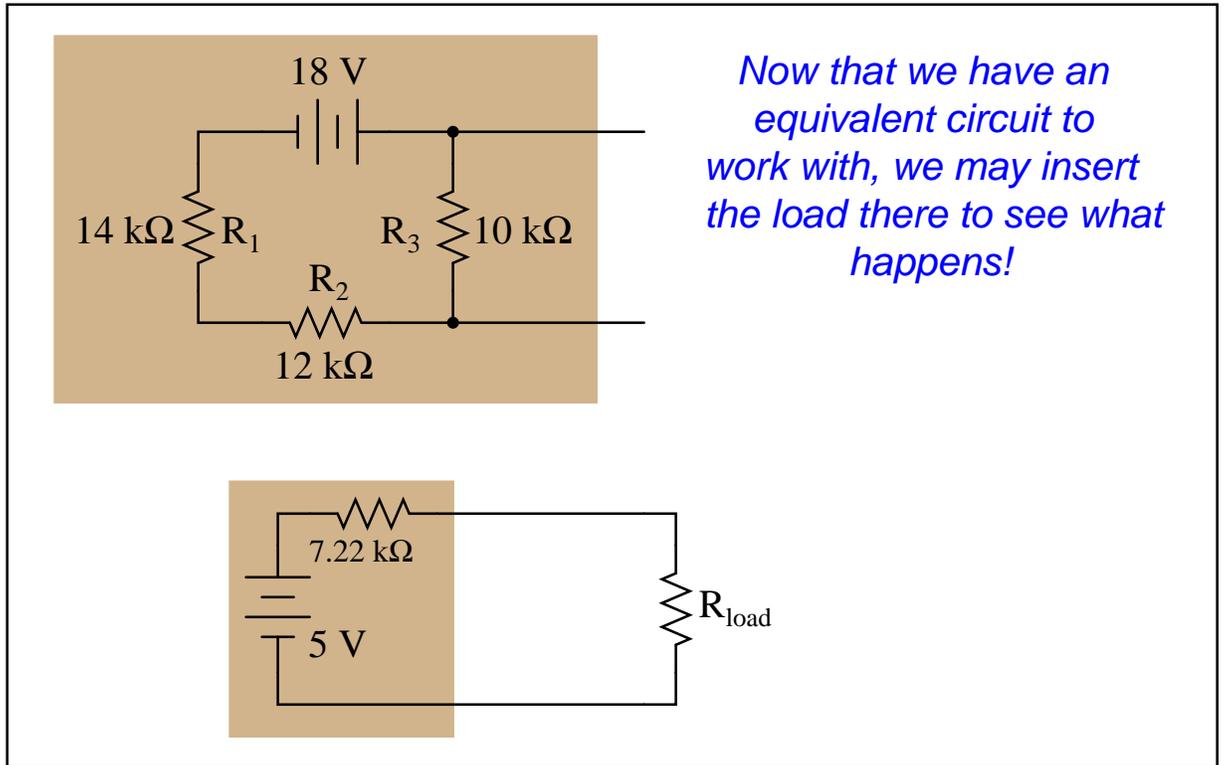


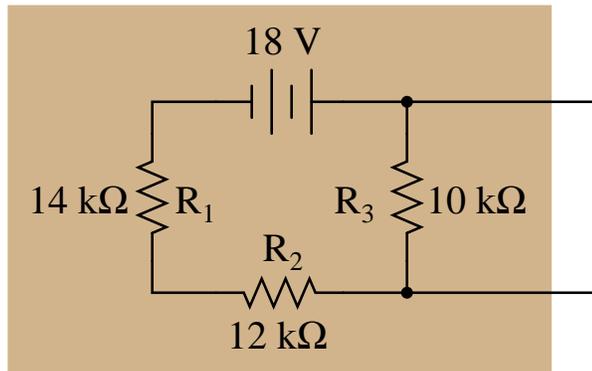




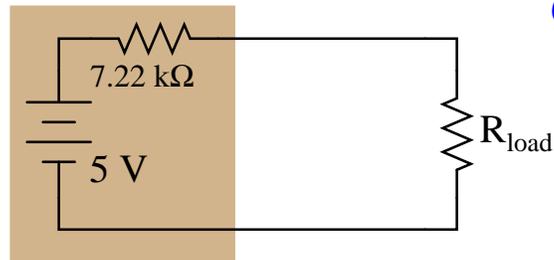




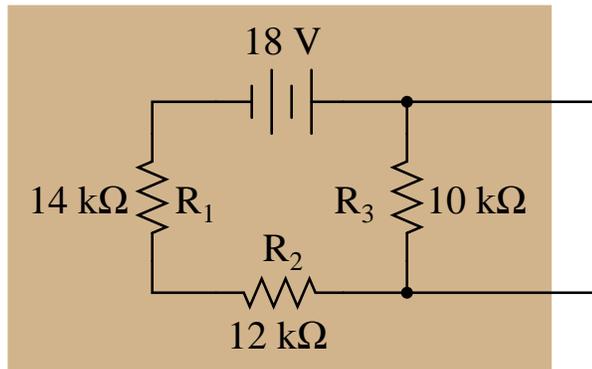




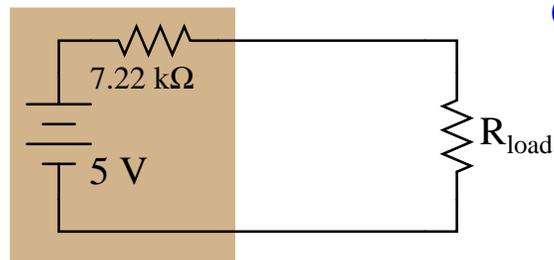
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!



Calculate:

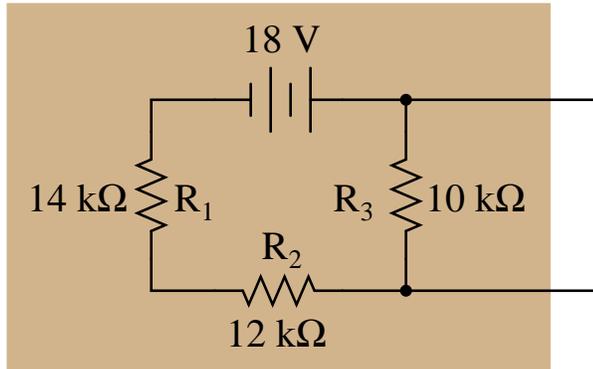


Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!

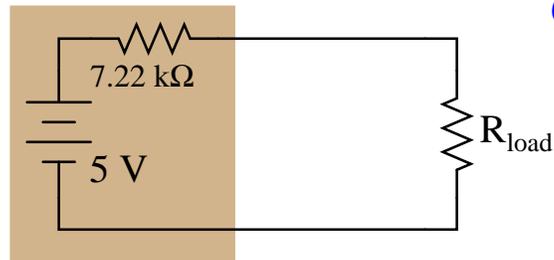


Calculate:

V_{load}



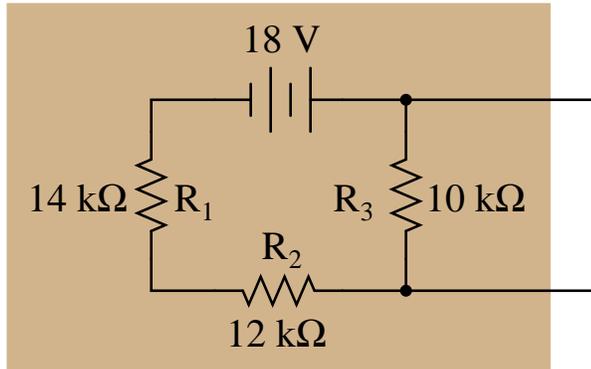
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!



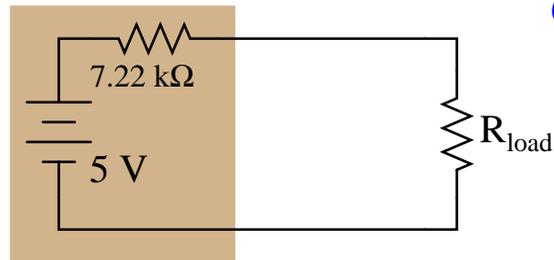
Calculate:

V_{load}

I_{load}



Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!

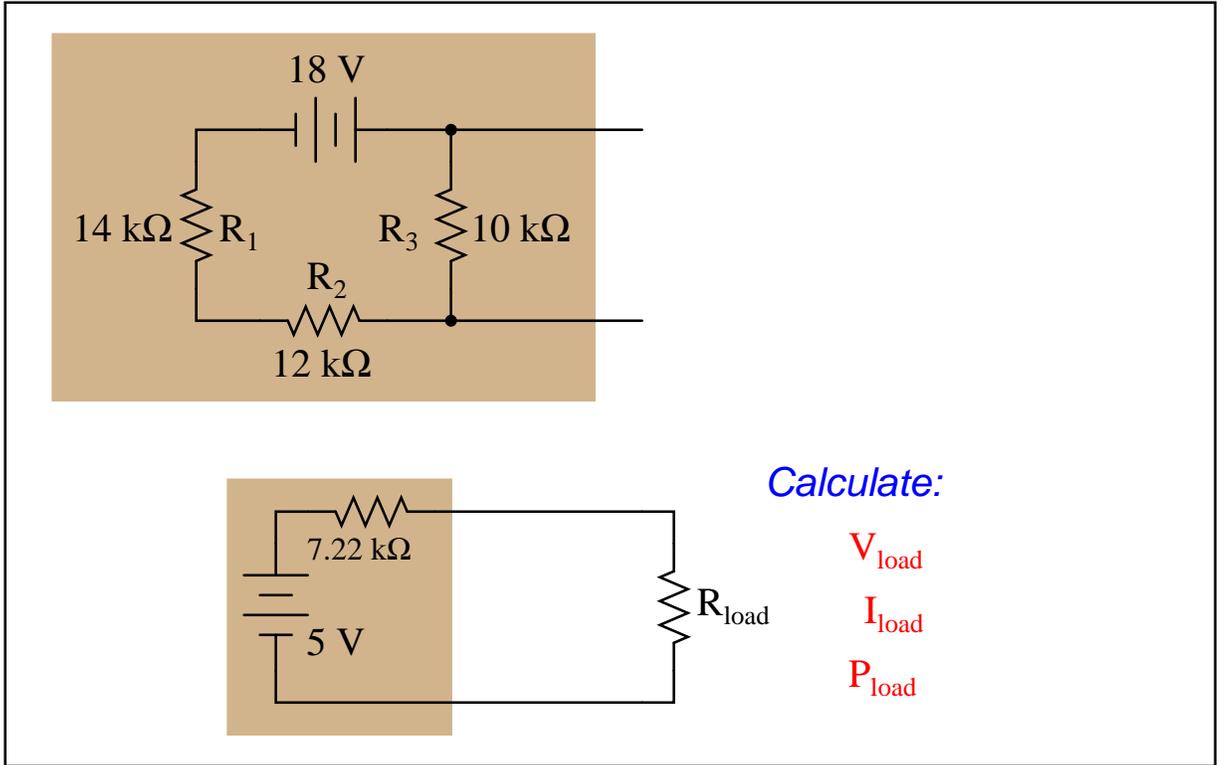


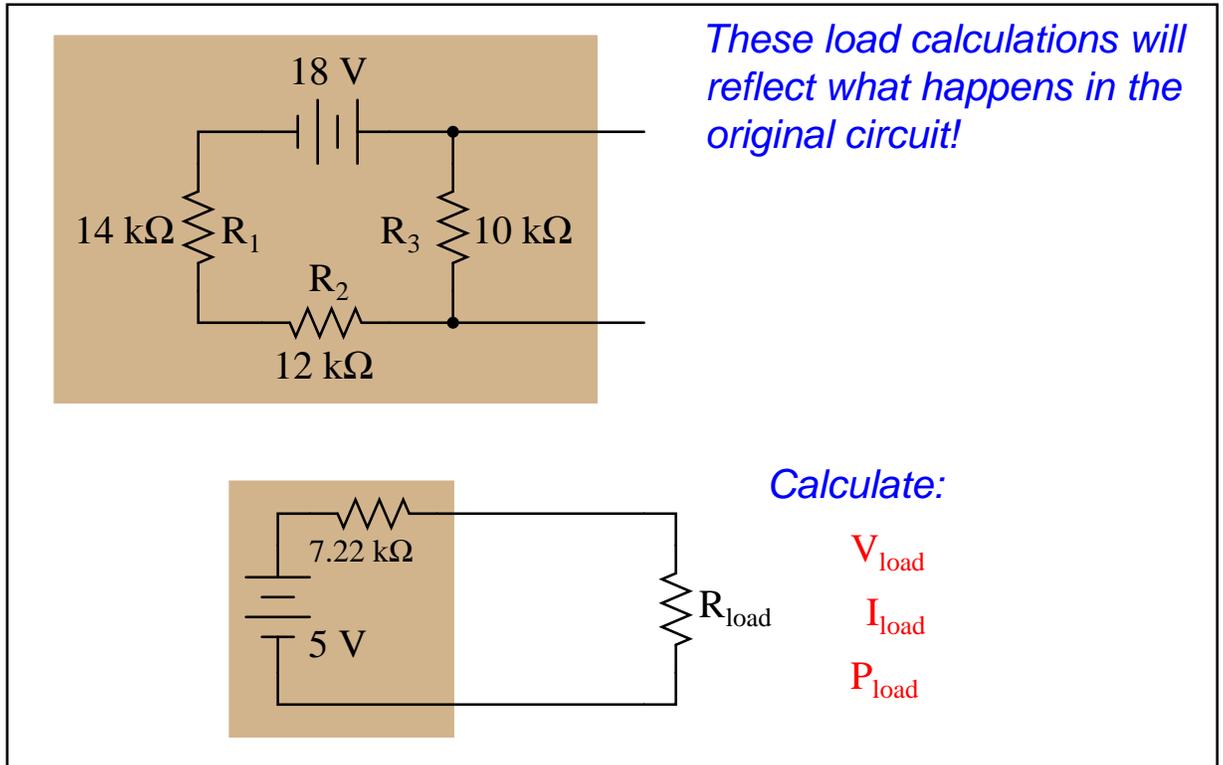
Calculate:

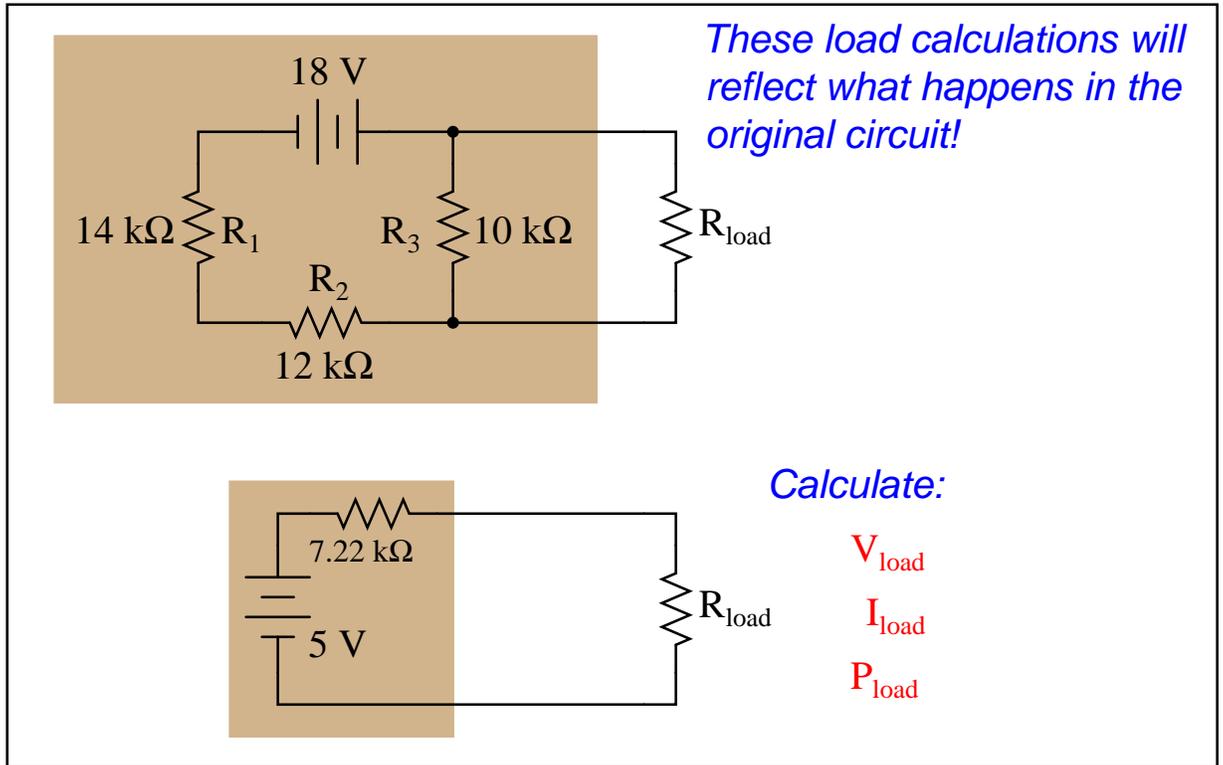
V_{load}

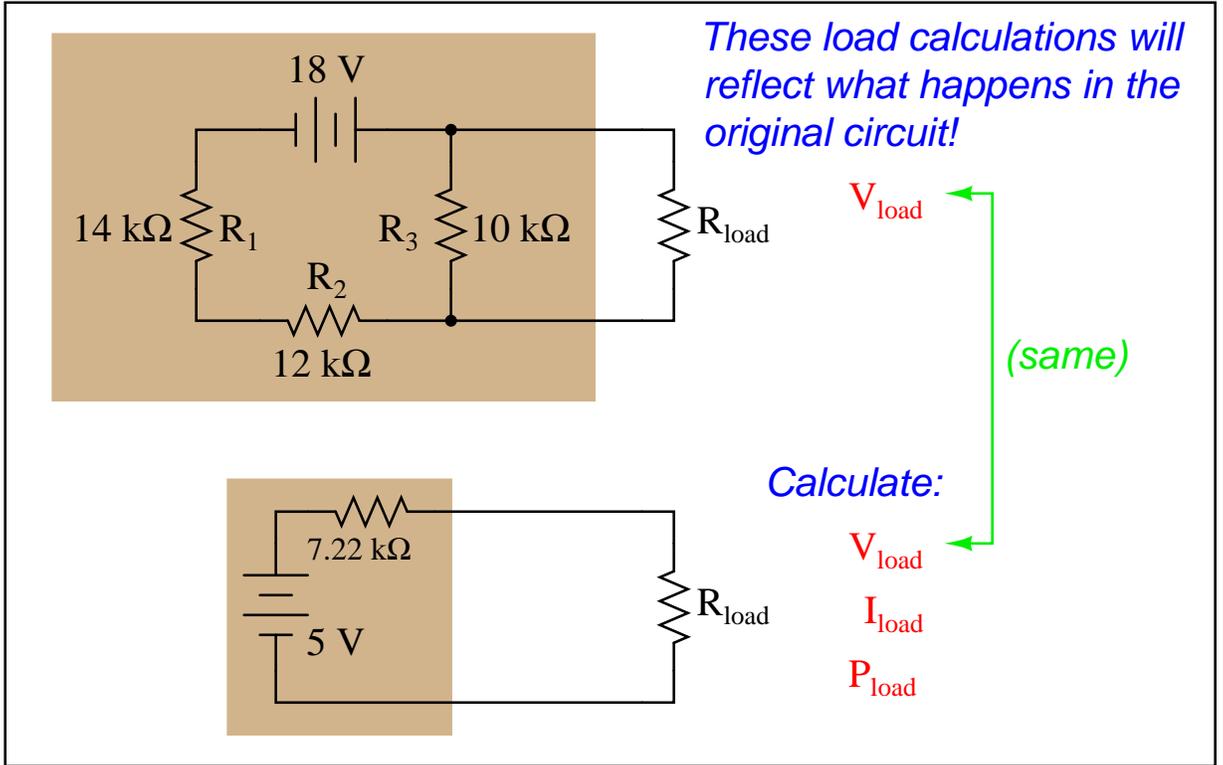
I_{load}

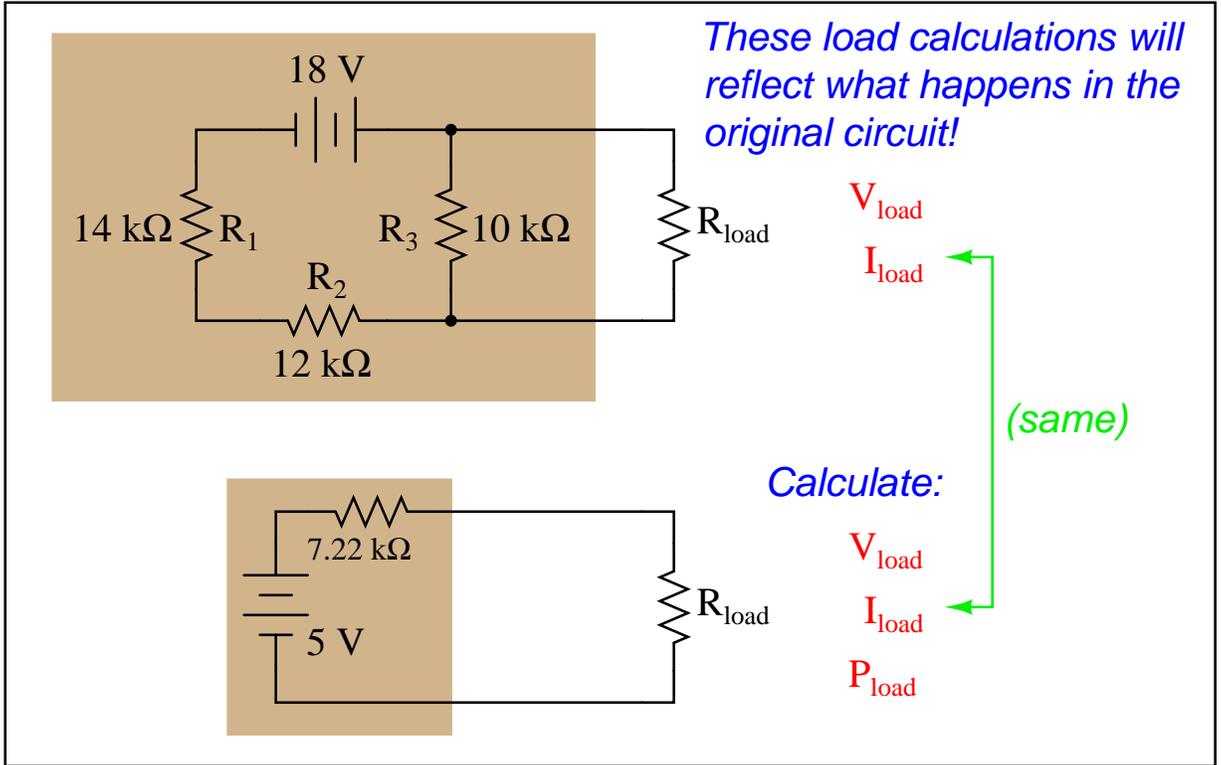
P_{load}

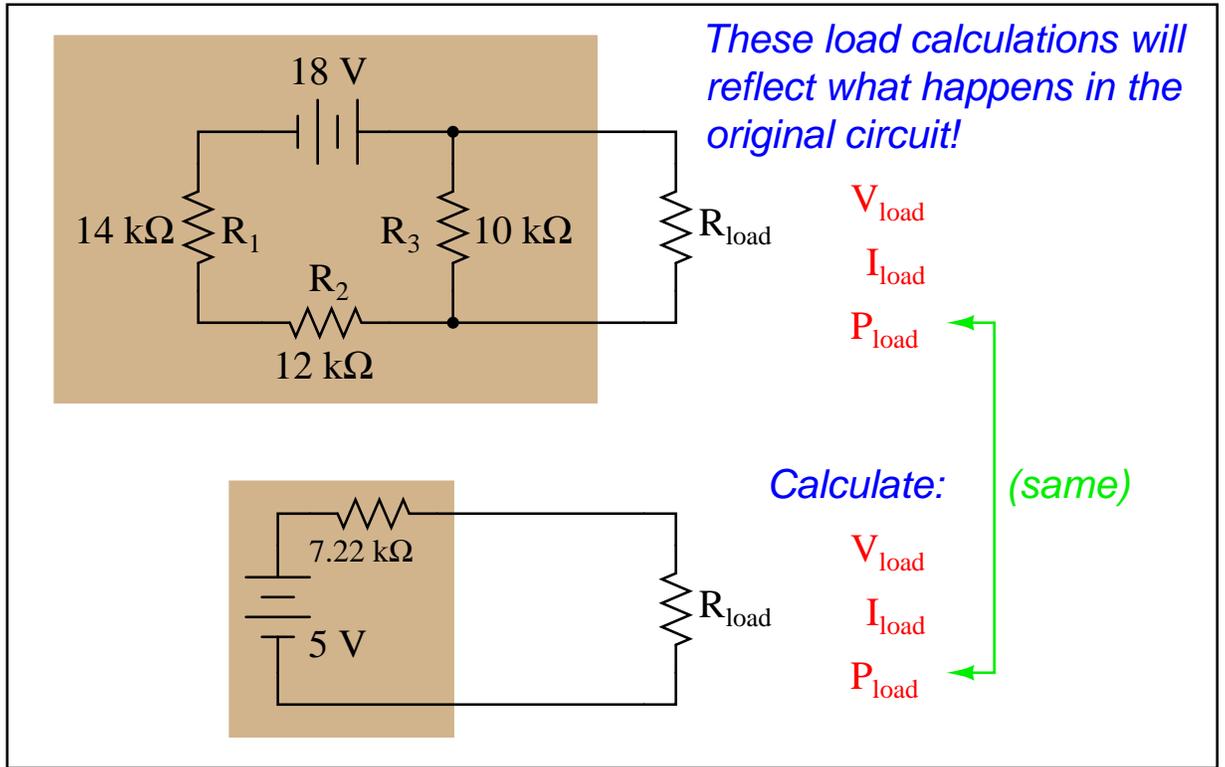


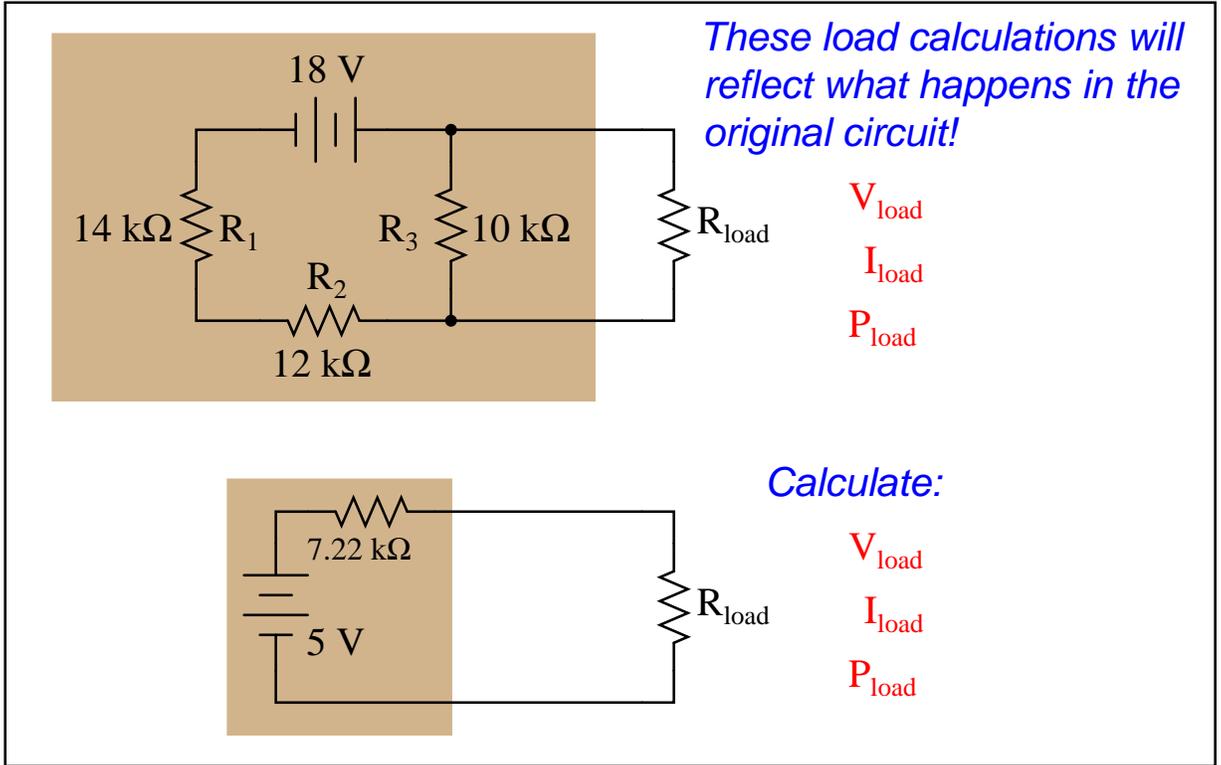


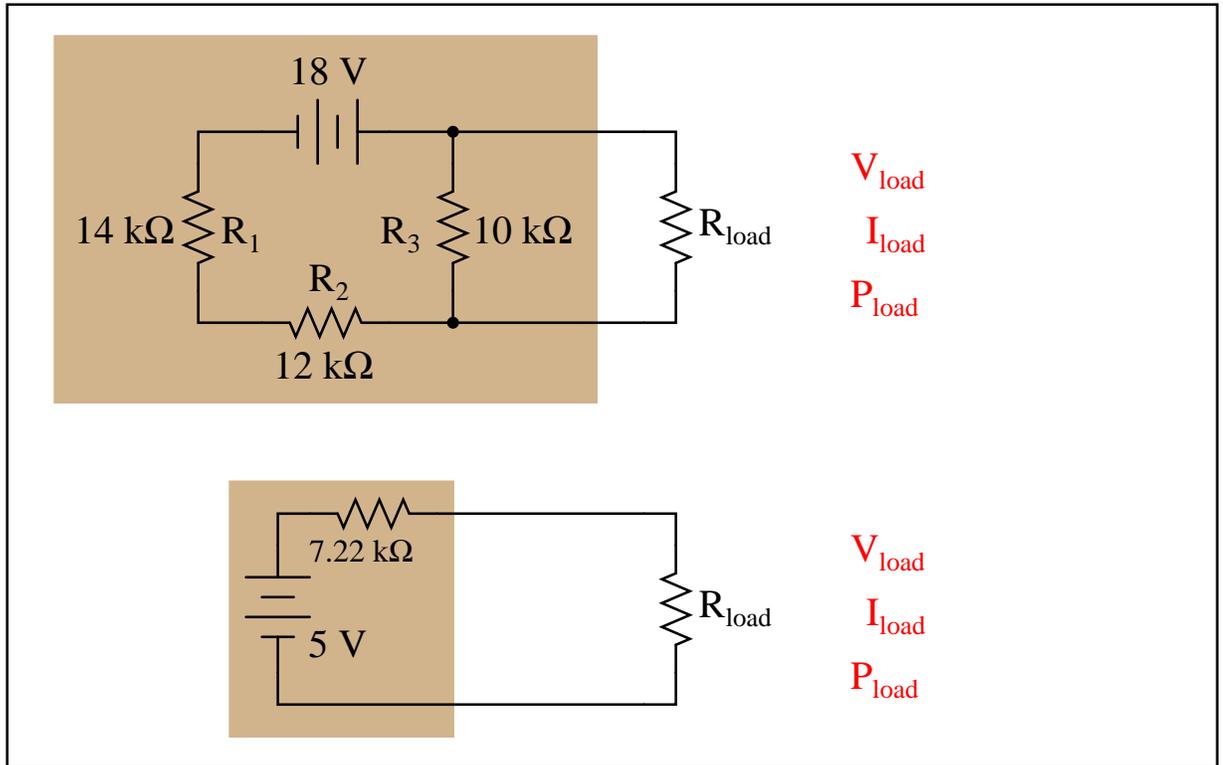


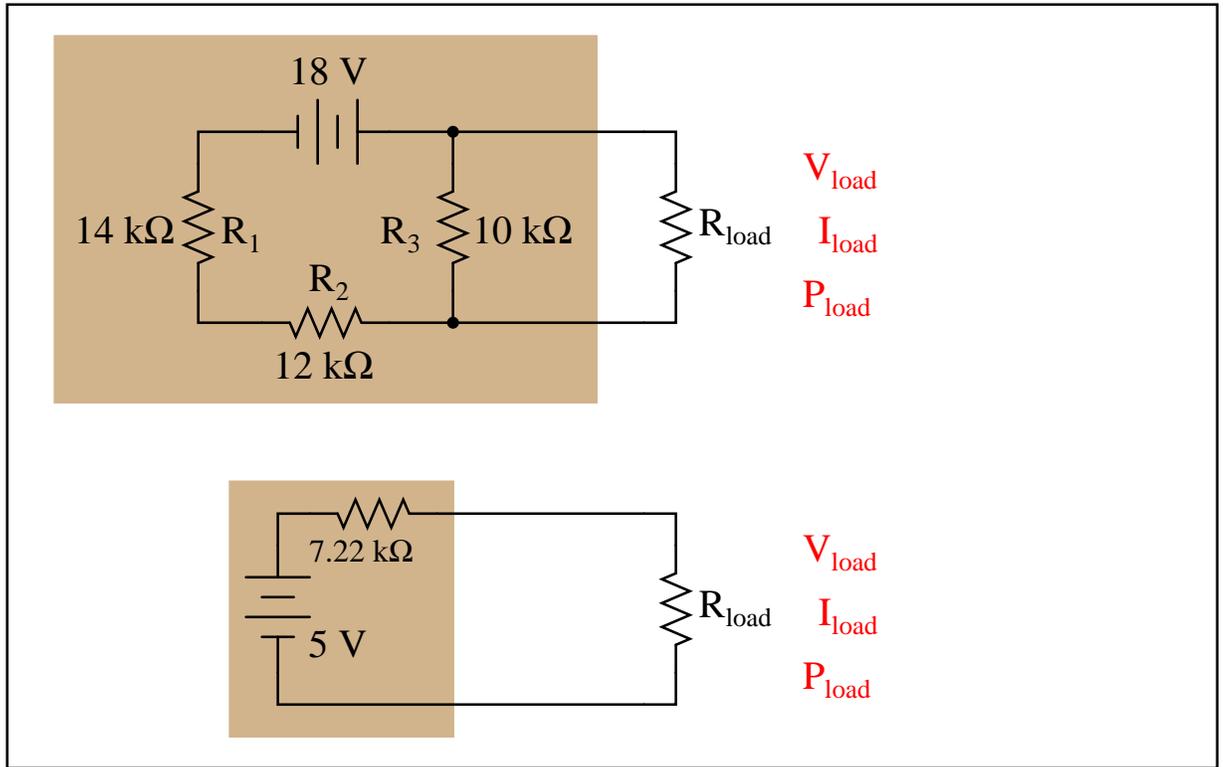


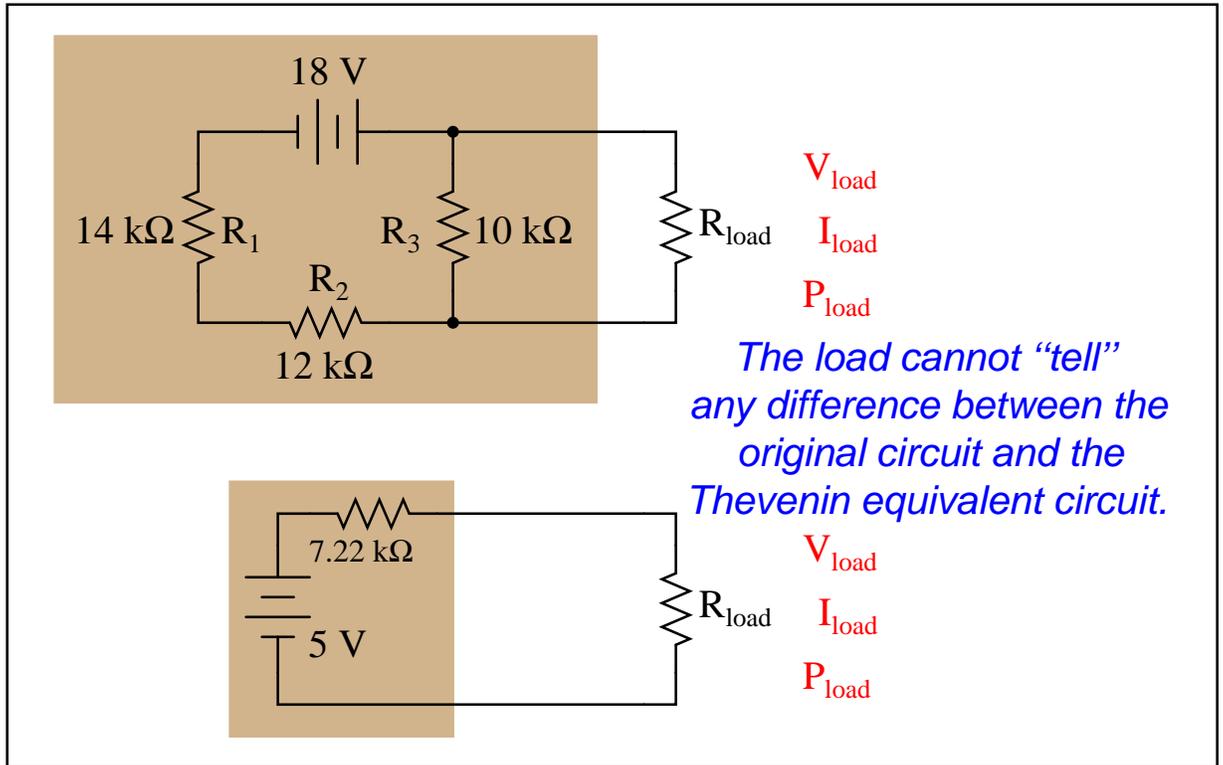












Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly **SUMMARIZE THE TEXT** in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Thought experiments as a problem-solving strategy

Electrical source

Electrical load

Series connection

Equivalent networks

Current source

Voltage source

Open

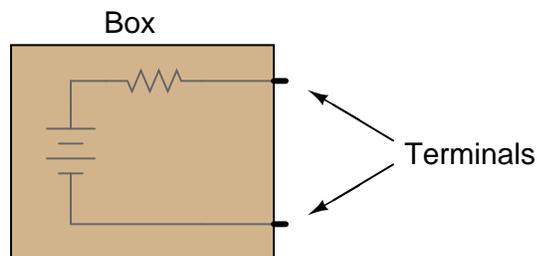
Short

Voltage divider

Re-drawing schematics as a problem-solving strategy

6.1.3 Testing a “black box” voltage source

Suppose you were handed a black box with two metal terminals on one side, for attaching electrical (wire) connections. Inside this box, you were told, was a voltage source (an ideal voltage source connected in series with a resistance):



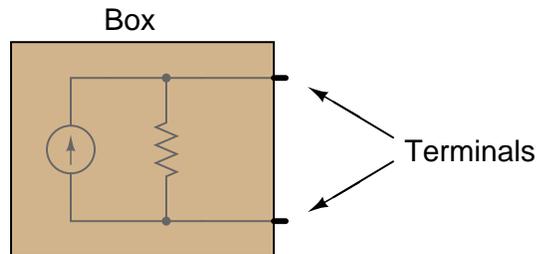
How would you empirically determine the voltage of the ideal voltage source inside this box, and how would you empirically determine the resistance of the series resistor? By “empirically” I mean determine voltage and resistance using actual test equipment rather than assuming certain component values (remember, this “black box” is sealed, so you cannot look inside!).

Challenges

- How does this empirical technique relate to the theoretical process of Thévenizing or Nortonizing a network?

6.1.4 Testing a “black box” current source

Suppose you were handed a black box with two metal terminals on one side, for attaching electrical (wire) connections. Inside this box, you were told, was a current source (an ideal current source connected in parallel with a resistance):



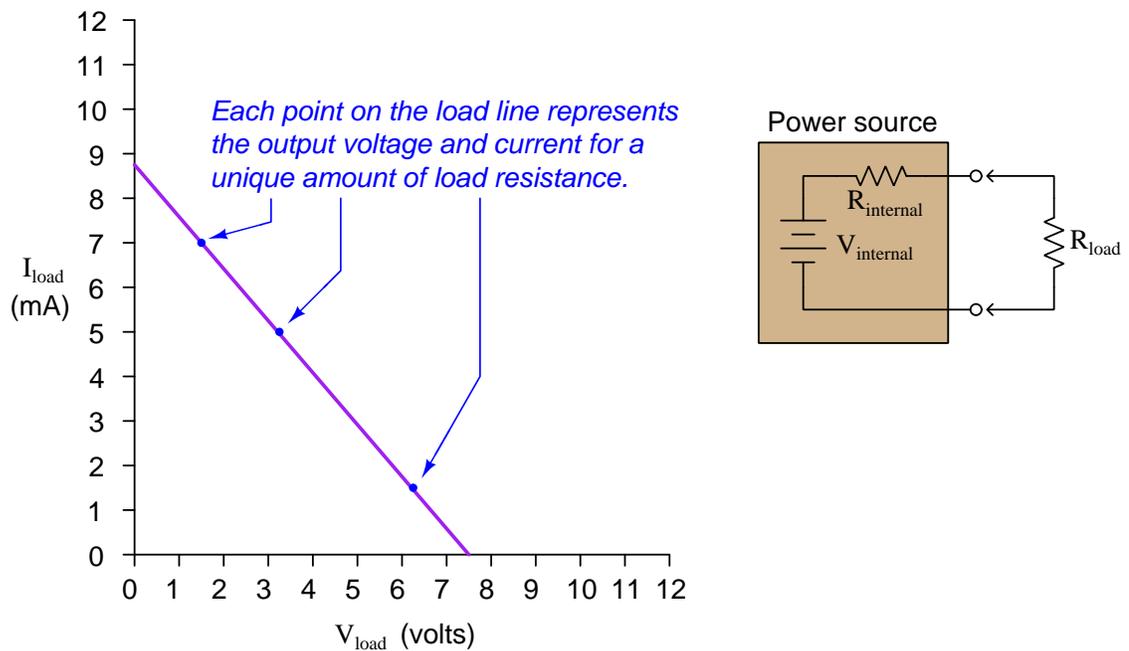
How would you empirically determine the current of the ideal current source inside this box, and how would you empirically determine the resistance of the parallel resistor? By “empirically” I mean determine current and resistance using actual test equipment rather than assuming certain component values (remember, this “black box” is sealed, so you cannot look inside!).

Challenges

- How does this empirical technique relate to the theoretical process of Thévenizing or Nortonizing a network?

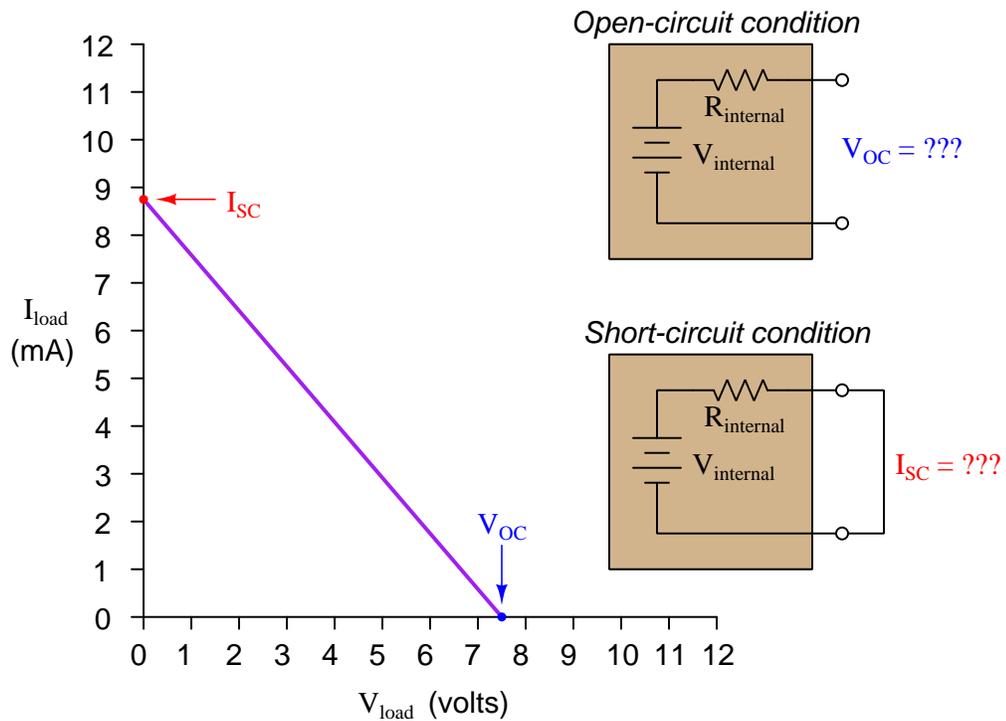
6.1.5 Load lines of equivalent sources

Load lines are special types of graphs used in electronics to characterize the output voltage and current behavior of different power sources:



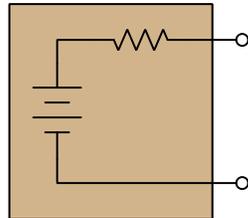
If we know that all the internal components of a power source are inherently *linear*, we know that the load line plot will indeed be a straight line. And, if we know the plot will be a straight line, all we need in order to plot a complete load line are *two* data points.

Usually, the easiest data points to gather for a circuit – whether it be a real circuit or an hypothetical circuit existing on paper only – is the *open-circuit* condition and the *short-circuit* condition. In other words, we see how much voltage the source will output with no load connected ($I_{load} = 0$ milliAmperes) and then we see how much current the source will output into a direct short ($V_{load} = 0$ Volts):



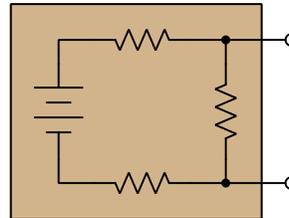
Suppose we have two differently-constructed power sources, yet both of these sources share the same open-circuit voltage (V_{OC}) and the same short-circuit current (I_{SC}). Assuming the internal components of both power sources are linear in nature, explain how we would know without doubt that the two power sources were electrically equivalent to one another. In other words, explain how we would know just from the limited data of V_{OC} and I_{SC} that these two power sources will behave exactly the same when connected to the same load resistance, whatever that load resistance may be.

How do we know these two power sources are completely equivalent to one another just from their equal open-circuit voltage and short-circuit current figures?



$$V_{OC} = 7.5 \text{ volts}$$

$$I_{SC} = 8.75 \text{ mA}$$



$$V_{OC} = 7.5 \text{ volts}$$

$$I_{SC} = 8.75 \text{ mA}$$

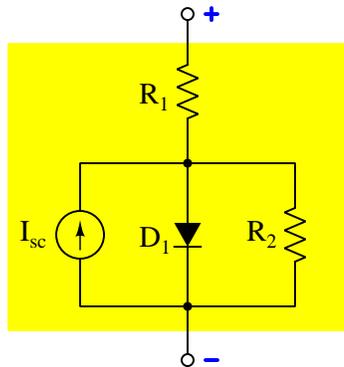
Challenges

- A line may be mathematically defined with just two points of data. Explain why this is fact is significant to this problem.

6.1.6 Thévenizing a solar cell

A photovoltaic *solar cell* is a semiconductor component designed to act as an electrical source when struck by sunlight. Its behavior is approximated by the lumped-component model shown below:

Solar cell model



The current output by source I_{SC} depends on the geometry of the solar cell and the amount of light striking it. The resistance values also depend on the geometry and chemical composition of the cell.

One of these modeling components – the *diode* labeled D_1 – has a non-linear characteristic. In other words, it does not exhibit a stable resistance value like the other modeling components. Although its real behavior is best described by the exponential *Shockley diode equation*, for the sake of our analysis here we may think of the diode as being either *on* or *off* depending on the voltage across its terminals: if the voltage is less than 0.7 Volts, this diode will be “off” (non-conducting); an attempt to raise its voltage above 0.7 Volts results in it turning “on” and maintaining a constant (regulated) voltage drop of 0.7 Volts as it conducts current. In other words, below 0.7 Volts it behaves as an open and at all other times it behaves as a constant 0.7 Volt device (e.g. a 0.7 Volt source).

Develop one Thévenin equivalent network for this solar cell assuming a condition where $V_{D1} < 0.7$ Volts, and another Thévenin equivalent network assuming $V_{D1} = 0.7$ Volts.

Challenges

- ???

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol⁻¹)

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^4$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared (m³/kg-s²)

Molar gas constant (R) = **8.314462618...** **Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin⁴** (W/m²·K⁴)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

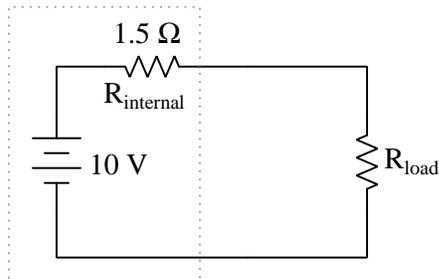
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

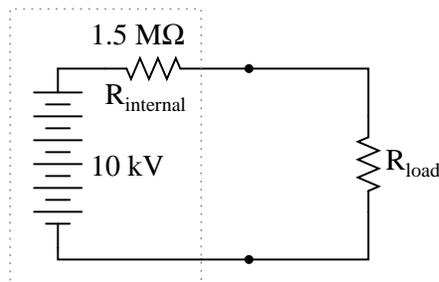
6.2.3 Testing sources under load

Calculate the voltage dropped across the load resistor, and the current through the load resistor, for the load resistance values of 1 k Ω , 2 k Ω , 5 k Ω , 8 k Ω , and 10 k Ω :



Do the “boxed” components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Now, do the same for this circuit:



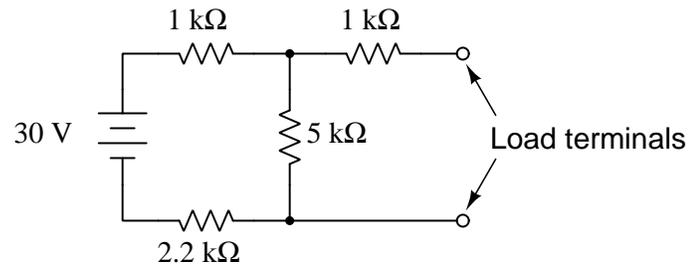
Do the “boxed” components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Challenges

- Would you say that voltage sources are typically characterized as having high internal resistances or low internal resistances? What about current sources? Explain your answers.

6.2.4 Thévenin and Norton equivalents of a single-source network

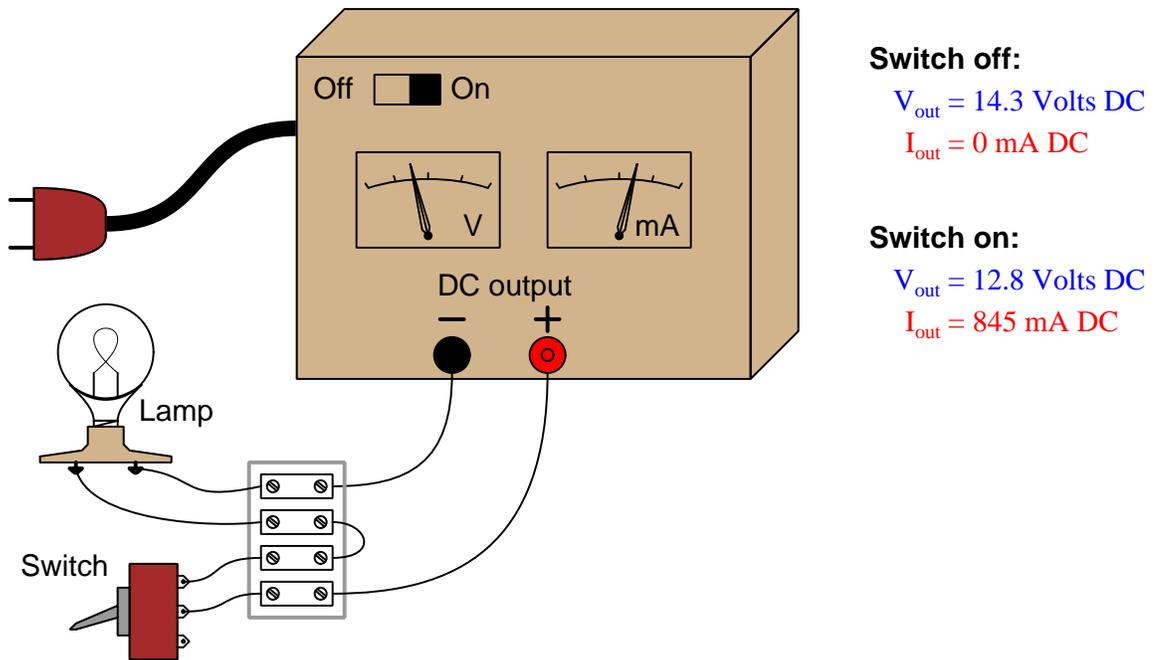
Reduce the following circuit to a Norton equivalent, and also to a Thévenin equivalent:

**Challenges**

- Explain why Thévenization and Nortonization are useful tools in circuit analysis.

6.2.5 Thévenin equivalent of an AC/DC power supply

Suppose you had an AC/DC power supply, which performed as follows (open-circuit and loaded test conditions):



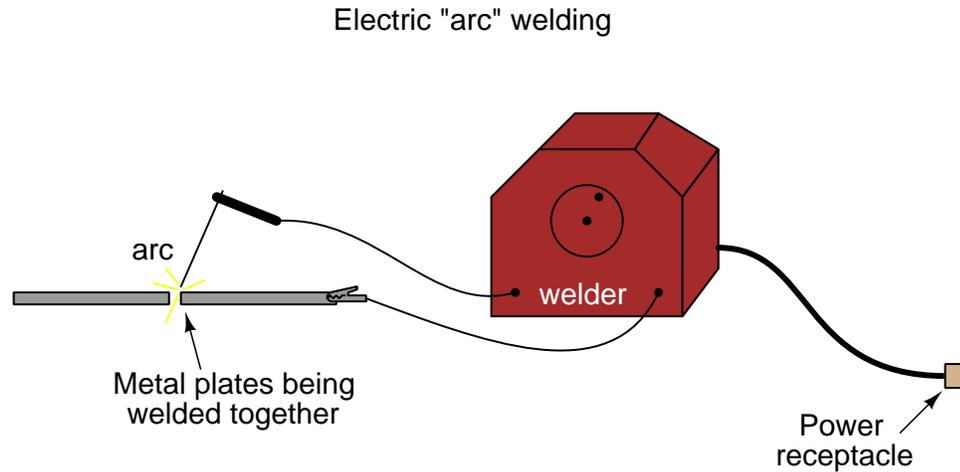
Draw a Thévenin equivalent network to model the behavior of this power supply.

Challenges

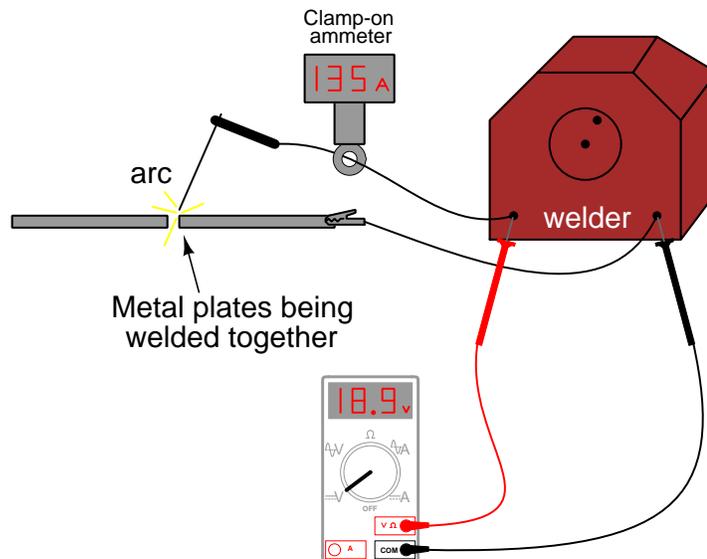
- Identify any assumptions we need to make about this power supply in order to apply Thévenin's Theorem to these measured values.

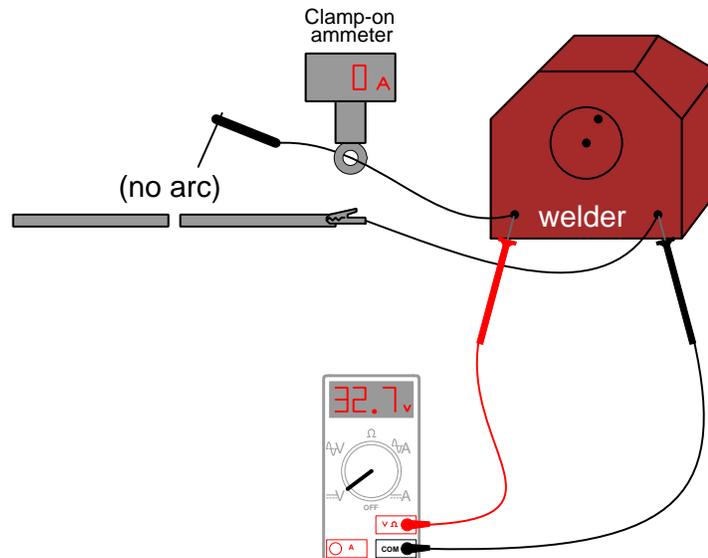
6.2.6 Norton equivalent of an electric arc welder

An *electric arc welder* is a low-voltage, high-current power source designed to supply enough electric current to sustain an arc capable of welding metal with its high temperature:



It is possible to derive a Norton equivalent network for an arc welder based on empirical measurements of voltage and current. Take for example these measurements, under *loaded* and *no-load* conditions:





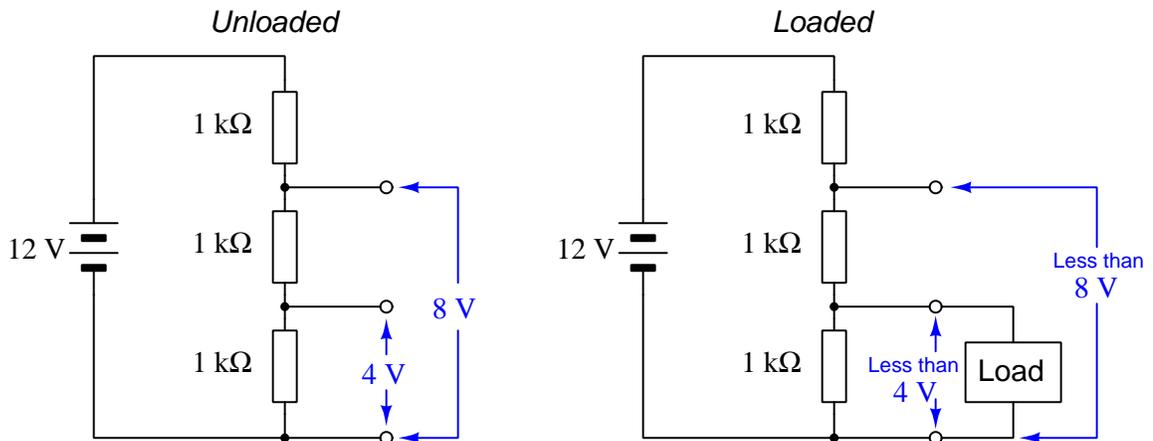
Based on these measurements, derive a Norton equivalent network for the arc welder.

Challenges

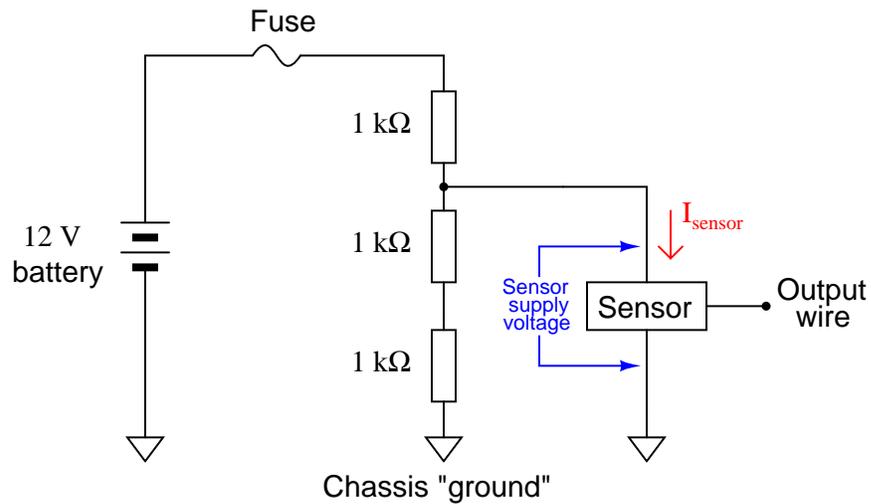
- Identify any assumptions we need to make about this arc welder in order to apply Thévenin's Theorem to these measured values.

6.2.7 Thévenizing a loaded voltage divider

Resistive voltage dividers are very useful and popular circuits. However, it should be realized that their output voltages “sag” under load:



Just how much a voltage divider’s output will sag under a given load may be a very important question in some applications. Take for instance the following application where we are using a resistive voltage divider to supply an engine sensor with reduced voltage (8 Volts) from the 12 Volt battery potential in the automobile:



If the sensor draws no current ($I_{sensor} = 0$ mA), then the voltage across the sensor supply terminals will be 8 Volts. However, if we were asked to predict the voltage across the sensor supply terminals for a variety of different sensor current conditions, we would be faced with a much more complex problem:

- $I_{sensor} = 0$ mA ; Sensor terminal voltage = 8 Volts
- $I_{sensor} = 1$ mA ; Sensor terminal voltage =
- $I_{sensor} = 2$ mA ; Sensor terminal voltage =
- $I_{sensor} = 3$ mA ; Sensor terminal voltage =
- $I_{sensor} = 4$ mA ; Sensor terminal voltage =
- $I_{sensor} = 5$ mA ; Sensor terminal voltage =

One technique we could use to simplify this problem is to reduce the voltage divider resistor network into a Thévenin equivalent network. With the three-resistor divider reduced to a single resistor in series with an equivalent voltage source, the calculations for sensor supply voltage become much simpler.

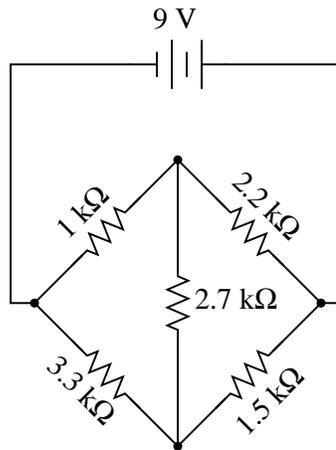
Show how this could be done, then complete the list of sensor supply voltages shown above.

Challenges

- If we are not supposed to let the sensor supply voltage to fall below 6.5 Volts, what is the maximum amount of current it may draw from this voltage divider circuit.
- Try solving for the sensor's terminal voltage without using Thévenin's Theorem, and describe the difficulty in doing so.

6.2.8 Unbalanced bridge circuit

Inspect the following circuit:



Note that it is not reducible to a single resistance and power source. In other words, it is *not* a series-parallel combination circuit. In fact, *no two resistors in this network are directly in series with each other or directly in parallel with each other!* And, while it is a bridge circuit, you are not able to simply analyze the resistor ratios because it is obviously not in a state of balance! If you were asked to calculate voltage or current for any component in this circuit, it would be a difficult task . . . unless you know either Thévenin's or Norton's Theorems, that is!

Apply either one of these theorems to the determination of voltage across the 2.2 kΩ resistor (the resistor in the upper-right corner of the bridge). Hint: consider the 2.2 kΩ resistor as the *load* in a Thévenin or Norton equivalent network.

Challenges

- What is the definition of a series network?
- What is the definition of a parallel network?

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

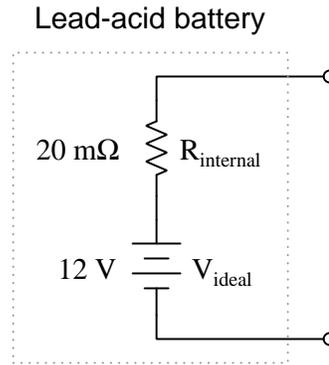
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

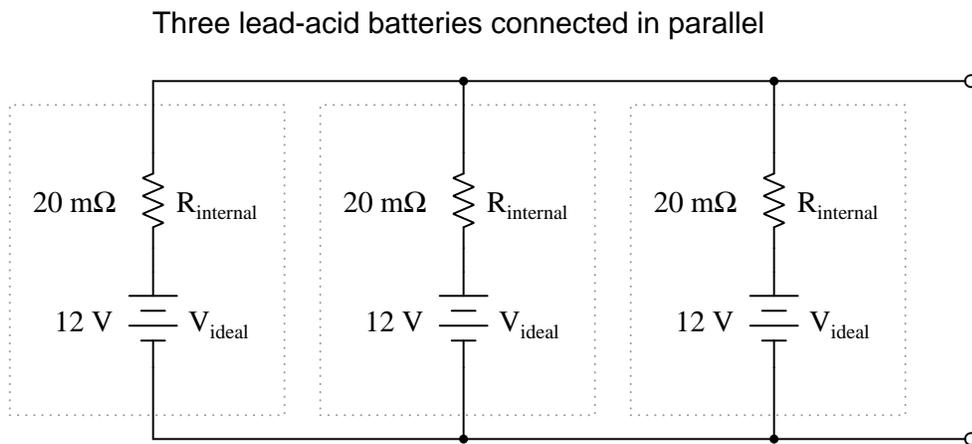
6.3.1 Fault current of a battery bank

Suppose a 12 Volt lead-acid battery has an internal resistance of 20 milli-Ohms ($20 \text{ m}\Omega$):



If a short-circuit were placed across the terminals of this large battery, the fault current would be quite large: 600 Amperes!

Now suppose three of these batteries were connected directly in parallel with one another:



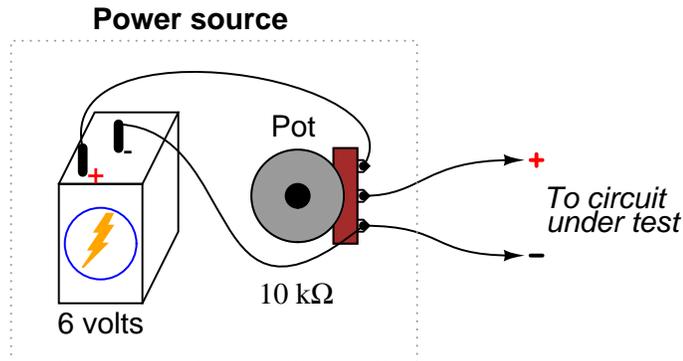
Re-calculate the fault current available at the terminals of the three-battery “bank” in the event of a direct short-circuit.

Challenges

- Explain what practical importance this question has for parallel-connected batteries, and how either Thévenin’s or Norton’s Theorems makes the concept easier to explain to someone else. What safety issues might be raised by the parallel connection of large batteries such as these?

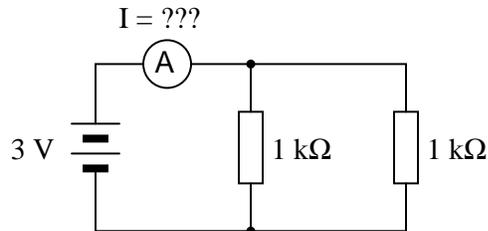
6.3.2 Variable-voltage power source

One day an electronics student decides to build her own variable-voltage power source using a 6 Volt battery and a 10 k Ω potentiometer:



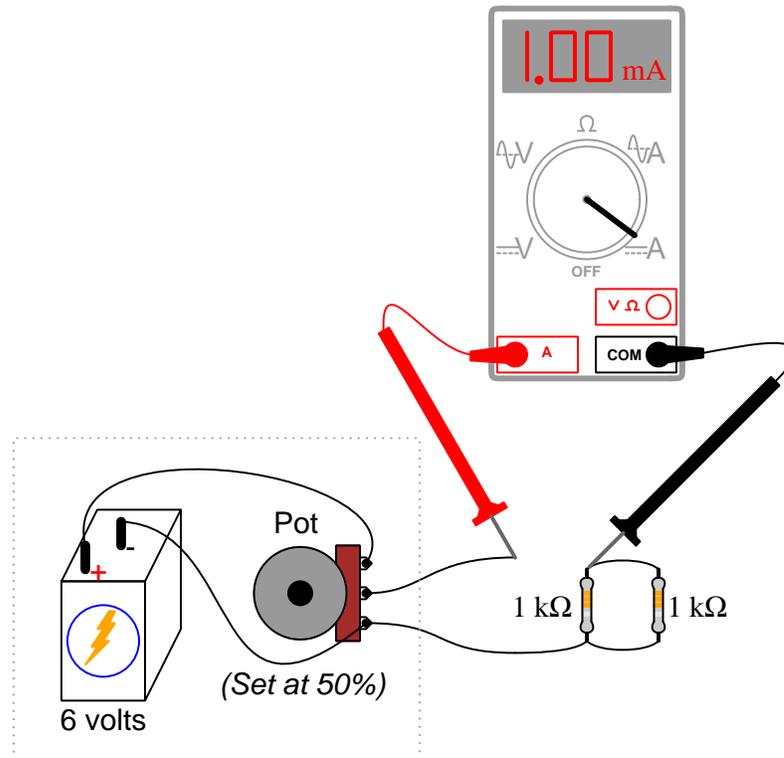
She tests her circuit by connecting a voltmeter to the output terminals and verifying that the voltage does indeed increase and decrease as the potentiometer knob is turned.

Later that day, her instructor assigns a quick lab exercise: measure the current through a parallel resistor circuit with an applied voltage of 3 Volts, as shown in the following schematic diagram.



Calculating current in this circuit is a trivial exercise, she thinks to herself: $3\text{ V} \div 500\ \Omega = 6\text{ mA}$. This will be a great opportunity to use the new power source circuit, as 3 Volts is well within the voltage adjustment range!

She first sets up her circuit to output 3 Volts precisely (turning the 10 k Ω potentiometer to the 50% position), measuring with her voltmeter as she did when initially testing the circuit. Then she connects the output leads to the two parallel resistors through her multimeter (configured as an ammeter), like this:



However, when she reads her ammeter display, the current only measures 1 mA, not 6 mA as she predicted. This is a very large discrepancy between her prediction and the measured value for current!

Why didn't her circuit behave as she predicted it would?

Challenges

- Explain what this student would have to do to use her adjustable-voltage power source circuit to properly demonstrate the lab circuit as assigned.
- Identify at least one circuit failure which would result in zero measured (ammeter) current.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge, critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' **Linux** and Richard Stallman's **GNU** project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of **Linux** back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient **Unix** applications and scripting languages (e.g. shell scripts, Makefiles, **sed**, **awk**) developed over many decades. **Linux** not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's Vim text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer **Vim** because it operates very similarly to **vi** which is ubiquitous on **Unix/Linux** operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the `MODEL` modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

`SPICE` is to circuit analysis as `TEX` is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

Creative Commons License

Creative Commons Attribution 4.0 International Public License

By exercising the Licensed Rights (defined below), You accept and agree to be bound by the terms and conditions of this Creative Commons Attribution 4.0 International Public License (“Public License”). To the extent this Public License may be interpreted as a contract, You are granted the Licensed Rights in consideration of Your acceptance of these terms and conditions, and the Licensor grants You such rights in consideration of benefits the Licensor receives from making the Licensed Material available under these terms and conditions.

Section 1 – Definitions.

a. **Adapted Material** means material subject to Copyright and Similar Rights that is derived from or based upon the Licensed Material and in which the Licensed Material is translated, altered, arranged, transformed, or otherwise modified in a manner requiring permission under the Copyright and Similar Rights held by the Licensor. For purposes of this Public License, where the Licensed Material is a musical work, performance, or sound recording, Adapted Material is always produced where the Licensed Material is synched in timed relation with a moving image.

b. **Adapter’s License** means the license You apply to Your Copyright and Similar Rights in Your contributions to Adapted Material in accordance with the terms and conditions of this Public License.

c. **Copyright and Similar Rights** means copyright and/or similar rights closely related to copyright including, without limitation, performance, broadcast, sound recording, and Sui Generis Database Rights, without regard to how the rights are labeled or categorized. For purposes of this Public License, the rights specified in Section 2(b)(1)-(2) are not Copyright and Similar Rights.

d. **Effective Technological Measures** means those measures that, in the absence of proper authority, may not be circumvented under laws fulfilling obligations under Article 11 of the WIPO Copyright Treaty adopted on December 20, 1996, and/or similar international agreements.

e. **Exceptions and Limitations** means fair use, fair dealing, and/or any other exception or

limitation to Copyright and Similar Rights that applies to Your use of the Licensed Material.

f. **Licensed Material** means the artistic or literary work, database, or other material to which the Licensor applied this Public License.

g. **Licensed Rights** means the rights granted to You subject to the terms and conditions of this Public License, which are limited to all Copyright and Similar Rights that apply to Your use of the Licensed Material and that the Licensor has authority to license.

h. **Licensor** means the individual(s) or entity(ies) granting rights under this Public License.

i. **Share** means to provide material to the public by any means or process that requires permission under the Licensed Rights, such as reproduction, public display, public performance, distribution, dissemination, communication, or importation, and to make material available to the public including in ways that members of the public may access the material from a place and at a time individually chosen by them.

j. **Sui Generis Database Rights** means rights other than copyright resulting from Directive 96/9/EC of the European Parliament and of the Council of 11 March 1996 on the legal protection of databases, as amended and/or succeeded, as well as other essentially equivalent rights anywhere in the world.

k. **You** means the individual or entity exercising the Licensed Rights under this Public License. **Your** has a corresponding meaning.

Section 2 – Scope.

a. License grant.

1. Subject to the terms and conditions of this Public License, the Licensor hereby grants You a worldwide, royalty-free, non-sublicensable, non-exclusive, irrevocable license to exercise the Licensed Rights in the Licensed Material to:

A. reproduce and Share the Licensed Material, in whole or in part; and

B. produce, reproduce, and Share Adapted Material.

2. Exceptions and Limitations. For the avoidance of doubt, where Exceptions and Limitations apply to Your use, this Public License does not apply, and You do not need to comply with its terms and conditions.

3. Term. The term of this Public License is specified in Section 6(a).

4. Media and formats; technical modifications allowed. The Licensor authorizes You to exercise the Licensed Rights in all media and formats whether now known or hereafter created, and to make technical modifications necessary to do so. The Licensor waives and/or agrees not to assert any right or authority to forbid You from making technical modifications necessary to exercise the Licensed Rights, including technical modifications necessary to circumvent Effective Technological Measures.

For purposes of this Public License, simply making modifications authorized by this Section 2(a)(4) never produces Adapted Material.

5. Downstream recipients.

A. Offer from the Licensor – Licensed Material. Every recipient of the Licensed Material automatically receives an offer from the Licensor to exercise the Licensed Rights under the terms and conditions of this Public License.

B. No downstream restrictions. You may not offer or impose any additional or different terms or conditions on, or apply any Effective Technological Measures to, the Licensed Material if doing so restricts exercise of the Licensed Rights by any recipient of the Licensed Material.

6. No endorsement. Nothing in this Public License constitutes or may be construed as permission to assert or imply that You are, or that Your use of the Licensed Material is, connected with, or sponsored, endorsed, or granted official status by, the Licensor or others designated to receive attribution as provided in Section 3(a)(1)(A)(i).

b. Other rights.

1. Moral rights, such as the right of integrity, are not licensed under this Public License, nor are publicity, privacy, and/or other similar personality rights; however, to the extent possible, the Licensor waives and/or agrees not to assert any such rights held by the Licensor to the limited extent necessary to allow You to exercise the Licensed Rights, but not otherwise.

2. Patent and trademark rights are not licensed under this Public License.

3. To the extent possible, the Licensor waives any right to collect royalties from You for the exercise of the Licensed Rights, whether directly or through a collecting society under any voluntary or waivable statutory or compulsory licensing scheme. In all other cases the Licensor expressly reserves any right to collect such royalties.

Section 3 – License Conditions.

Your exercise of the Licensed Rights is expressly made subject to the following conditions.

a. Attribution.

1. If You Share the Licensed Material (including in modified form), You must:

A. retain the following if it is supplied by the Licensor with the Licensed Material:

i. identification of the creator(s) of the Licensed Material and any others designated to receive attribution, in any reasonable manner requested by the Licensor (including by pseudonym if designated);

ii. a copyright notice;

- iii. a notice that refers to this Public License;
- iv. a notice that refers to the disclaimer of warranties;
- v. a URI or hyperlink to the Licensed Material to the extent reasonably practicable;

B. indicate if You modified the Licensed Material and retain an indication of any previous modifications; and

C. indicate the Licensed Material is licensed under this Public License, and include the text of, or the URI or hyperlink to, this Public License.

2. You may satisfy the conditions in Section 3(a)(1) in any reasonable manner based on the medium, means, and context in which You Share the Licensed Material. For example, it may be reasonable to satisfy the conditions by providing a URI or hyperlink to a resource that includes the required information.

3. If requested by the Licensor, You must remove any of the information required by Section 3(a)(1)(A) to the extent reasonably practicable.

4. If You Share Adapted Material You produce, the Adapter's License You apply must not prevent recipients of the Adapted Material from complying with this Public License.

Section 4 – Sui Generis Database Rights.

Where the Licensed Rights include Sui Generis Database Rights that apply to Your use of the Licensed Material:

- a. for the avoidance of doubt, Section 2(a)(1) grants You the right to extract, reuse, reproduce, and Share all or a substantial portion of the contents of the database;
- b. if You include all or a substantial portion of the database contents in a database in which You have Sui Generis Database Rights, then the database in which You have Sui Generis Database Rights (but not its individual contents) is Adapted Material; and
- c. You must comply with the conditions in Section 3(a) if You Share all or a substantial portion of the contents of the database.

For the avoidance of doubt, this Section 4 supplements and does not replace Your obligations under this Public License where the Licensed Rights include other Copyright and Similar Rights.

Section 5 – Disclaimer of Warranties and Limitation of Liability.

a. Unless otherwise separately undertaken by the Licensor, to the extent possible, the Licensor offers the Licensed Material as-is and as-available, and makes no representations or warranties of any kind concerning the Licensed Material, whether express, implied, statutory, or other. This includes, without limitation, warranties of title, merchantability, fitness for a particular purpose, non-infringement, absence of latent or other defects, accuracy, or the presence or absence of errors,

whether or not known or discoverable. Where disclaimers of warranties are not allowed in full or in part, this disclaimer may not apply to You.

b. To the extent possible, in no event will the Licensor be liable to You on any legal theory (including, without limitation, negligence) or otherwise for any direct, special, indirect, incidental, consequential, punitive, exemplary, or other losses, costs, expenses, or damages arising out of this Public License or use of the Licensed Material, even if the Licensor has been advised of the possibility of such losses, costs, expenses, or damages. Where a limitation of liability is not allowed in full or in part, this limitation may not apply to You.

c. The disclaimer of warranties and limitation of liability provided above shall be interpreted in a manner that, to the extent possible, most closely approximates an absolute disclaimer and waiver of all liability.

Section 6 – Term and Termination.

a. This Public License applies for the term of the Copyright and Similar Rights licensed here. However, if You fail to comply with this Public License, then Your rights under this Public License terminate automatically.

b. Where Your right to use the Licensed Material has terminated under Section 6(a), it reinstates:

1. automatically as of the date the violation is cured, provided it is cured within 30 days of Your discovery of the violation; or

2. upon express reinstatement by the Licensor.

For the avoidance of doubt, this Section 6(b) does not affect any right the Licensor may have to seek remedies for Your violations of this Public License.

c. For the avoidance of doubt, the Licensor may also offer the Licensed Material under separate terms or conditions or stop distributing the Licensed Material at any time; however, doing so will not terminate this Public License.

d. Sections 1, 5, 6, 7, and 8 survive termination of this Public License.

Section 7 – Other Terms and Conditions.

a. The Licensor shall not be bound by any additional or different terms or conditions communicated by You unless expressly agreed.

b. Any arrangements, understandings, or agreements regarding the Licensed Material not stated herein are separate from and independent of the terms and conditions of this Public License.

Section 8 – Interpretation.

a. For the avoidance of doubt, this Public License does not, and shall not be interpreted to, reduce, limit, restrict, or impose conditions on any use of the Licensed Material that could lawfully

be made without permission under this Public License.

b. To the extent possible, if any provision of this Public License is deemed unenforceable, it shall be automatically reformed to the minimum extent necessary to make it enforceable. If the provision cannot be reformed, it shall be severed from this Public License without affecting the enforceability of the remaining terms and conditions.

c. No term or condition of this Public License will be waived and no failure to comply consented to unless expressly agreed to by the Licensor.

d. Nothing in this Public License constitutes or may be interpreted as a limitation upon, or waiver of, any privileges and immunities that apply to the Licensor or You, including from the legal processes of any jurisdiction or authority.

Creative Commons is not a party to its public licenses. Notwithstanding, Creative Commons may elect to apply one of its public licenses to material it publishes and in those instances will be considered the “Licensor.” Except for the limited purpose of indicating that material is shared under a Creative Commons public license or as otherwise permitted by the Creative Commons policies published at creativecommons.org/policies, Creative Commons does not authorize the use of the trademark “Creative Commons” or any other trademark or logo of Creative Commons without its prior written consent including, without limitation, in connection with any unauthorized modifications to any of its public licenses or any other arrangements, understandings, or agreements concerning use of licensed material. For the avoidance of doubt, this paragraph does not form part of the public licenses.

Creative Commons may be contacted at creativecommons.org.

Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

23 October 2024 – added a new Technical References section on applying Thévenin’s and Norton’s Theorems to time-delay circuits.

8 October 2024 – fixed the R_{Th} error in equivalent network #3 of the “Example: Thévenizing series-parallel networks” section of the Case Tutorial chapter.

2 October 2024 – added another “Challenging concepts” bullet-item to the list in the Introduction chapter.

15-17 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors. Also added active-reading suggestions to the Tutorial chapter on analyzing the example equivalent networks, as well as changed the phrase “equivalent circuit” to “equivalent network”.

5 March 2024 – added notes to images `anim_thevenin_24` and `anim_thevenin_25` explaining the use of the voltage divider equation to calculate V_{R3} .

10 October 2023 – converted a Case Tutorial section into a new (regular) Tutorial section, focusing on equivalent electrical networks using a “sealed box” analogy.

12 June 2023 – added Derivations and Technical References chapter, with a section showing a derivation of Millman’s Theorem.

6-7 Mar 2023 – minor edits to the Tutorial for readability, including capitalizing the word “Theorem” when used with Thévenin’s or Norton’s.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

10-11 October 2022 – minor additions to the Tutorial chapter, providing alternative conceptualizations for the disabling of current sources and voltage sources. Also fixed a typographical error (“short-circuit voltage” should have been “short-circuit current”) courtesy of Kobe Wessels. Also made minor additions to the Introduction chapter. Also corrected a mathematical error in one of the instructor comments.

28 February 2022 – added some page breaks to the “Example: Thévenizing series-parallel networks” Case Tutorial section for cleaner appearance.

15-17 February 2022 – minor additions to the Introduction and Tutorial chapters, as well as some edits to illustrations in the “Norton equivalent of an electric arc welder” Quantitative Reasoning question. Also added a Conceptual Reasoning question on Thévenizing a solar cell, and added some new Case Tutorial sections.

12 October 2021 – added “A” and “B” terminal labels to image_2038 to make it clear that the two equivalent networks are referencing the terminals shown in the original network. Also, colored the DMM digits shown in image_3524 and image_3513 and image_3514.

23 September 2021 – minor edits to a schematic diagram in the “Thévenizing a loaded voltage divider” Quantitative Reasoning question.

16 September 2021 – added section to Tutorial elaborating on the practical applications of these theorems.

28 August 2021 – commented out or deleted empty chapters, as well as added more questions.

23 March 2021 – added Case Tutorial sections showing a multi-source network being Thévenized and Nortonized.

12-13 October 2020 – minor additions to the Introduction chapter, and minor additions to instructor notes.

1 October 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

27 September 2020 – added a Case Tutorial chapter.

7 May 2020 – completed the Foundational concepts section.

30 April 2020 – added many questions.

12 September 2019 – continued writing Tutorial.

11 September 2019 – continued writing Tutorial.

6 September 2019 – continued writing Tutorial.

5 September 2019 – document first created.

Index

- Active reading, 25, 26
- Adding quantities to a qualitative problem, 148
- Annotating diagrams, 147

- Bridge network, 39

- Checking for exceptions, 148
- Checking your work, 148
- Code, computer, 155

- Dimensional analysis, 147

- Edwards, Tim, 156
- Equivalent networks, 24

- Graph values to solve a problem, 148
- Greenleaf, Cynthia, 117

- Heavy versus light load, 22
- How to teach with these modules, 150
- Hwang, Andrew D., 157

- Identify given data, 147
- Identify relevant principles, 147
- Impedance, 35
- Instructions for projects and experiments, 151
- Intermediate results, 147
- Inverted instruction, 150
- Isosceles Triangle Theorem, 3

- Kirchhoff's Voltage Law, 40
- Knuth, Donald, 156

- Lampert, Leslie, 156
- Limiting cases, 148
- Load, heavy versus light, 22

- Maximum Power Transfer Theorem, 35
- Metacognition, 122

- Millman's Theorem, 44
- Moolenaar, Bram, 155
- Murphy, Lynn, 117

- Norton equivalent source, 44
- Norton's Theorem, 26, 27

- Ohm's Law, 24, 30, 31, 42, 44
- Open, 31
- Open-source, 155

- Parallel, 39
- Potentiometer, 14, 37
- Problem-solving: annotate diagrams, 147
- Problem-solving: check for exceptions, 148
- Problem-solving: checking work, 148
- Problem-solving: dimensional analysis, 147
- Problem-solving: graph values, 148
- Problem-solving: identify given data, 147
- Problem-solving: identify relevant principles, 147
- Problem-solving: interpret intermediate results, 147
- Problem-solving: limiting cases, 148
- Problem-solving: qualitative to quantitative, 148
- Problem-solving: quantitative to qualitative, 148
- Problem-solving: reductio ad absurdum, 148
- Problem-solving: simplify the system, 147
- Problem-solving: thought experiment, 3, 30, 147
- Problem-solving: track units of measurement, 147
- Problem-solving: visually represent the system, 147
- Problem-solving: work in reverse, 148

- Qualitatively approaching a quantitative problem, 148

- Reading Apprenticeship, 117

- Reading, active, 25, 26
- Reductio ad absurdum, 148–150
- Rheostat, 37

- Schoenbach, Ruth, 117
- Scientific method, 122
- Series, 39
- Short, 31
- Simplifying a system, 147
- Socrates, 149
- Socratic dialogue, 150
- SPICE, 117
- Stallman, Richard, 155

- Thévenin's Theorem, 26, 27
- Theorem, Isosceles Triangle, 3
- Thought experiment, 3, 30, 147
- Torvalds, Linus, 155

- Units of measurement, 147

- Visualizing a system, 147
- Voltage divider formula, 38

- Work in reverse to solve a problem, 148
- WYSIWYG, 155, 156