

Dynamic fluids

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This worksheet introduces the basic concepts of gases and liquids in motion.

Questions

Question 1

Water flowing through an 8-inch pipe at an average velocity of 5 feet per second enters a narrower section of pipe (3 inches), changing its velocity accordingly. Calculate the water's average velocity in this narrower section of pipe.

[file i04796](#)

Question 2

Crude oil flows at an average velocity of 14 feet per second through a pipe with an internal diameter of 7.92 inches. Calculate the flow rate of this oil in units of gallons per minute, as well as barrels per day.

[file i04797](#)

Question 3

Crude oil flows at an average velocity of 10 feet per second through a pipe with an internal diameter of 5.95 inches. Calculate the flow rate of this oil in units of gallons per minute, as well as barrels per day.

[file i04785](#)

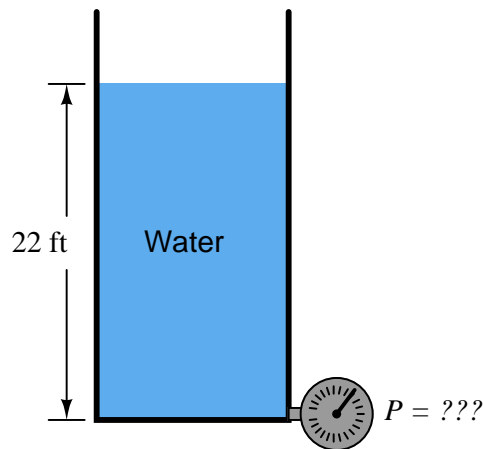
Question 4

Calculate the average velocity of gasoline flowing through a 6-inch pipe at a flow rate of 180 GPM.

[file i04788](#)

Question 5

Use Bernoulli's equation to calculate the hydrostatic pressure at the bottom of this water storage tank:



Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

Where,

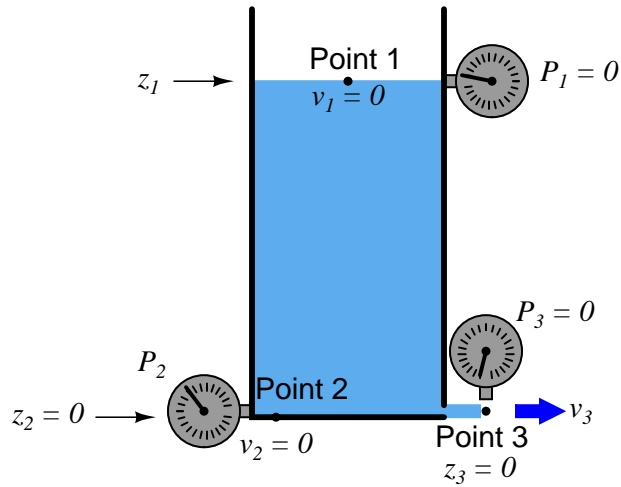
$$\rho = 1.94 \text{ slugs/ft}^3 \text{ (for water)}$$

$$g = 32.2 \text{ ft/s}^2$$

[file i02987](#)

Question 6

If we poke a hole at the bottom of a vessel containing an otherwise static column of liquid, we have an application for Bernoulli's equation at three different points in the system:



Note that point 1 has no velocity or pressure, but it does have elevation (height); point 2 has no velocity or elevation, but it does have pressure; and point 3 has no elevation or pressure, but it does have velocity.

Use Bernoulli's equation to write a new equation relating the elevation of point 1 with the pressure of point 2 and the velocity of point 3, and then solve for the velocity at point 3 in terms of the other two points' non-zero variables.

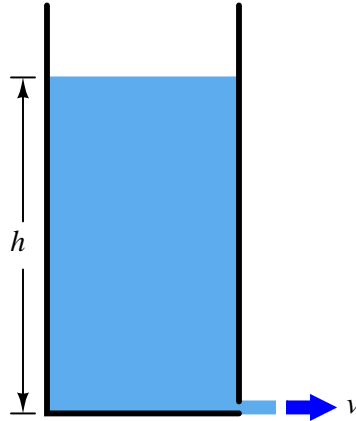
Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

[file i00446](#)

Question 7

Almost one century prior to Daniel Bernoulli's famous equation (1738), an Italian named Evangelista Torricelli (1643) discovered that the velocity of a liquid stream exiting a vessel was proportional to the square root of the height of the liquid column inside the vessel:



$$v = \sqrt{2gh}$$

Where,

v = Liquid stream velocity, in meters per second (m/s)

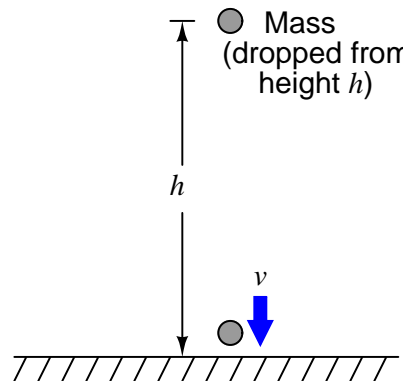
g = Acceleration of gravity, in meters per second squared (m/s^2)

h = Height of liquid column, in meters (m)

An interesting aspect of Torricelli's discovery was that this velocity did not depend at all on the density of the liquid. In other words, the velocity of a mercury stream would be the same as the velocity of a water stream so long as the two liquids' column heights were equal.

It should come as no surprise that Torricelli was a student of Galileo, the man who discovered that the velocity of a falling object did not depend upon the mass of the object (neglecting the effect of air friction).

Use algebra to prove falling objects follow the same basic rule, namely that free-fall velocity is a function of height alone and not mass (neglecting air friction):



Potential energy of mass at height h (before falling) = mgh

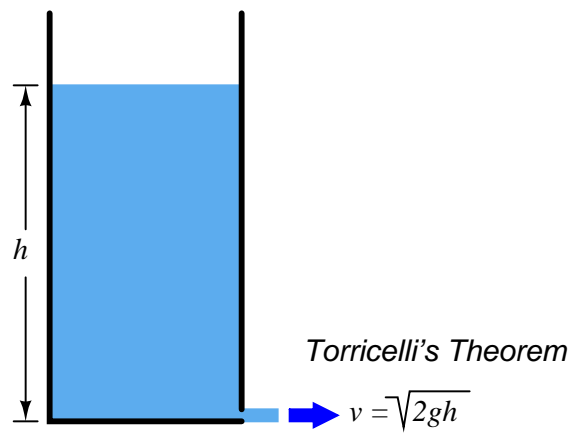
Kinetic energy of mass just before it hits the ground = $\frac{1}{2}mv^2$

Do you notice a similarity between the Torricelli's formula and the one you derived from the potential and kinetic energy equations?

[file i00447](#)

Question 8

Torricelli's Theorem describes the velocity of a liquid exiting the bottom of a vessel as a function of liquid height within the vessel, and also the acceleration of Earth's gravity (g):



We also know that the hydrostatic pressure generated by a vertical column of liquid follows this formula:

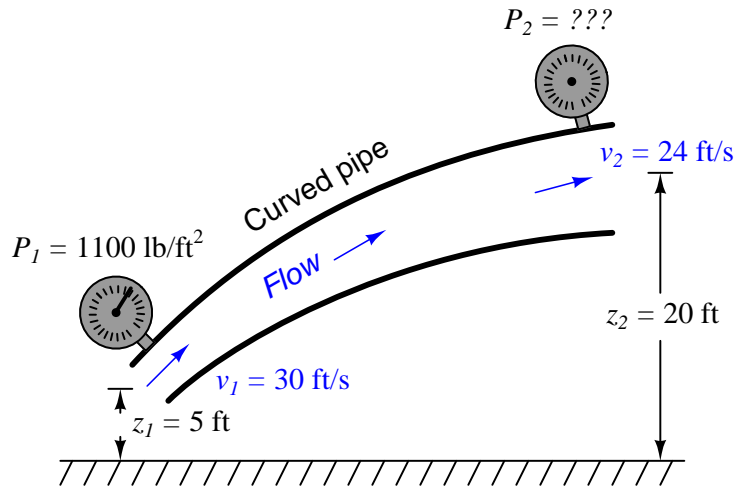
$$P = \rho gh$$

Combine this formula with Torricelli's Theorem to express fluid velocity through an orifice as a function of *pressure* rather than of liquid *height*.

[file i02747](#)

Question 9

Calculate the pressure at the discharge end of this pipe (P_2), assuming water as the fluid (with a mass density $\rho = 1.94$ slugs/ft³), 32.2 ft/s² as the acceleration of gravity (g), and frictionless flow (no pressure loss due to friction):



Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

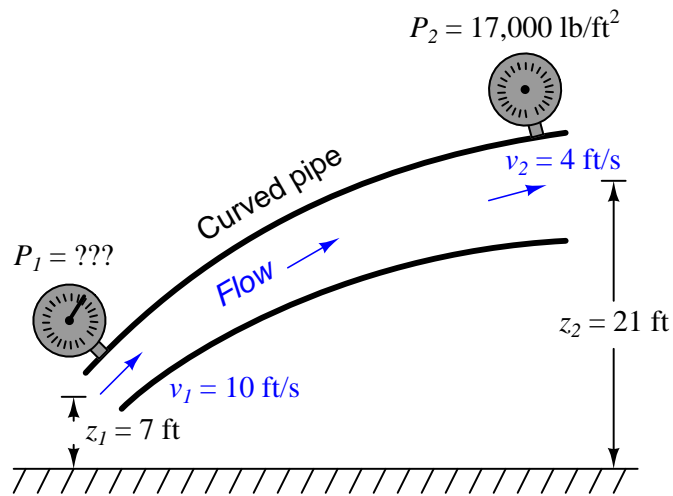
Suggestions for Socratic discussion

- One way students commonly fail to arrive at the correct answers with Bernoulli's Law calculations is by using incompatible units of measurement. Show how all the units of measurement provided to you in this question are compatible in their given forms, with no need for conversion.

file i00450

Question 10

Calculate the pressure at the discharge end of this pipe (P_2), assuming water as the fluid (with a mass density $\rho = 1.94$ slugs/ft³), 32.2 ft/s² as the acceleration of gravity (g), and frictionless flow (no pressure loss due to friction):



Express your answer in units of PSI as well as PSF (pounds per square foot).

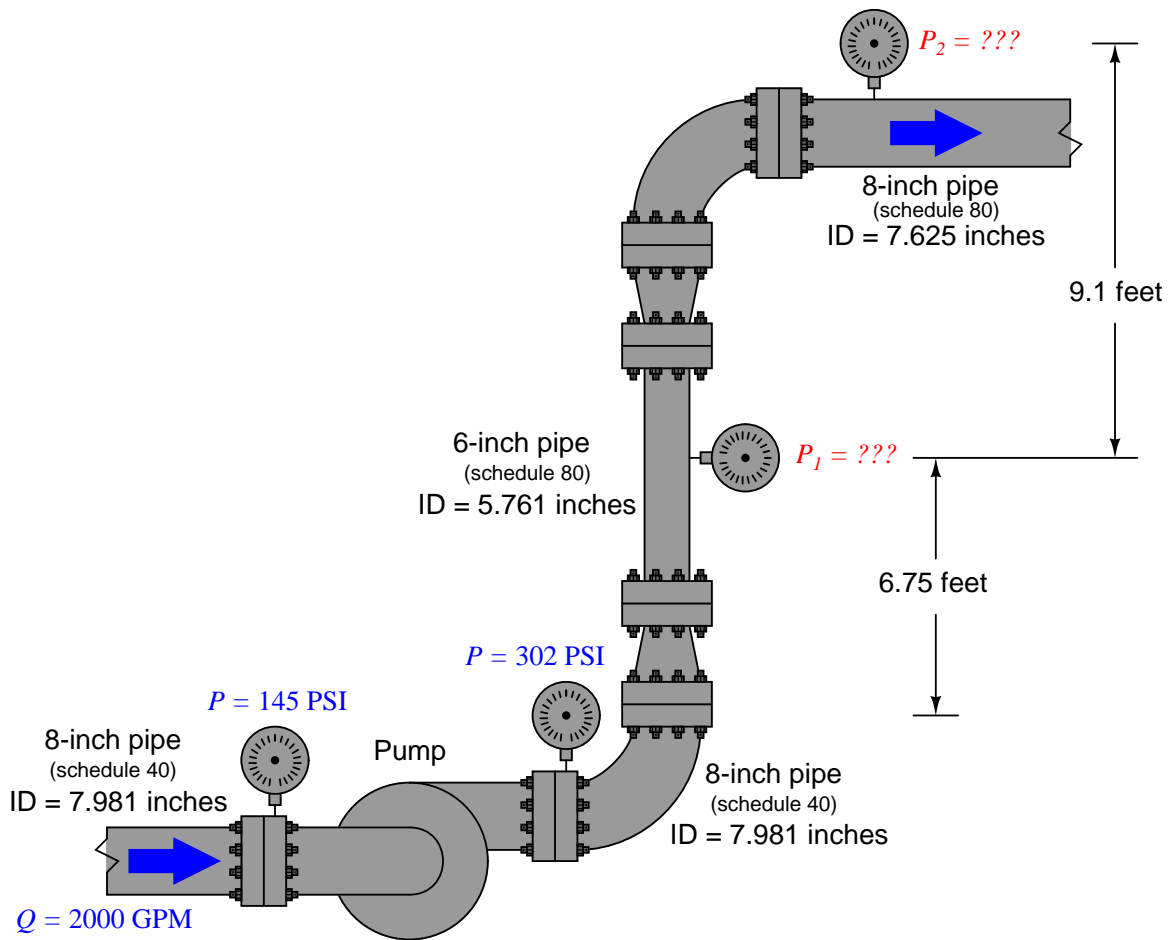
Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

file i02978

Question 11

Calculate the pressures P_1 and P_2 , assuming a fluid mass density of 1.72 slugs per cubic foot:

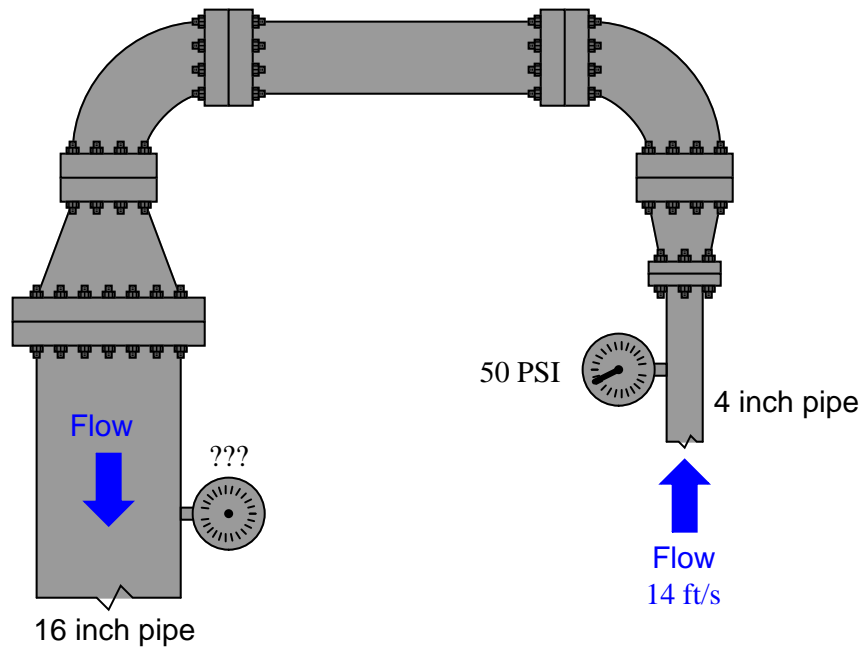


Also, comment on whether or not Bernoulli's equation could be used to compare the suction and discharge pressures of the pump, being that those two pressures (145 and 302 PSI) are measured on the same size pipe, with the same flow rate, and very similar elevations (heights).

file i00457

Question 12

The following illustration shows a portion of water piping from an overhead view, looking down toward the ground (a “birds-eye” view). The pipe itself is completely level (parallel) with the ground, so that all points along the pipe centerline are at the same height:



The inlet pressure gauge shows 50 PSI, and the velocity of the water entering through the 4 inch pipe is known to be 14 feet per second. Both pressure gauges are fixed at the centerline of the pipe, and are thus at the exact same height. Calculate the pressure registered at the outlet gauge (on the 16 inch pipe section) in units of PSI, assuming inviscid (frictionless) flow throughout, and a mass density for water of $\rho = 1.94$ slugs/ft³.

Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

file i02979

Question 13

Elevation (z) and pressure (P) readings are taken at two different points in a piping system carrying liquid benzene ($\gamma = 56.1 \text{ lb/ft}^3$):

$$z_1 = 50 \text{ inches} \qquad z_2 = 34 \text{ inches}$$

$$P_1 = 70 \text{ PSI} \qquad P_2 = 69 \text{ PSI}$$

Calculate the fluid velocity at point 2 (v_2) if the velocity at point 1 is known to be equal to 5 feet per second ($v_1 = 5 \text{ ft/s}$).

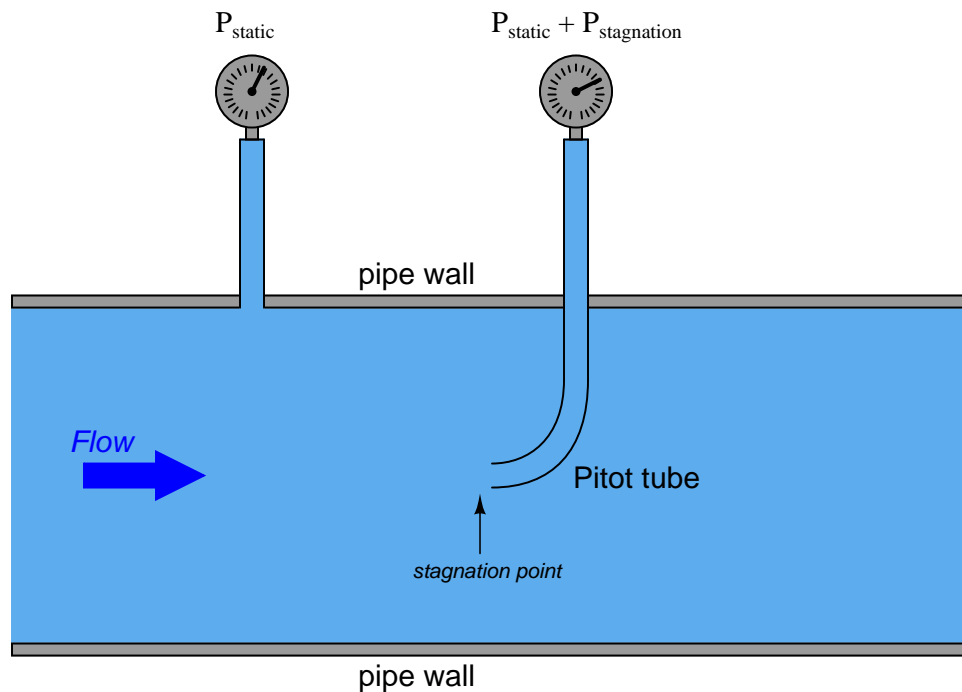
Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

[file i02988](#)

Question 14

As fluid flows past a stationary object such as a *Pitot tube*, the fluid immediately in front of the tube comes to a full stop. This is called a *stagnation point*, and the pressure resulting from the complete loss of velocity at the stagnation point is called the *stagnation pressure*.



Manipulate Bernoulli's equation to show how this stagnation pressure is determined by fluid velocity (v).

[file i02981](#)

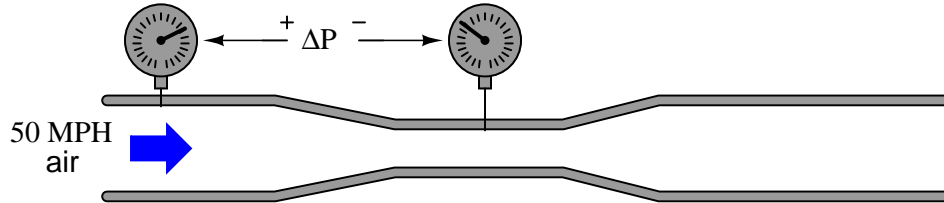
Question 15

Calculate the pressure developed by a Pitot tube measuring air speed at 50 MPH, at sea level ($\rho_{air} = 0.00235$ slugs/ft³).

Also, how much pressure will the Pitot tube develop at twice the air speed (100 MPH)?
[file i02982](#)

Question 16

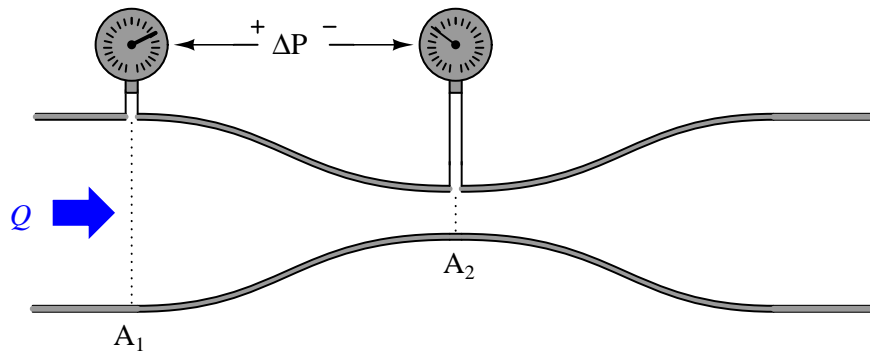
Calculate the differential pressure developed by an open venturi tube measuring air speed at 50 MPH, at sea level ($\rho_{air} = 0.00235$ slugs/ft³), where the throat diameter is one-half that of the entrance diameter:



Also, how much pressure will the venturi tube develop at twice the air speed (100 MPH)?
[file i02984](#)

Question 17

From Bernoulli's equation, develop a formula for calculating volumetric flow rate (Q) given differential pressure drop ΔP between two flow streams with differing cross-sectional areas (A_1 and A_2). Assume an incompressible fluid ($\rho = \text{constant}$) flowing along a level path ($z_1 = z_2$), and recall that volumetric flow rate is equal to the product of cross-sectional area and fluid velocity ($Q = Av$).



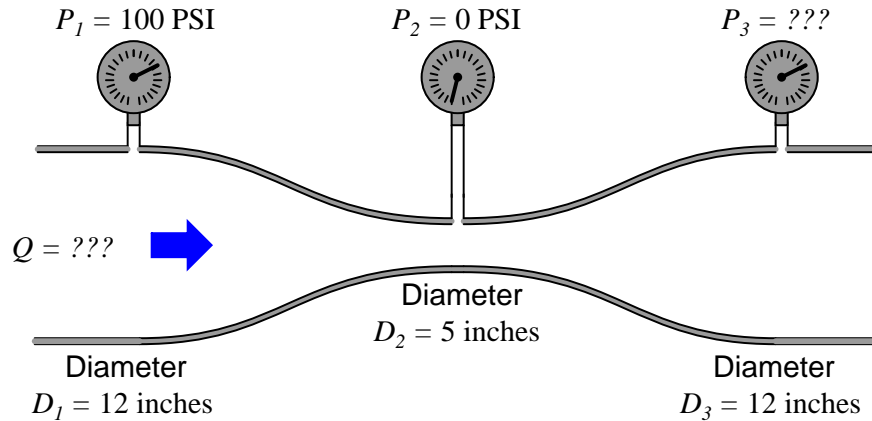
Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

[file i02983](#)

Question 18

Calculate the flow rate necessary (in units of gallons per minute) to create exactly 0 PSI gauge pressure at the throat of this venturi, and also calculate the (ideal) pressure at the third gauge. Assume a liquid with a mass density (ρ) of 1.951 slugs per cubic foot:



Furthermore, determine what will happen to the pressure at the throat if the flow rate increases beyond this rate, assuming all other factors remain unchanged?

[file i00444](#)

Question 19

Compare and contrast *laminar* versus *turbulent* flow regimes with regard to the following criteria:

- Which regime creates the least amount of *drag* (frictional energy losses) through a long length of pipe?
- Which regime is better for mixing fluids together in a piping system?
- Which regime is better for ensuring thorough reaction between chemical reactants in a piping system?
- Which regime is preferable inside of a heat exchanger, to ensure maximum heat transfer?

Examine the following formulae for calculating Reynolds number:

To calculate Reynolds number given English units (gas flow):

$$\text{Re} = \frac{(6.32)\rho Q}{D\mu}$$

Where,

Re = Reynolds number (unitless)

ρ = Mass density of gas, in pounds (mass) per cubic foot (lbm/ft³)

Q = Flow rate, standard cubic feet per hour (SCFH)

D = Diameter of pipe, in inches (in)

μ = Absolute viscosity of fluid, in centipoise (cP)

To calculate Reynolds number given specific gravity instead of density (liquid flow):

$$\text{Re} = \frac{(3160)G_f Q}{D\mu}$$

Where,

Re = Reynolds number (unitless)

G_f = Specific gravity of liquid (unitless)

Q = Flow rate, gallons per minute (GPM)

D = Diameter of pipe, in inches (in)

μ = Absolute viscosity of fluid, in centipoise (cP)

Now, qualitatively identify which direction each variable in the formula must change (e.g. *increase* versus *decrease*) in order to promote turbulence in a fluid stream, all other factors remaining unchanged.

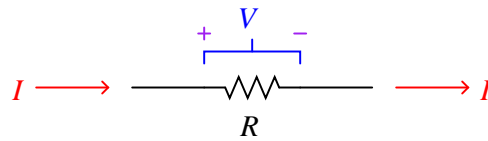
Suggestions for Socratic discussion

- For any given fluid *velocity*, Reynolds number will decrease if the pipe diameter decreases. Knowing this, explain why we see D in the denominator of these fractions rather than in the numerator.

file i01299

Question 20

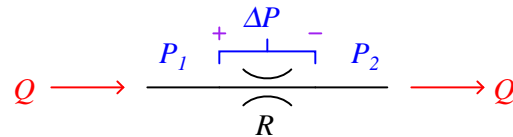
One of the most fundamental relationships in the study of electricity is *Ohm's Law*. This mathematical expression relates the flow of electric charge (I , which we call *current*) to electromotive potential (V , which we call *voltage*) and opposition to charge flow (R , which we call *resistance*):



Arrows point in direction
of conventional flow

$$I = \frac{V}{R} \quad (\text{For all conditions})$$

The relationship between **liquid** flow rate, pressure drop, and “resistance” for a piping restriction (orifice, throttling valve, pipe bend, etc.) is not as simple. No single mathematical expression is sufficient to predict flow rates for all conditions:

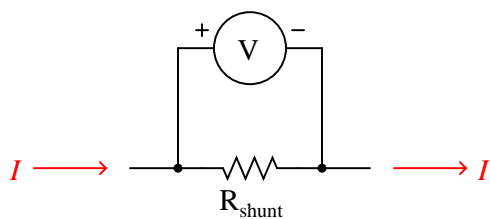


$$Q = \frac{P_1 - P_2}{R_1} = \frac{\Delta P}{R_1} \quad (\text{For laminar flow conditions only})$$

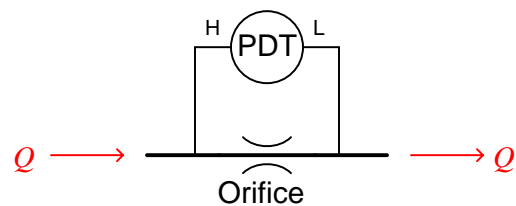
$$Q = \sqrt{\frac{P_1 - P_2}{R_2}} = \sqrt{\frac{\Delta P}{R_2}} \quad (\text{For turbulent flow conditions only})$$

Shunt resistors may be used as electric current-measuring elements, producing a voltage drop in precise proportion to the current through it. All we need to know is the shunt resistance, and we may infer current by measuring voltage. In a similar manner, orifices may be used as liquid flow-measuring elements, producing a pressure drop that varies with the amount of flow passing through it:

Measuring electric current



Measuring fluid flow

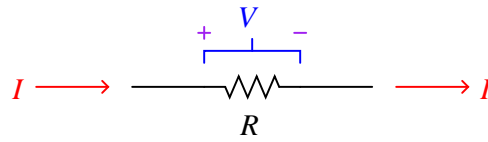


Given the “Ohm’s Law” equations shown for liquid flow, identify what we would have to know before using an orifice as an accurate liquid flow-measuring device, and how we would be able to obtain that information. Also identify the type of sensing instrument we would need to measure the pressure, so we could infer flow rate from its reading.

[file i00086](#)

Question 21

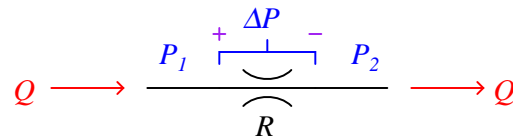
One of the most fundamental relationships in the study of electricity is *Ohm's Law*. This mathematical expression relates the flow of electric charge (I , which we call *current*) to electromotive potential (V , which we call *voltage*) and opposition to charge flow (R , which we call *resistance*):



Arrows point in direction of conventional flow

$$I = \frac{V}{R} \quad (\text{For all conditions})$$

The relationship between **gas** flow rate, pressure drop, and “resistance” for a piping restriction (orifice, throttling valve, pipe bend, etc.) is not as simple. No single mathematical expression is sufficient to predict flow rates for all conditions:



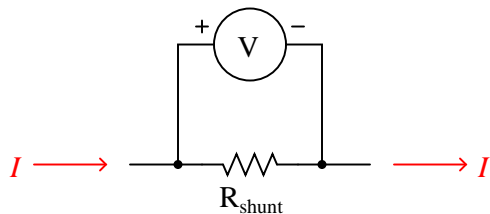
$$Q = \frac{P_1 - P_2}{R_1} = \frac{\Delta P}{R_1} \quad \text{Subsonic velocity with small } \Delta P$$

$$Q = \sqrt{\frac{P_1 - P_2}{R_2}} = \sqrt{\frac{\Delta P}{R_2}} \quad \text{Subsonic velocity with moderate } \Delta P$$

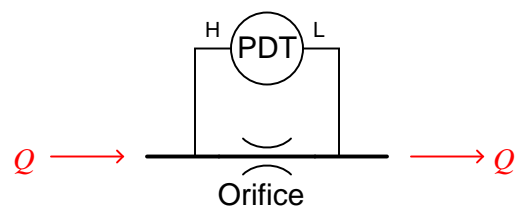
$$Q = \sqrt{\frac{P_2(P_1 - P_2)}{R_3}} = \quad \text{Subsonic velocity with large } \Delta P$$

Shunt resistors may be used as electric current-measuring elements, producing a voltage drop in precise proportion to the current through it. All we need to know is the shunt resistance, and we may infer current by measuring voltage. In a similar manner, orifices may be used as gas flow-measuring elements, producing a pressure drop that varies with the amount of flow passing through it:

Measuring electric current



Measuring fluid flow



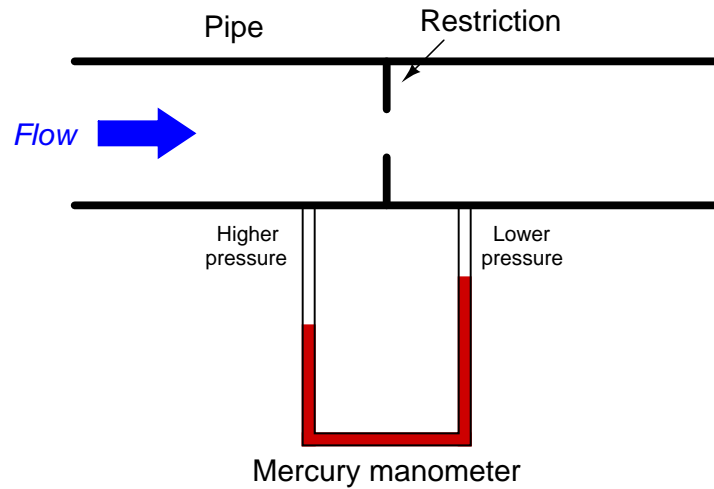
Given the “Ohm’s Law” equations shown for gas flow, identify what we would have to know before using an orifice as an accurate gas flow-measuring device, and how we would be able to obtain that information.

Also identify the type of sensing instrument(s) we would need to measure the pressure, so we could infer flow rate from its reading.

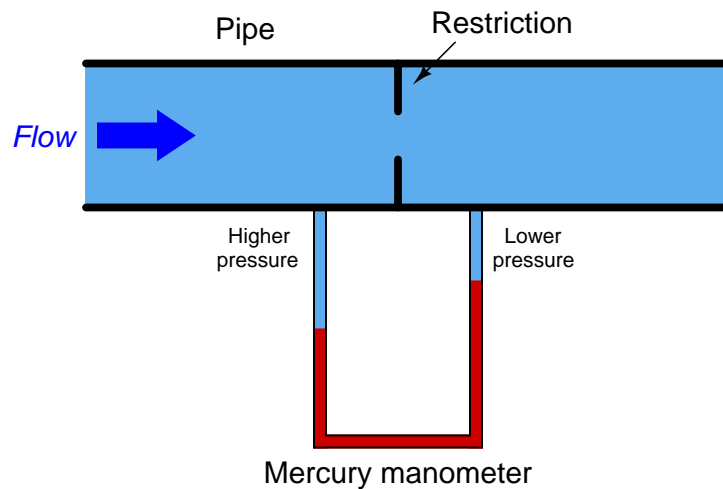
[file i00087](#)

Question 22

A manometer may be used to measure differential pressure across a restriction placed within a pipe. Pressure will be dropped as a result of flow through the pipe, making the manometer capable of (indirectly) measuring flow:



In the example shown above, the fluid moving through the pipe is air, and the manometer uses mercury as the indicating liquid. If we try to measure the flow rate of a *liquid* such as water using the same technique, though, we will find that the manometer does not register quite the way we might expect:

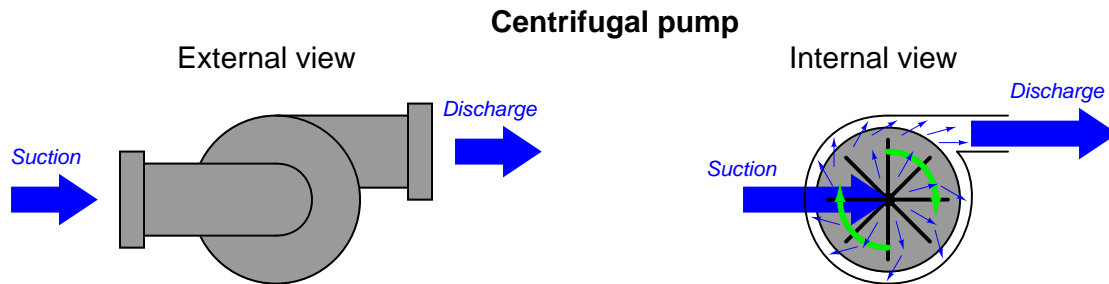


That is to say, given the exact same amount of differential pressure generated by the restriction, the manometer will register differently than if it was measuring air pressure. Determine whether the manometer will register falsely high or falsely low, and also *why* it will do so.

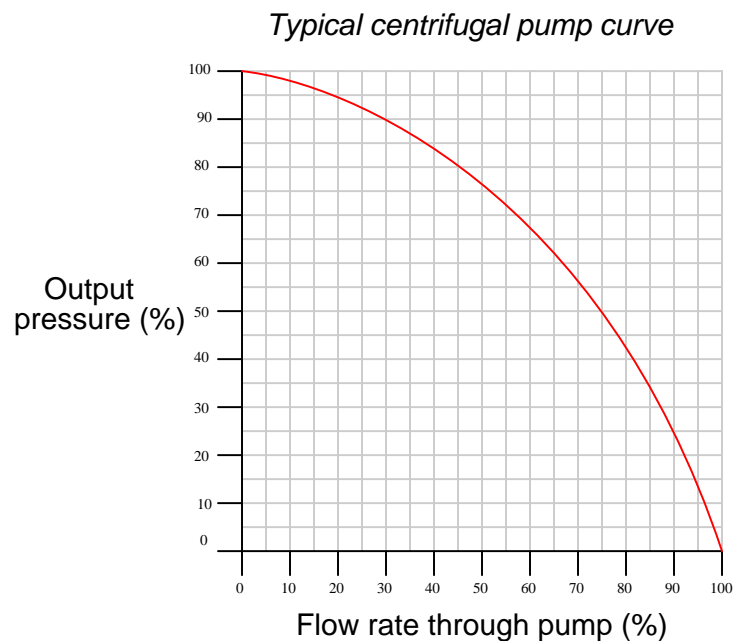
[file i00796](#)

Question 23

A *centrifugal pump* works by spinning a disk with radial vanes called an “impeller,” which flings fluid outward from the center of the disk to the edge of the disk. This kinetic energy imparted to the fluid translates to potential energy in the form of pressure when the fluid molecules strike the inner wall of the pump casing:



The performance of a centrifugal pump is often expressed in a special graph known as a *pump curve*. A typical centrifugal pump curve appears here, traced for one particular shaft speed:



Examine this pump curve, and explain in your own words what it tells us about the performance behavior of this pump when turned at a constant speed.

Suggestions for Socratic discussion

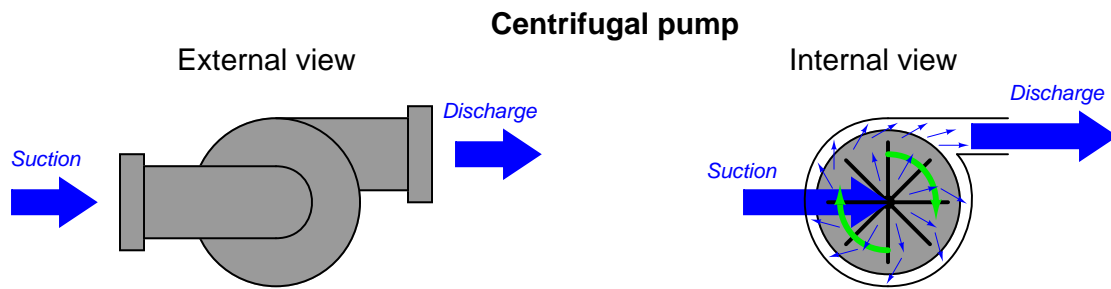
- One way to describe the operation of a centrifugal pump is to say it generates discharge pressure by converting kinetic energy into potential energy. Elaborate on this statement, explaining exactly where and how kinetic energy gets converted to potential energy. Hint: this might be easier to answer if you consider the “limiting case” of maximum discharge pressure described by the pump curve, where flow is zero and pressure is maximum.
- Appealing to the conversion of energy between kinetic and potential forms, explain *why* discharge pressure for a centrifugal pump falls off as flow rate increases.

- The pump curve shown assumes a constant rotational speed for the pump's impeller. How would the pump curve be modified if the pump were rotated at a slower speed?

file i01407

Question 24

A *centrifugal pump* works by spinning a disk with radial vanes called an “impeller,” which flings fluid outward from the center of the disk to the edge of the disk. This kinetic energy imparted to the fluid translates to potential energy in the form of pressure when the fluid molecules strike the inner wall of the pump casing:



The energy conveyed by the liquid exiting the discharge port of this pump comes in two forms: *pressure head* and *velocity head*. Ignoring differences in elevation (height), we may apply Bernoulli's equation to describe this fluid energy:

$$\text{Fluid Energy at discharge port} = \frac{\rho v^2}{2} + P$$

Where,

Fluid Energy = expressed in units of pounds per square foot, or PSF

P = Gauge pressure (pounds per square foot, or PSF)

ρ = Mass density of fluid (slugs per cubic foot)

v = Velocity of fluid (feet per second)

When the discharge port is completely blocked by an obstruction such as a closed valve or a blind, there is no velocity at the port ($v = 0$) and therefore the total energy is in the potential form of pressure (P). When the discharge port is completely unobstructed, there will be no pressure at the port ($P = 0$) and therefore the total energy is in kinetic form ($\frac{\rho v^2}{2}$). During normal operation when the discharge experiences some degree of resistance, the discharge fluid stream will possess some velocity as well as some pressure.

Assuming that the fluid molecules' maximum velocity is equal to the speed of the impeller's rim, calculate the discharge pressure under these conditions for a pump having an 8 inch diameter impeller spinning at 1760 RPM and a discharge port of 2 inches diameter, with water as the fluid (mass density $\rho = 1.94$ slugs per cubic foot) and assuming atmospheric pressure at the suction port:

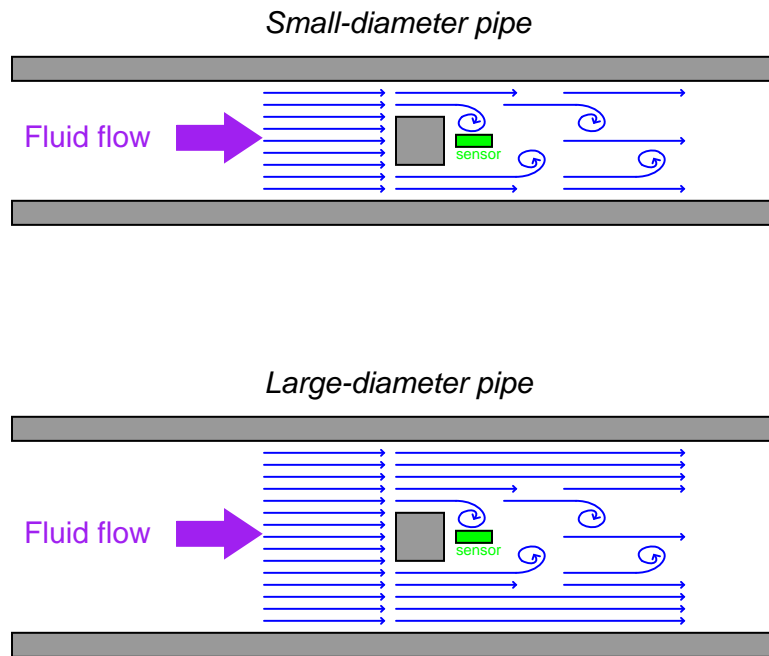
- Discharge flow = 0 GPM ; $P =$ _____ PSI
- Discharge flow = 100 GPM ; $P =$ _____ PSI
- Discharge flow = 200 GPM ; $P =$ _____ PSI
- Discharge flow = 300 GPM ; $P =$ _____ PSI
- Discharge flow = 400 GPM ; $P =$ _____ PSI
- Discharge flow = 500 GPM ; $P =$ _____ PSI

Next, calculate the maximum flow rate out of the pump with a completely open discharge port ($P = 0$).

[file i02588](#)

Question 25

Suppose two water pipes of different diameter both have blunt objects (“bluff bodies”) in the paths of their respective water flows. A pressure sensor device located near each of the bluff bodies measures the frequency of the vortices produced:



If the bluff bodies in both pipes have the same physical dimensions, and the vortex shedding frequencies are the same in both scenarios, which pipe carries a greater volumetric flow rate of water? Or, do they carry the same amount of flow? Why or why not??

[file i00495](#)

Question 26

An important numerical constant related to the von Kármán effect is the *Strouhal number*. Explain what this number means, and why its constant (unchanging) value is important to flow-measuring instruments based on the von Kármán effect.

[file i00492](#)

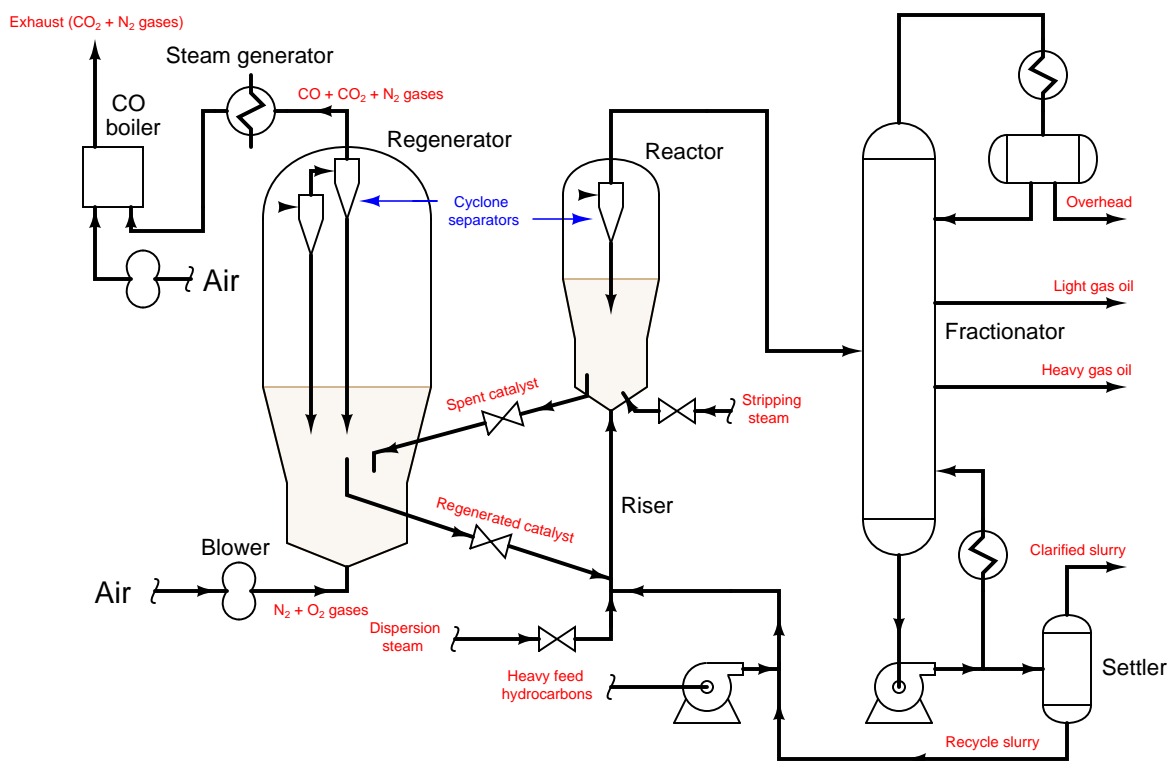
Question 27

Suppose a stream of water flowing through a 10-inch diameter pipe passes by a 1-inch-wide blunt object installed in the middle of that pipe, and generates a series of von Kármán vortices having a frequency of 4.167 Hz. Calculate the flow rate through the pipe in gallons per minute (GPM).

[file i00493](#)

Question 28

This process flow diagram shows a Fluid Catalytic Cracking (FCC) unit, used in oil refineries to convert heavy oils into feedstocks for making gasoline and other high-value fuels:



The word “fluid” in this unit’s name refers to the fact that the catalyst used to promote the chemical cracking reaction is a fine powder, which is circulated in the reactors and pipes along with the hot hydrocarbon vapors as though it were a fluid itself.

An FCC unit process should be designed so that the flow regime inside the *riser* pipe is highly turbulent. Explain why this is, and identify which design parameters of the riser might be controlled to ensure turbulent flow.

An FCC unit process should be designed so that the flow regime inside the *settler* vessel is laminar. Explain why this is, and identify which design parameters of the vessel might be controlled to ensure laminar flow.

Multiple *cyclone separators* are used at the tops of the reactor and regenerator vessels to separate powdered catalyst from hydrocarbon vapors, returning the catalyst powder to the vessels and letting the hydrocarbon vapors move on to other processes. Explain how a cyclone separator works.

Suggestions for Socratic discussion

- Identify which reactions in this process are *exothermic* and which are *endothermic*, and explain why for each case based on a simple analysis of the reactants and products.

[file i02748](#)

Answer 1

The Continuity equation relates volumetric flow rate to pipe area and average velocity, assuming a constant fluid density:

$$Q = A_1 v_1 = A_2 v_2$$

Given the same flow rate in both sections of pipe, the relationship between pipe area and velocity is as such:

$$\frac{v_2}{v_1} = \frac{A_1}{A_2}$$

Since area is proportional to the square of the diameter (or radius), we may express the ratio of velocities as a ratio of squared diameters:

$$\frac{v_2}{v_1} = \left(\frac{d_1}{d_2}\right)^2$$

This being the case, we may solve for the velocity in the narrower section of pipe:

$$v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2$$

$$v_2 = 5 \text{ ft/s} \left(\frac{8 \text{ in}}{3 \text{ in}}\right)^2 = 35.56 \text{ ft/s}$$

Answer 2

The volumetric flow rate for any fluid is equal to the flow velocity (v) multiplied by the cross-sectional area of the pipe (A):

$$Q = Av$$

We already know the velocity, so what we need to do now is calculate the pipe's flowing area. The pipe's internal diameter of 7.92 inches gives us this cross-sectional area:

$$A = \pi r^2$$

$$A = \pi \left(\frac{7.92}{2} \right)^2 = 49.27 \text{ in}^2$$

Converting the velocity into units of *inches* per minute before multiplying will give us a flow rate in cubic inches per minute:

$$\left(\frac{14 \text{ ft}}{\text{s}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = 10080 \text{ in/min}$$

Calculating volumetric flow rate:

$$Q = Av$$

$$Q = (49.27 \text{ in}^2)(10080 \text{ in/min}) = 496593 \text{ in}^3/\text{min}$$

Converting volumetric flow rate units:

$$\left(\frac{496593 \text{ in}^3}{\text{min}} \right) \left(\frac{1 \text{ gal}}{231 \text{ in}^3} \right) = 2149.8 \text{ GPM}$$

$$\left(\frac{2149.8 \text{ gal}}{\text{min}} \right) \left(\frac{1 \text{ bbl}}{42 \text{ gal}} \right) \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) \left(\frac{24 \text{ hr}}{1 \text{ day}} \right) = 73706 \text{ bbl/day}$$

Answer 3

The volumetric flow rate for any fluid is equal to the flow velocity (v) multiplied by the cross-sectional area of the pipe (A):

$$Q = Av$$

We already know the velocity, so what we need to do now is calculate the pipe's flowing area. The pipe's internal diameter of 5.95 inches gives us this cross-sectional area:

$$A = \pi r^2$$

$$A = \pi \left(\frac{5.95}{2} \right)^2 = 27.81 \text{ in}^2$$

Converting the velocity into units of *inches* per minute before multiplying will give us a flow rate in cubic inches per minute:

$$\left(\frac{10 \text{ ft}}{\text{s}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = 7200 \text{ in/min}$$

Calculating volumetric flow rate:

$$Q = Av$$

$$Q = (27.81 \text{ in}^2)(7200 \text{ in/min}) = 200196.4 \text{ in}^3/\text{min}$$

Converting volumetric flow rate units:

$$\left(\frac{200196.4 \text{ in}^3}{\text{min}} \right) \left(\frac{1 \text{ gal}}{231 \text{ in}^3} \right) = 866.7 \text{ GPM}$$

$$\left(\frac{866.7 \text{ gal}}{\text{min}} \right) \left(\frac{1 \text{ bbl}}{42 \text{ gal}} \right) \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) \left(\frac{24 \text{ hr}}{1 \text{ day}} \right) = 29713.8 \text{ bbl/day}$$

Answer 4

The Continuity equation relates volumetric flow rate to pipe area and average velocity, assuming a constant fluid density:

$$Q = Av$$

Velocity may be determined by manipulating this equation and plugging in the known quantities in the proper units:

$$v = \frac{Q}{A}$$

First, we need to convert the flow rate into units of cubic inches per minute, and calculate area in square inches:

$$Q = \left(\frac{180 \text{ gal}}{\text{min}} \right) \left(\frac{231 \text{ in}^3}{1 \text{ gal}} \right) = 41580 \text{ in}^3/\text{min}$$

$$A = \pi r^2 = \pi(3 \text{ in})^2 = 28.27 \text{ in}^2$$

Now we may calculate average velocity:

$$v = \frac{Q}{A} = \frac{41580 \text{ in}^3/\text{min}}{28.27 \text{ in}^2} = 1470.6 \text{ in}/\text{min}$$

Answer 5

$$P = 1374.3 \text{ lb}/\text{ft}^2 = 9.544 \text{ PSI}$$

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2 = z_3 \rho g + \frac{v_3^2 \rho}{2} + P_3$$

$$z_1 \rho g + 0 + 0 = 0 + 0 + P_2 = 0 + \frac{v_3^2 \rho}{2} + 0$$

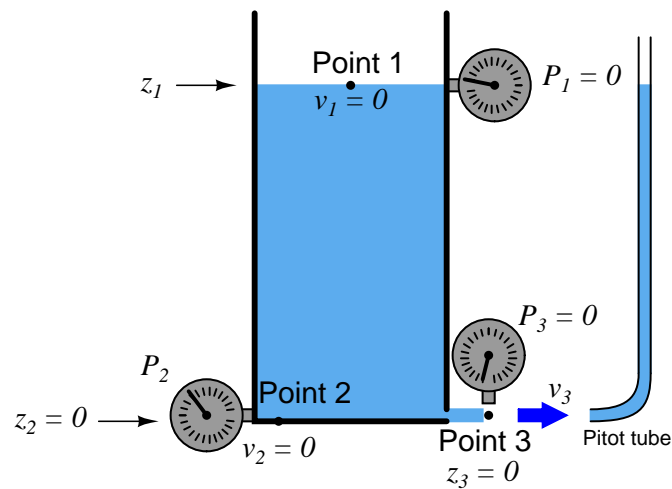
$$z_1 \rho g = P_2 = \frac{v_3^2 \rho}{2}$$

$$v_3 = \sqrt{2gz_1}$$

$$v_3 = \sqrt{\frac{2P_2}{\rho}}$$

The Pitot tube converts the outlet stream's velocity head ($\frac{v^2 \rho}{2}$) into a stagnation pressure head (P), then into an elevation head ($z\rho g$).

Challenge question: explain why a *Pitot tube* placed in the path of the outlet stream generates a liquid column equal in height to z_1 :



There is more than just a similarity here – the two equations are absolutely identical! This means the velocity of the liquid stream is equal to the final velocity of a falling object, if the liquid column height is the same as the object's drop height.

Follow-up question: are the units of measurement specified for Torricelli's Theorem specific to this form of the equation, or can we use different units of measurement with the exact same equation?

In order to combine these two formulae together, we need to identify the common variable. In this case, it is *height*. Note that g is not really a variable, but rather a *constant* so long as the location is on the surface of planet Earth.

Now that we know what the common variable is, we may manipulate the hydrostatic pressure formula to solve for that variable, so that we will have something to substitute into Torricelli's Theorem and finally have an formula solving for velocity (v) in terms of pressure (P):

$$P = \rho gh$$

$$h = \frac{P}{\rho g}$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2g \left(\frac{P}{\rho g} \right)}$$

$$v = \sqrt{2 \left(\frac{P}{\rho} \right)}$$

$$v = \sqrt{\frac{2P}{\rho}}$$

(Alternatively . . .)

$$v = \sqrt{2} \sqrt{\frac{P}{\rho}}$$

Note that P actually refers to the amount of *differential* pressure across the opening, since Torricelli's Theorem assumes a discharge into atmospheric pressure as well as a vented tank (atmospheric pressure on top of the liquid).

Head	Calculation at low end	Value
$z_1 \rho g$	(5 ft) (1.94 slugs/ft ³) (32.2 ft/s ²)	312.34 lb/ft ²
$v_1^2 \rho / 2$	(30 ft/s) ² (1.94 slugs/ft ³) / 2	873.00 lb/ft ²
P_1	(value already given)	1100 lb/ft ²
Total	312.34 lb/ft ² + 873.00 lb/ft ² + 1100 lb/ft ²	2285.34 lb/ft²

Head	Calculation at high end	Value
$z_2 \rho g$	(20 ft) (1.94 slugs/ft ³) (32.2 ft/s ²)	1249.36 lb/ft ²
$v_2^2 \rho / 2$	(24 ft/s) ² (1.94 slugs/ft ³) / 2	558.72 lb/ft ²
P_2		??? lb/ft ²
Total	1249.36 lb/ft ² + 558.72 lb/ft ² + ??? lb/ft ²	2285.34 lb/ft²

$$P_2 = 477.26 \text{ lb/ft}^2$$

It is tempting to alter Bernoulli's Equation to handle measurements in inches rather than feet (especially the annoying unit of pressure measurement: pounds per square *foot*, rather than PSI). However, caution must be exercised when attempting this, because there is more to it than simply converting feet into inches every place you see "ft" in the equation.

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

There is the unit of "feet" lurking inside the unit of "slugs" which must also be accounted for. Here is the standard weight-mass-gravity equation relating slugs to pounds:

$$W = mg$$

$$[\text{lb}] = [\text{slug}] \left[\frac{\text{ft}}{\text{s}^2} \right]$$

If we re-write the unit analysis equation to show slugs as a compound unit, we see that "feet" lurks within:

$$[\text{lb}] = \left[\frac{\text{lb} \cdot \text{s}^2}{\text{ft}} \right] \left[\frac{\text{ft}}{\text{s}^2} \right]$$

Thus, expressing g in inches per second squared would require us to invent a new unit of mass (lb · s² per in) instead of slugs (lb · s² per ft).

 Answer 10

$$P_2 = 17,793 \text{ lb/ft}^2 = 123.56 \text{ PSI}$$

 Answer 11

$$P_1 = 296.77 \text{ PSI} \quad P_2 = 293.27 \text{ PSI}$$

Bernoulli's equation assumes no gain or loss of energy between the two locations compared, and so it *cannot* be used to contrast the pump's suction and discharge pressures. The pump is a machine that adds energy to the fluid going through it, and so the assumption of equal (total) energy between the incoming and outgoing flow streams is not correct.

Answer 12

$$P_{out} = 51.315 \text{ PSI}$$

Note: with a pipe diameter ratio of 4:1 (out:in), the exit velocity will be *16 times* slower than the inlet velocity $(1:4)^2 = (1:16)$.

Answer 13

$$v_2 = 16.57 \text{ ft/s}$$

Note that the two pressures are given in units of PSI (not pounds per square *foot*), and that the two heights are given in inches instead of feet. Also, $\rho_{benzene} = 1.753 \text{ slugs/ft}^3$.

Answer 14

Bernoulli's equation:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

Assuming no change in height (z) is involved:

$$\frac{v_1^2 \rho}{2} + P_1 = \frac{v_2^2 \rho}{2} + P_2$$

Knowing that P_1 is the static pressure and that P_2 is equal to $P_{static} + P_{stagnation}$:

$$\frac{v_1^2 \rho}{2} + P_{static} = \frac{v_2^2 \rho}{2} + P_{static} + P_{stagnation}$$

$$\frac{v_1^2 \rho}{2} = \frac{v_2^2 \rho}{2} + P_{stagnation}$$

Knowing that v_2 is zero at the stagnation point:

$$\frac{v_1^2 \rho}{2} = P_{stagnation}$$

Therefore, $P_{stagnation} = \frac{1}{2} v^2 \rho$

Answer 15

$$P \text{ at } 50 \text{ MPH} = 1.215 \text{ inches H}_2\text{O}$$

$$P \text{ at } 100 \text{ MPH} = 4.859 \text{ inches H}_2\text{O}$$

Answer 16

$$\Delta P \text{ at } 50 \text{ MPH} = 18.22 \text{ "W.C.}$$

$$\Delta P \text{ at } 100 \text{ MPH} = 72.88 \text{ "W.C.}$$

Answer 17

Assuming no difference in height (z):

$$\frac{v_1^2 \rho}{2} + P_1 = \frac{v_2^2 \rho}{2} + P_2$$

$$P_1 - P_2 = \frac{v_2^2 \rho}{2} - \frac{v_1^2 \rho}{2}$$

$$\Delta P = \frac{\rho}{2} (v_2^2 - v_1^2)$$

$$\frac{2\Delta P}{\rho} = v_2^2 - v_1^2$$

$$\text{If } Q = Av \text{ then } v = \frac{Q}{A}$$

$$\frac{2\Delta P}{\rho} = \left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2$$

$$\frac{2\Delta P}{\rho} = \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}$$

$$\frac{2\Delta P}{\rho} = \frac{Q^2 A_1^2}{A_1^2 A_2^2} - \frac{Q^2 A_2^2}{A_1^2 A_2^2}$$

$$\frac{2\Delta P}{\rho} = Q^2 \frac{A_1^2 - A_2^2}{A_1^2 A_2^2}$$

$$Q^2 = \left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right) \left(\frac{2\Delta P}{\rho}\right)$$

$$Q = \sqrt{\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

Where,

Q = Volumetric flow rate (ft³/s)

A_1 = Large flow area (ft²)

A_2 = Small (throat) flow area (ft²)

ΔP = Differential pressure drop (lb/ft²)

ρ = Mass density of fluid (slugs/ft³)

Answer 18

$$Q = 16.822 \text{ ft}^3/\text{sec or } 7550.29 \text{ GPM}$$

If the flow rate increases beyond 7550.29 GPM, the pressure at P_2 will decrease further, creating a vacuum.

Answer 19

- Which regime creates the least amount of *drag* (frictional energy losses) through a long length of pipe?
Laminar
- Which regime is better for mixing fluids together in a piping system? *Turbulent*
- Which regime is better for ensuring thorough reaction between chemical reactants in a piping system?
Turbulent
- Which regime is preferable inside of a heat exchanger, to ensure maximum heat transfer? *Turbulent*

All other factors being equal, turbulence will be promoted by the following:

- Increasing flow rate (Q)
- Increasing fluid density (ρ or G_f)
- Decreasing pipe diameter (D)
- Decreasing viscosity (μ)

Answer 20

Here, what we do not know about the flow-measurement scenario is the flow regime (laminar or turbulent), and also what the “resistance” of the fluid restriction is. Of course, the Reynolds number for our flowstream will indicate its regime status, and the R factor for the orifice may be either determined experimentally or derived from orifice equations (available in any exhaustive reference book).

The proper pressure-sensing instrument to use for fluid flow is a *differential pressure* instrument, such as a DP cell or DP gauge, or perhaps even a mercury manometer.

Answer 21

Here, what we do not know about the flow-measurement scenario is the flow regime (laminar or turbulent), and also what the “resistance” of the fluid restriction is. Of course, the Reynolds number for our flowstream will indicate its regime status, and the R factor for the orifice may be either determined experimentally or derived from orifice equations (available in any exhaustive reference book).

The proper pressure-sensing instrument to use for fluid flow is a *differential pressure* instrument, such as a DP cell or DP gauge, or perhaps even a mercury manometer. In the case of large pressure drops, it may also be important to measure downstream pressure (P_2) with reference to atmosphere.

Answer 22

The manometer will register falsely high, showing greater differential pressure than what is actually there. If you are having difficulty figuring this out, imagine if the liquid moving through the pipe was just as dense as the mercury within the manometer: what would *that* do to the mercury in the manometer given any applied ΔP ? In other words, set up a *thought experiment* with absurdly (simple) conditions and then look for patterns or trends which you may generalize for any condition.

Challenge question: derive a mathematical correction factor for interpreting the manometer’s indication to yield true inches of mercury ΔP .

Answer 23

This graph relates pressure output versus liquid flow rate for a centrifugal-style pump operating at a constant rotational speed.

The velocity of the fluid molecules will be equal to the rim speed of the impeller, which is the circumference of the impeller multiplied by its rotational speed:

$$\left(\frac{1760 \text{ rev}}{\text{min}}\right) \left(\frac{8\pi \text{ in}}{\text{rev}}\right) = 44233.6 \text{ in/min} = 61.436 \text{ ft/s}$$

This velocity lets us calculate the velocity head at the impeller's rim. If we assume the water enters the pump with no pressure, this velocity head should be the *only* energy the water possesses at the impeller rim:

$$\text{Fluid Energy at impeller rim} = \frac{\rho v^2}{2} = \frac{(1.94)(61.436)^2}{2} = 3661.1 \text{ PSF} = 25.42 \text{ PSI}$$

This figure of 25.42 PSI will be the blocked-discharge pressure, where 100% of the fluid's kinetic energy is translated into pressure as it finds no place to flow and its velocity stagnates to zero.

Conversely, if we imagine a situation where the discharge port is completely unblocked to achieve zero discharge pressure, the fluid velocity exiting the port will be approximately equal to the impeller rim velocity. Applying this velocity to the Continuity equation to calculate volumetric flow at the 2-inch diameter discharge port:

$$Q = Av$$

$$Q = \pi r^2 v$$

$$Q = (\pi)(1^2)(44233.6) = 138964 \text{ in}^3/\text{min} = 601.6 \text{ GPM}$$

Therefore, the maximum flow rate of this pump at zero discharge pressure will be approximately 600 gallons per minute.

At any flow rate between zero and maximum, the combined sum of velocity and pressure heads at the pump discharge must be equal to the maximum head at the impeller rim (3661.1 PSF equivalent). Therefore:

$$3681.9 = \frac{\rho v^2}{2} + P$$

$$P = 3661.1 - \frac{\rho v^2}{2}$$

Using the Continuity equation to calculate discharge velocity at 100 GPM (23100 in³/min), and then Bernoulli's equation to calculate discharge pressure:

$$v = \frac{Q}{A} = \frac{23100}{\pi} = 7352.96 \text{ in/min} = 10.21 \text{ ft/s}$$

$$P = 3661.1 - \frac{(1.94)(10.21^2)}{2} = 3458.8 \text{ PSF} = 24.02 \text{ PSI}$$

Using the Continuity equation to calculate discharge velocity at 200 GPM (46200 in³/min), and then Bernoulli's equation to calculate discharge pressure:

$$v = \frac{Q}{A} = \frac{46200}{\pi} = 14705.9 \text{ in/min} = 20.42 \text{ ft/s}$$

$$P = 3681.9 - \frac{(1.94)(20.42^2)}{2} = 3256.4 \text{ PSF} = 22.61 \text{ PSI}$$

Using the Continuity equation to calculate discharge velocity at 300 GPM (69300 in³/min), and then Bernoulli's equation to calculate discharge pressure:

$$v = \frac{Q}{A} = \frac{69300}{\pi} = 22058.9 \text{ in/min} = 30.64 \text{ ft/s}$$

$$P = 3681.9 - \frac{(1.94)(30.64^2)}{2} = 2750.6 \text{ PSF} = 19.10 \text{ PSI}$$

Using the Continuity equation to calculate discharge velocity at 400 GPM (92400 in³/min), and then Bernoulli's equation to calculate discharge pressure:

$$v = \frac{Q}{A} = \frac{92400}{\pi} = 29411.8 \text{ in/min} = 40.85 \text{ ft/s}$$

$$P = 3681.9 - \frac{(1.94)(40.85^2)}{2} = 2042.5 \text{ PSF} = 14.18 \text{ PSI}$$

Using the Continuity equation to calculate discharge velocity at 500 GPM (115500 in³/min), and then Bernoulli's equation to calculate discharge pressure:

$$v = \frac{Q}{A} = \frac{115500}{\pi} = 36764.8 \text{ in/min} = 51.06 \text{ ft/s}$$

$$P = 3681.9 - \frac{(1.94)(51.06^2)}{2} = 1132.0 \text{ PSF} = 7.861 \text{ PSI}$$

Summarizing these calculated results:

- Discharge flow = 0 GPM ; $P = \underline{25.42}$ PSI
- Discharge flow = 100 GPM ; $P = \underline{24.02}$ PSI
- Discharge flow = 200 GPM ; $P = \underline{22.61}$ PSI
- Discharge flow = 300 GPM ; $P = \underline{19.10}$ PSI
- Discharge flow = 400 GPM ; $P = \underline{14.18}$ PSI
- Discharge flow = 500 GPM ; $P = \underline{7.861}$ PSI

If we were to plot these flow and pressure data points, we would have a *pump curve* for this centrifugal pump.

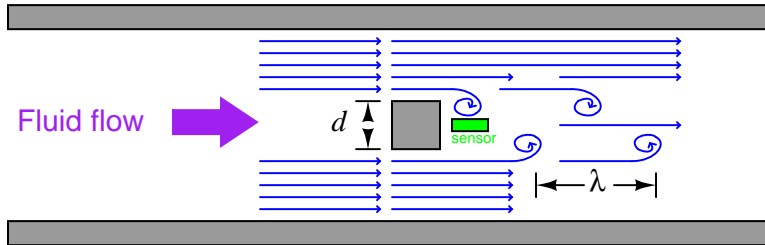
Answer 25

The large pipe carries a greater volumetric rate of water flow than the small pipe.

Since the vortex shedding frequency is proportional to the fluid *velocity*, we know that the flow velocities in both cases must be the same (given identical bluff body geometries). However, since the larger pipe has a greater cross-sectional area, an identical velocity equates to a greater *volume* rate of water moving past the bluff body and sensor.

Answer 26

The Strouhal number (approximately equal to 0.17) is the ratio of a blunt object's width to the distance between successive fluid vortices spilling off of the side of that object.



$$\frac{d}{\lambda} = 0.17$$

In essence, the Strouhal number tells us that the *wavelength* (λ) of vortex “waves” is always constant given a bluff body of constant width. Therefore, the frequency of these waves is directly proportional to the velocity of the fluid (and thus the volumetric flow rate in a pipe of constant cross-sectional area).

It should also be noted that wavelength (λ), wave velocity (v), and wave frequency (f) are related to each other by the following equation:

$$v = f\lambda$$

Dimensional analysis helps prove this is true:

$$\left[\frac{\text{meters}}{\text{second}} \right] = \left[\frac{\text{cycles}}{\text{second}} \right] \left[\frac{\text{meters}}{\text{cycle}} \right]$$

My choice to use “meters” here is arbitrary. The relationship is true regardless of length unit (feet, inches, centimeters, miles, cubits, whatever!).

Answer 27

$$Q = 500.04 \text{ GPM}$$

We may derive an exact formula by using a *proportionality factor* (k) and the specific set of values used to answer the quantitative question. We know that flow rate will be proportional to the product of frequency, bluff body width, and the square of pipe diameter¹:

$$Q \propto fwD^2$$

We may express this as an exact equation by adding a factor k :

$$Q = kfwD^2$$

Solving for k :

$$k = \frac{Q}{fwD^2}$$
$$k = \frac{500.04}{(4.167)(1)(10)^2} = 1.2$$

Therefore,

$$Q = 1.2fwD^2$$

Where,

- Q = Flow rate, in gallons per minute (GPM)
- f = Vortex shedding frequency, in Hertz (Hz)
- w = Bluff body width, in inches (in)
- D = Diameter of pipe, in inches (in)

One really neat advantage of calculating k in this manner is that it incorporates all the necessary unit conversions performed in the original solution of $Q = 500.04$ GPM. Now, all we do is enter f in Hertz, w in inches, and D in inches, and we automatically get an answer for Q in units of gallons per minute.

Answer 28

Turbulence in the riser is desirable to ensure thorough mixing of all the reactants, to expedite the cracking reaction. High flow rates and a narrow pipe ensure the necessary turbulence.

Laminar flow is desirable inside the settler, to allow catalyst powder to settle to the bottom and be sent back to the reactor rather than be carried off to another processing unit. Large cross-sectional area for the flowing slurry ensures the flow will drop to a laminar regime once inside the settler vessel.

Cyclone separators introduce the incoming flow tangential to the circumference of the separator, letting centrifugal force pin solid particles to the separator wall. This causes the particles to lose kinetic energy and fall out the bottom of the separator.

¹ Using qualitative analysis, we can tell that Q increases proportionally with increases in f and with increases in w . We can also tell that Q increases proportionally with the *square* of increases in D . Just imagine doubling each of these variables, one at a time (while holding the others constant) and determine the effect on Q .